Motion's Observation Through Light's Signals (I)

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Abstract

A novel view of space, time, and inertial frames that maintains the absolute nature of time is presented. One but arbitrary inertial frame say *S*, is considered stationary and identified with the absolute space, while all other inertial frames are moving relative to *S*. The constancy of the light's velocity in free space within each inertial frame is postulated and employed to link time durations measurements to geometric distance. The geometric distance in the chosen stationary frame plays the decisive role in the determination of time and distance in all inertial frames. A unique time prevails in all inertial frames, but distance between a moving object in *S* and a stationary observer in *S* is identified by the optical length of a light trip from the object to the observer; this distance functions as the geometric distance in the frame in which the object is at rest when the latter frame is considered stationary. The arbitrariness of the chosen stationary frame guarantees that all inertial frames are equivalent, and according the physical laws are the same in all. The so-called scaling transformations which relate the geometric distances in *S* and in a moving frame are derived and applied to explain the Doppler's effect and the lifetime of meta-stable particles phenomenon. The quantitative predicted Doppler's effect, which is in a striking agreement with the Ives-Stilwell experimental results, coincides with the relativistic prediction for longitudinal motion, but yet predicts complete absence of a traverse effect. The direction of the light trip is observed from a moving frame to be tilted from its direction in the stationary frame by the aberration angle; a fact which is employed to explain the phenomenon of stellar aberration. The true status of the Lorentz transformations as an equivalent form of the scaling transformation is illuminated. In a forthcoming part of this work, a second type of scaling transformations corresponding to given beginning and end of a light's trip in a stationary frame is derived and employed to explain the Michelson and Morley experiment, the Michelson and Gale experiment, and the Sagnac effect. The translative nature of the latter effect is explored and studied in detail. The pioneer anomaly which can be explained by Euclidation of optical measurements will be discussed separately.

Key words: contiguity, universal space, scaling transformations, illusive Lorentz transformations.

1.Introduction

The Newtonian conceptions of space and time are modified to incorporate observations through light's signals. The absolute Newtonian nature of distance and time is carried over to what we call proper time and proper distance. In contrast with the geometric distance (and geometric time) which are frame dependent, the proper time and distance are absolute. The proper distance between an object and an observer, which depends on their relative velocity, is determined in terms of the familiar geometric distance by means of what we call the scaling transformations. Our novel conception of space and time is characterized by the following:

(i)Time measurements are linked to spatial measurements through a constant light's velocity, which is postulated to hold within every inertial frame.

(ii) One arbitrary inertial frame S is considered stationary and identified by the absolute space, while all other inertial frames are moving relative to S. The absolute space may be corresponded with any other inertial frame, say s, without any bearing on the transformations that relate geometric distances in S and s regardless of which frame we choose to be stationary.

Based on the above requirements and the implicit assumption regarding the Euclidean nature of the absolute space we employ the geometric distance in the stationary frame to endow any other inertial frame s, through light's signals, with distance and time intervals such that the speed of light is also c within s.

The chart of logic that leads to the transformations between the geometric distances in the two frames can be summarized as follows:

- We start by identifying one arbitrary inertial frame S, which is initially coordinated using a given unit of geometric length, with the absolute space. Utilizing the postulate that light propagates rectilinearly in all directions within (inside) S at a constant speed c we set up a global time in S using the familiar procedure of clock synchronization [1]. A frame such as has been described will be called a *timed or universal frame*.
- If b is source of light moving at a constant speed $\vec{u}$ in S then the scaling theory determines the *optical or proper duration* t of a light's trip $(b\ \ at\ B \in S \rightarrow O \in S)$ in terms of its *geometric length* $|\overrightarrow{BO}| = R$. It is t what is measured for the latter trip in the universal frame S and in any other inertial frame. We shall see that the optical duration t is generally different from the geometric duration of the trip in S, which is by definition $T \equiv R/c$; they are equal however when the source of light is at rest in S.
- A basic concept of the scaling theory is that time flows equably in all inertial frames, and in particular in S and in the frame s in which the source of light b is stationary. If o is an s-observer which is contiguous to the S-observer O when light arrives at O, then the light's trip $(b \in s\ at\ B \in S \rightarrow O \in S)$ in S is the same as the light's trip $(b \in s\ at\ B \in S \rightarrow O \in S\ and\ o \in s)$, which occurs conclusively within s, and hence its duration is the same in S and s. When looking at this trip from the stationary S the Galilean law of velocity addition is employed to calculate its optical or proper duration t which must be the same in s. In order to get velocity of light also c in s, we should associate with this trip the optical length $r = ct$ in s. Because the source of light b is at rest in s, the geometric length of the latter trip in s, when s is considered stationary, is identical to its optical length r.
- In the same way by which the geometric length R in S of the above light trip gives rise to the optical length r, which is identified by the geometric length of the trip in s, the geometric length r of the virtual trip $(B\ \ at\ b \rightarrow o)$ in s, with B is the source of light, should induce when s is considered the stationary frame an optical length of the trip in s that is equal to its geometric length R in S. It useful to stress that geometric duration and length of a light's trip are simply two equivalent measures of the same quantity which is either length or duration.
- Since every inertial frame can be identified with the absolute space, the transformations we seek should be the same whether we identify S or s by the absolute space.

The implementation of the above view of space and time yields an anisotropic transformations, called the scaling transformation of first type (STI), between the geometric and optical characters of a light trip in S, or equivalently, between the geometric distances in S and s. The STI will be employed to explain

the meta-stable particles' lifetime phenomenon, Doppler's effect, drag effect and aberration. The scaling transformations of second type (STII) are concerned with the case in which the beginning and the end of the trip are known in *S* and in *s* from start. Although neat explanations, based on the scaling theory, of the drag effect, Sagnac effect, Michelson and Morley experiment, Michelson and Gale experiment, Doppler's effect, and the stellar aberration had already appeared in earlier works [2V,VI,VIII,IX,X], more profound explanations of these effects and experiments will be presented here and in forthcoming parts of this work.

2. Global Time

Consider an arbitrary inertial frame $S \equiv OXYZ$. The coordinates system is assumed to be already calibrated using a given unit of length, say *LS*. The existence of the Cartesian system of coordinates *OXYZ* in *S* requires an implicit assumption that the geometry of the space is Euclidean [3]. The geometric distance between two points $A \in S$ and $B \in S$, or geometric length of a rod *AB* stationary in *S*, refer to the result L_{AB} obtained by laying the unit of length *LS* in *S* along the rod repeatedly from one end till reaching the other by multiples and fractions of *LS*. This can be done rationally (or it had already been done) and takes no time. If $R = R_g.LS$ is the geometric distance of the point *B* from *O,* then the dimensionless quantity R_g is the radial coordinate of *B*.

The unit of time, though arbitrary, is chosen as the duration, say "second", between two consecutive ticks (or readings) of identical clocks that run at synchrony with each other. Contemplating in the last statement, we may be astounded by the fact that we really have defined nothing concerns the world outside the clocks [4]. In order that a unit of time, say a second, bears a meaning as far as motion in *S* is concerned, *it should be correlated to what can happen during "a second" in the world outside the clocks* [4], and more precisely, it should quantify the amount of the spatial displacement intrinsic to some *reference physical phenomenon*, such as the propagation of light from an arbitrary point in *S*, or be related to a free reference (spherical) body that is not translating in *S* but rotating about its axis [2XII]. *A "second" must thus be corresponded with (and actually could be measured by) the distance traveled by light within S during the period of a second in the former case, and with the angle at which the reference body rotates relative to the remote universe in the latter* [2XII]. *Time measurements therefore must be reducible to specific types of spatial displacement's measurements.*

Employing the postulate that light propagates rectilinearly within the inertial frame *S* in all directions at a constant velocity *c,* synchronization of the clocks in *S* can be materialized in a measurable meaning. Indeed, we can now proceed with the Newtonian view and imagine that as soon as *S* is furnished with a system of coordinates through geometrical means, a system of synchronized timing is immediately established with respect to one timer, say $O \in S$. This means that, in the same way we envisage rationally the assignment of a triplet $(R, \emptyset, \theta)$ to each point *B* in *S*, we can also imagine that a timer can be placed at each point $B \in S$ which is synchronized with $O \in S$ and runs uniformly at the same rate as the master timer, and accordingly with all other timers. Indeed, due to the latter postulate a *global timing* in *S* can be practically established, with the notion of an "instant T_0" has a global meaning in *S*, in the sense that if an event takes place at $B_0(R_0, \emptyset_0, \theta_0)$ at T_0 then it will be detected at $B(R, \emptyset, \theta)$ through a light signal emanating from B_0 and arriving at *B* at the instant $T = T_0 +$

$\|\vec{R}_0 - \vec{R}\|/c$. Thus every S observer B assigns to the event of light's emission the same instant $T_0 = T - r/c$, where r is his spatial separation from B_0 and T is the time read at the clock B when light is received. It follows that the concept of time arrow -past, present, and future- has a global meaning in S, and any two or more S observers have the same temporal ordering of the events monitored by them. In particular, the notions of simultaneity and non-simultaneity are well-defined global concepts in S.

It is emphasized that Newton's global time was assumed to be readable at each point of space [2XII]. The synchrony of all point-wise timers was partially circumvented through appealing to a universal timer formed by the fixed stars in the firmament. This seems to be a generalization of the approximately uniform global time set up in the region from which almost all our observations are conducted, namely the earth surface. The earth's global time is induced by the configuration of the firmament relative to the earth. It is also stressed that no synchronization in the real sense is to be done in order an inertial frame becomes timed.

Synchronization in a uniformly rotating frame, or more accurately, in the part from which observations are conducted, can be achieved without appealing to light's signals [2XII]. An approximate example of this is the earth's surface. The existence of a global time in non-inertial frames motivates the following definition:

Synchronous frames: A frame s, not necessarily inertial, is said to be synchronous if it is endowed with a global time. In other words, the frame s can be furnished by a system of clocks that remain synchronous according to a specific criterion not requiring necessarily light signals.

3. Distance and Simultaneity by Contiguity

We proceed here to closely model the Newtonian conceptions of absolute space and time in a measurable way, but with observation through light signals is still discarded.

Assume that the inertial frame $S \equiv OXYZ$ is synchronous. The following discussion which will be confined to the X-axis is valid allover S. Suppose that $S \equiv X'OX$ is furnished with a lattice of points $\{X_n.LS = \pm n.LS,\ n = 0,1,2,\dots\}$ with LS is the length of a bar which we choose a unit of distance in S. Let $s \equiv oxyz$ be an inertial frame in standard configuration with S and translating uniformly relative to S. According to the Newtonian concepts, the length of a rigid rod is the same when measured from any inertial frame. This applies in particular to the unit length rod LS, which accordingly enjoys the same *identity* in all inertial frames. Any two points in s can be thought of as the ends of a rigid rod in s, and the distance between them, say $u.LS$, will be the same in s and S.

