Gravitation and Electromagnetism Unified

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The theoretical and experimental reasons leading to the unification of electromagnetism and gravitation are discussed The conservation of momenta is explored and is shown that it applies in the same way both to the binding of the electron to the nucleus of an atom as well as to the planets around the sun. This idea, in fact, unifies gravitation with electromagnetism. A model based on our theory is proposed, which solves the Pioneer anomaly problem.

PACS : 03.50.-z – Classical field theories, 12.10.-g, 04.50.+h - unified field theories; alternative theories of gravity, 12.10.Kt - Unification of couplings; mass relations; **Key words**: four-vectors, electromagnetism, gravity

"*understanding mass allows us to address very fundamental issues about unification and gravity*"
Frank Wilczec
Nobel Prize 2004 Winner

1. Introduction

Several years ago, the present author hypothesized that the constitution of all Nature is electromagnetic. In particular, this author holds that the elementary particles of matter, such as the proton and electron, are constituted of photons in circular orbits. Such model or postulate explains a number of particle parameters. Additionally, such hypothesis constitutes the epistemological basis over which all of the physical phenomena should be explained and simplified.

In previous papers [1], the present author was able to apply, his new non-associative algebra for four-vectors, to classical electromagnetism. It was demonstrated that all the physical variables considered, such as the electric and magnetic fields, scalar and vector potentials, the charge-currents, energy-momentum and the action, satisfy the homogeneous wave equation. Such results provide a strong argument that confirms the above mentioned hypothesis and suggests what the orientation for further investigations should be.

In the mentioned paper I stated that "the use of four-vectors allows discerning constants, variables and relations, previously unknown to Physics, which are needed to complete and make coherent the theory." Such intention was fully demonstrated, although it is impossible for a single author to cover all of the possible physical applications, consequently, the reader is urged to familiarize with these tools and hypothesis, in order to contribute to their development.

Within the same epistemological basis, the application of four-vectors to both electromagnetism and gravitation is performed in the present paper.

It is found that both gravity and electromagnetism satisfy the same equations, which further confirms the hypothesis that matter has electromagnetic constitution. Its applicability to a few experimental results is shown.

The uniform applicability of the same equations both to electromagnetism as well as to gravitation means that both phenomena are unified.

The objective in this paper is to show that the same energy-momentum equation explains and allows computing the values for the binding energy of two electromagnetic particles, such as the proton and electron, which constitute the atom of hydrogen, as well as for the gravitational binding energy between two planetary bodies. Moreover, the preliminary analysis carried-out in section 5 of the present paper suggests that the mass-equivalent binding energy carries the long sought-after explanation of the gravitational problem called "Pioneer anomaly", which, according to Anderson et al [2] "Currently, we find no mechanism or theory that explains the anomalous acceleration."

2. The Four-Vectors

The four-vectors, according to the present author [3], are four-dimensional numbers of the form:

$$A_\mu = A^\mu = \mathbf{e}a_t + \mathbf{i}\,a_x + \mathbf{j}\,a_y + \mathbf{k}\,a_z \tag{1}$$

or, assuming that the order of the basis elements, $\mathbf{e},\mathbf{i},\mathbf{j},\mathbf{k}$, is the one indicated, those basis elements can be suppressed and included implicitly, with a notation similar to a vector or 4D point:

$$A_\mu = A^\mu = (a_t, a_x, a_y, a_z) \tag{2}$$

The four-vectors will be denoted with an upper-case letter having a Greek super- or sub-index. This avoids confusing the four-vectors with the three-dimensional vectors, which will be denoted with bold letters. Such indexed notation resembles the tensor notation and could be used for compatibility with previous knowledge, but this author does not recommend interpreting it as a range of values for summation.

The t, x, y and z, as sub- or super-indices of the elements, should be interpreted as the coordinate associated to the respective element of the four-vector.

The three-dimensional vectors consist of only the three spatial elements of a four-vector. We will represent them also by using the abbreviated form from expression (2), namely with comma-separated elements and implicit basis elements $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.

