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AL
UNIVERSITĂŢII "POLITEHNICA" DIN TIMIŞOARA

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NEW ASPECTS OF A GLOBAL INTEGRATION THEORY

Marius BORNEAS

ABSTRACT. Some new aspects of a global integration theory are presented, mainly by a Lagrangean formalism. The fundamentals are outlined, then electromagnetic and gravitational phenomena are discussed. The Lagrangean with higher derivatives is decomposed, and elementary equations are derived.

1. Introduction.

The idea of integrating all countenances of the physical world by a theory with higher derivatives, a global integration theory, has been initiated some time ago [1,2], and developed in a number of works. In this paper we present some aspects of the theory in a new way, by mainly a Lagrangean formalism.

Any physical theory starts from some hypotheses, even if these are hidden. Beyond the philosophical hypotheses referring to the physical world, we begin as the first hypothesis with the principle of background nonpreferentiality, stating, that basically there is no preference in the physical nature. That means a background, primordial symmetry exists.

In accord with the concept of the concept of the primordial symmetry the second hypothesis is that origin of all physical phenomena is a unique primordial entity (PE). In other words all physical phenomena, space, time, fields etc. are globally integrated in the PE. The multitude of nature`s countenances is the result of symmetry reforming.

Considering that PE includes space and time, it must have a geometric character, and in view of the basic symmetry, its geometry is Euclidean. In continuation, we envisage the observable physical phenomena as passages from one point to another in PE, covering all intermediary elementary paths.

This proposition has to be given a mathematical formulation. It is most natural to advance as the fundamental mathematical hypothesis the expression

$$\delta\int_1^2 d\sigma = 0 \qquad (1.1)$$

where $d\sigma$ is the path element.

2.Fundamentals.

Defining PE by a set of Euclidean coordinates α_ε (ε= 1,2,3,…), the law (1.1) reads

$$\delta\int_1^2 \left[\sum_\varepsilon (d\alpha_\varepsilon)^2\right]^{1/2} = 0 \ . \qquad (2.1)$$

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The next step is to make a correspondence between PE and physical phenomena, reached by symmetry breaking.

Of course one observable countenance of PE is space- time. But space- time may be in general non-Euclidean, therefore one must consider in general non-Euclidean cross sections in PE containing the lines given by (2.1).

If β_ρ are intrinsic coordinates on the cross section, one has

$$\delta\int_1^2\left(\sum_{\rho\tau}\hat{\gamma}_{\rho\tau}d\beta_\rho d\beta_\tau\right)^{1/2}=0 \tag{2.2}$$

where

$$\hat{\gamma}_{\rho\tau}=\sum_{\varepsilon}\frac{\partial\alpha_\varepsilon}{\partial\beta_\rho}\frac{\partial\alpha_\varsigma}{\partial\beta_\tau} \tag{2.3}$$

($\rho, \tau = 1,2,3,\ldots$)

The differentials $d\beta_\rho$ are obtained by passage to the limit of the increments $\Delta\beta_\rho$. These increments must be related to some actual magnitudes in connection with the description of the physical phenomena. If we denote generically by b_ς these magnitudes, the most natural and simplest relation is

$$\Delta\beta_\rho=e_\rho\left(\prod_\varsigma\Delta b_\varsigma\right)_\rho \quad . \tag{2.4}$$

Here e_ρ are operators acting in specific way on each magnitude on the right hand side, and revealing the observable quantities which appear in various cases. These operators represent a kind of informations for the forming of the observables.

Now the basic law (2.2) can be written

$$\delta\int_1^2\left[\sum_{\rho\tau}\hat{\gamma}_{\rho\tau}\,e_\rho\left(\lim\prod_\varsigma\Delta b_\varsigma\right)_\rho\ e_\tau\left(\lim\prod_\eta\Delta b_\eta\right)_\tau\right]^{1/2}=0\,. \tag{2.5}$$

As first symmetry breaking we separate the magnitudes b_ζ into two parts, Ξ_m related to independent variables and Ω_φ related to dependent variables. Eq. (2.5) then can be written

$$\delta\int_1^2\left[\sum_{\rho\tau}\hat{\gamma}_{\rho\tau}e_\rho\left(\lim\prod_m\Delta\Xi_m\prod_\varphi\Delta\Omega_\varphi\right)_\rho\ e_\tau\left(\lim\prod_n\Delta\Xi_n\prod_\psi\Delta\Omega_\psi\right)_\tau\right]^{1/2}=0 \tag{2.6}$$

($m+\varphi = \varsigma$).

