Finding Magic Numbers for Heavy and Superheavy Elements

Roger A. Rydin

Associate Professor Emeritus of Nuclear Engineering, University of Virginia

626 Cabell Avenue, Charlottesville, VA 22903-2011

e-mail: rarydin@earthlink.net

For at least sixty years, scientists have known that certain numbers of protons or neutrons in nuclei formed closed shells of some kind, producing additional stability to nuclei that possess these properties. The most stable nuclei, or nuclei exhibiting enhanced stability, are called doubly magic. Only recently, Lucas has explained that the magic numbers are really composites of several sub shells filling, rather than being single shells. Protons are charged particles, and by Coulomb repulsion they try to get as far away from each other as possible, hence tending to occupy the outer regions of nuclei. Neutrons, being uncharged but possibly polarizable, tend to occupy both outer and inner shells and to possibly increase the number in an outer shell when the nuclei are heavy in a similar way to electrons filling in inner shells in the Lanthanide and Actinide series.

Using a simple modification of Lucas' geometrical packing scheme, individual candidates for new magic proton numbers and new magic neutron numbers have been identified. Amazingly, these new magic numbers correspond to the experimentally identified superheavy element distribution to a very large extent. As an added bonus, the newly suggested magic numbers correspond to the long lived Thorium and Uranium isotopes, and to the Fermium isotopes, which may help explain the shape of the Peninsula of Heavy isotopes. They also suggest going back to reassess somewhat lighter isotopes in the Continent. This has been done for several isotopes, and magic effects do indeed appear.

1. Introduction

In the late 1950s, I took two semesters of Nuclear Physics for Engineers, taught by Robert A. Dudley and Robley D. Evans. We used Evan's book, **The Atomic Nucleus** [1], which was arranged in a very logical manner, covering information about nuclei in the order of: charge; radius; mass; moments, parity and statistics; isotopic abundance; systematics of stable nuclei; binding energy; nuclear forces; nuclear models; and nuclear reactions. After this came: cross sections; radioactive decay; spectra; charged particle slowing down; gamma ray effects; and statistics. Evans was an experimentalist, whose research centered on measuring the effects of radium in the body. As such, his book was heavily centered on experimental data, and what it said about the nucleus. He discussed the theory of the time, but treated it as work in progress, subject to change and not cut in stone. This was before development of the Standard Model.

Two of the things that fascinated me the most were: 1) the Semi-Empirical Binding Energy formula, which fit the BE/A data for A>30 but missed the low A peaks {see Figures 1 and 2}; and 2) the "Magic Numbers" that led to extra stable isotopes and such things as the "double hump" fission product curve, delayed neutrons and fission product poisons. The Binding Energy formula led to the mass parabolas determining beta and positron/EC decay. Two things that disturbed me the most were: 1) the idea that nucleons were rapidly moving around inside the nucleus in shells or orbits of some kind; and 2) alpha particles were emitted in alpha decay by "magically" tunneling through the Coulomb potential barrier in a statistical manner via wave mechanics.
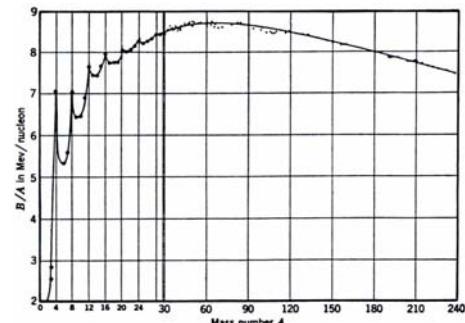
Fig. 1. Binding Energy per Nucleon vs. A
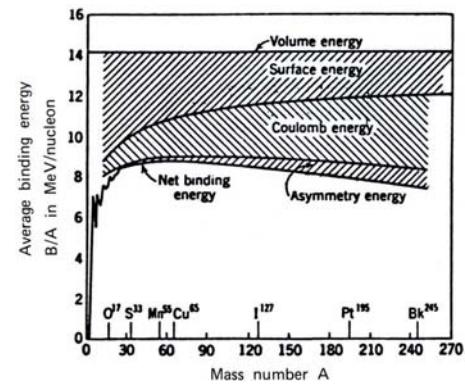
Fig. 2. Binding Energy per Nucleon Contributions