3. Basic Operations and Four-Vectors

We are going to assume that the components of the four-vectors used in this paper are real numbers.

The *sum* of two four-vectors is another four-vector, where each component has the sum of the corresponding argument components:

$$A^{\mu} + B^{\mu} = \mathbf{e}(a_t + b_t) + \mathbf{i}(a_x + b_x) + \mathbf{j}(a_y + b_y) + \mathbf{k}(a_z + b_z) \quad (3)$$

The *difference* of two four-vectors is defined similarly:

$$A^{\mu} - B^{\mu} = \mathbf{e}(a_t - b_t) + \mathbf{i}(a_x - b_x) + \mathbf{j}(a_y - b_y) + \mathbf{k}(a_z - b_z) \quad (4)$$

Given two four-vectors A_μ and B^μ the *four-vector product* is defined as:

$$\begin{aligned} A_\mu\ B^\mu = &\ \mathbf{e}(a_t b_t + a_x b_x + a_y b_y + a_z b_z) \\ &+ \mathbf{i}(-a_t b_x + a_x b_t + a_y b_z - a_z b_y) \\ &+ \mathbf{j}(-a_t b_y - a_x b_z + a_y b_t + a_z b_x) \\ &+ \mathbf{k}(-a_t b_z + a_x b_y - a_y b_x + a_z b_t) \end{aligned} \quad (5)$$

The space between the four-vectors will be used to denote the product. Using the notation of three-dimensional vector analysis we obtain a shorthand expression for the product. Regarding $\mathbf{i}, \mathbf{j}, \mathbf{k}$ as unit vectors in a Cartesian coordinate system, we interpret the four-vector A^μ as comprising the scalar a and the vector part $\mathbf{a} = \mathbf{i}\, a_x + \mathbf{j}\, a_y + \mathbf{k}\, a_z$. The four-vector is written in the simplified form $A^\mu = (a, \mathbf{a})$. With this notation, the product (5) is expressed in the compact form:

$$A_\mu\ B^\mu = (a\,b + \mathbf{a}\cdot\mathbf{b},\ -a\,\mathbf{b} + \mathbf{a}\,b + \mathbf{a}\times\mathbf{b}) \quad (6)$$

where the usual rules for vector sum as well as dot and cross products are being invoked.

The product of a four-vector by itself produces a result different from zero only in the first or 'scalar' component, which is identified as the norm of the four-vector. In this sense it is similar to the dot product in vector calculus:

$$A_\mu\, A^\mu = (a_t^{\ 2} + a_x^{\ 2} + a_y^{\ 2} + a_z^{\ 2}, 0, 0, 0) \quad (7)$$

The *velocity four-vector* of a static particle is

$$U^\mu = (i\,c, 0, 0, 0) \quad (8)$$

Where i is the imaginary unit, $\sqrt{-1}$, here and in the following equations.

The velocity of the particle, boosted with the velocity vector $\mathbf{v} = \mathbf{i}\, v_x + \mathbf{j} v_x + \mathbf{k}\, v_z$, is defined as:

$$U^\mu = \gamma(v)(i\,c, v_x, v_y, v_z) \quad (9)$$

or, concisely

$$U^\mu = \gamma(v)(i\,c, \mathbf{v}) \quad (10)$$

Where i is the imaginary unit, $\sqrt{-1}$, here and in the following equations, $v = \sqrt{v_x^{\ 2} + v_y^{\ 2} + v_z^{\ 2}}$ and the $\gamma(v)$ factor is:

$$\gamma(v) = 1/\sqrt{1 - v^2/c^2} \quad (11)$$

Therefore, the product of the velocity four-vector by itself gives:

$$U_\mu\, U^\mu = (-c^2, 0, 0, 0) \quad (12)$$

Equations (8) and (12) show that the velocity of a particle is equal to speed of light. This reflects and is justified by our hypothesis that the particles of matter are constituted of photons. The velocity of a boosted particle, such as in equation (9), is a point in the surface of a four-dimensional sphere whose radius is equal to the speed of light.