The independent variables being the same for all dependent variables, one can write eq. (2.6) in the from

$$\delta\int_1^2\left[\left(e\lim\prod_m\Delta\Xi_m\prod_r\Delta\Xi_r\right)\sum_{\rho\tau}\hat{\gamma}_{\rho\tau}e_\rho\left(\lim\prod_\varphi\Delta\Omega_\varphi\right)_\varphi\ e_\tau\left(\lim\prod_\psi\Delta\Omega_\psi\right)_\tau\right]^{1/2}=0. \tag{2.7}$$

Introducing the independent variables by the relation

$$\left(e \lim \prod_m \Delta\Xi_m \, \Delta\Xi_n \right)^{1/2} = \prod_j dx^j \tag{2.8}$$

one can write (2.7) in the form

$$\delta \int_1^2 \prod_j dx^j \mathsf{L} = 0 \tag{2.9}$$

which represents an action principle, with

$$\mathsf{L} = \int \left[\sum_{\rho\tau} \hat{\gamma}_{\rho\tau} e_\rho \left(\lim \prod_\varphi \Delta\Omega_\varphi \right)_\varphi e_\tau \left(\lim \prod_\psi \Delta\Omega_\psi \right)_\tau \right]^{1/2} = 0 \tag{2.10}$$

the Lagrangean density, or simply the "Lagrangean".

From the above decomposition it results that this Lagrangean comprises the "content" of the spacetime frame.

3. The universal field.

Now we advance another hypothesis, asserting that the "content" is determined by a universal and nonlical field – say $\overline{\overline{W}}$ [1,2]. This field is introduced through the quantities Ω_φ, functionals depending in general on $\overline{\overline{W}}$, on the independent variables, and on the covariant higher order derivatives of $\overline{\overline{W}}$ with respect to the independent variables.

Thus eq. (2.9) looks like

$$\delta \int_1^2 \prod_j dx^j \mathsf{L} \, (x^j, W_{r(k)j\ldots}) = 0 \tag{3.1}$$

where are the components of $\overline{\overline{W}}$, $W_{r(k)j\ldots}$ are the K- th order covariant derivatives of W_r with respect to x^j, etc.

The Lagrangean L is invariant under any formal transformation of $\overline{\overline{W}}$, because this field being universal, it can interact only with itself, and any transformation cannot lead to a gauge field, only to an expression of W_r.

A general formalism with an equation with higher derivatives has been developed in ref. [3,4].

The variations to the limit of the quantities are composed of two parts: the intrinsic variations on PE croos section – say $\tilde{d}$, and a more sophisticated variations including the passage to another cross section – say $\bar{d}$. Thus one can write (2.7) as

$$\delta \int_1^2 \prod_j dx^j \left\{ \sum_{\rho\tau} \hat{\gamma}_{\rho\tau} e_\rho \left[\prod_\varphi \left(\tilde{d}\Omega_\varphi + \bar{d}\Omega_\varphi \right) \right]_\rho e_\tau \left[\prod_\psi \left(\tilde{d}\Omega_\psi + \bar{d}\Omega_\psi \right) \right]_\tau \right\}^{1/2} = 0 \,. \tag{3.2}$$

The Lagrangean can be set in a from easier to handle if one writes [5].

$$\sum_{\rho\tau}\hat{\gamma}_{\rho\tau}e_\rho\left[\prod_{\varphi}\left(\tilde{d}\Omega_\varphi+\bar{d}\Omega_\varphi\right)\right]_\rho e_\tau\left[\prod_{\psi}\left(\tilde{d}\Omega_\varphi+\bar{d}\Omega_\psi\right)\right]_\tau=\left\{\sum_{\rho}\chi_\rho\left[\prod_{\varphi}\left(\tilde{d}\Omega_\varphi+\bar{d}\Omega_\varphi\right)\right]_\rho\right\}^2 \qquad (3.3)$$

with the condition

$$\chi_\rho\chi_\tau+\chi_\tau\chi_\rho=\hat{\gamma}_{\rho\tau}\left(e_\rho e_\tau+e_\tau e_\rho\right). \qquad (3.4)$$