2. Magic Numbers

Wikipedia [2], "In nuclear physics, a magic number is a number of nucleons (either protons or neutrons) such that they

are arranged into complete shells within the atomic nucleus. The seven most widely recognized magic numbers as of 2007 are:

2, 8, 20, 28, 50, 82, and 126 (for neutrons)

Atomic nuclei consisting of such a magic number of nucleons have a higher average binding energy per nucleon than one would expect, and are hence more stable against nuclear decay."

3. Superheavy Elements

From Wikipedia [2] (with minor modifications), "The unusual extra stability of isotopes having magic numbers implies that extremely heavy transuranic elements can be produced that are not subject to the short half life radioactive decay normally associated with high atomic numbers (as of 2007, the longest-lived, known isotope among all of the superheavy elements between Z = 110 and 120 lasts only 12 minutes, and the next longest lasts 22 seconds). Superheavy isotopes with magic numbers of nucleons are said to exist in an "island" of stability. Unlike the magic numbers 2 to 126, which are realized in spherical nuclei, theoretical calculations predict that superheavy nuclei can be deformed. Before this possibility was considered, higher magic numbers, such as 184, were predicted based on simple calculations that assumed spherical shapes." The superheavy nuclei decay primarily by alpha decay, with spontaneous fission as an additional decay mode. Areas of stability for deformed nuclei have been predicted around N = 152, 164 and 172 [3].

From Gupta [4], "The superheavy elements are traditionally considered to be those that lie above element 103 (Lawrencium, Lr), the last of the actinides. Starting with Rutherfordium (Rf), element 104, these elements are sometimes referred to as the super-transactinides. Collectively, they represent the very top end of the Periodic Table of Elements and a study of their properties is intrinsically linked to an understanding of the physics and chemistry at the limit of stability in mass and charge. The limitation on the number of chemical elements possible remains a long standing question."

"Due to the rapid increase of the repulsive Coulomb forces between the protons, the number of chemical elements is limited by fission. This macroscopic behavior is governed by shell effects, without which the nuclear chart may end near Element 106 (Seaborgium, Sg). There is evidence to suggest that nuclei can survive beyond the macroscopic limit, far into the trans-uranium region, where the necessary balance between the nuclear force and the Coulomb force is achieved only through shell stabilization. Superheavy elements are hypothesized to exist near the next (predicted) double shell closure above Lead where they may have surprisingly long half-lives, maybe even on the order of millions of years. This postulate has fuelled vigorous research in the field, thereby earning it the reputation of being a search for the next "magic" shell, a Holy Grail of contemporary physics. On the way to this region of extra stability, deformed regions exhibiting stronger binding to varying degree are also suggested." In fact, there is no experimental data as yet to support the hypothesis that new superheavy doubly magic nuclei will be very much more stable

than their neighbors. Magic simply reduces the Q value available for decay, and correspondingly increases the half life.

"Over the past decades, different theories have been put forward in an attempt to uncover the physics of this elusive mass region. Microscopic-Macroscopic (MM) theories traditionally involve a number of parameters and assume prior knowledge of densities and single particle potentials around the mass region of interest. They predict the next "magic" shell at Z = 114 and N = 184."

"Both non-relativistic (e.g. Skyrme-Hartree, Fock-Bogliubov) theory and relativistic microscopic mean field models (RMF) predict probable closures at Z = 114 and 120. The important spin-orbit term is incorporated manually in the non-relativistic theories whereas it emerges naturally within the relativistic formalism. RMF theory utilizes a smaller number of parameters which are obtained through a chi-square fit to the ground state properties of doubly magic and a few open shell spherical nuclei. Taking into account pairing effects, relativistic theory predicts additional shell closures around Z ~ 108 - 110; N ~ 162 and possibly N ~ 172, apart from the ones at N = 184 and Z = 114. Interestingly, it is seen that predictions of new "magic" numbers depend on the combination of both N and Z." Indeed, just looking at the plot of the known heavy and superheavy nuclei given in Fig. 1, the N/Z ratio for the most stable isotopes needs to be somewhere in the vicinity of 1.54, which only allows certain combinations of magic numbers to come into play.