But how can we judge practically that two copies of LS, the first is stationary in S and the second is moving, say stationary in s, have the same length? In the synchronous frame S the answer is simple: if the ends of the moving rod occupies at an instant of time $T_0 = 0$ the points $A \in S\ and\ B \in S$ then the distance between the latter points in S should be LS. Because of the absence of synchronized clocks, or global time, in s, the reasoning we have just applied in S, seems to break down in s when a rod LS that is stationary in S is considered. This is because s is not yet endowed with a global time. We shall show however that this reservation is not necessary and that the same clocks employed to read time in S are also qualified to indicate the same instants of time in s. In fact we shall demonstrate that the absoluteness of time follows from the absoluteness of length.

Consider a rod *ob* of length *u.LS* stationary in *s*. Because length is absolute, the length of this rod is also *u.LS* in *S*. Suppose that at an instant $T_0 = 0$ in *S* the rod occupies the interval $[O, B] \equiv [0, u] \subset X'OX$, with the points $o \in s$ and $b \in s$ are contiguous to $O \in S$ and $B \in S$ respectively. The latter points can be imagined to be the ends of a rod *OB* stationary in *S*. The frame *s* admits that the contiguity of $o \in s$ and $O \in S$ [or instead, *b* and *B*] signifies the same instant of time in both frames, and he has no objection to denote this instant, as *S* did, by $T_0 = 0$, but he may doubt that the contiguity of *b* and *B* [the contiguity of *o* and *O*] took place at the instant of contiguity of *o* and *O* [*b* and *B*]. To eliminate this doubt we assume the contrary: (the contiguity of *o and O*) took place before (after) (the contiguity of *b and B*). In the first (second) case, *s* will find the length of *OB* less (greater) than *u.LS*, which is a contradiction, since length is absolute. It follows therefore that if it was found at $T_0 = 0$ in *S* that ($o \in s$ is contiguous to $O \in S$) and ($b \in s$ is contiguous to $B \in S$) then the same compound event takes place at the same instant in *s*, which we denote by $t = T_0 = 0$. Since at the instant $T_0 = 0$ in the synchronous frame *S* there corresponds to *every* point $B \in S$ a contiguous point $b \in s$, all clocks in *s* must read when *o* is contiguous to *O* the same instant of time $t = 0$. Therefore, when *o* and *O* are contiguous, we have

(3.1) $\quad X = x, Y = y, Z = z, T = t = 0 \; everywhere \; in \; S \; and \; s,$

with (X, Y, Z) and (x, y, z) are the coordinates of an arbitrary contiguous points $B \in S$ and $b \in s$ respectively.

Suppose that the frame $s \equiv x'ox$ is also furnished by a lattice of points $\{x_n.LS = n.LS: n = 0, \pm 1, \pm 2, \dots\}$, and let's redefine the unit of time *TS* in *S* by the period during which a point x_n of *s* which is at the instant $T_0 = 0$ contiguous to X_n moves to become contiguous to the point $X_{n+1} = x_n + 1$. By the concept of inertial frames, and because length is absolute, the last relation applies to every lattice point *n* of *s* which moves to the lattice point $n + 1$ of *S*. The new state of contiguity corresponds to the displacement $LS = TS$ of each point of *s*. Note that the unit of time in *S* has been defined now through spatial displacement of *s* relative to *S;* it can be specified either by the period *TS* during which an *s*-object is displaced by *LS* in *S*, or by the difference in readings of two clocks in *S*, separated by the distance *LS*, when the same *s*-object passes by. Thus time and distance have the same dimension. In a similar way to what was proven earlier, there corresponds to the new instant of time $T.TS \equiv \Delta T.TS = 1.TS$ in *S* an instant of time $t.TS \equiv \Delta t.TS$ in *s* at which the new state of contiguity is also realizable in *s*, and which results from displacing each lattice point *n* of *S*, initially contiguous to the lattice point *n* in *s*, by the same magnitude *LS* but in the opposite direction to become contiguous to the point $n - 1$ in *s*. But as determined in *s*, $LS = \Delta t.TS$. Comparing the last two expressions of the equal displacements we get $\Delta t.TS = 1.TS$. Since *LS*, and accordingly $\Delta T.TS \equiv TS$ can be chosen arbitrarily, we permanently have $\Delta T = \Delta t$, It follows therefore that *accepting length as absolute results in time flowing equably in S and s.*

The following remarks help to illuminate the concept of absolute time by contiguity:

-To each instant of time T_0 in *S* there corresponds a unique state of contiguity between *S* and *s* which is characterized by the following: all events of the form $(b \in s \; is \; contiguous \; to \; B \in S)$ are simultaneous in *s* and in *S*. This defines a unique instant of time in *s* which is conveniently denoted by T_0 (though it may be denoted by another number). A new state of contiguity corresponding to relative displacement

$$|\Delta X|_1 . LS = |\Delta x|_1 . LS = LS$$

defines a unit of time TS in both frames, and corresponds to the instant of time $(T_0 + 1)TS$ in S and in s. If T is any real number, then the relative displacement

$$|\Delta X| . LS \equiv T|\Delta X|_1 . LS = T|\Delta x|_1 . LS = T . LS = T . TS$$

corresponds to the period of time $T . TS$ elapsing in both frames.

Thus there corresponds to each given arbitrary instant of time T in the synchronous frame S a unique instant of time T in s, which signifies the same instant of time in both frames. This implies in particular that simultaneity is absolute, in the sense that it is frame independent. An instant of time T in both frames is fully meaningful and may be identified by a unique state of simultaneous contiguity of the points of s and S as realized in both frames.

- Since the frame s yields itself to synchrony by means of contiguity to the synchronous frame S, we may consider both frames as equivalent in terms of which is a hypothesis and which is a conclusion. In other words, it makes no difference to the result whether we start from S or from s as being synchronized by hypothesis and then conclude that other frame is also synchronized by means of contiguity. It follows that one system of synchronized clocks in one frame will be sufficient to determine time in both frames. Thus and regardless of his state of motion, any observer registers the time shown on the S-clock which is just contiguous to him. Of course, it makes no harm to imagine an additional s-system of clocks with each clock is always at synchrony with the S-clock that is contiguous to it, or each registering $T + T_0$ where T_0 is constant. What matters really is that the S- and s-clocks register the same period ΔT of time.
- In practical applications it is convenient to take $LS = 1\ meter,$ and define the unit of time "$TS = a\ second$" by the period taken by light to travel the distance $c\ meters = 3 \times 10^8 m.$ Thus $1\ meter = second/c.$ The numerical value of light's velocity in these units is $c = second/meter.$ Now if the frame s is displaced u meters in a second then $\Delta X\ meters = u\ meters = \frac{u}{c} seconds.$ In T seconds the frame s is displaced by $\Delta X\ [m] = u\left[\frac{m}{s}\right] T[s],$ which is the familiar expression of displacement using the familiar units.
- The induction of time and distance in an inertial frame s through its state of contiguity with the synchronous frame S amounts operationally to the following: Assume that at T_0 as determined in S, the points $a \in s$ and $b \in s$ are contiguous to $A \in S$ and $B \in S$ respectively. Now

-We define the time reading at an every point $b \in s$ in s by the reading of the contiguous clock at $B \in S$.

-We define geometric distance $d(a, b)$ between a and b in s by $D(A, B)$, where D is the geometric distance in S.

The scaling theory retains equal time readings for contiguous clocks but modifies the second Newtonian requirement to incorporate a constant speed of light.

4. Timed Inertial Frame- Universal Space

Let us consider the set of all inertial frames. It is clear what motion of a frame with respect to another means, but what needs elaboration is that the concept of a frame being at rest. The latter concept requires the existence of an *independent entity* with respect to which the state of being at rest is referred. This entity is reminiscent of Newton's absolute, or physical, space; it corresponds to *the physical space when referred to a frame set up by a force-free (i.e. far from matter) observer and not rotating relative to the fixed stars. Any given frame S*

defined by the latter statement is an inertial frame that can be identified by Newton's absolute space, and thus considered stationary, while all other inertial frames are then moving relative to S, and accordingly relative to the fixed stars. Recalling that light propagates within the stationary frame S at a constant velocity c, *the frame S which is already furnished by a coordinate system through geometric measurement can be endowed with a global time, with synchronization is carried out in the familiar way* [1]. *All other frames which are moving with respect to S derive their global time from S by contiguity*. We thus define a *timed frame S by a stationary inertial frame in which a global time has been set up.* Since the state of being stationary can be assigned to any inertial frame, the absolute, or physical, space in its Newtonian sense as the *unique standard of rest* has to be abandoned, or else, modified to admit identification with any inertial frame of fixed stars, *but one at a time*. However, *the role of the absolute space as a unique standard of orientations is retained*. The latter requirement is essential to single out inertial frames from rotating frames. The physical space which can then be corresponded by one arbitrary stationary frame S will be referred to as "the *universal space*"; it is universal because every observer participating in any observation has agreed to consider it as the standard of absolute rest, and has yielded to project the global timing in S, by contiguity on his own frame.

We may think ideally of a timed frame as any laboratory S, sufficiently far from all matter, and not rotating with respect to the remote universe. The unit of length - a meter - which serves to set up coordinates $OXYZ$ in the laboratory, serves also, when combined with the constancy of the speed of light within S, to define a unit of time and to synchronize all timers in S. The system of coordinates and synchronized timing in the laboratory can be extended indefinitely. With all other frames employing the S timing and admitting S as the standard of rest, S becomes a timed inertial frame.

Starting from a timed inertial frame S a global time can be set up by contiguity in any other inertial frame s. Consequently *one system of clocks* in S is sufficient to determine time in S and in any other inertial frame. The last statement implies that simultaneous events in S are also so in any other inertial frame. It is important however, to note that the frame S is an arbitrary inertial frame, in the sense that one should be able at any stage to view the other frame s, if inertial, as the timed inertial frame, and thus identifiable with the universal or physical space.

Global time in a timed frame S is compatible with geometric measurements. Indeed, when we say that the length of a rod that is stationary in S, or the geometric distance between its two ends A and B in S, is L, we mean that have we measured this length by a calibrated ruler, or by a light signal and two synchronized clocks situated at A and B, the two results will be L. In the second type of measurement, the length of the rod is $L = cT$, where T is the period taken by light to cross this rode from one end to another regardless of which end we choose as the initial point of the light signal. We will thus refer to L and T appearing in the latter equation as geometric length and geometric duration of either of the light trips $(A \in S \rightarrow B \in S)$, or $(B \in S \rightarrow A \in S)$. Benefiting from the compatibility of global time with geometric measurements in S, geometric distance in S, and in particular coordination of S, can be carried out either by (i) employing geometric measurements to determine the length of a baseline through the origin O, and then measuring the distance between any point $B \in S$ and O, by

triangulation, or (ii) using a clock at O to determine half the period of the return light trip $(O \to B \to O)$, say T; the sought distance will be $R = |OB| = cT$.