Now the Lagrangean is

$$\mathsf{L}=\int\sum_{\rho}\chi_\rho\left[\prod_{\varphi}\left(\tilde{d}\Omega_\varphi+\bar{d}\Omega_\varphi\right)\right]_\rho \qquad (3.5)$$

and we can write eq. (3.2) in the from

$$\delta\int_1^2\prod_j dx^j\sum_\rho\chi_\rho\left[\prod_j\left(\tilde{d}\Omega_\varphi+\bar{d}\Omega_\varphi\right)\right]_\rho=0. \qquad (3.6)$$

From the expression of Ω_φ one can write the general from

$$\Omega_\varphi=\sum_{rKj\ldots}a_\varphi^{rKj\ldots}W_{r(K)j\ldots}+\sum_{rsKLj\ldots k\ldots}b_\varphi^{rsKLj\ldots k\ldots}\quad W_{r(K)j\ldots}\quad W_{s(L)k\ldots}+\Omega_\varphi^c \qquad (3.7)$$

where Ω_φ^c is a possible more complicated expression.

The quantities $\tilde{d}\Omega_\varphi$ indicating the intrinsic variations on a cross section can be writte in the usual way

$$\tilde{d}\Omega_\varphi=\sum_{rKj\ldots k}\left(\frac{\partial\Omega_\varphi}{\partial W_{r(K)j\ldots}}DW_{r(K)j\ldots}+\frac{\partial\Omega_\varphi}{\partial x^\chi}dx^k\right) \qquad (3.8)$$

where D is the covariant differential.

On the other hand the more sophisticated variations cannot be expressed as in (4.2) because there is a change of cross section, which is not described by a simple differential. Instead of $DW_{r(K)j\ldots}$ we introduce [6] the symbol $\Delta_k W_{r(K)j\ldots}dx^k$,and we write in general from

$$\bar{d}\Omega_\varphi=\sum_{rKj\ldots kl}\left(\frac{\partial\Omega_\varphi}{\partial W_{r(K)j\ldots}}\Delta_\mu W_{r(K)j\ldots}dx^k+\frac{\partial\Omega_\varphi}{\partial x^\chi}dx^l\right). \qquad (3.9)$$

In view of (3.8), (3.9), and bearing in mind the invariance under space time transformations, the Lagrangean in (3.5) reads

$$\mathsf{L}=\int\sum_{\rho Kj\ldots k}x_\rho\left[\prod_\varphi\left(\frac{\partial\Omega_\varphi}{\partial W_{r(K)j\ldots}}DW_{r(K)j\ldots}+\frac{\partial\Omega_\varphi}{\partial W_{r(K)j\ldots}}\Delta_k W_{r(K)j\ldots}dx^k\right)\right]_\rho. \qquad (3.10)$$

Taking into account the usual experience evidenciating the three-dimensionality of space, it has been shown [1,2,5] that one can make $\varphi = 1,2.$ Thus the Lagrangean may be written

$$
\begin{aligned}
\mathsf{L} = \int \sum_{\substack{\rho rsKL \\ i\ldots j\ldots kl}} \chi_\rho \Bigg[\left(\frac{\partial \Omega_1}{\partial W_{r(K)i\ldots}} DW_{r(K)i\ldots} \frac{\partial \Omega_2}{\partial W_{s(L)j\ldots}} DW_{s(L)j\ldots} \right) + \\
+ \left(\frac{\partial \Omega_1}{\partial W_{r(K)i\ldots}} \Delta_k W_{r(K)i\ldots} dx^k \frac{\partial \Omega_2}{\partial W_{s(L)j\ldots}} \Delta_l W_{s(L)j\ldots} dx^l \right) + \\
+ \left(\frac{\partial \Omega_1}{\partial W_{r(K)i\ldots}} DW_{r(K)i\ldots} \frac{\partial \Omega_2}{\partial W_{s(L)j\ldots}} \Delta_l W_{s(L)j\ldots} dx^l + \right. \\
\left. + \frac{\partial \Omega_1}{\partial W_{r(K)i\ldots}} \Delta_k W_{r(K)i\ldots} dx^k \frac{\partial \Omega_2}{\partial W_{s(L)j\ldots}} DW_{s(L)j\ldots} \right) \Bigg].
\end{aligned}
\tag{3.11}
$$