4. Momentum Four-vector

Not even the Nobel Prize winning physicists [4] have a clear picture of the whereabouts of energy and momentum, after the more than one hundred years of the hodge-podge Einstein made with the concepts of relativity, in particular, and Physics in general. The physicists have not had the proper mindset to correct and simplify the theory and they have just have compounded the problem, creating, for example, an enormous zoo of particles, in many cases imaginary.

The mass-energy equation is given as $E = m\,c^2$ [5].

The first question we should ask is whether the particles of light, the photons, have zero mass. Elsewhere the present author has argued that the photons do have mass, contrary to what most physicists believe [4]. The reason lays in the electromagnetic conception of Nature. As we have assumed that mass is electromagnetic, and mass has energy, by the same reason photons have energy (and mass). Therefore, the present author considers that it is a severe error in current Physics to believe in particles without mass. Thus, it is incorrect, for example, to say that "The mass of ordinary matter is the embodied energy of more basic building blocks, themselves lacking mass." [4]

There are many physicists who believe that the mass changes with velocity. This is the first problem with this mass-energy equation. The fact is that mass is an invariant. The second error is to consider that the energy of a moving body is given by $E = \gamma(v)\, m\, c^2$ [4 (Appendix A)], where $\gamma(v)$ is the Lorentz contraction factor.

What the present author believes is that, in the first place, there is no need of the concept of energy. The only four-vector that the physicists should be using is the momentum four-vector. There is no energy four-vector. Nevertheless, for easier understanding and comparison with classical concepts, this paper will be written using some concepts referring to energy. The second problem with the equation, for a moving body, is that it represents only a fraction of the whole information. Energy is equivalent to mass and, therefore, is invariant. The energy shown in the equation $E = \gamma(v)\, m\, c^2$ represents only part of the energy or, better, momentum, of the moving body and is not invariant.

The *momentum four-vector*, for the present author, is defined simply as the mass, m, of the particle multiplied by the four-velocity. In the following, and without loss of generality, let us assume that the velocities occur only in the x direction:

$$P^{\mu} = m\, \gamma(v)(i\, c, v, 0, 0) \tag{12}$$

The magnitude of the momentum for this body can be obtained, as in the case of velocities, multiplying the four-momentum by itself. This proves that the momentum is also an invariant and equal to $m\, c$. If energy were dependent on velocity then there would be no energy conservation.

4. The Momentum between two Particles

We define a four-vector as a set of four quantities, which transforms like the coordinates t, x, y and z. This representation has been enormously successful in Physics, although, at first sight, it would seem that the time does not mixes with the spatial coordinates, the electrical charges with the currents or the energy with the momenta.

Throughout this paper we will try to stick to the $(-,+,+,+)$ signature convention, with a Minkowski metric.

Let us assume two physically coupled bodies, such as the electron and proton in a hydrogen atom, or the sun and Mercury in a planetary system. Let M and m represent their respective masses. The internal momentum of each of the static bodies is:

$$\begin{aligned} P_M^{\mu} &= M\, (i\, c, 0, 0, 0) \\ P_m^{\mu} &= m\, (i\, c, 0, 0, 0) \\ P_a^{\mu} &= a\, (i\, c, 0, 0, 0) \end{aligned} \tag{13}$$

Experimentally we know that the two bodies cannot continue to exist as a static system. Therefore, we have to include, in addition, the mass, a, which has the internal momentum given by the last of the equations (13). The total momentum, P_s^{μ}, of the static system, is computed as the sum of the three four-momenta (13):

$$P_s^{\mu} = (a + m + M)\,(i\, c, 0, 0, 0) \tag{14}$$

Actually, the mass a does not exist as such, but as an equivalent binding momentum between the particles. Such binding momentum is divided among the two bodies in the manner that is computed below.

This means that the two bodies need to be in motion. Let us assume that the velocity of the body of mass m is v and the velocity of the body of mass M is w.