5. Universality of Physical Laws

Since any inertial frame can be identified with the physical space, the description of the physical world from any inertial frame, with measurements are conducted within this frame, should be the same. For geometric length within any inertial frame, copies of a rod in one frame can be transported to all frames and act as a unit of geometric length. *Or instead, the equivalent of this rod's length in wavelengths of the stationary emission of a specific spectral line can be used as a unit of length within any frame.* The unit of time is then the same in all inertial frames. Under this arrangement, any physical experiment in a frame S yields the same results of a similar experiment conducted within another frame s. The sameness of physical laws when formulated within any inertial frame will be referred to as the *universality of physical laws*.

The above paragraph does not contradict what was asserted that there is only one stationary frame at a time. In fact as long as the configuration of any physical system in an inertial frame S is determined through measurements within S, then as far as S is concerned any other inertial frame adopted by another set of observers is illusionary; the frame S have a direct access for measurements pertaining to any physical system, and the observers in any other inertial frame s, are as if not existing. The same thing is true for the s observers for whom the frame S can be dismantled, with their measurements are not affected. The frames which we have described with measurements of spatial and time intervals are carried out within each frame, are called *independent inertial frames*. If for instance two spaceships S ad s, employing the same unit of length, are employed as inertial frames then any experiment conducted within each will yield the same results.

It is only when the same physical phenomenon is observed through light (or electromagnetic) signals from two different frames, then either frame, but not both, can be considered timed and identifiable with the universal space while the other is moving in the universal space.

6. Absolute Light's Trips in the Universal Space

If a source of light b has an arbitrary vector velocity $\vec{u}$ relative to the inertial frame $S \equiv OXYZ$, we may choose without loss of generality the velocity vector in the direction of the X-axis, for we may always rotate the S-axes so that the X-axis is in the direction of $\vec{u} = u\vec{\imath}$, where $\vec{\imath}$ is the unit vector of the X-axis. Let $s \equiv oxyz$ be an inertial frame whose axes are parallel to those of S, and moving with respect to S at a constant velocity $\vec{u} = u\vec{\imath}\ (u > 0)$, so that the source b is stationary in s. We endow the frames S and s with systems of spherical coordinates $(R, \theta, \emptyset)$ and $(r, \theta', \emptyset')$ respectively, with θ (θ') is the azimuth angle between the X-axis (x-axis) and the radius vector $\vec{R}$ $(\vec{r})$. The latitude angles $\emptyset$ and $\emptyset'$ will be suppressed because of the axial symmetry of the motion about the X-axis (Fig.(6.1)).

Assume that the source of light b which is stationary in s emits, when at $B(R, \theta, \emptyset)$ in S, a spherical pulse of light. When light arrives at O, it reaches also an s-observer whom we choose the origin o of s. Two S and s observers who are contiguous when hit by the pulse are called *conjugate observers*. Similarly, two sources, each emitting a pulse of light when contiguous, are called *conjugate sources*.

The situation we have displayed has the following features:
(i) In a given frame S, a source of light b is moving. Or equivalently, in the inertial frame s in which the source is stationary, an inertial observer O, attached to a frame S, is moving at velocity $(-\vec{u})$.
(ii) Light is emitted from b when at $B \in S$.
(iii) On arriving at $O \in S$ light arrives at the conjugate observer $o \in s$. While O is already given, o emerges at the instant light arrives at O; it is the s-observer that is contiguous to O when light is received at O and hence by o. But we may equally imagine that when light arrives at $o \in s$ it also arrives at $O \in S$, and thus $o \in s$ is already known while $O \in S$ is known when light arrives at o.

The S frame can be considered at rest throughout the light's trip which starts from (*B when occupied by the source b*) and ends up at (*O and o*), while b is moving at velocity $\vec{u} = u\vec{\imath}$ in S. Also the s frame can be claimed the stationary frame during the duration of the trip which starts from the point (*b when was at B*) and ends up at (*o and O*), while S is moving at velocity $-\vec{u} = -u\vec{\imath}$. Since each frame is entitled to claim itself stationary, and thus identifiable with the universal space, while the other is moving, all observers (the S and s observers) accept that light emanated "at the same time" from one and the "same point" in the universal space and ended at the same time at the same point.

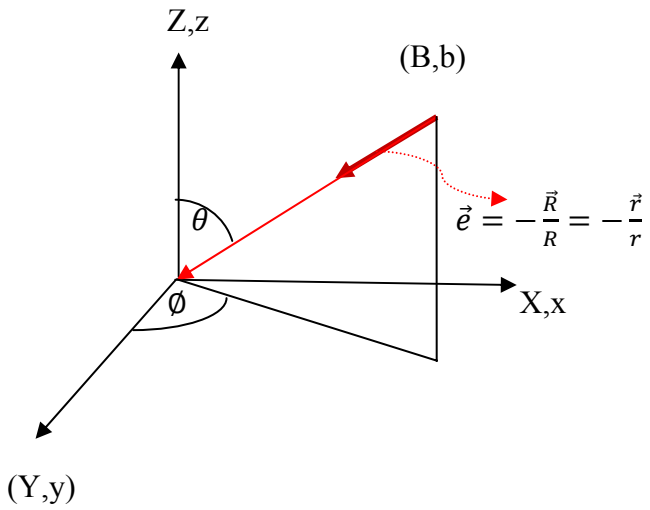


Fig.(6.1). The path of the trip ($b\ at\ B \rightarrow O\ and\ o$) in the universal space whether identified by S or by s .

To elaborate, the phrase ($b \in s$ *when at* $B \in S$) defines in each frame a pair composed of a location and an instant of time, or what we shall call a *universal point*, and denote by ($b\ at\ B$). Thus *a true source of light gives rise to a universal point, which is a frame independent entity that embodies the same instant of time in both frames together with an S- and s-locations that are coincident in the universal space at the instant of contiguity of b and B*. Similarly the end point of the pulse in one frame determines a conjugate end in the other frame, and accordingly, another universal point. It follows that *all observers concede to the fact that there is one and the same trajectory in the universal space associated with a given light's trip*, which starts from ($b\ at\ B$) and ends at ($O\ and\ o$) $\equiv (O, o)$. In other words, the single pulse traces a universal straight path connecting the universal points (*b at B*) and (*O,o*). The last fact is valid whether S or s is considered stationary and thus identified by the universal space.

A direct consequence of the last statement is the following: *If* $(\theta, \emptyset)$ *are the directional angles of the path in S when considered stationary and* $(\theta', \emptyset')$ *are its directional angles in s when s is considered stationary, then*

(6.1) $$\theta = \theta', \quad \emptyset = \emptyset'.$$

The velocity of the source, or equivalently the relative velocity of S and s, does not appear in the last relations. This implies that, had the frame S been replaced by another frame S_2 in standard configuration with the former, the directional angles of the path would not change: $\theta_2 = \theta' = \theta, \quad \emptyset_2 = \emptyset' = \emptyset$.

7. The Anisotropic Scaling Transformations of the First Type

Let b be a source of light moving in an inertial frame $S \equiv OXYZ$ at a constant velocity $\vec{u}$, with the X-axis of S is taken along $\vec{u} = u\vec{\imath}$ $(u > 0)$. Let s be an inertial frame which is moving relative to S at a constant velocity $\vec{u} = u\vec{\imath}$, and hence the light's source b is stationary in s. Now, we set out to determine the transformations which allows for each frame, S or s, to be considered stationary while the other is moving.

Assume that when at $B \in S$ the source b emits a pulse of light. When the pulse arrives at the point (or observer) $O \in S$, it arrives also at its s-conjugate point (or observer) $o \in s$, which is contiguous to $O \in S$ at the moment the pulse hits O (or when the pulse arrives at $o \in s$ it also arrives at its S-conjugate $O \in S$). We choose now the axes of s such that $s \equiv oxyz$ are in standard configuration with $S \equiv OXYZ$. Each of the conjugate observers O and o is entitled to consider his frame stationary relative to the fixed stars and thus identifiable with the universal space while the other frame is moving relative to his own frame. Each observer, O and o, assigns to the pulse path the same beginning (B, b) and the same end (O, o). In other words the pulse follows a universal path connecting the universal points (B, b) and (O, o), and the direction of the path is the same when looked at from the stationary frame whether it was S or s. In each frame, when considered stationary, the pulse propagates along a direction determined by a unit vector $\vec{e}$, with

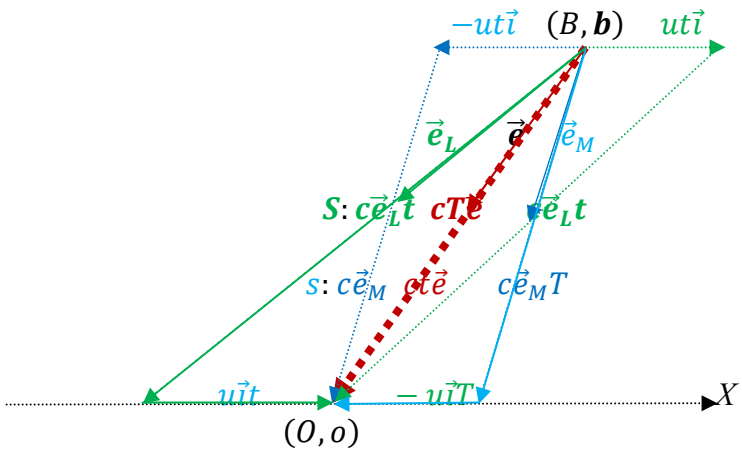


Fig (7.1). The view of S (s) is in green (blue)

(7.1) $$\vec{e} = \frac{\overrightarrow{BO}}{\|\overrightarrow{BO}\|} \ (in\ S) \equiv \frac{\overrightarrow{bo}}{\|\overrightarrow{bo}\|} \ (in\ s)$$

If S is chosen the stationary frame, then the *geometric length* $\|\overrightarrow{BO}\| = R$ of the trip $(b\ at\ B \to O\ and\ o)$ in S is employed to induce corresponding length $r = \|\overrightarrow{bo}\|$ and duration $t = r/c$ in s such that r and t have the status of geometric

length and time in *s*, when *s* is the stationary frame, in the same way *R* and *T* have in *S*.

By the Galilean law of velocity addition, the velocity of the pulse in *S* is the vector sum of its velocity in *s* and the velocity $\vec{u}$ of its emitter. However the pulse emanating from $(b\ at\ B)$ and arriving at *O* should had been ejected in a direction $\vec{e}_L$ in *S* such that the resultant velocity $c\vec{e}_L + u\vec{\imath}$ is along the unit vector $\vec{e}$. The duration $t = r/c$ taken by the pulse to arrive at *O* is given by the quotient of its displacement vector $\overrightarrow{BO} = -\vec{R} = R\vec{e}$ and its velocity $c\vec{e}_L + u\vec{\imath}$ in *S*, i.e.