4. Variations on a cross section of PE

Let us consider that part of the Lagrangean containing only the variations a given cross section in PE. From (3.11) we have

$$
\mathsf{L}_I = \int \sum_{\substack{\rho rsKL \\ j\ldots k\ldots}} \chi^I_\rho \left(\frac{\partial \Omega_1}{\partial W_{r(K)j\ldots}} DW_{r(K)j\ldots} \frac{\partial \Omega_2}{\partial W_{s(L)k\ldots}} DW_{s(L)k\ldots} \right)_\rho . \tag{4.1}
$$

At this level, because always approximations give rise to significant descriptions of some aspects of the physical phenomena, we restrict our study by neglecting the rapid variation of the field W_r and its selfinteraction , and we propose the simple expression

$$
\Omega^I_\varphi = \sum_r a^r_\varphi W_r . \tag{4.2}
$$

Thus (4.1) reads

$$
\mathsf{L}_I = \int \sum_{\rho rsjk} \chi^I_\rho \left[a^r_1 \left(\frac{\partial W_r}{\partial x^j} + \Gamma_r \right) dx^j \, a^s_2 \left(\frac{\partial W_s}{\partial x^k} + \Gamma_s \right) dx^k \right]_\rho \tag{4.3}
$$

where Γ_r represents the part containing the connections.

Introducing an arbitrary parameter ω, multiplying and dividing by $(d\omega)^2$, and denoting

$$Y^{jk} = \int \left(d\omega^2\right) \frac{dx^j}{d\omega} \frac{dx^k}{d\omega} \tag{4.4}$$

one can write

$$\mathsf{L}_I = \sum_{\rho rsjk} \chi_\rho^I \left[Y^{jk} a^r \left(\frac{\partial W_r}{\partial x^j} + \Gamma_r \right) a^\rho \left(\frac{\partial W_s}{\partial x^k} + \Gamma_s \right) \right]_\rho \tag{4.5}$$

where we made $a_1^r = a_2^r = a^r$ and $a_1^s = a_2^s = a^s$ for obvious symmetry reasons.

We separate L_I into two parts, one without the connections, the other containing them:

$$\mathsf{L}_A = \sum_{\rho rsjk} \chi_\rho^A \left(Y^{jk} a^r \frac{\partial W_r}{\partial x^j} a^s \frac{\partial W_s}{\partial x^k} \right)_\rho \tag{4.6}$$

and

$$\mathsf{L}_B = \sum_{\rho rsjk} \chi_\rho^B \left[Y^{jk} \left(a^r \frac{\partial W_r}{\partial x^j} a^s \Gamma_s + a^r \Gamma_r a^s \frac{\partial W_s}{\partial x^k} \right) + \left(a^r \Gamma_r a^s \Gamma_s \right) \right]_\rho . \tag{4.7}$$

5.Electromagnitism

The form of L_A suggests to relate it with the electromagnetic potential. We separate from (4.6) the expression

$$\mathsf{L}_{A(0)} = \sum_{\rho rj} \chi_\rho^{A(0)} \left(Y^{jj} a^r \frac{\partial W_r}{\partial x^j} a^r \frac{\partial W_r}{\partial x^j} \right)_\rho . \tag{5.1}$$

Taking in consideration as independent variables only the four spacetime coordinates x^λ $(\lambda = 1,2,3,4)$, extracting from ρ the corresponding four dimensions, and using four components of $\overline{\overline{W}}$, one can write

$$\mathsf{L}_{em} = \sum_{\lambda\mu} Y^{\lambda\lambda} \frac{\partial \left(\chi_\mu^{A(0)a^\mu} W_\mu \right)}{\partial x^\lambda} \frac{\partial \left(\chi_\mu^{A(0)a^\mu} W_\mu \right)}{\partial x^\lambda} . \tag{5.2}$$