The boosted momenta of the bodies are computed as was suggested in equation (12):

$$\begin{aligned} P_{M'}^{\mu} &= m\, \gamma(v)\,(i\, c, v, 0, 0) \\ P_{m'}^{\mu} &= M\ \ \gamma(w)(i\, c, w, 0, 0) \end{aligned} \tag{15}$$

The total momentum of the boosted system, P_b^{μ}, is the sum of momenta of the two boosted bodies:

$$P_b^{\mu} = P_{M'}^{\mu} + P_{m'}^{\mu} = m\gamma(v)\,(i\, c, v, 0, 0) + M\ \gamma(w)(i\, c, w, 0, 0) \tag{16}$$

As it is assumed that all the internal momentum of the mass a is used to set the system in motion, then the total momentum of the static system becomes equal to the corresponding total momentum of the moving system. This is achieved by equating separately each of the components of the four-vectors P_s^{μ} and P_b^{μ}:

$$(a + m + M) = m\gamma(v) + M\ \gamma(w) \tag{17}$$

$$0 = m\gamma(v)\, v + M\ \gamma(w)\, w \tag{18}$$

Thus, we have obtained two equations of motion, where the only unknowns are the velocities v and w.

Pretty simple, right? The origin of mass, the missing mass or mass defect, atomic physics, electromagnetism, gravitation—it's all there!

If the masses of the two bodies are known, the velocity v can be computed as:

$$v = \frac{\sqrt{a\,(a + 2m)(a + 2M)(a + 2m + 2M)}}{a^2 + 2(a + m)(m + M)}\, c \tag{19}$$

The solution for w is symmetric to this one and obtained by interchanging the masses M and m.

Solving this equation for the binding mass, a as a function of the speed v:

$$a = (\gamma(v) - 1)m - M + \sqrt{(\gamma(v)^2 - 1)m^2 + M^2} \quad (20)$$

If we multiply everything by the speed of light, then the first term in the right-hand side is recognized as the relativistic kinetic momentum (or energy) of the particle m.

The first term within the square root also has a relativistic interpretation. It is the momentum of motion of the particle m, which could be re-expressed as $\gamma(v)\ m\ v$.

A slightly different form of the same equation (20), is the following:

$$a = -(m+M) + \left(m^2 + M^2 + 2\frac{m^2 v^2}{c^2 - v^2} + \ldots \frac{2mc}{c^2 - v^2}\sqrt{M^2 c^2 + (m^2 - M^2)v^2} \right)^{1/2} \quad (21)$$

So, equation (20) multiplied by the square of the speed of light provides the binding energy as a function of the masses of the particles and the speed of the smaller one.

Let us apply formula (19) to compute the velocity of the electron in the first circular Bohr orbit around the proton in the hydrogen atom. In such case, m is the mass of the electron and M is the mass of the proton. The mass a is equivalent to the ionization energy of the hydrogen atom (about 13.6 eV). Using the most accurate data known by the author, the velocity obtained by applying formula (19) is: $v = 2187.052\,\mathrm{km/sec}$ (the amount of binding energy is computed, classically, multiplying Rydberg's constant by Planck's constant divided by the speed of light). This speed has a difference of about $639\,\mathrm{m/sec}$ or about 0.03% error with respect to the value computed by multiplying the classical value of the fine structure constant by the speed of light ($v = 2187.691\,\mathrm{km/sec}$).

This is a large error. Our interest at the moment is not to blame for this error to CODATA, NIST or whatever values of the constants, but to examine the theory, which is, as explained above, very intuitive and elementary. Later a new value for the fine structure constant will be suggested.

Let us continue now by trying to apply the same equation (19) to the planetary system. If we succeed in this endeavor it will mean that we are in the correct track for unifying gravitation with electromagnetism. At least this equation would be shared by both phenomena.

We need to compute, first, the energy that the planet has for being moving around the sun. As a first approximation let us compute the part of the energy a allocated to the planet so it does not collapse towards the sun. This is just the kinetic energy.