(7.2) $$R\vec{e} = (c\vec{e}_L + u\vec{\imath})\, r/c = (\vec{e}_L + \beta\vec{\imath})r,$$

where $\beta = u/c$.Thus the *geometric length of the trip in S*, which is *R*, has given rise to *the optical length* $r = ct$ *of the trip in S*. The length *r* can be looked on as the *geometric length of the trip in s* because *b* is stationary in *s*, and *r* therefore must induce for a source *B* an optical distance $R = cT$ in *s* which should be identical to its geometric distance *R* in *S*. Since *B* is moving in *s* at velocity $(-\vec{u})$, the velocity in *s* of the pulse that emanates from the virtual source *B* is the sum of its velocity in *S* and the velocity $(-\vec{u})$ of its emitter. The pulse emanating from *B* should have then been ejected in a direction $\vec{e}_M$ in *s* such that the resultant velocity $c\vec{e}_M - u\vec{\imath}$ is along $\vec{e}$. The duration $T = R/c$ taken by the pulse to arrive at *o* is given by the quotient of the displacement $\overrightarrow{bo} = -\vec{r} = r\vec{e}$ of the pulse as seen in *s* and its velocity $c\vec{e}_M - u\vec{\imath}$ in *s*, i.e.

(7.3) $$r\vec{e} = (c\vec{e}_M - u\vec{\imath})\, R/c = (\vec{e}_M - \beta\vec{\imath})R.$$

Whether *b* or *B* was the source, we start only with one quantity, *R* if *S* is the stationary frame or *r* if *s* is the stationary frame (but not both), which is already geometrically measured whereas the other quantity is induced in the other frame by the relations (7.2) and (7.3). It follows therefore that it is sufficient to know the ratio $r/R = \Gamma(\beta, \theta)$ to determine both quantities, regardless of which had been measured geometrically, or equivalently, which frame was considered stationary.

Dividing the equations (7.2) and (7.3) side to side we obtain

(7.4) $$\frac{R}{r} = \frac{\vec{e}_L + \beta\vec{\imath}}{\vec{e}_M - \beta\vec{\imath}}\frac{r}{R}\ .$$

Or

(7.5) $$\Gamma(\beta, \theta)^2 = (\frac{r}{R})^2 = \frac{\vec{e}_M - \beta\vec{\imath}}{\vec{e}_L + \beta\vec{\imath}}.$$

By equations (7.2) and (7.3) the vectors appearing in the numerator and dominator on the right hand-side of the last equation are both along $\vec{e}$. Setting

(7.6) $$\vec{e}_L + \beta\vec{\imath} = k\vec{e}, \quad \vec{e}_M - \beta\vec{\imath} = k'\vec{e}\ ,$$

we get

(7.7) $$\vec{e}_L = k\vec{e} - \beta\vec{\imath}, \quad \vec{e}_M = k'\vec{e} + \beta\vec{\imath}.$$

Taking the norms of both sides in each equation (7.7) we get

$$1 = k^2 + \beta^2 - 2\beta k(\vec{\imath}.\vec{e}) = k^2 + 2\beta cos\theta\ k + \beta^2,$$
$$1 = k'^2 + \beta^2 + 2\beta k'(\vec{\imath}.\vec{e}) = k'^2 - 2\beta cos\theta\ k' + \beta^2.$$

Solving for *k* and k' we obtain

$$k = -\beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta},$$
$$k' = \beta cos\theta + \sqrt{1 - \beta^2 sin^2\theta}.$$

Dividing the latter equations side to side gives

$$\Gamma(\beta,\theta)^2 = \left(\frac{r}{R}\right)^2 = \frac{k'}{k} = \frac{\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{-\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}}$$
$$= \frac{(\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta})^2}{1-\beta^2},$$

which yields the *scaling factor* $\Gamma(\beta,\theta)$ given by

(7.8) $$\Gamma(\beta,\theta) = \frac{r}{R} = \frac{\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{\sqrt{1-\beta^2}}.$$

It is easily seen that

(7.9a) $$\Gamma(0,\theta) = \Gamma(\beta,\pi/2) = 1,$$

(7.9b) $$\Gamma^{-1}(\beta,\theta) = \Gamma(-\beta,\theta) = \Gamma(\beta,\pi-\theta).$$

Moreover, for β fixed and positive, $\Gamma(\beta,\theta)$ is a monotonically decreasing function with $\theta \in [0,\pi]$, and

(7.9c) $$\Gamma(\beta,0) = \sqrt{\frac{1+\beta}{1-\beta}} > \Gamma(\beta,\theta) > \sqrt{\frac{1-\beta}{1+\beta}} = \Gamma(\beta,\pi) \qquad 0 < \theta < \pi$$

The scaling transformations are therefore

(7.10a) $$\mathrm{r} = \Gamma(\beta,\theta)R, \qquad t = \Gamma(\beta,\theta)T, \qquad \emptyset = \emptyset', \qquad \theta = \theta'.$$

These can be written in terms of the angle $\theta = \measuredangle(\vec{u},\vec{R})$ between the velocity of the source *b* in *S* and the radius vector as follows:

(7.10b) $$\vec{r} = \Gamma(\beta,\theta)\vec{R},\ t = \Gamma(\beta,\theta)T.$$

Another explicit forms that hold for arbitrary relative orientations of axes of *S* and *s* are the following:

(7.10c) $$\frac{\vec{r}}{\vec{R}} = \frac{t}{T} = \Gamma(\beta,\theta),$$

(7.10d) $$r\vec{e} = \frac{\vec{\beta}.\vec{R} + \sqrt{R^2 - (\vec{\beta}\times\vec{R})^2}}{\sqrt{1-\beta^2}}\vec{\mathrm{e}},$$

supplemented by $R = cT$ and $r = ct$.

8. The Active View in Interpreting the Scaling Transformations

The active view corresponds to the frame *S* taken from start as the timed frame and thus identifiable with the universal space with its global timing is valid in every other inertial frame. Any non *S*-observer yields to the fact that his frame is moving in the universal space *S*. Consider the light trip

$$(b(r,\theta,\emptyset)\ at\ B(R,\theta,\emptyset)) \rightarrow (O\ and\ o)$$

in which *b* is a true source of light. The transformations (7.10) can be understood in the timed inertial frame *S* in either of the following ways:

(i)It determines in *the timed frame S the ratio between the characters (length and duration) of the true light trip* $(b\ at\ B \rightarrow O)$ *and the corresponding characters of the light trip* $(B \rightarrow O)$*, whether B was a true or a virtual source.*

(ii) It determines *the optical or proper distance r from O of the moving body b in terms of the geometric (which is also optical) distance R of a conjugate body B that is stationary in S.*

(iii) It determines *the distance r between the moving body b when at B and an observer* $O \in S$ *in terms of its geometric distance* $\|\overrightarrow{BO}\| = R$. If *b* heads towards *O*, then the duration it takes to arrive at *O* is r/u.

In all above interpretations, the geometric distance $\|\overrightarrow{BO}\| = R$ is already known, whereas the values r and t are what S measures for the length and duration of the true light trip $(b\ at\ B \to O)$, or equivalently, for the distance $\|\overrightarrow{bO}\| = r = ct$. If the theory is correct, these measured values must be related to the known geometric data R and T by the transformations

(8.1) $$r = \Gamma(u,\theta)R, \quad t = \Gamma(u,\theta)T = \Gamma(u,\theta)R/c.$$

If $T_0 = 0$ is read on the clock B at the instant of emission, then the time read on the clock O at the instant of light reception is

(8.2) $$t = \Gamma(u,\theta)\, R/c = r/c.$$

The quantity t [r] which is actually measured for the duration [length] of the light trip $(b\ at\ B \to O)$ is called its *optical or proper duration* [*length*]. The quantity $R/c = T$ appearing in (8.2) represents the duration that light takes from B to O were B a true source. The S system of clocks alone is sufficient of course to specify the characters of the trip $(b\ at\ B \to O)$ since the readings of the clocks B and O of the events of light's emission and reception respectively determine these characters. Moreover, as evidenced by (8.2), *only the geometric distance R between B and O is sufficient to determine the duration of the trip* $(b\ at\ B \to O)$, provided the velocity of the source b in S is already given. Thus the relations (7.10) give rise to transformations within the same frame S, between the *geometric length* $R = cT$ (*or geometric distance R*) *and the optical length* $r = ct$ (*or proper distance r*). The directional coordinates $(\emptyset, \theta)$ of the true and virtual trips are obviously the same. When the pulse arrives at O the source b occupies a point $b' \in S$ with $\overrightarrow{Bb'} = \vec{u}t$. The angle $\delta = \measuredangle\left(\vec{R}, \overrightarrow{Ob'}\right)$ is calculated from (7.7(i)); it is given by $\sin\delta = \beta \sin\theta$.

Alternatively, the transformations (7.10) hold within a *timed inertial frame s*, with B is a true source while b can be a true or a virtual source. Here, R is the optical (or proper) distance from o of a true source B, which is moving at velocity $-u\vec{\imath}$ in s, and r is its geometric distance from o. In this case the expression of the optical length in terms of the geometric length is obtained just by interchanging r and R in (7.10) (or (8.1)) and replacing β by $-\beta$, to obtain

(8.3a) $$\vec{R} = \Gamma(-\beta,\theta)\vec{r} = \vec{r}/\Gamma(\beta,\theta),$$

which is identical to the forms (7.10).

The interpretation of the scaling transformations when the light trip is specified in one reference frame is called the *active view*. In the active view therefore, the specification of the characters of light's trip $(b\ at\ B \to o\ and\ O)$ is realized through two trips of which one trip is certainly true while the other can be true or virtual. When only one true source is present then the optical quantities belong to the true trip whereas the geometric quantities characterize the virtual one. Only one frame in the active view is necessary for full determination of the optical characters of a light's trip, and the latter coincide with its geometric characters if the source is at rest in that frame. Moreover, no ambiguity arises regarding units, because the same units in one frame, namely in S, are used when considering the characters of the trips $(B \to O)$ and $(b \to O)$.

The Case of Two Trips: We consider here the case in which b and B are both true sources. We have here in addition to the previous true light's trip $(b\ at\ B \to O)$ another true trip $(B\ when\ b \to O)$. It is clear that it makes no difference to the transformations between R [T] and r [t] in S if B was also a true source. We recall of course that the frame S can be considered permanently stationary. The current

case digresses from the case of a single trip in that, there are two pulses arriving at O at two different instants, T and t. Both trips follow the *same path* in S, namely the straight segment connecting (B when occupied by b) and O, but with

(8.4) $$t - T = (\Gamma(u,\theta) - 1)T$$

time difference in arrival at O.

