A Bohr-type assessment of the Quantum Hall Effect

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28 April 2010, *filename PostQHE1.tex, fwh*

Abstract

Arguments of Cooper type screening and Bohr's angular momentum quantization yields a joint expression for integer and fractional Quantum Hall effect while confirming the Doll-Fairbank flux quantum $h/2e$ over h/e.

When Klaus von Klitzing at al. [1] discovered their surprising transitions between normal- and quantum Hall effects, it was correctly inferred that random drift patterns of conduction might be converting into cyclotron electron states that might be moving more easily in the 2-dimensional interaction space of the Mosfet sample. In fact the proximity of adjacent layers provides a Cooper-type screening accounting for a highly reduced interaction between electrons in the same cyclotron orbit. An indeed observed induced super-conductivity of the sample current hints at effects of this nature. However, rather than calling Schrödinger's equation to assess cyclotron quantum states, we shall call instead on a methodology that gave Bohr the stationary states of hydrogen. Let r be the cyclotron radius, m the electron mass, the magnetic induction B is perpendicular the sample's 2-dimensional interaction space, e electronic charge, ω circular frequency and $h = 2\pi\hbar$ Planck's constant. The equation of motion gives for each electron that may now be taken to be in the same simple circular orbit, it is

$$mr\omega^2 = er\omega B \qquad \rightarrow \qquad \omega = \frac{e}{m} B \,. \tag{1}$$

The Bohr condition

$$mr^2\omega = n\hbar = n\frac{h}{2\pi} \,; \qquad n = 1, 2, 3, \ldots \,. \tag{2}$$

The cyclotron energy per electron E_p is the sum of its kinetic energy and its interaction energy with the B field

$$E_\mathrm{p} = \frac{1}{2} m r^2 \omega^2 + \pi r^2 B e \omega / 2\pi\,. \tag{3}$$

Replacing B in Eq. (3) by using Eq. (1) gives:

$$r^2 B e \omega / 2\pi = \frac{1}{2} m r^2 \omega^2\,.$$

The following mechanical, magnetic and quantum energy expressions E_p are equivalent:

$$E_\mathrm{p} = m r^2 \omega^2 = r^2 B e \omega = n \hbar \omega\,. \tag{4}$$

It follows from the last two expression in Eq. (4) that the flux intercepted by the orbital charge equals

$$\Phi = \pi r^2 B = n \frac{h}{2e}\,. \tag{5}$$

Note that flux quantum $h/2e$ is the one experimentally verified by Doll et al. [2] and Fairbank at al. [3]; it is half the value of the one suggested by Aharonov-Bohm [4].

Eqs. (1) to (5) give the ingredients needed for the quantum Hall effect culminating in a quantized Hall impedance Z_H = Hall voltage over Hall current. The Hall voltage is the flux passing the contact sensor of the voltage circuit and the Hall current is the charge passing per unit time Z_H = cyclotron flux/cyclotron orbital charge. For an ordered quantum situation the number of identical cyclotrons simultaneously passing the Hall sensor and the transition speed cancel. Hence a single one of the cyclotron sets the stage for the flux over charge ratio assuming s electrons per orbit. We have from Eq. (5)

$$Z_\mathrm{H} = \frac{n}{s} \frac{h}{2e^2} \qquad n \text{ and } s \text{ integers})\,. \tag{6}$$

For n and s integers, Eq. (6) reproduces a result suggested in a 1982 article [5] in which the Hall impedance was identified as the ratio of two integrals, Aharonov-Bohm and Gauss-Ampère, holding the position of period integrals in de Rham Cohomology [6]. Eq. (6) simultaneously describes the integer effect $n = 1$ and the fractional effect $n > 1$, which is the somewhat artificial quantum distinction ensuing from using Schrödinger methods. So the morale of this little pre-1925 intermezzo shows us how a highbrow topology-based assessment can be reproduced by a lowbrow Bohr-like assessment of the quantum Hall effect. It perhaps shows us how the highbrow approach is not way out and devoid of reality whereas a presumed lowbrow path brings as surprisingly closer to physical reality. It is up to the assigned protectors of the quantum faith whether it is sound policy pursuing understanding of highly ordered systems with tools designed to assess statistics.

References

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