The geometry of quantum mechanics[1]

Abstract

The formalism of quantum mechanics is shown to exhibit properties characteristic of a classical three body problem consisting of nucleus, electron, and photon. Its two mathematically equivalent formulations, matrix and wave mechanics, represent two of the three possible field superposition states. Geometry is also used to show how the space of quantum mechanics is related to the space-time of the special and general theories of relativity.

1.0 Introduction

Quantum mechanics describes the inner workings of atoms and molecules during the emission and absorption of radiation. Its mathematical methods were not selected because they help us to understand how microscopic processes function, but because they enable us to make predictions within a set of well-defined conditions. In fact, on the surface they are so strange that it seems pointless to try to understand them. Moreover, for no apparent reason there are two very different but mathematically equivalent formulations of quantum mechanics, matrix mechanics and wave mechanics. The consensus among physicists is that we should simply apply the mathematics without questioning the physical mechanisms behind them. However, in the discussion that follows it will be seen that the mathematics exhibits a geometric pattern that is characteristic of the intersection of three vector fields. It will then be evident that the formalism of quantum mechanics could not take any other form.

2.0 Theoretical foundations

At first glance the physics seems simple enough. We want to analyze the hydrogen atom consisting of an electron orbiting a proton. However, an undisturbed hydrogen atom does not provide very interesting subject matter. In order to actually study atomic structure we must disturb the atom by exciting the electron into a higher orbital and then recording what happens as it decays. If the energy discharge were classical in nature the electron would spiral into the nucleus while dissipating energy continuously to the environment. The reason this does not happen is that energy is discrete rather than continuous. As the electron falls to a lower orbital it emits energy in a tiny bundle, or photon. Quantum mechanics treats emission as a two-body problem in which the electron creates a photon during decay as an independent step. However, this does not take into account the possibility that the photon is *an integral part of the emission process*. If the photon is created during excitation so that it is present before emission occurs then the decay process describes the relationship of three bodies and the quantum mechanical formalism must reflect that.

In classical mechanics the three-body problem consists of point particles that interact according to time-independent gravitational potentials. Although the two-body problem can be rigorously defined, it has proven impossible to extend these methods to the three-body problem by combining the effects of the three two-body problems it contains. Much of the complexity in classical problems is due to the continuous and unrestricted motion of the bodies in three spatial dimensions and time. However, in quantum mechanics space and time have very different meanings so that this aspect of the problem can be greatly simplified. Suppose that the excited hydrogen atom is composed of three field sources and emission is determined by their spatial relationships to each other. Then the position of the nucleus relative to an observer (laboratory frame) and the orientation of the electron and photon with respect to it may be disregarded. We can ignore the requirements of classical mechanics that the particle motions be described and instead direct our analysis towards defining spatial and temporal difference measurements among the three field sources. This simplification allows the electromagnetic three-body problem to be solved using the usual method of combining the effects of three two-body problems.

3.0 Quantum mechanical formalism

3.1 The first two-body superposition state

The continuous attributes of matter such as field are described mathematically by means of functions. Functions take numbers as inputs and then process the numbers according to specific mathematical instructions to produce a second number. In the classical case of a massive body, for example, we say that the gravitational potential V is a *function* of the distance r and write it in the form V(r). In the case of a two particle gravitational system the input number r is inserted into Newton's law for gravitational potential yielding a value for the force. However, if there are three functions we cannot simply combine all three to obtain a single function and then insert physical variables into it to obtain solutions. Instead we must separate the functions into pairs and then apply the third function/field in a separate step. In the classical case of continuous functions and generalized coordinates a solution is not possible. However, functions are not continuous in quantum mechanics so the number of possibilities available is limited and an exact solution is possible.

The electron and proton vector fields are first combined yielding a series of concentric shells around the nucleus that extend to infinity, which are the electron orbitals. Each pair of shells/orbitals represents both electron transition and photon emission. It is convenient to organize the pairs into rows and columns such that each component in the array, or matrix, represents a single energy value. Each energy value corresponds to the third vector field, or photon. The infinite series of photons that is obtained forms a set of solutions. The set of solutions is infinite and denumerable, and in contrast to the classical problem all are exact. The energy matrix is expressed discontinuously while the time coordinate determines phase in terms of continuous time coordinates. However, the motion of the electron cannot be described using these coordinates because they do not refer to an origin. Thus quantum mechanics is non-relativistic due to our choice of a coordinate system. It seems more complex than classical mechanics because the simplest observable electromagnetic interaction involves not two, but three distinct bodies. When it is correctly interpreted as a three-body problem we see that it is actually simpler than classical interactions.

3.2 The second two-body superposition state

We may also combine the properties of the three bodies by superimposing vector fields in a different order. Thus if the photon and electron are combined they form a "wave function" that includes properties of both field sources. The fixed potential of the nucleus is then applied to the wave function to obtain solutions. On the other hand, the wave function may be used to describe a varying potential as in the case of scattering problems. The force between electron and mass points is quantized and takes the form of a photon. The sinusoidal fields of the photon provide for a variable contribution given by the time dependent equation. The equivalence of wave mechanics with matrix mechanics is an indication that forces act symmetrically so that measurements may be performed relative to the nucleus or to the electron. The symmetry present in microscopic forces is also evident macroscopically and has been observed at the classical level using Tesla coils[2]. Thus electromotive force is more than simple vector addition. It is an embodiment of energy that manifests properties independently of the two interacting field sources. The fact that wave functions are used in scattering problems shows that all forces are quantized and include both discrete and continuous (wave-like) properties. We do not perceive electron and photon as separate components of a wave function because it is impossible to observe a particle independently of forces and the forces used to detect the electron must be transmitted through the intermediation of photons[3]. Conventional methods conceive of the wave function as a single entity because we are conditioned from experience to think of elementary interactions classically, as only involving two particles.

