

ASPDEN'S EARLY LAW OF ELECTRODYNAMICS

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Abstract

A law of electrodynamics which was formulated by H. Aspden in the late 50's is examined, and a new field system based on this law is investigated. Maxwell's equations are not affected by this, but the Lorentz force law is modified, and the existence of a new type of radiation is considered.

1 Introduction

In 1959, Harold Aspden proposed a new law of electrodynamics ([1], see also [2]) which consisted of introducing a new term in the familiar empirically derived formula. This new term integrates to zero when closed circuital currents are involved. Thus, since Maxwell, Ampere, and Biot and Savart relied on closed circuits in their experiments it is understandable that such a term could have been missed. (See [5], p. 174 and [6], p. 87.) Later, in 1969, Aspden revised his law of electrodynamics by multiplying his new term by a certain mass ratio [4]. This law was interesting, but it had as a corollary that entropy could be reversed, something this author cannot accept.

Specifically, Aspden maintains that the force on a charged particle p having charge q and with velocity \vec{v} is not in general given by

$$F = q\vec{v} \times \vec{B} \quad (1)$$

in a magnetic field \vec{B} , and in particular not in the case where

$$\vec{B} = (\mu_o/4\pi) q' \frac{\vec{v}' \times \vec{r}}{r^3} \quad (2)$$

is due to a charged particle p' having charge q' and velocity \vec{v}' , with \vec{r} the separation vector from p' to p . He shows that an additional force component must be added to the right hand side of Eq. (1) in the two particle case, i.e., when \vec{B} is given by Eq. (2). This component is

$$\vec{G} = -\frac{\mu_o qq'}{4\pi r^3} (\vec{v}' \cdot \vec{r}) \vec{v}. \quad (3)$$

2 Preliminary Discussion

Note that we can write

$$\vec{G} = q\varphi(\vec{r})\vec{v}$$

where

$$\varphi(\vec{r}) = -\frac{\mu_o}{4\pi} q' \frac{(\vec{v}' \cdot \vec{r})}{r^3}. \quad (4)$$

Evidently, φ is analogous to \vec{B} and \vec{E} . We exploit the latter analogy in the next section by computing the Laplacian of φ which will turn out to be closely related to Poisson's equation.

3 Computing the Laplacian

First we change our point of view slightly and consider instead two charged particles the case where we have a finite but large distribution of (moving) charged particles each of which can be analyzed as above. Let \vec{J}' be the current density of this system and let V' be a volume of space bounded by a smooth surface S' such that the above distribution vanishes near S' and everywhere outside S' . Let 0 be a point in space, let dv' be a volume element in space, and let \vec{r}' be the vector from 0 to dv' . Then Eq. (4) becomes

$$\varphi(\vec{r}) = -\frac{\mu_o}{4\pi} \int_{V'} \frac{(\vec{r} - \vec{r}') \cdot \vec{J}'(\vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

where \vec{r} is a vector from 0 to the point p at which φ is required. Note that we have the vector identity

$$\begin{aligned} \frac{\vec{J}'(\vec{r}') \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} &= \vec{J}'(\vec{r}') \cdot \nabla' \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) \\ &= \nabla' \cdot \frac{\vec{J}'(\vec{r}')}{|\vec{r} - \vec{r}'|} - \frac{1}{|\vec{r} - \vec{r}'|} \nabla' \cdot \vec{J}'(\vec{r}'). \end{aligned}$$

Thus we may use the divergence theorem to obtain

$$\nabla \varphi(\vec{r}) = -(\mu_o/4\pi) \left\{ \int_{S'} \frac{\vec{J}'(\vec{r}') \cdot \vec{n} da'}{|\vec{r} - \vec{r}'|} - \int_{V'} \frac{\nabla' \cdot \vec{J}'(\vec{r}') dv'}{|\vec{r} - \vec{r}'|} \right\}$$

where \vec{n} is the unit normal to the surface S' (pointing outwards) and da' is an element of area S' . Note that the surface integral vanishes because of our assumption that the charge distribution vanishes near S' . Thus we have

$$\nabla \varphi(\vec{r}) = -(\mu_o/4\pi) \int_{V'} \frac{(\nabla' \cdot \vec{J}'(\vec{r}')) (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv',$$

and so

$$\nabla^2 \varphi(\vec{r}) = \mu_o \nabla \cdot \vec{J}'(\vec{r}).$$

4 Poisson's Equation

The equation of continuity asserts that

$$\frac{\partial \rho'}{\partial t} + \nabla \cdot \vec{J}' = 0$$

where ρ' is the charge density corresponding to J' . Thus we have

$$\nabla^2 \varphi(\vec{r}) = -\mu_o \frac{\partial \rho'(\vec{r})}{\partial t}. \quad (5)$$

This is closely related to Poisson's equation

$$\nabla^2 U = -\rho/\epsilon;$$

in fact, if $\lambda(\vec{r})$ is a solution to

$$\nabla^2 \lambda(\vec{r}) = -\rho'(\vec{r})/\epsilon_o,$$

then

$$\mu_o \epsilon_o \frac{\partial \lambda(\vec{r})}{\partial t} = \frac{1}{c^2} \frac{\partial \lambda(\vec{r})}{\partial t}$$

is a solution to Eq. (5), and conversely.