Thus the magnitude

$$\chi_\mu^{A(0)} a^\mu W_\mu = A_\mu \tag{5.3}$$

can be interpreted as the electromagnetic potential. Then if the arbitrary parameter in (4.4) is such that integral $Y^{\lambda\lambda}$ has the same value for every λ, (5.2) can be written

$$\mathsf{L}_{em} = -\sum_{\lambda\mu} C \frac{\partial A_\mu}{\partial x^\lambda} \frac{\partial A_\mu}{\partial x^\lambda} \quad . \tag{5.4}$$

Let us introduce the electromagnetic tensor

$$T_{\lambda\upsilon} = \partial_\lambda A_\upsilon - \partial_\upsilon A_\lambda \tag{5.5}$$

$$T_{11} = T_{22} = T_{33} = T_{44} = 0,\quad T_{12} = \mu_0 H_3,\ T_{13} = -\mu_0 H_2,\ T_{14} = -(i/c)E_1,\ T_{23} = \mu_0 H_1,$$
$$T_{24} = -(i/c)E_2\ ,\ T_{34} = -(i/c)E_3,$$

and the antisymmetric components; $E_1, E_2, E_3, H_1, H_2, H_3$ represent the electric and magnetic fields $\vec{E}$ and $\vec{H}$.

Using the Lagrangean (5.4), and with help of the Lorentz condition, the Euler-Lagrange equations lead to Maxwell`s equations for the electromagnetic field without charges:

$$\sum_{\lambda\upsilon\mu} \partial_\upsilon T_{\lambda\mu} = 0$$
$$\sum_\upsilon \frac{\lambda}{\mu_0} \partial_\upsilon T_{\lambda\upsilon} = 0 \tag{5.6}$$

or in the form (x^4=ict)

$$\begin{cases} \nabla \vec{H} = \varepsilon_0 \partial_t \vec{E} \\ \nabla \vec{E} = 0 \\ \nabla \vec{H} = 0 \\ \nabla \vec{E} = -\mu_0 \partial_t \vec{H} \end{cases} \tag{5.7}$$

It is remarkable that the Lagrangean (5.1) is liable to give rise to equations including other fields linked with the electromagnetic field.

Let us add a fifth independent variable such that j=1,2,3,4,5, and put $x^5 = i\varsigma\varphi$. Extracting now five dimensions from φ and using five components of $\overline{\overline{W}}$, one can write

$$\mathsf{L}_F = \sum_{jk} Y^{jj} \frac{\partial\left(\chi_k^{A(F)} a^k W_k\right)}{\partial x^j} \frac{\partial\left(\chi_k^{A(F)} a^k W_k\right)}{\partial x^j}. \tag{5.8}$$

The magnitude

$$B_k = \chi_k^{A(F)} a^k W_k \tag{5.9}$$

represents an extended potential and eq. (5.6) reads

$$\mathsf{L}_F = K \frac{\partial B_k}{\partial x^j}\frac{\partial B_k}{\partial x^j}. \tag{5.10}$$

Introducing the tensor

$$I_{jk} = \partial_j B_k - \partial_k B_j \tag{5.11}$$

with the components

$I_{11} = I_{22} = I_{33} = I_{44} = I_{55} = 0,\ I_{12} = \mu_0 H_3,\ I_{13} = -\mu_0 H_2,\ I_{14} = -(i/c)E_1,\ I_{15} = -iF_1, I_{23} = \mu_0 H_1$
$I_{23} = \mu_0 H,\ I_{24} = -(i/c)E_2,\quad I_{25} = -iF_2,\ I_{34} = -(i/c)E_3,\ I_{35} = -iF_3,\ I_{45} = -iF_4$

and the antisymmetric components ; F_1, F_2, F_3, are the components of the field $\vec{F}$, $F_4 = iF_0$

In a similar way as above we obtain

$$\begin{cases} \sum_{jkl} \partial_l I_{jk} = 0 \\ \sum_j \frac{1}{\mu_0} \partial_j I_{jk} = 0 \end{cases} \tag{5.12}$$

which lead to the equations

$$\begin{cases} \nabla \vec{H} = \varepsilon_0 \partial_t \vec{E} + (\mu_0 \varsigma)^{-1} \partial_\varphi \vec{F} \\ \nabla \vec{E} = c\varsigma^{-1} \partial_\varphi F_0 \\ \nabla \vec{H} = 0 \\ \nabla \vec{E} = -\mu_0 \partial_t \vec{H} \end{cases} \tag{5.13}$$

These are Maxwell`s equations with supplementary terms.