Whereas N = 184 is yet to be reached, the emerging experimental support is encouraging. Currently, elements up to Z = 118 have been artificially synthesized and efforts are on to create Z = 120 in the laboratory. Most recently, alpha decay chains assigned to the parents 293,294117 have been reported from Dubna. The most neutron rich element synthesized is still about 7 neutrons away from the "magic" N = 184."
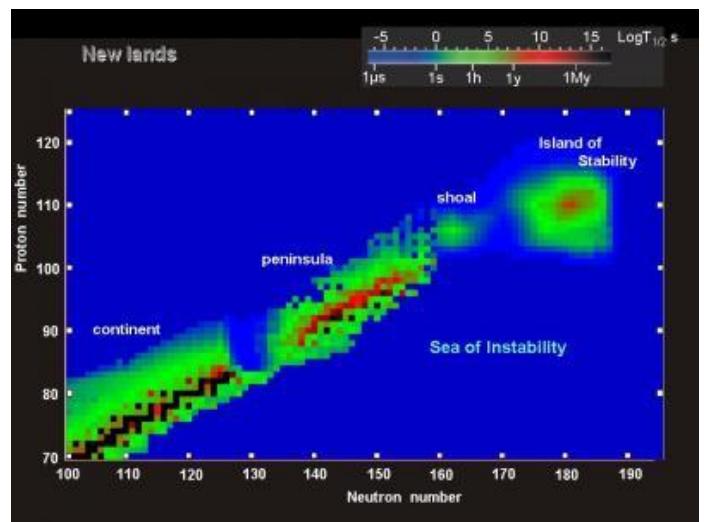

Fig. 3. Isotopes in the Sea of Instability [5]

4. Electrodynamic Model of the Nucleus [6]

The nuclei binding energy data were qualitatively fit by the old Semi-Empirical Binding Energy formula, which was an attempt to combine the liquid drop model and the quantized nuclear shell model. For more than 40 years, no theory was put for-

ward that could quantitatively explain why all of these ideas worked.

Based upon early experimental and theoretical work done by Compton [7–9] and his student Bostick [10], a new qualitative explanation for these phenomena has been obtained by Lucas [6]. Protons and neutrons are each represented by small charged ring magnets, as suggested by X-ray scattering experiments on electrons [11], and these nucleons are then arranged as symmetrically as possible in three dimensional space so that the electrodynamic forces between them attain *static balance*. Geometrical packing follows some electrodynamic constraints, so the pattern is not completely arbitrary. The neutron is known to have an internal charge distribution, so it can polarize and orient its positive and negative ends to a position of torque balance. Lucas' new model, having static positioning of nucleons, along with his new Semi-Empirical Binding energy formula that fits A<30, answers *both* of my early concerns about the nucleus! Alpha decay is a vibration process leading to an unstable configuration, just like fission.

With this model, Lucas [12] predicts all the magic number shell closings for neutrons and protons and explains why they have the values we know. Using a similar model for atoms, he also predicts the periodic table [13] and shows why nuclear shells are different from atomic shells. For Pb-208, the protons occupy the outermost two rings of 32 and 50, giving the magic number of 82, while the neutrons occupy five different rings of 8, 18, 18, 32, and 50 that add up to 126. The protons get as far away from each other as possible, which is why they occupy the outer shells.

Between these two proton shells lies a neutron shell, where the neutrons are polarized sideways. That shell acts like a force decoupler and gives the nucleus its liquid drop properties corresponding to the vibration-induced liquid drop model of fission. The rest of the neutrons polarize away from the protons, and the innermost shell is left empty as the positive ends of the neutrons also get as far away from each other as possible. It is the rearrangement and emptying of inner shells when outer shells can be completed that accounts for the difference between nuclear magic numbers and the atomic magic numbers that correspond to the noble gases.