9. Lifetime of Meta Stable Particles

The $\mu -$ meson particles are generated at an altitude of $X = 60km$ and move at velocity v close to that of light. Even if these particles have the velocity of light, it can travels during its short lifetime ($\tau \approx 2.10^{-6}s$) only the distance $d \approx c\tau = 0.6km$, which is just 0.01 of the distance from the earth surface. According to active view (iii) the distance of an μ −meson particle generated at an altitude X and approaching the earth surface shrinks to a value

$$x = \Gamma(\beta,\pi)X = \sqrt{\frac{1-\beta}{1+\beta}}X.$$

In order to reach the earth surface the particle should possess a velocity v such that

$$\sqrt{\frac{1-\beta}{1+\beta}}X < v\tau \approx c\tau \approx 0.6.$$

Setting $X=60$ and solving for β yields $\beta > 0.9998$, which is a tangibly probable range in the speed distribution of such particles. Because of the approximation we have made the result obtained is a rough estimate for the range of β.

10. The Active View Through Geometric and Proper (or Optical) Units

We may think of the transformations (7.10) as *setting up in S a unit of proper (or optical) length* $pLS(u,\theta)$ *associated with a trip* ($b\ at\ B \to O$) *in terms of the unit of geometric length LS, whereas the numerical values of the geometric and proper (or optical) lengths are the same in S.* This means that, if the geometric distance $\|\overrightarrow{BO}\|$ is $nlen.LS$, then the length of the light's trip ($b\ at\ B \to O$) is $nlen.pLS(u,\theta)$. The unit of optical length $pLS(u,\theta)$ of the light's trip ($b\ at\ B \to O$) which is determined in terms of LS depends on the velocity of the source and its orientation relative to $\overrightarrow{OB} \equiv \vec{R}$. *The latter assertions remain unchanged regardless of the nature of the source B, true or virtual.* Let's call the length $\|\overrightarrow{BO}\| = R$ of the trip ($B \to O$) in S whether true or virtual, the geometric length of every light trip ($b\ at\ B \to O$), and denote the units of geometric length (time) in S by LS (TS), and the associated units of optical length (duration) by $pLS(u,\theta)$ ($pTS(u,\theta)$). We have already asserted that the geometric and optical lengths (durations) of the trip have the same numerical value *nlen* (*ntim*). Consider now a light's trip ($b\ at\ B \to O$) of unit geometric length in S, i.e. $\|\overrightarrow{BO}\| = SL$. Setting $R = SL$ and $r = pSL(u,\theta)$ in (7.10) yields

(10.1) $$\frac{pLS(u,\theta)}{LS} = \frac{pTS(u,\theta)}{TS} = \frac{\Gamma(u,\theta)}{1}.$$

Now, there corresponds to the trip ($b\ at\ B' \to O$) with geometric length $R' = nlen\,.LS$, the optical length $r' = \Gamma(u,\theta)R'$ which is expressible in the form we have asserted:

(10.2) $$r' = \Gamma(u,\theta)(nlen.LS) = nlen.\big(LS.\Gamma(u,\theta)\big) = nlen.pLS(u,\theta)$$

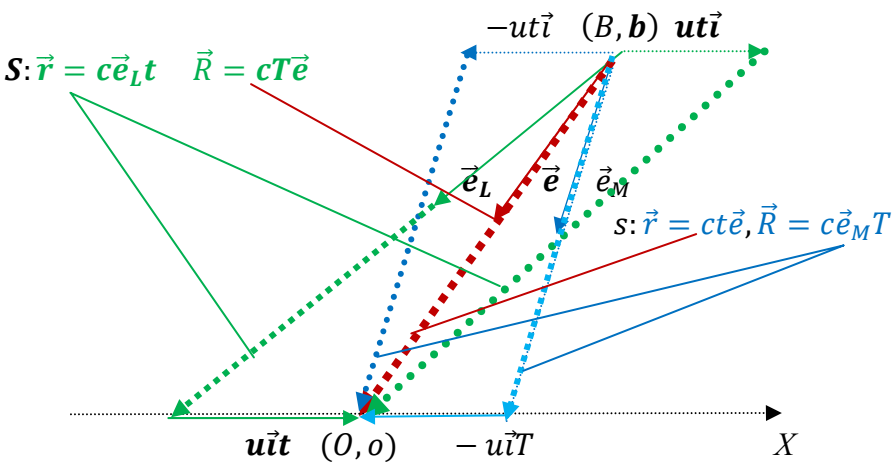
The geometric and optical lengths of the trip are equal if and only if $\Gamma(u,\theta) = 1$, which amounts to either $u = 0$, or $\theta = \pi/2$.

On recalling that the proper distance of a moving body b when at B from an observer O corresponds to its optical distance from O, we stress that the above discussion amounts to inducing a proper distance $\|\overrightarrow{bo}\| = r$ in terms of the geometric distance $\|\overrightarrow{BO}\| = R$.

If the geometric unit of length in S is taken the wavelength λ_0 of the stationary emission of a given spectral line, and if $\|\overrightarrow{BO}\| = R$ accommodates *nlen* wavelengths then the proper length $\|\overrightarrow{bo}\| = r$ accommodates also *nlen* of the optical unit of length $pLS = \Gamma(u,\theta)\lambda_0$.

11. Viewing the Light Trip in the Universal Space -The Passive View

The way in which we derived the scaling transformations allows for the identification of the universal space with either frame S or s while the other frame is moving. The relations (7.10) express the relations between what is measured in S (in s) when chosen the stationary frame to what is measured in s (in S) when chosen stationary; it is valid whether b or (exclusive) B was the true source. If b was the true source of light, then R (r) is it geometric (proper or optical) distance from O in S. And since the source is stationary in s, r is also its geometric distance from o in s. If B is the true source then r (R) is its geometric (proper or optical) distance from o in s, and R is also its geometric distance from O in S. In any case, the proper or optical length (duration) of the light's trip, which coincides with the geometric length (duration) in the frame in which the source is stationary, is the same in both frames. This guarantees that time flaws equably in both frames and sets up accordingly a proper distance between the source and the observer in the frame in which it is moving. We may thus look on (***b*** *at B*) as one point in the universal space with *one optical, or proper distance,* from (*O and* o), and interpret (7.10) as *defining units of geometric length and time in one frame from the counter units in the other; whereas the optical characters of a true trip are absolute in the universal space.* The absolute characters of the trip, which concern its length and duration, must be the same in both frames.



We denote the unit of geometric length (time) in S and s by LS (TS) and ls (ts) respectively. Setting $R = LS, r = ls$ and $T = TS, t = ts$ in (7.10), we obtain

$$\frac{ls}{LS} = \frac{ts}{TS} = \frac{\Gamma(u,\theta)}{1}, \tag{11.1}$$

with $\vec{u} = u\vec{\imath}$ is the velocity of s relative to S. The latter relation indicates that the unit of length (time) in s and S must comply with the ratio $1 : 1/\Gamma(u,\theta)$. Units of

length (time) in the frames *s* and *S* that obey the ratio $\Gamma(u,\theta):1$ will be referred to as *universal units.*

Since the optical length of a given light trip is absolute in the universal space, we must have

(11.2a) $$R_c.LS = r_c.ls,$$

(11.2b) $$T_c.LS = t_c.ts,$$

where R_c and T_c (r_c and t_c) are the length and duration of the trip as read in *S* (*s*) respectively. From (11.1) and (11.2) we have

(11.3) $$\frac{R_c}{r_c} = \frac{T_c}{t_c} = \frac{\Gamma(u,\theta)}{1}.$$

According to the last relations the observed characters in *S* are simply the *S* equivalents (means using *S* optical units) of the corresponding observed characters in *s*, and vice versa. It is noted that the relations (11.1) and (11.3) *are valid whether b or B was the source of light, and whether S or s was considered stationary.*

We consider now how the transformations (11.3) can be employed, and what available data are there.

Suppose that $b \in s$ is the source of light.

If ***s* is chosen the stationary** (or timed) frame then the geometric unit of length *ls* is already given and the optical length of the light trip is identical to its geometric length in *s*, and hence $r_c = r_g$ (and $t_c = t_g$) is an available data. The relations (11.3) determine the characters of the light trip as observed by an observer *O* moving at velocity $-\vec{u}$ using his unit of geometric length $LS(u,\theta) = ls/\Gamma(u,\theta)$, which is direction dependent; it is given by $R_c = r_g\Gamma(u,\theta)$.

Suppose that *b* is a source of monochromatic light of a characteristic wavelength λ_0 as measured in *s*, and that the path $(b\ at\ B) \to (o\ and\ O)$, which is of length $r_g\ in\ s\ and\ R_c$ in *S* measures *n* wavelengths. If λ is the wavelength as measured in *S*, then by (11.3), $n\lambda = R_c = \Gamma(u,\theta)r_g = \Gamma(u,\theta)n\lambda_0$, or $\lambda = \Gamma(u,\theta)\lambda_0$, which is the Doppler effect.

If ***S* is the timed inertial** frame then *LS* is given, and the situation encountered can be looked on as a body *b* moving in *S*. Now, by the active view on one hand, the optical distance (to be denoted here by R_c) in *S* is $\Gamma(u,\theta)$ times the geometric distance. i.e., $R_c = R_g\Gamma(u,\theta)$. On the other hand, $R_c = r_c\Gamma(u,\theta)$, by (11.3). Comparing the last two relations we have

(11.4) $$r_c = R_g$$

which is an available data. It is to be noted that r_c is measured in *s* through measuring the light trip length by the unit $ls(u,\theta) = LS.\Gamma(u,\theta)$, whereas R_g is the distance as measured in *S* had the source been stationary in *S* using of course his unit of length *LS*. In other words *s* reads for *b* what *S* reads for the conjugate source *B* but each using his own units.

12. The Doppler Effect

Let *s* be an inertial frame that is moving relative to *S* at velocity $\vec{u} = u\vec{\imath}$ $(u > 0)$, and consider a stationary source of light *b* in *s* that radiates monochromatic light of a characteristic wavelength $\lambda_0.ls$ where *ls* is the unit of length in *s*. Let $o \in s$ be another point in *s*, and suppose that the path bo accommodates *n* wavelengths. i.e. $|bo| = r = n.\lambda_0.ls$. Imagine that the source starts radiating when at $B \in S$ and that $O \in S$ is contiguous to $o \in s$ when light arrives at *o*. The path's length $|BO| = R$ accommodates then *n* wavelengths $\lambda.LS$, where *LS* is the unit of length in *S*.

According to the passive view the length of the trip $(b\ at\ B \to O\ and\ o)$ is absolute, and $n\lambda_0.ls = n\lambda.LS$, which yields

(12.1) $$\frac{\lambda}{\lambda_0} = \frac{ls}{LS} = \Gamma(u,\theta),$$

by (11.1). The wavelength as observed in *S* is therefore

(12.2) $$\lambda = \lambda_0\Gamma(u,\theta).$$

By (7.9c),

(12.3) $$\lambda < \lambda_0\ for\ \pi \geq \theta > \frac{1}{2}\pi, \qquad \lambda > \lambda_0\ for\ \frac{1}{2}\pi > \theta \geq 0,$$
$$\lambda = \lambda_0\ for\ \theta = \frac{1}{2}\pi\,.$$

These correspond to the body approaching the observer in the first case, receding from the observer in the second, and moving at right angle to the position vector of the body in the third case.