The determination of the fields $\vec{F}$ and F_0 is given by the equations

$$\begin{cases} \nabla \vec{F} = -c^{-1} \partial_t F_0 \\ \nabla \vec{F} = -\mu_0 (\varsigma)^{-1} \partial_\varphi \vec{H} \\ \partial_n F_0 + c^{-1} \partial_t F_n - (c\varsigma)^{-1} \partial_\varphi E_n = 0 \end{cases} \tag{5.14}$$

(n = 1,2,3).

6. Gravitation

The Lagrangean L_B contains the connections. In order to express these connection, making again $\lambda = 1,2,3,4$, x^λ being as indicated the spacetime coordinates, we consider a Riemannian metric, i. e.

$$ds^2 = \sum_{\lambda\mu} g_{\lambda\mu} dx^\lambda dx^\mu \tag{6.1}$$

where $g_{\lambda\mu}$ is the symmetric fundamental metric tensor and the connections are given by the Christoffel symbol. It seems that this is the metric nearest to observations.

It is clear that the components of the fundamental metric tensor constitute general coordinates in the Lagrangean L_B.

All physical magnitudes are generated, as asserted, by the components of the field $\overline{\overline{W}}$, so are $g_{\lambda\mu}$ quantities too, and furthermore, we have to build other similar tensors. The action of χ_ρ^B on the W_r components allows to write

$$\chi_\rho^B (W_r)_\rho = W_{\rho r}^\chi \tag{6.2}$$

Extracting as before from ρ the four dimensions corresponding to the four dimensions of spacetime, and using four components of $\overline{\overline{W}}$, we have

$$W_{\rho r}^\chi = W_{\lambda\mu}. \tag{6.3}$$

Expliciting the connections in (4.7) one obtains

$$\begin{aligned}\mathsf{L}_\mathrm{B} = \sum_{\eta\theta\chi\lambda\mu\vartheta\xi\pi} Y^{\eta\theta} a^{x\lambda} a^{\mu\upsilon} \Bigg[\frac{\partial W_{\chi\lambda}}{\partial x^\eta} \left(-\Gamma_{\mu\theta}^\xi W_{\xi\vartheta} - \Gamma_{\vartheta\theta}^\xi W_{\mu\xi} \right) + \\ + \frac{\partial W_{\mu\upsilon}}{\partial x^\theta} \left(-\Gamma_{\chi\eta}^\pi W_{\pi\lambda} - \Gamma_{\lambda\eta}^\pi W_{\pi\eta} \right) + \left(-\Gamma_{\mu\theta}^\xi W_{\xi\vartheta} - \Gamma_{\vartheta\theta}^\xi W_{\mu\xi} \right)\left(-\Gamma_{\chi\eta}^\pi W_{\pi\lambda} - \Gamma_{\lambda\eta}^\pi W_{\pi\eta} \right) \Bigg].\end{aligned} \tag{6.4}$$

Owing to the presence of the Christoffel symbol in (6.4) we interpret this part as the Lagrangean related to the gravitational field. Splitting L_B into two parts we have

$$\mathsf{L}_\mathrm{F} = \sum_{\eta\theta\chi\lambda\mu\vartheta\xi\pi} Y^{\eta\theta} a^{x\lambda} a^{\mu\upsilon} \left(-\frac{\partial W_{\chi\lambda}}{\partial x^\eta} W_{\xi\upsilon} \Gamma_{\mu\theta}^\xi - \frac{\partial W_{\chi\lambda}}{\partial x^\eta} W_{\eta\xi} \Gamma_{\upsilon\theta}^\xi - \frac{\partial W_{\mu\upsilon}}{\partial x^\theta} W_{\chi\eta} \Gamma_{\chi\lambda}^\pi - \frac{\partial W_{\mu\upsilon}}{\partial x^\theta} W_{\chi\lambda} \Gamma_{\chi\eta}^\pi \right) \tag{6.5}$$