The basic Lucas filling structure has eight cycles, but many of the cycles for the heavier nuclei are empty. The structure from the inside to the outside is the pattern: 2, 8, 18, 18, 32, and 50. The purpose of the extra shells is not obvious, since 6 cycles could do the job for all of the stable isotopes, especially if the outer shells can at some point continue to fill to the next number like the inner electron shells fill in for the Lanthanide and Actinide series (i.e., if an inner 18 shell fills to 32 and an inner 32 shell fills to 50 at some point). His original assignments for the known doubly magic nuclides, when the table is reduced to 6 cycles, would be as shown in Table 1, with a tentative assignment for Ni-48 and Tin-100 using the logic given above that the protons tend to stay to the outside while neutrons fill in towards the center. That is why Calcium-40 in the Lucas model uses shells of 14 and 6 protons instead of 18 and 2 protons to get to the proton magic number of 20. This is a small subset of his complete table.

	A	Z	N	N_1	P_1	N_2	P_2	N_3	P_3	N_4	P_4	N_5	P_5	N_6	P_6
He	4	2	2	2	2										
O	16	8	8			8	8								
Ca	40	20	20			6	6	14	14						
Ca	48	20	28	2		8	6	18	14						
Ni	48	28	20		2	6	8	14	18						
Sn	100	50	50									18	18	32	32
Sn	132	50	82			6		8		18		18	18	32	32
Pb	208	82	126			8		18		18		32	32	50	50

Table 2. Modified Lucas' "Rule" Assignments for Doubly Magic Isotopes

5. Extension to Possible Superheavy Combinations [13]

$$
\begin{aligned}
Z = 50 + 32 + \quad 8 &= 90 \\
Z = 50 + 32 + \quad 8 + 2 &= 92 \\
Z = 50 + 32 + 18 &= 100 \\
Z = 50 + 32 + 18 + 2 &= 102 \\
Z = 50 + 32 + 18 + 8 &= 108 \\
Z = 50 + 32 + 18 + 8 + 2 &= 110 \\
Z = 50 + 32 + 18 + 18 &= 118 \\
Z = 50 + 32 + 18 + 18 + 2 &= 120
\end{aligned}
\tag{1}
$$

We use the six cycle pattern, with the protons filling in from the outside, and the neutrons filling in between the proton shells towards the center. If we then expand the neutron shells as the nucleus becomes larger and more attraction is needed to stabilize the nucleus, we obtain a geometrical pattern of new shell fillings. This leads to the following possible next "magic" Z numbers

beyond 82, where only the outer shells are first filled as the protons get as far away from each other on average as possible.

It is ambiguous to say that all of these are truly magic, and that some new magic numbers may be only two protons apart for Z. The new semi-empirical binding energy fit developed by Lucas [6] makes no such distinction, so we are really speaking about added binding when sub shells are filled. Lucas uses a "surface" term which counts only the neutrons and protons in the outer sub shell which is near the nuclear surface. He does not have a symmetric "asymmetry" term of the form $(N - Z)^2$. His "pairing" term includes only the sum of the unpaired protons and neutrons. He combines the "asymmetry" and "magic" terms as $(\# \text{ paired neutrons} - \# \text{ paired protons})^2$. The nucleus is becoming crowded, and the number of possible fillings of six shells is becoming limited.

$$
\begin{aligned}
N &= 50 \quad\quad\; + 32 + 32 + \;\;8 \quad\quad = 140 \\
N &= 50 \quad\quad\; + 32 + 32 + \;\;8 + 2 = 142 \\
N &= 50 + 50 + 32 \quad\quad + 18 + 2 = 158 \\
N &= 50 + 50 + 32 + 32 \quad\quad\quad\; = 164 \quad\quad (2)\\
N &= 50 + 50 + 32 + 32 + \;\;8 \quad\;\; = 172 \\
N &= 50 + 50 + 32 + 32 + 18 \quad\; = 182 \\
N &= 50 + 50 + 32 + 32 + 18 + 2 = 184
\end{aligned}
$$

Following the same general scheme also leads to the following possible next "magic" N numbers beyond 126, normally leaving the central 2 shell empty. Here we fill in the next outer shell to a new maximum, to give the nucleus more binding power against the additional outer shell protons in the heavier nuclei.