We compare here the quantitative Doppler's effect as determined by the scaling theory

(12.4) $$\lambda = \gamma\left(\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}\right)\lambda_0,$$

with the relativistic formula [1]

(12.5) $$\lambda_E = \gamma(1+\beta cos\theta)\lambda_0,$$

where $\gamma = 1/\sqrt{1-\beta^2}$, and the relativistic predicted wavelength has been denoted by λ_E to distinguish it from the wavelength λ predicted by the scaling theory. It is clear that the predictions of the two theories coincides for longitudinal motion. Indeed

(12.6a) $$\lambda(\theta=\pi) = \Gamma(\beta,\pi)\lambda_0 = \sqrt{\frac{1-\beta}{1+\beta}}\lambda_0 = \lambda_E(\theta=\pi),$$

(12.6b) $$\lambda(\theta=0) = \Gamma(\beta,0)\lambda_0 = \sqrt{\frac{1+\beta}{1-\beta}}\lambda_0 = \lambda_E(\theta=0).$$

The predictions of the two theories become qualitatively distinct for $\theta = \pi/2$. In this case the relativistic formula (12.5) predicts a red shift given by

(12.7) $$\lambda = \gamma\lambda_0 = \frac{\lambda_0}{\sqrt{1-\beta^2}},$$

whereas the relation (12.4) reduces to

(12.8) $$\lambda = \Gamma(\beta,\pi/2)\lambda_0 = \lambda_0,$$

which contrary to the relativistic prediction, shows that there is *no traverse Doppler's effect.*

In spite of the absence of traverse Doppler's effect in the scaling theory, the prediction of the theory are in excellent agreement with the results of the Ives-Stilwell experiment [5,6]. To specify the goal of the experiment, we denote the wavelengths associated with approaching and receding sources by λ_a and λ_r respectively. The Ives-Stilwell experiment was designed to measure the shift [5,6,7]

(12.9) $$\Delta\lambda = \tfrac{1}{2}(\lambda_a + \lambda_r) - \lambda_0.$$

In the relativistic theory

(12.10) $$\lambda_{Er} = \gamma(1+\beta cos\theta)\lambda_0, \quad \lambda_{Ea} = \gamma(1-\beta cos\theta)\lambda_0,$$

and the shift in wave length is

(12.11) $$\Delta\lambda_E = (\gamma-1)\lambda_0 \approx \frac{1}{2}\beta^2\lambda_0.$$

In the scaling theory

(12.12) $$\lambda_r = \gamma(\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}\,),$$

(12.13) $$\lambda_a = \gamma(-\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}),$$

and the wavelength shift is

(12.14) $$\Delta\lambda = \left(\gamma\sqrt{1-\beta^2 sin^2\theta} - 1\right)\lambda_0 \approx \frac{1}{2}\beta^2 cos^2\theta\lambda_0 = (\Delta\lambda_E)cos^2\theta.$$

The last relation shows that the scaling theory predicts in general a smaller shift than the relativistic one, and the two prediction coincide for $\theta = 0$ or $\theta = \pi$. In Ives-Stilwell's experiment a small concave mirror is set at an angle $\theta = 7°$with the ions velocity to reflect the emitted radiation backwards. As (12.14) shows, the relativistic prediction should be scaled by a factor $cos^2 7° \approx 0.985$ producing accordingly a smaller shift, and the predicted shifts by the scaling theory can be closer to the experimental observations only when the observed shifts are less than the relativistic predictions.

The following table displays some of the predictions of the special theory of relativity and the scaling theory together with the observed shift in Ives and Stilwell experiment, all measured in angstrom.

The relativistic prediction $\Delta\lambda_E = \frac{1}{2}\lambda_0\beta^2$	*observed shift* $(Ives - Stilwell)$	*The Scaling predictions* $\Delta\lambda = \frac{1}{2}\lambda_0\beta^2 cos^2\theta$
0.0202	0.0185	0.0198
0.0243	0.0225	0.0239
0.0280	0.0270	0.0275
0.0360	0.0345	0.0354
0.0478	0.0470	0.0470
0.0670	0.0670	0.0660
0.0686	0.0675	0.0675
0.0869	0.0900	0.0856

13. Galileozation of Optical Measurements

Let S be a timed inertial frame in which a source of light b is moving at a velocity $\vec{u} = u\vec{\imath}\,(u > 0)$. The inertial frame S is stationary relative to the fixed stars and it is endowed with a global time. Every other inertial frame S' will then be moving relative to the fixed stars with velocity that is equal to its translational velocity with respect to S. Time in S' is induced from time S by contiguity.

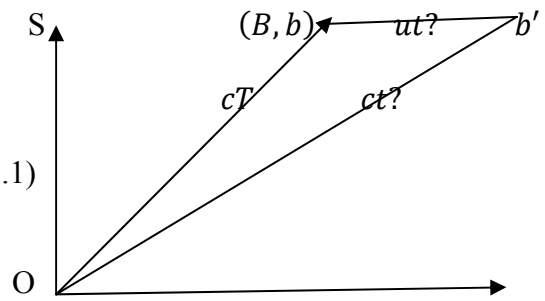


Fig.(13.1)

Assume that at an instant of time $t = 0$ corresponding to b at $B \in S$ the source b emits a pulse of light that arrives at the observer $O \in S$ after a period t. The pulse follows the path $(B \in S \to O \in S)$ whose true (or proper) time length is given by

(13.1) $$t = \Gamma(\beta, \theta)T,$$

where T is the geometric time length of the path $\overrightarrow{BO}$ with geometric length $gl(B,O) = R = cT$. When light arrives at O the source occupies a new position $b' \in S$. The question is that: when light arrives at O, can we envisage $b' \in S$ at geometric distances ut and ct from B and O respectively? In other words, is it ct and ut the present, or "*now*", geometric distances of the source from O and B respectively? If the answer is no, then is it possible to find a relation between ut and ct on one hand and their respective counter geometric lengths $d' = gl(B,b')$ and $r' = gl(O,b')$ respectively? The posed question can have an affirmative answer if and only if the rules of Euclidean trigonometry apply to the triangle $Bb'O$ with sides

(13.2a) $$\overrightarrow{BO} = cT\vec{e},\ \ \overrightarrow{Bb'} = ut\vec{\imath},\ \ \overrightarrow{b'O} = ct\vec{e}_L,$$

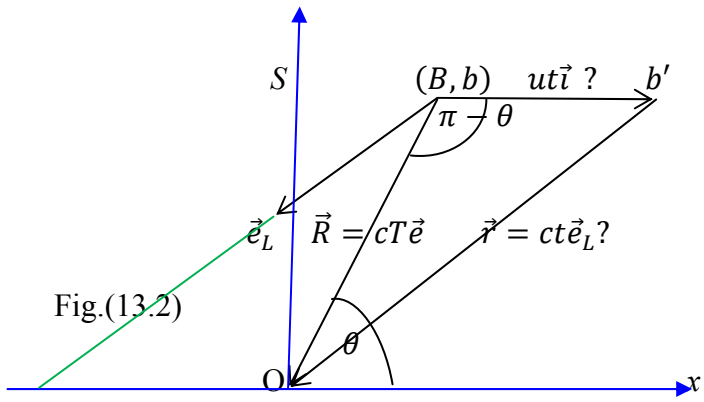


Fig.(13.2)

or equivalently to the vector sum

(13.2b) $$cT\vec{e} = ut\vec{\imath} + ct\vec{e}_L,$$

and yet the resulting relation between t and T are the same as that prescribed by the scaling transformations. By the law of cosines in Euclidean trigonometry we have

$$(ct)^2 = (cT)^2 + (ut)^2 + 2ucTtcos\theta,$$

or

$$(1-\beta^2)t^2 - 2\beta Tcos\theta.t - T^2 = 0,$$

which yields

(13.3) $$t = \frac{\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{1-\beta^2}T.$$

The latter equation can be written in the form

(13.4) $$t = G(\beta,\pi-\theta)T = \frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}}T,$$

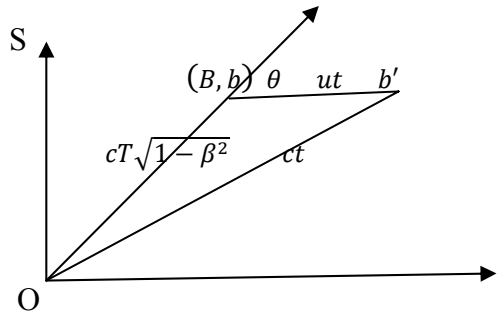
where

(13.5) $$G(\beta,\pi-\theta)t = \frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}},$$

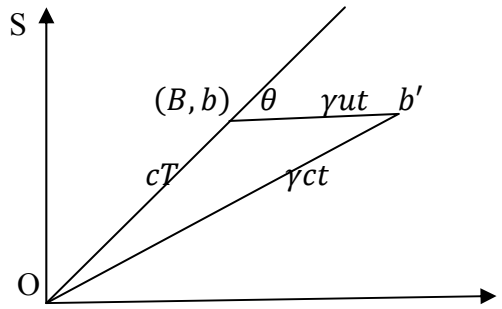
is the Galilean factor.

The relation (13.3) show that the scaling transformations (13.1) cannot be satisfied by the sides' lengths of the triangle $Bb'O$ as given by (13.2), and our question accordingly has a negative answer. However, the scaling transformations (13.1) can be written in either of two equivalent forms that can be reconciled with (13.4):

(13.6i) $$t = G(\beta,\theta)\left(\sqrt{1-\beta^2}T\right),$$



(13.6ii) $$ct' \equiv \frac{ct}{\sqrt{1-\beta^2}} = G(\beta,\theta)cT.$$



In both forms we have the vector sum relation (or Galilean transformation)

$$\overrightarrow{BO} = \overrightarrow{Bb'} + \overrightarrow{b'O}$$

holds, but with the geometric length of its left hand-side, namely cT, is contracted by $\gamma^{-1} = \sqrt{1-\beta^2}$ in the first form, and the length of each vector on the right hand-side, namely *ut* and *ct*, is expanded by γ in the second form. The first form indicates that to envisage, when light reaches *O,* the source b at $b' \in S$ with distances *ut* and *ct* from *B* and *O* respectively, the initial distance cT of the source from *O* should b contracted by γ^{-1}. Alternatively, the distances *ct* and *ut* should be expanded by γ, if the second form is to be applied. It is important to note that, in both views, only geometric distances are liable either to contraction (in the first view) or expansion (in the second view), while the true time *t* remains intact.

The Galieozation process discussed above will be used in a subsequent work to explain the pioneer anomaly, at least partially.