$$\mathsf{L}_\mathrm{G} = \sum_{\eta\theta\chi\lambda\mu\vartheta\xi\pi} Y^{\eta\theta} a^{x\lambda} a^{\mu\upsilon} \left(W_{\xi\upsilon} W_{\pi\lambda} \Gamma_{\mu\theta}^\xi \Gamma_{\chi\eta}^\pi + W_{\xi\vartheta} W_{\pi\lambda} \Gamma_{\mu\theta}^\xi \Gamma_{\lambda\eta}^\pi + W_{\mu\xi} W_{\pi\lambda} \Gamma_{\vartheta\theta}^\xi \Gamma_{\chi\eta}^\pi + W_{\mu\vartheta} W_{\chi\pi} \Gamma_{\vartheta\theta}^\xi \Gamma_{\lambda\eta}^\pi \right) \tag{6.6}$$

(it has been admitted that $\Gamma_{\upsilon\theta}^\xi$ commute with $W_{\mu\xi}$).

Compresing the quantities Y, a, and W in shorthand notations [7], L_G can be written

$$\begin{aligned}\mathsf{L}_\mathrm{G} = \sum_{\eta\theta\chi\lambda\mu\upsilon\xi\pi} \Big[G^I (\eta\theta\chi\mu\xi\pi) \Gamma_{\mu\theta}^\xi \Gamma_{\chi\eta}^\pi + G^{II} (\eta\theta\chi\mu\xi\pi) \Gamma_{\mu\theta}^\xi \Gamma_{\lambda\eta}^\pi + G^{III} (\eta\theta\chi\mu\xi\pi) \Gamma_{\upsilon\theta}^\xi \Gamma_{\chi\eta}^\pi + \\ G^{IV} (\eta\theta\chi\mu\xi\pi) \Gamma_{\vartheta\theta}^\xi \Gamma_{\lambda\eta}^\pi \Big].\end{aligned} \tag{6.7}$$

Changing some indices and presuming the symmetry

$$G^{III} + G^{IV} = -G^{I} - G^{II} = G \tag{6.8}$$

L_G gets the expression

$$L_G = \sum_{\eta\theta\lambda\mu\xi\pi} G(\eta\theta\lambda\mu\xi\pi)\left[\Gamma^{\xi}_{\mu\theta}\Gamma^{\pi}_{\lambda\eta} - \Gamma^{\xi}_{\eta\theta}\Gamma^{\pi}_{\lambda\eta}\right] \tag{6.9}$$

Consider now the well known relation [8]

$$\delta_g \int dx(-g\ )^{1/2} R = \delta_g \int dx \sum_{\eta\theta\lambda\mu} (-g)^{1/2} g^{\lambda\mu}\left[\Gamma^{\eta}_{\mu\theta}\Gamma^{\theta}_{\lambda\eta} - \Gamma_{\eta\theta}\Gamma^{\theta}_{\mu\lambda}\ \right] \tag{6.10}$$

where R is the spacetim scalar curvature

$$R = \sum_{\lambda\mu} g^{\lambda\mu} R_{\lambda\mu} \tag{6.11}$$

$R_{\lambda\mu}$ is the Riemann tensor and δ_g means variation with respect to $g_{\lambda\mu}$.

From the set of brackets in (6.9) with different indices we chose that one which becomes identic with the bracket in (6.10), provided G vanishes for $\xi \neq \eta$ and $\pi \neq \theta$. If the components of G with indices λ and μ are all equal, one can write

$$\delta_g \int dx L_G = \delta_g \int dx \sum_{\eta\theta\lambda\mu} G(\eta\theta)\left[\Gamma^{\eta}_{\mu\theta}\Gamma^{\theta}_{\lambda\eta} - \Gamma^{\eta}_{\theta\eta}\Gamma^{\theta}_{\lambda\mu}\right]. \tag{6.12}$$

With the identification

$$G(\eta\theta) = (-g)^{1/2} g^{\eta\theta} \tag{6.13}$$

from (6.10) and (6.12) we have

$$\delta_g \int dx(-g)^{1/2} R = \delta_g \int dx L_G \tag{6.14}$$

It is well known that the equation

$$\delta_g \int dx(-g)^{1/2} R = 0 \tag{6.15}$$

leads to Eunstein`s equation of gravitation in the absence of the energy-matter tensor

$$R_{\lambda\mu} - \frac{1}{2} g_{\lambda\mu} R = 0\,. \tag{6.16}$$

According to (6.10) it is obvious that

$$\delta_g \int dx \mathsf{L}_G = 0 \qquad (6.17)$$

leads to the same equation.