To summarize we may now describe a hydrogen atom's emission process as follows: In the Schroedinger picture the time dependence of the system is described by a wave function made up of the superposed fields of photon and electron. The wave function is subjected to the influence of a time independent operator, the nucleus, to obtain solutions. On the other hand, in the Heisenberg picture a state vector is formed by combining the time independent vector fields of electron and nucleus. This yields an infinite number of possible solutions, or probability amplitudes, as expressed by a state vector in a complex linear vector space. Time dependence as determined by the phase is in the operator, or photon. When the operator is applied to the state vector we obtain an infinite number of values for the physical variable, the eigenvalues. Thus two distinct formulations of quantum mechanics are obtained by combining the vector fields in different orders.

4.0 Conclusion

Discussions of quantum mechanics often refer to the complex linear vector space used to define probability amplitudes as though it were a real space. Thus it has been stated that a particle traveling from one point to another occupies all possible paths, or that the wave function collapses at speeds greater than that of light. However, when quantum mechanics is correctly formulated as a three body problem we see that the unusual properties of an imaginary space are not to be regarded as having actual physical meaning. The imaginary space of quantum mechanics is created when two of the three vector fields involved in emission are combined. It disappears when the third field is added to give a solution in real space.

Because the mathematics of quantum mechanics is based upon experiment there is no doubt that it is correct, thus forming an essential part of our description of nature. However, it would be premature to assume that the essence of nature is expressed in the mathematics. The way the mathematics is used and applied provides an underlying meaning that is distinct from that of classical mechanics. In quantum mechanics we do not define position as the distance relative to an origin as required by the laws of classical mechanics. Instead space refers to the spacing between particles. Because it is not subject to the laws of ordinary classical mechanics its space is referred to as absolute. Therefore quantum mechanics does not refer to ordinary mechanics, but rather to the internal microscopic processes that give rise to material structure. These arguments are reinforced because its formalism exhibits a geometric pattern that is characteristic of three interacting entities. Thus quantum mechanics defines an internal space-time geometry due to material structure that is distinct from the external geometry in which its component field sources reside. This introduces the possibility that rather than reconciling the differences between quantum mechanics and relativity theory by joining their geometries into a single mathematical formalism we must provide a framework within which each one can be developed individually.

1 This interpretation of quantum mechanics was developed by incorporating many different points of view into a single coherent picture. The following resources were the most useful:

a J. Mehra & H. Rechenberg, *The Historical Development of Quantum Theory*, Vol. II (NY: Springer, 1982-1988)
b Robert B. Lindsay, *Physical Mechanics*, (van Nostrand, 1961), p. 3.
c W. Heisenberg (1925), *Z Phys* **33**, 879. Reprinted in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics*, (Amsterdam, 1967)
d Heisenberg (1927), in J.A. Wheeler & W.H. Zurek (eds.), *Quantum Theory and Measurement*, (Princeton, 1983), p. 62.
e M. Jammer, *The Conceptual Development of Quantum Mechanics* 2nd ed. (NY: Tomash, 1989)
f M. Born, *The Born-Einstein Letters* (Walker, 1971)
j A. Einstein (1917), in B.L. van der Waerden (ed.) *Sources of Quantum Mechanics*, (Amsterdam, 1967), p. 63.
k P.A.M. Dirac *Phys Zshrift Sov U* 3, 1 (1933) in *Selected Papers on Quantum Electrodynamics*, ed. J. Schwinger (NY: Dover, 1958)
l R.P. Feynman, *Lectures on Physics*, (Addison-Wesley, 1965)
m R.P. Feynman, *Rev Mod Phys* **20**, 267 (1948) in *Selected Papers on Quantum Electrodynamics*, ed. J. Schwinger (NY: Dover, 1958)
n J.D. Jackson, *Classical Electrodynamics*, 3rd edn., (Wiley, 1999)
o R.H. Dicke, "Coherence in spontaneous radiation processes", *Physical Review* **93**, 99 (1954)
p E.L. Andrews, *Optics of the Electromagnetic Spectrum*, (Prentice Hall, 1960)
q N. Mukunda, *The World of Bohr and Dirac* (Wiley Eastern Ltd., 1993).
r J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, (Cambridge, 1987).
s A. Pais, *Niels Bohr's Times*, (Oxford, 1991)
t A. Pais, *Subtle is the Lord* (Oxford, 2005).
u Schiff, *Quantum Mechanics*, (NY: 1968).

2

Illustration 1: Tesla coil spark discharges

The following description appeared on a Tesla coil web page on August 21, 1996. "A phenomenon which is especially evident in the streamer discharge to a grounded target in small tabletop systems is where a portion of the thin streamer seems to be twice as bright as the rest (majority) of the streamer. This effect is often near dead center in the length of the streamer, but sometimes is much closer to the target end. I've seen the same thing in streamers to ground, and on some occasions (more rarely) in streamers to air. On those to ground it almost seems that the individual streamers coming from the ground and toroid are faint, but then seem to reinforce each other somewhere in between once the spark jumps the full distance. Now, one would think that once a discharge path was formed, the current flow would be the same all along the path, resulting in a relatively uniform degree of spark intensity all along the path."

3 In the case of general relativity the field sources are the mass points and intermediation takes the form of a field geometry.