14. Observing the Ray's Direction From two Frames - Aberration Angle

Let M be an inertial frame which is moving relative to the stationary frame S at a constant velocity $\vec{v} = v\vec{\imath}, (v > 0)$. The light's trip $(b \in s\ at\ B \in S \rightarrow O \in S)$ takes place along the segment *BO.* The direction of the light trip $(b \in\ s\ at\ B \in S) \rightarrow (O \in S\ and\ m \in M)$ in *M*, where *m* is an *M* observer that is contiguous to *O* when light is received is determined by two *M*-points through which light passes in its course to (*m* when at *O*). One of these points can be taken *m* itself, where the other *p*, which can be imagined a small ring through which the light passes, is any *M*-point that lies on the light path *BO* when light passes through, say $P \in S$. I.e. $p \in M$ is contiguous to $P \in S$, when light passes through *p.*

When light arrives at (m *and* O), the point $p \in M$ through which light passes, occupies a position $p' \in S$ at distances vt and ct from P and O (and hence m). From the geometry of fig.(14.1), in which $\delta \equiv \angle(\overrightarrow{OP}, \overrightarrow{Op'})$, we obtain

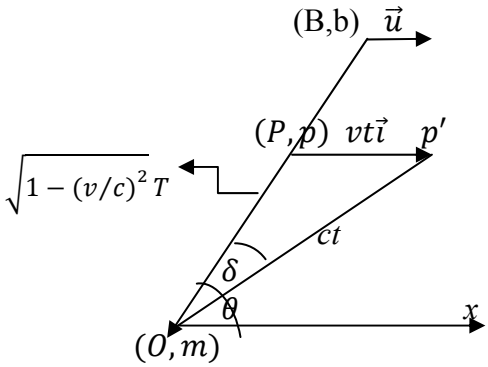


Fig (14.1)

(14.1) $$\frac{|\overrightarrow{Pp'}|}{\sin\delta} = \frac{|\overrightarrow{p'O}|}{sin\theta} \qquad i.e. \qquad \frac{vt}{\sin\delta} = \frac{ct}{sin\theta},$$

which yields the "aberration" angle δ in S and M by

(14.2) $$sin\delta = \frac{v}{c} sin\theta$$

In down to earth language, if p is the objective lens of a telescope in M (fig.(14.2)), then its ocular occupies when light enter p a position o' such that $\overrightarrow{o'O} = \overrightarrow{Pp'} = vt$, and when light is received, $o'p$ coincides with Op'.

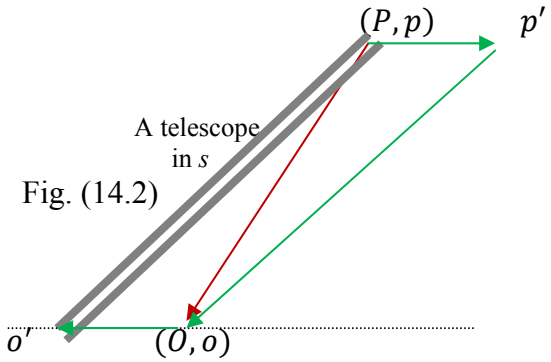


Fig. (14.2)

Fig.(14.2) light propagation in M through a telescope.

Thus the direction of the ray is observed in M to be tilted from a fixed direction $\overrightarrow{OP} \equiv \overrightarrow{OB}$ in the universal space S by an angle δ given by (14.2), where $\theta \equiv \angle(OX, \overrightarrow{OB}) \in [0, \pi]$ is the angle between the vector velocity $\vec{v}$ of p (or M) and the fixed direction (or radius vector of P) $\overrightarrow{OP}$. The angle δ is called the aberration angle. The velocity of M in S, the fixed direction $\overrightarrow{OB}$ in S, and the tilted direction $\overrightarrow{OP}$ all lie in a plane called *the aberration plane*. It is important to note that the velocity $u\vec{\imath}$ of the source b has no effect at all on the aberration angle, and the velocity appearing in (14.2) is the relative velocity between S and M. In the case discussed in the pervious section the velocity of the source is the same as the velocity of the frame s which is commoving with it.

15. The Illusive Lorentz Transformations

We have shown in the last section that the light which follows in reality the path ($B \to O$ *and* o) in the timed frame S is indeed seen in the moving frame

s to follow the path ($b' \in S \rightarrow o\ and\ O$), which is also agreed on in S. To adopt the s view, in s and in S, we have, as prescribed in (13.6i), to contract R in (13.2b) by γ^{-1} to obtain $\sqrt{1-\beta^2}\, R\vec{e} = \left(r\vec{e}_L + vt\vec{i}\right)$, or

(15.1) $$\vec{R} = \gamma(\vec{r} + vt\vec{i})\ ,$$

where we have set $\vec{R} \equiv R\vec{e}\ and\ \vec{r} \equiv r\vec{e}_L$. This look like one of the Lorentz transformations, but it is radically different in meaning, since r is related to t by $r = ct$. In fact we may formally obtain Lorentz transformations if we choose B and b' (Fig.(13.2)) the origins of systems of coordinates S and s and supplement (15.1) with its dual relation

(15.2) $$T\vec{e} = \gamma\left(t\vec{e}_L + vr\vec{i}/c^2\right).$$

It is easily verified that

(15.3) $$R^2 - c^2T^2 = r^2 - c^2t^2.$$

But again both sides are equal to zero. For the special case in which the observers (O and o) are on the line of motion of the source b we get the Lorentz transformations:

(15.4) $$X = \gamma(x + vt), \qquad Y = y = 0, \qquad Z = z = 0, \qquad T = \gamma(t + vx/c^2).$$

The generalized Lorentz transformations, and its special case (15.4), differ radically in its meaning from the common interpretation of Lorentz transformations. Indeed

-There is only one time, namely the true time t, which is read by in the timed frame S and prevails also in s, while the geometric time length T is no more than a different measure of the geometric distance in S.

- Because $R = cT\ and\ r = ct$ the two claimed dual relations are in fact one relation, which is *the corresponding scaling transformations*. Indeed, the second relation results from the first through multiplying by c and setting $vt = vr/c$. It follows that both sides in (15.3) vanish. Setting $r = ct$ in the (15.1) we calculate $t = \Gamma(\beta, \theta)\, R/c$.

- $\vec{R}\ and\ \vec{r}$ here are the position vectors of conjugate S and s observers O and o in (S, with origin B) and (s, with origin b') *when hit by the light pulse*. O and o are of course any conjugate S and s observers. In conjugate coordinate systems (S,O) and (s,o) which are contiguous when hit by light, we have to replace $\vec{R}$ and $\vec{r}$ in (15.1) and (15.2) by their negatives to obtain

$$\vec{R} = \gamma(\vec{r} - vt\vec{i}), \quad T\vec{e} = \gamma\left(t\vec{e}_L - vr\vec{i}/c^2\right).$$

Here $\vec{R}\ and\ \vec{r}$ are the position vectors in S (and in s) of B and b respectively. The latter relations reduce of course to one relation, which is the corresponding scaling transformations.

-There is only one and the same pulse which takes a duration t in the timed frame S, and accordingly in s, to arrive at (O and o), and there is no ground to suppose two durations T in S and t in s as it is assumed when deriving the Lorentz transformations [3]. In fact if B is a true source of light conjugate to the source b then light *will not* arrive at O from both sources at the same time. In any case the Lorentz invariant (15.3) is by our arguments identically zero, and the identification of the physical space by Minkowski space as it adopted in relativity theory is a big unfounded claim.

16. Combination of the Velocity of Light with the Velocity of its Emitter

The Galilean Factor

We have found in section 13 that the rules of the Euclidean trigonometry applied to the triangle OBb', with sides yields

(16.1) $$r' \equiv \frac{r}{\sqrt{1-\beta^2}} = G(\beta, \pi-\theta)R = \frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}}R,$$

with $r = ct$, $R = cT$. The Galilean factor

(16.2) $$G(\beta,\theta) = \frac{\Gamma(\beta,\pi-\theta)}{\sqrt{1-\beta^2}}$$

has the properties

(i) $$G(0,\theta) = 1,$$

(ii) $$G(\beta,0) = \frac{1}{1+\beta}, \qquad G(\beta,\pi) = \frac{1}{1-\beta}, \qquad G\left(\beta,\tfrac{1}{2}\pi\right) = \frac{1}{\sqrt{1-\beta^2}}$$

(iii) $$G(-\beta,\theta) = G(\beta,\pi-\theta) = \frac{\beta cos\theta + \sqrt{1-\beta^2 sin^2\theta}}{1-\beta^2} = \frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}}$$

(iv) $$G(\beta,\theta)G(-\beta,\theta) = \frac{1}{1-\beta^2}$$

(vi) $$G(\beta,\theta)G(\beta,\pi-\theta) = \frac{1}{1-\beta^2} = (G(\beta,\pi/2))^2$$

(vii) $$G^{-1}(\beta,\theta) = (1-\beta^2)G(-\beta,\theta)$$

Velocity Addition: Let's return to the relation (7.2)

(16.3) $$cT\vec{e} = (c\vec{e}_L + v\vec{\imath})t,$$

which expresses the Galilean law of velocity addition applied provisionally in *S* to the velocity of a pulse and its emitter. This formulae do not conform to the rules of Euclidean geometry unless *R* is contracted by the factor γ^{-1}. On carrying out this scaling we obtain the formula

(16.4) $$T\vec{e} = (\vec{e}_L + \beta\vec{\imath})\frac{t}{\sqrt{1-\beta^2}},$$

Substituting for *t* from $t = \Gamma(\beta,\theta)T$ yields

(16.5) $$\vec{e} = (\vec{e}_L + \beta\vec{\imath})\frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}} = G(-\beta,\theta)(\vec{e}_L + \beta\vec{\imath})$$

Taking the cross product of both sides by $\vec{e}$ yields

(16.6) $$\vec{e}\times\vec{e}_L = -\vec{e}\times\beta\vec{\imath},$$

which gives

(16.7) $$\sin\delta = \beta\sin\theta,$$

where $\delta = \angle(\vec{e}_L,\vec{e}) = \angle(\vec{e},\vec{e}_M)$. The relation (16.5) determines the *law of combination of the velocity of light signal, which is c, with the velocity of its emitter. The resulting velocity is c, and the direction of the resulting pulse is tilted from that of the original one by the aberration angle* δ. *The resulting velocity* $c\vec{e}$ *is along a vector* $\vec{e}$ *inbetween the vectors* $v\vec{\imath}$ *and* $\vec{e}_L$ *and makes an angle* δ *with the latter.* The relation (16.5) can be written in a more convenient form

(16.8) $$c\vec{e}_L = (1-\beta^2)G(\beta,\theta)c\vec{e} - v\vec{\imath},$$

Some special cases of addition of the velocities of a light signal and its emitter are listed here. In all cases the resulting velocity is *c* of course. We assume in all cases that $v > 0$.