For the known data of the Earth, the square of its orbital velocity, v, is given by $v^2 = G\,Ms / AU$, where G is Newton's gravitational constant, Ms is the mass of the sun and AU is one astronomical unit or the distance Sun-Earth. With these values we find the kinetic energy of the Earth: $T = \frac{1}{2} Me\, v^2$. Therefore, the binding mass is

$$a = \frac{Me\, v^2}{2c^2} \quad (22)$$

By replacing the previous values in equation (19) we recover the speed v of the Earth around the sun.

As you should have noticed, it is not very useful to compute the velocity of orbiting by using as data the kinetic energy, which requires the same velocity we wish to compute. However, what we have proved is that the equation (19) works as intended and, in fact, applies without changes both to electromagnetism and to gravitation. The only remaining problem would be to express the velocity v as a function of the binding energy and solve for a as a function of the masses of the intervening particles.

From equation (22) solve for the velocity v, for a body of mass m:

$$v = \sqrt{\frac{2a}{m}}\,c \quad (23)$$

If Physics had the correct value for the speed v, equation (23) would identical to equation (19). As there is a discrepancy, this means that we have to change the formulae for kinetic energy so as to obtain the right-hand side of equation (19).

Expanding in series the equations (20) or (21), the terms that seem more relevant are (the right-hand side must be divided by the square of the speed of light to obtain mass):

$$a = \frac{1}{2} m v^2 + \frac{1}{2}\frac{m^2}{M} v^2 + \frac{3}{8} m \frac{v^4}{c^2} + O(v^4) \quad (24)$$

The first and third terms can be attributed to the kinetic energy. Special Relativity also allows computing such terms, by expanding in series the expression for relativistic kinetic energy: $T = m \cdot c^2 \cdot \left(1/\sqrt{1 - v^2/c^2} - 1\right)$. The rest of this expansion, in addition to the mentioned two terms, is negligible to explain the Le Verrier anomaly of the precession of Mercury's orbit. In particular, the second term of our expression (24) does not have a corresponding term in the relativistic series expansion. For the case of Mercury the second term in expression (24) evaluates to $6.55 \cdot 10^{25} \cdot joule$.

This result is about 4.1% times larger than the value predicted by General Relativity with the expression [6 (Eq. 5.36)]:

$$\frac{3 \cdot G \cdot M \cdot L^2}{c^2 m r^3} \quad (25)$$

where r is the distance Sun-Mercury and $L = m r v$ is the angular momentum of the planet. Therefore, replacing this value of L in (25):

$$3m\frac{G\cdot M}{r}\frac{v^2}{c^2} \tag{26}$$

As was shown above, the quotient $G\cdot M/r$, represents the square of the velocity of the planet. Consequently, all this expression can be simplified to:

$$3m\frac{v^4}{c^2}\approx 6.289\cdot 10^{25}\cdot joule \tag{27}$$

It is interesting to note that the term with the leading value in our computation is of order v^2, whereas in GR the analogous term, shown in the expression (27), used to simulate this effect, is of order v^4.

As a second application, let us compute the electron velocities around the nucleus of the chemical elements, with equation (19), taking as data the experimentally measured 1*s* orbital binding energies [7].

The velocities thus computed form just about a line, as a function of the atomic number, Z (number of protons in the nucleus), shown in the following figure:

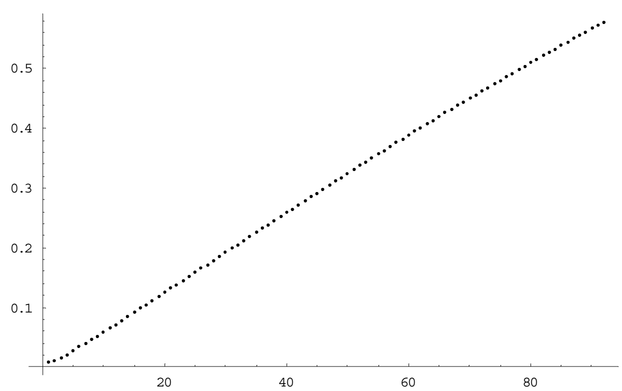


Fig. 1 - Speed, in "c" units, computed with eq. (19)