Note that these purely geometrical shell fillings include all the theoretically proposed new magic numbers except $Z = 106$ and 114. What is new is that many more possible magic numbers and combinations come to light that have not been examined before against available nuclear data in the medium and heavy nuclei range. We already have many isotopes in the heavy nuclei zone with half lives of millions of years! There may be more at work here besides magic. Neutrons act as glue to hold the protons together, and the optimum amount of glue corresponds to N/Z approximately equal to 1.54.

Most presently known experimental data on the heavy and superheavy elements is shown in Figure 1 as peninsulas, shoals and islands in the Sea of Instability. We see that the lower end of the Peninsula is near $Z = 90$ and $N = 140$, while the upper end of the Peninsula is near $Z = 100$ and $N = 158$. This represents most of the long-lived red-black band. The low end of the green area is near $Z = 82$, while the upper end of the green area is around $Z = 108$. Can all this be coincidence, since no fancy theoretical modeling was used, only a geometrical pattern of filling 3D space in a symmetric way?

The Shoal is near $Z = 108$, and it lies between $N = 158$ and 164. The Shoal is probably part of the Peninsula. The Island of Stability is centered with a red area near $Z = 108$ and $N = 182$, while Z lies between $Z = 102$ and 116, and N lies between $N = 172$ and $N = 184$. The highest Z discovered Isotope to date is $Z = 118$, although $Z = 120$ is being sought.

6. Examination of the Peninsula

Since magic numbers were never before proposed for Z between 82 and 126, and the composite sub shell nature of the currently accepted magic numbers was unrecognized, it becomes pertinent to examine what consequences new single and double magic might have on the pattern of known isotopes in the Peninsula area of heavy nuclei. The available data are summarized in the Table of Isotopes [14], and newer data in the ENSDF database.

The next two proposed doubly magic nuclei using the new shell closing scheme, correspond to Thorium-230, taking $Z = 90$, $N = 140$, and Thorium-232, at $N = 142$! There are 19 known isotopes of Thorium, ranging from Th-218 to Th-237. Th-218 has a half life of nanoseconds, and then the half lives range up to minutes for Th-226 and days for Th-227. These isotopes come mostly from alpha decay of heavier isotopes. Th-229, one isotope short of doubly magic has a half life of 7340 years. Th-230 has a half life of 75,380 years, followed by Th-231 at 25 hours. Th-232 has a half life of 1.4E10 years. The heaviest four Thorium isotopes up to Th-237 then have half lives of days to minutes. These results seem to confirm the assertion that Th-230 and Th-232 are doubly magic, if long half lives are the primary criteria. Perhaps relatively long half lives is a better criterion. Both have an N/Z of approximately 1.54 which helps stability. The most proton rich isotopes have the shortest half lives, so having extra protons is worse than having extra neutrons.

Next, we examine Uranium which is also possibly magic at $Z = 92$. There are 20 known Uranium isotopes, ranging from U-222 to U-242. The first ten have half lives from microseconds to days. U-232 is possibly doubly magic with $N = 140$, and has a half life of 68 years. U-233 has a half life of 1.6E5 years, followed by possibly doubly magic U-234 with $N = 142$ at 2.45E5 years. U-234 is a known longer-lived exception to the Seaborg spontaneous fission correlation for even-Z, even A nuclei [15]. However, the next two isotopes are also long lived, U-235 has a half life of 7.15E5 years and U-236 has a half life of 2.4E7 years. The next isotope has a half life of days, while U-238 has the longest half life of 4.5E9 years and $N/Z = 1.52$. The last three isotopes have half lives from days to minutes. Obviously, more than magic is involved in this pattern, although magic is clearly playing an important part.