-The value $\theta = \pi$, which corresponds to the same direction for the emitter's and the pulse's velocities, gives

(16.11a) $$c\vec{e}_L = (1-\beta^2)G(\beta,\pi)c\vec{e} - v\vec{\imath} = (1+\beta)c\vec{\imath} - v\vec{\imath} = c\vec{\imath},$$

(16.11b) $$\delta = 0.$$

-The value $\theta = 0$ corresponds to opposite directions for the velocities of the emitter and the pulse. It gives

(16.12a) $$c\vec{e}_L = (1-\beta^2)G(\beta,0)c\vec{e} - v\vec{\imath} = (1-\beta)c(-\vec{\imath}) - v\vec{\imath} = -c\vec{\imath},$$

(16.12b) $$\delta = 0.$$

-The value $\theta = \pi/2$, which corresponds to perpendicular directions of the emitter and the pulse, gives

(16.13a) $$c\vec{e}_L = (1-\beta^2)G(\beta,\pi/2\,)c\vec{e} - v\vec{\imath} = \sqrt{1-\beta^2}c\vec{e} - v\vec{\imath},$$

(16.13b) $$\delta = sin^{-1}\beta.$$

The relation (16.13a) affirms also that

$$c = |c\vec{e}_L| = \left|\sqrt{1-\beta^2}c\vec{e} - v\vec{\imath}\right|.$$

17. Stellar Aberration

Due to the earth's orbital motion around the sun the apparent position of a star changes throughout the year. The change in the direction of the starlight because of the earth orbital motion constitutes the phenomenon of stellar aberration which was discovered by Bradley [10] in 1727. Bradley explained stellar aberration using the corpuscular model of light [9,10]. However, it was necessary to explain this phenomenon on the basis of the wave theory of light at a time the latter stood supreme. Indeed, one can account for aberration effect in terms of waves traveling through the ether, provided the ether remains completely undisturbed by the earth's motion [9,11]. The relativistic explanations presented in most textbooks seem lacking consistency and comprehensiveness. Perhaps, the odd feature [12,9] of the relativistic model of aberration is to consider the relative velocity between a terrestrial observer and a star always equal to the orbital velocity of the earth around the sun. It seems inconsistent [9,12] to accept that the relative velocity between a distant celestial object and a terrestrial observer should be high in order to account for the red shift effect, and should be just the earth's orbital velocity, when it comes to aberration. Indeed, the star velocity, or more rigorously the relative velocity between a star and the earth does not show at all in the observed effect of aberration. In this work we shall discuss aberration on the bases of the scaling theory and show that it is free of the inconsistency we have just pointed to.

Stellar Aberration

Consider a "distant" star *b*, in the sense that the radius of the earth's orbit is negligible in comparison with the distance of the star from the sun. In this context, the phrase "the vicinity of the sun at some instant T_0" will mean the region of space containing the sun and the earth and whose dimensions remain negligible in comparison with the distance of the star *b* from the sun throughout a long period of time (centuries). In the stationary inertial frame $S \equiv NXYZ$ with origin at the sun "*N*", the motion of *b* will have no observable effect on its location in *S* during a relatively long period of time (centuries), and in particular, on the latitude angle Θ, between $\overrightarrow{Nb}$ and the ecliptic, which remains almost unchanged. In vicinity of the sun the rays sent out from *b* and received by all *S* observers in this region are practically parallel, and the star appears to all these observers at the same latitude Θ. Let $M \equiv Exyz$ be a frame of reference co-moving with the Earth in its orbital motion, and whose axes remain parallel to the axes of the inertial frame *S*. i.e. geocentric frame.

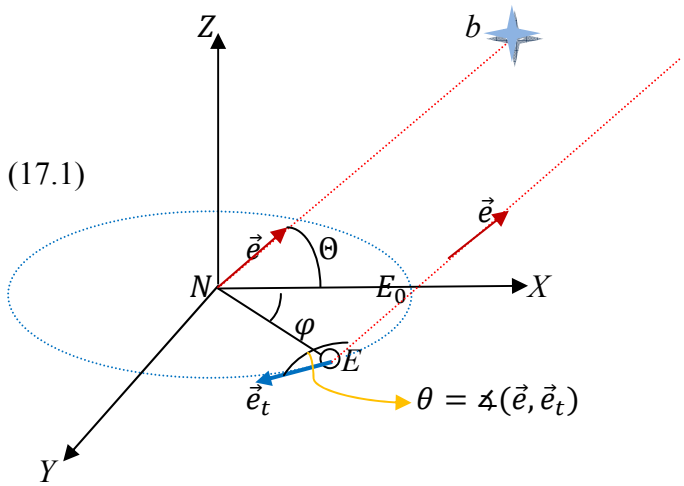
All *S*-observers see the rays from the star *b* throughout the year coming along the negative direction of the vector $\vec{e}$ which is the unit vector of $\overrightarrow{Nb}$. A

telescope op on the earth surface, with the ocular o and objective lens p sees the star only if it was oriented along a direction $\overrightarrow{op}$ which is tilted from the fixed direction $\overrightarrow{Nb}$ in S by an angle

(17.1) $$\sin\delta = \frac{v}{c}\sin\theta$$

with $\vec{v}$ is the instantaneous orbital velocity of the Earth around the Sun and θ is the angle between this vector and the vector $\overrightarrow{Nb} \parallel \overrightarrow{OP}$ (Figs. (14.2)). In other words θ is the angle in the aberration plane between the fixed direction $\overrightarrow{Nb}$ in S and the instantaneous vector velocity of the earth.

Fig. (17.1)



The vector velocity $\vec{v} = v\vec{e}_t$ of the earth around the sun, with $\vec{e}_t$ is the unit tangent vector to the Earth's orbit, rotates approximately uniformly with an angular velocity $\omega = 2\pi/year$ in S. From this follows that the aberration plane containing this rotating vector, the fixed vector $\overrightarrow{Nb}//\overrightarrow{Eb}$, and the tilted direction $\overrightarrow{op}$ rotates in S with angular velocity $\omega = 2\pi/year$. We choose the axes of S so that the Z axis is perpendicular to the ecliptic and the XZ plane comprises the star b. The zero of timing in S (and M) is chosen to correspond to the closest position E_0 of the Earth to the star, i.e. when Earth is on the X-axis, and thus its velocity is perpendicular to the XZ plane. In the frame S, and within the approximations imposed by the meaning of "distant star", the unit vector $\vec{e}$ of the negative direction of the incoming ray and the tangent vector $\vec{e}_t$ of the earth orbit are

(17.2a) $$\vec{e} = (cos\Theta, 0, sin\Theta)$$

(17.2b) $$\vec{e}_t = \left(\cos\left(\frac{\pi}{2} + \omega t\right), \sin\left(\frac{\pi}{2} + \omega t\right), 0\right) = (-sin\omega t, cos\omega t, 0)$$

respectively, and hence the cosine of the angle $\theta \equiv \angle(\vec{e}, \vec{e}_t)$ between the earth's vector velocity and the negative direction of the incoming ray is

(17.3) $$cos\theta = \vec{e}.\vec{e}_t = -cos\Theta\ \sin\omega t.$$

From (17.1) we have

(17.4) $$sin^2\delta = \left(\frac{v}{c}\right)^2 (1 - cos^2\Theta\ \sin^2\omega t) = \left(\frac{v}{c}\right)^2 (1 - cos^2\Theta\ \sin^2\varphi),$$

with $\varphi = \omega t$ is the polar angle in the ecliptic of the radius vector $\overrightarrow{NE}$ connecting the sun N and the earth E, i.e., $\varphi = \angle(\overrightarrow{NE_0}, \overrightarrow{NE})$. For a given star the altitude angle Θ is fixed, and the relation (17.4) determines the aberration angle δ in terms the polar angle φ of the earth's position at its orbit in the ecliptic, or equivalently, at any instant throughout the years. Identifying $sin\,\delta$ by δ, we find from (17.4) the angle between two vision lines of a star separated be six months period:

(17.5) $$2\delta = 2\,\frac{v}{c}(1 - \cos^2\Theta\,\sin^2\varphi)^{1/2}$$

The aberration angle attains its maximal values δ_{max}, determined by

(17.6) $$\delta^2_{max} = (\frac{v}{c})^2$$

for $\sin^2\varphi = 0$, which corresponds to $\varphi = 0\ or\ \varphi = \pi$, and its minimal values, given by,

(17.7) $$\delta^2_{min} = (\frac{v}{c})^2 sin^2\Theta$$

for $\varphi = \frac{1}{2}\pi\ or\ \varphi = \frac{3}{2}\pi$. Recalling that the telescope in *s* is tilted by δ towards the earth's orbital velocity vector, we deduce that the observed altitude of the star in (fig.(17.1)) is greatest for $\varphi = \frac{1}{2}\pi$ and least for $\varphi = \frac{3}{2}\pi$. The altitude of the star at $\varphi = 0$ or $\varphi = \pi$ remains unchanged because the telescope is tilted horizontally.

If the star *b* is in the ecliptic then the relation (17.4) reduces to

(17.8) $$sin\,\delta = \frac{v}{c}\cos\omega t = \frac{v}{c}\cos\varphi,$$

and the aberration occurs in the ecliptic attaining its maximal value (17.6) at $\varphi = 0, \pi$, and minimal value $\delta = 0$ at $\varphi = \frac{1}{2}\pi, \frac{3}{2}\pi$.

Observing Aberration from an Earth's Satellite

Suppose that a satellite is orbiting the earth in a low circular orbit that lies in the ecliptics. During the orbital period of the satellite, say less than two hours, the frame $M \equiv Exyz$ (which does not rotate relative to distant stars) is almost stationary, and the position of a distant star *b* appears fixed. This defines an approximately fixed direction $\overrightarrow{Eb}$ in *M* along which the star *b* is seen for two hours. Now, an argument identical to that presented when studying stellar aberration can be carried out here, with the earth replacing the sun and the satellite replacing the earth, leading to an aberration angle in the satellite frame and in *M* given by

$$sin\,\delta_{sat} = \frac{v_{sat}}{c} sin\,\theta$$

where $v_{sat}$is the orbital velocity of the satellite relative to $M \equiv Exyz$ and $\theta$is the angle between the negative direction of the incoming ray and the instantaneous vector velocity of the satellite. After half a period ($< 1\ hour$) the velocity of the satellite in *s* reverses direction, and with it the aberration angle, resulting in an angle

$$2\,\delta_{sat} \approx 2\frac{v_{sat}}{c} sin\,\theta$$

between the lines of sight to the star at two observations from the satellite separated by half a period. For $v_{sat} \approx 7.5km/s$ and $\theta = \frac{\pi}{2}$,

$$2\,\delta_{sat} = \frac{1}{4}.2\delta \approx \frac{1}{4}(41.25") \approx 10.31''$$

Gratitude

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