The linear fit of this quasi-line, assuming zero for the constant term and Z as the independent variable, is:

$$v=0.006398186\cdot Z\cdot c \tag{28}$$

Notice that, for $Z=1$, classical Physics predicts that the speed of light must be multiplied by the fine structure constant in order to obtain the speed of the electron in the hydrogen atom. Here we have found that the value needed for such constant is very different with respect to the classical value of the fine structure constant, namely:

$$\begin{aligned}&\text{classical:}\quad \alpha=0.007297353\\&\text{computed:}\alpha=0.006398186\end{aligned} \tag{29}$$

The differences, between the speed computed with the linear function (28) minus the speed computed with equation (19), have a rather continuous curve, with a minimum error around -0.005 and a maximum around 0.010 [in "*c*" units], as shown in the following graph:

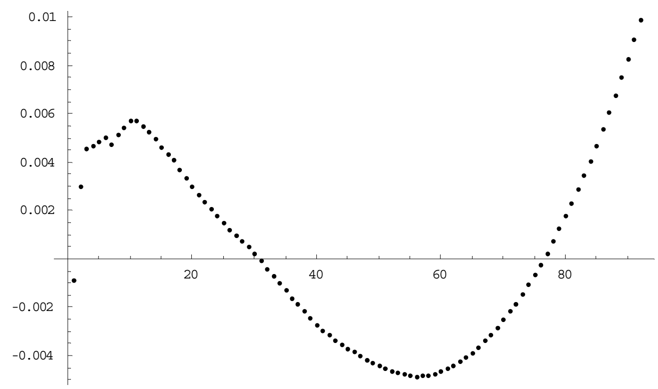


Fig. 2 - Differences between eq. (28) minus eq. (19)

This shows that the linear model is a rather good fit for the speeds of the electrons around the nucleus. However, there is a second order effect, evidenced by the differences shown in Fig. 2, which depends on some peculiar property of the chemical elements that is not revealed by the direct application of our equation (19). For example, the observed binding energy for oxygen is of 543.1 eV and the energy recomputed with the equation (20), using the linear-speed model (28), produces the value 670.76 eV, which is close to the binding energy for the ionized oxygen O_{VI} [8].

In another example, the binding energy for iron is reported as 7112 eV and our model computes 7220.8 eV. Several iron ions show absorption spectra dominated by 1*s* resonances close to the second value [9].

Still a third example that reinforces our confidence in equation (28) refers to the known hydrogen transition line $Ly\alpha=1215.67\cdot 10^{-10}m$ [10], which converts to 10.20 eV. Confront this with the 10.46 eV computed by inserting the speed $0.0063982\cdot c$ for hydrogen (Z=1) from equation (28) into equation (20).

Now, let us compare our results with the 1s Slater-type function

The speed of the 1s electron for gold (Z=79), computed with equation (28), is $v=0.505457\cdot c$. Replacing into equation (20), with a mass $M=79$ proton masses, returns $a=81222\,\text{eV}$. This figure compares favorably with the experimental energy ($a=80725\text{eV}$). On the other hand, the Slater's theory [11] provides much larger values, namely: $a=84913.2\,\text{eV}$ for the non-relativistic case and between $a=90994\text{eV}$ and $a=93460\,\text{eV}$ for the relativistic computations. It is possible to obtain much better results if we fit the speeds not only with a line, as in equation (28), but also fitting the differences of Fig. 2 with a polynomial.

5. Explaining the Pioneer anomaly problem

One question that the reader might put forward is whether the binding mass-energy is affected gravitationally in any form. The answer seems to be affirmative.