5 A Brief Discussion

It has no doubt become clear to the reader that φ really describes a new (scalar) field different from an electric or magnetic field but closely related to both. We wish to emphasize that the theory of electric and magnetic *fields* is unaffected by what we have discussed above; Maxwell's equations stand as they are. The only change is that the formula for the force on a charged particle in a magnetic field contains an additional new term Eq. (3).

6 The Advantage of Introducing φ

The question comes up: Why introduce the scalar field φ . The answer is that by introducing φ , we can speculate on the existence of other related fields which should be implied by φ 's existence. We will give one example.

Let $\vec{A}_i = \nabla\varphi$. Then \vec{A}_i is an irrotational vector and we have

$$\nabla \cdot \vec{A}_i = \nabla^2 \varphi = -\mu_0 \frac{\partial \rho}{\partial t}$$

and

$$\nabla \times \vec{A}_i = 0.$$

We are reminded by this of Maxwell's equations which in the case of only an electric field \vec{E} obtained from the gradient of a scalar field reduce to

$$\nabla \times \vec{E} = 0$$

and

$$\nabla \cdot \vec{E} = \nabla \cdot (\vec{D}/\epsilon_0) = \rho/\epsilon_0.$$

Next we note that any vector field $\vec{C}(\vec{r})$ can be written as a sum of an irrotational vector field $\vec{C}_i(\vec{r})$ and a solenoidal vector field $\vec{C}_s(\vec{r})$, i.e., we have

$$\vec{C} = \vec{C}_i + \vec{C}_s$$

and

$$\nabla \times \vec{C}_i = \nabla \cdot \vec{C}_s = 0.$$

Also, if $\lim_{r \rightarrow \infty} \vec{C}(\vec{r}) = 0$, then we can make the decomposition unique by specifying that $\lim_{r \rightarrow \infty} \vec{C}_i(\vec{r}) = \lim_{r \rightarrow \infty} \vec{C}_s(\vec{r}) = 0$. This suggests that we let $\vec{A} = \vec{A}_i + \vec{A}_s$ where \vec{A}_s is a solenoidal vector field whose properties will now be specified. We assume that $\lim_{r \rightarrow \infty} \vec{A} = \lim_{r \rightarrow \infty} \vec{A}_s = 0$ (since we have $\lim_{r \rightarrow \infty} \vec{A}_i = 0$ because we are restricting our attention to finite but large charge distributions). Now assume for a moment that isolated magnetic poles exist (as, for example, Dirac has suggested might be the case); then, clearly, we have a scalar field φ' with

$$\nabla^2 \varphi' = -\epsilon_0 \frac{\partial \rho'}{\partial t}.$$

(Here we have changed notation slightly; we mean the prime to denote the magnetic analog of the electric quantity. Thus ρ' is the magnetic analog of ρ for example; i.e., the magnetic charge distribution.)

Thus if $\vec{A}_i = \nabla\varphi'$, we can speculate on the existence of vector fields \vec{A}_i and \vec{A}'_s with $\vec{A}' = \vec{A}'_i + \vec{A}'_s$ and $\lim_{r \rightarrow \infty} \vec{A}'_s = 0$. But, in the event that isolated magnetic poles do not exist, φ' and A' still could if we set

$$\nabla \times \vec{A}' = \nabla \times \vec{A}'_s = k_1 \frac{\partial \vec{J}}{\partial t} + k_2 \frac{\partial \vec{A}}{\partial t}, \quad (6)$$

$$\nabla \times \vec{A} = \nabla \times \vec{A}_s = -k_3 \frac{\partial \vec{A}'}{\partial t}, \quad (7)$$

$$\nabla \cdot \vec{A}' = \nabla \cdot \vec{A}'_i = 0, \quad (8)$$

and

$$\nabla \cdot \vec{A} = \nabla \cdot \vec{A}_i = -\mu_0 \frac{\partial \rho}{\partial t}, \quad (9)$$

where k_1 , k_2 , and k_3 are constants of the required dimensionality to make the units right. Then we would have changes in \vec{A} inducing changes in \vec{A}' and hence changes in φ inducing changes in φ' and vice versa.

The alert reader will notice that if our speculations leading to the three equations (6)-(8) are correct, the four equations (6)-(9) as a set would seem to imply the existence of $\vec{A} - \vec{A}'$ waves in analogy with electro-magnetic waves, and one would expect that such waves would be generated by the cosmos if they exist. Evidently these waves could be detected by measuring small velocity fluctuations of moving charged particles (e.g., alpha particles emitted by a radioactive substance).

References

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