There are a number of other long lived isotopes that are heavier than Uranium, and which also need to be examined. The pattern of Neptunium and Plutonium isotopes is somewhat similar to the pattern for Uranium, except that the long lived isotopes do not closely correspond to magic numbers. The most striking observation is that the long lived isotopes all lie at a ratio of N/Z near 1.54. This represents the middle of the N versus Z plot shown in Fig. 3.

Fermium is element 100, which is at the next possible magic Z number. It has 19 known isotopes. Fm-242 has a half life of 0.8 milliseconds, and isotopes up to Fm-250 have half lives of seconds to minutes. Isotopes up to Fm-256 have half lives of hours to days. The longest lived isotope is Fm-257 with a half life of 100 days. The next higher isotope is Fm-258, which would possibly be doubly magic at $N = 158$ with a spontaneous fission half life of 37μs. This is an obvious exception, and it is followed by two more isotopes with half lives of seconds and milliseconds. The process of fission is physically different from alpha decay, so that may be part of the explanation.

Nobelium is element 102, which is at the next possible magic Z number. It has 12 known isotopes. No-250 has a half life of 0.25 milliseconds. Isotopes ranging from No-251 to No-257 have half lives in the range of seconds to minutes. No-259 has the longest half life of 58 minutes, and then the heavier isotopes are in the range of milliseconds, with No-260 possibly being doubly magic at $N = 158$, but it decays by spontaneous fission.

For both Fermium and Nobelium, the longest lived isotope is odd A and one short of doubly magic but more stable to spontaneous fission, while the doubly magic isotope is unstable to spontaneous fission. Perhaps magic is the reason for the existence of so many isotopes of Fermium and Nobelium.

7. Reconsideration of Middle Range Isotopes

Since new possible magic neutron numbers below 126 are suggested by the proton number sequence, it is useful to examine a few suggested isotopes to see if magic effects become evident. If we take N = 90, 100 and 118 as possible new magic neutron numbers, and use a ratio N/Z = 1.54 to obtain the corresponding Z, we are led to examine the isotopes Cerium at Z = 58, Dysprosium at Z = 66, and Osmium at Z = 76 as representative candidates. These isotopes were examined in detail [14].

If magic numbers for neutrons can also be extended downwards, then perhaps there exist lower magic Z numbers as well, for example, Z = 32 + 18 + 8 = 58 which is Cerium, Z = 32 + 18 + 18 = 68 which is Erbium, and Z = 32 + 18 + 18 + 8 = 76 which is Osmium. Hence, two of the above examined middle range isotopes may also be magic in Z, and have doubly magic N partners.

Cerium has about 20 known isotopes, going from ^{133}Ce to ^{151}Ce. The major stable isotope is ^{140}Ce, magic at N = 82, accounting for almost 90% of natural Cerium. It is interesting that ^{148}Ce, at N = 90 with a half life of 56 seconds, and ^{150}Ce at N = 92 with a half life of 4 seconds are among the heaviest Cerium isotopes known, suggesting that their very existence is due to magic closing of secondary shells.

Dysprosium is another very interesting element, having almost 30 known isotopes ranging from ^{141}Dy to ^{169}Dy. Five isotopes, from ^{160}Dy to ^{164}Dy, comprise most of the naturally stable isotopes. However, ^{158}Dy, with a magic N = 92, and ^{156}Dy, with a magic N = 90, are also stable. ^{148}Dy, with a magic N = 82, has a half life of 3.1 minutes, while the other lighter Dysprosium isotopes have half lives of the order of seconds. On the heavy end of the isotope sequence are ^{166}Dy, with a magic N = 100 and a half life of 81.6 hours, and ^{168}Dy, with a magic N = 102 and a half life of 8.7 minutes. The very large number of Dysprosium isotopes meeting conditions of possible magic numbers seems more than coincidental.