Using some data from the Pioneer spacecrafts reported by Nieto and Anderson [12 (Table 2)] we are able to compute the rough average speed of each craft by dividing the distance to the sun in astronomical units (AU) by their travel time. Such velocity is not accurate since the distances are in the line of sight to the sun and not the lengths of the arcs traveled by the Pioneers.

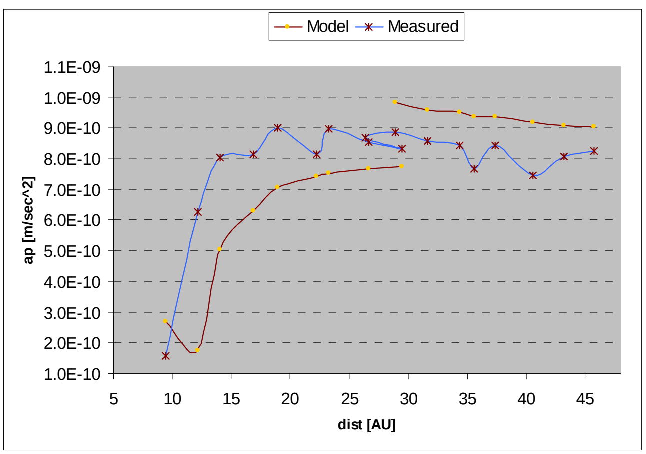
However, the results are very encouraging and provide a model that can be refined with more precise data.

Let us estimate, the proportion between the acceleration attributed to the binding mass and the mass of the spacecraft, by using only the first term of equation (24). Divide the square of the velocity of the body by twice the square of the speed of light.

It is known that the square of the velocity of a body divided by its orbital distance is acceleration. For this case, the distances are the same both for the mass of the spacecraft as well as for its kinetic energy so it is simplified:

$$\frac{a_p}{m} = \frac{v^2}{2c^2} \tag{28}$$

The figure below shows, in the y-axis, the acceleration in meters per second in each second and in the x-axis the distances in AU's. The blue line is the measured data and the brown lines correspond to the models for the Pioneer 11 (curve mostly below the blue line) and Pioneer 10 (above the blue line):



As can be seen, our model is a rather good predictor of the accelerations, even without using an accurate data for velocities. The discontinuity found at about 30 AU's corresponds to the two different spacecrafts, which, obviously, have two different orbits. These differences should be reduced with more accurate data of velocities.

6. Conclusions

Maybe it is not very humble to say it, but it is the firm conviction of the present author that his hypothesis of the electromagnetic constitution of matter provides a framework to explain all physical phenomena.

The present essay was prepared just to continue providing some evidence to confirm the previous paragraph. The reader should refer to the previous paper of this author [1], where he presented a successful reformulation of four-vectors and of electromagnetic theory.

"As is often the case in science, once a dramatic leap forward is taken and all the facts click into place, then it seems obvious that what has been discovered is true." [13]

The binding energy in the atoms can be quantized by inserting the needed number of protons, Z, in the nucleus. Also, the principal quantum number has not been included but must, necessarily, be given by the fact that the electrons occupy only levels or orbits with an integer number of wavelengths.

It is noteworthy that, despite the very wrong values produced by Slater's theory, the paper where it appeared [14] has been one of the most often cited papers in a 22-year period [15]. This reference remarks: "The oldest paper was published in 1930 in *Physical Review*. J.C. Slater, Harvard University, Cambridge, Massachusetts, described a simple method for approximating wave functions and energy levels of atoms and ions. The paper was cited about 1,200 times from 1961 to 1982. It received 35 citations in 1983, indicating that physicists not only still rely on Slater's atomic shielding constants but also feel compelled to explicitly cite the primordial reference more than 50 years later."

Nieto and Anderson conclude that: "[U]nderstanding the Pioneer anomaly, no matter what turns out to be the answer, will be of great value scientifically." [12] The present author considers that his model of gravitational attraction of the binding masses is reasonable and predicts rather well the measured phenomena. This is compounded by the applications to atoms and planets, which provide an all-encompassing theory.

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