Next, we look at Osmium, which has more than 30 known isotopes ranging from ^{164}Os to ^{196}Os. The naturally occurring Osmium isotopes lie between ^{192}Os and ^{187}Os. Among the lightest Osmium isotopes are ^{166}Os, with a magic N = 90 and a half life of 7.1 seconds, and ^{168}Os, with a magic N= 92 and a half life of 2.2 seconds, below which the half lives are very short. The next interesting isotopes are ^{176}Os, with a magic N = 100 and a half life of 3.6 minutes, and ^{178}Os, with a magic N = 102 and a half life of 5 minutes. Somewhat surprising are ^{184}Os, with a magic N = 108 and a half life of 5E13 years, and ^{186}Os, with a magic N = 110 and a half life of 2E15 years. Finally, on the heavy end we have ^{194}Os, with a magic N = 118 and a half life of 6 years, and ^{196}Os, with a magic N = 120 and a half life of 35 minutes. Again, the very large number of Osmium isotopes meeting conditions of possible magic numbers seems more than coincidental. The neutron rich isotopes are also more stable than the proton rich isotopes.

Finally, we examine Lead, which is magic at Z = 82. There are 33 known Lead isotopes between ^{182}Pb and ^{214}Pb. The three stable isotopes are ^{206}Pb, ^{207}Pb and doubly magic ^{208}Pb at N = 126. However, possibly doubly magic ^{202}Pb at N = 120 has a half life of 5.25E4 years, and doubly magic ^{200}Pb at N = 118 has a half life

of 21.5 hours, after which the half lives are shorter. Nonetheless, on the light side, doubly magic ^{192}Pb at N = 110 has a half life of 3.5 minutes and doubly magic ^{190}Pb at N = 108 has a half life of 1.2 minutes, after which the half lives are seconds or less. The lightest doubly magic isotope is ^{182}Pb at N = 100 with a half life of 55 milliseconds. On the heavy end, if N = 128 is also magic, then ^{210}Pb with a half life of 22.3 years is also doubly magic, after which the half lives become much shorter, on the order of minutes.

We are forced to conclude that the closing of new sub shells is an important factor in allowing the existence of so many of these isotopes.

As a final thought, if 58 is also a magic number, we would possibly obtain a much better fit to the double hump fission distributions using a base of (28, 58) for the light fragment and (58, 82) for the heavy fragment. Thus we would only have to share 6 extra protons and 4 extra neutrons between them for U-235 + n fission, rather than 14 and 12

8. Conclusion

Finding new "magic" numbers as geometrical combinations of only six shell fillings makes sense qualitatively, and gives results not far from similar shell closings predicted with complicated theoretical models. The theoretical models use fits to known structure and other measured data. Indeed, the newly suggested doubly magic numbers are a combination of the different ways that protons and neutrons act, plus the possibility of symmetrically filling 3D space. These new combinations need to be confirmed.

In a not too surprising way, the next possible shell closings are somewhat different for protons and neutrons, because of the different way they react to forces. There are many more ways of filling a shell with less spacing between the magic numbers. This is because the nucleus is becoming filled with nucleons, requiring more difficult force balances, while the possibilities for filling shells are becoming restricted. Experimental evidence indicates that the nuclear density in the centers of heavy nuclei decreases with increasing size [12], indicating spread towards the outside.

When we speak of a sub shell, it is not a spherical surface located at a fixed radius from the center of the nucleus. Rather, it is an annular band in which the nucleons are located as symmetrically as possible in order to produce a static force balance. The protons and neutrons in a given band can geometrically overlap to some extent, and the position and thickness of a band can vary as the nucleus gets larger. In principle, these positions can be verified using a computer code such as developed by Boudreaux and Baxter [16], who used such a code to verify a few light nuclei. Unfortunately, the code would have to be modified to add polarization, re-dimensioned to treat the heavy nuclei, and the iterative variational procedure would have to be verified for numerical stability.

Acknowledgements

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