In view of the composition of G one can see from (6.13) that $g_{\eta\theta}$ is determined by combinations of the magnitudes $W_{\lambda\mu}$.The energy- matter tensor, and perhaps other terms, coult be deduced from the remaining parts of the total Lagrangean.

$$\delta_g \int dx(\mathsf{L}_F + \mathsf{L}_G) = 0 \qquad (6.18)$$

one can find more general equations of gravitation, e.g.[9.10]

7. Mixed variations.

The second part of L in (3.11) is

$$\mathsf{L}_{II} = \int \sum_{\substack{\rho rsKL \\ \lambda...\mu...\vartheta\xi}} \chi_\rho^{II} \left(\frac{\partial \Omega_1}{\partial W_{r(K)\lambda...}} \Delta_\vartheta W_{r(K)\lambda....} dx^\vartheta \frac{\partial \Omega_2}{\partial W_{s(L)\mu...}} \Delta_\xi W_{s(L)\mu...} dx^\xi + \right.$$
$$+ \frac{\partial \Omega_1}{\partial W_{r(K)\lambda...}} D\ W_{r(K)\lambda....} \frac{\partial \Omega_2}{\partial W_{s(L)\mu...}} \Delta_\xi W_{s(L)\mu...} dx^\xi + \qquad (7.1)$$
$$\left. \frac{\partial \Omega_1}{\partial W_{r(K)\lambda...}} \Delta_\vartheta W_{r(K)\lambda....} dx^\vartheta \frac{\partial \Omega_2}{\partial W_{s(L)\mu...}} D\ W_{s(L)\mu...} \right).$$

In this study we drop from the derivatives the part containing the connexions, because the gravitation has been considered in the previous chapter. Also for natural simplicity (Ockham`s razor) we drop the complicated part Ω_φ^c from (3.7), and we have

$$\Omega_\varphi^{II} = \sum_{rK\lambda...} a_\varphi^{rK\lambda...} \partial_{\lambda...}^K W_r + \sum_{rsKL\lambda...\mu...} b_\varphi^{rsKL\lambda...\mu...} \left(\partial_{\lambda...}^K W_r\right) \left(\partial_{\mu...}^L W_s\right) \qquad (7.2)$$

$\left(\partial_{\lambda...}^K W_r = \partial^K W_r / \partial x^\lambda ...\right)$, and

$$\frac{\partial \Omega_\varphi^{II}}{\partial\left(\partial_{\lambda...}^K\right) W_r} = a_\varphi^{rK\lambda...} + \sum_{rK\lambda...} \hat{b}_\varphi^{rsKL\lambda...\mu...} \partial_\lambda^K W_r \qquad (7.3)$$

($\hat{b}$ are combinations of b).

Introducing also $Y^{\vartheta\xi}$ defined previously, the Lagrangean L_{II} has the particular form

$$\mathsf{L}_{\Delta} = \sum_{\substack{\rho KLMNpqrs \\ \eta\lambda\ldots\mu\ldots\vartheta\ldots\xi\ldots}} \chi_{\rho}^{\Delta}\Big[\overline{A}_{KL}^{rs\eta\lambda\ldots\mu\ldots}\left(\partial_{\eta\lambda\ldots}^{K+1}W_r\right)\left(\partial_{\mu\ldots}^{L}W_s\right) + A_{KL}^{rs\lambda\ldots\mu\ldots}\left(\partial_{\lambda\ldots}^{K}W_r\right)\left(\partial_{\mu\ldots}^{L}W_s\right) +$$

$$+\overline{B}_{KLM}^{qrs\eta\lambda\ldots\mu\ldots\vartheta\ldots}\left(\partial_{\eta\lambda\ldots}^{K+1}Wq\right)\left(\partial_{\mu\ldots}^{L}W_r\right)\left(\partial_{\vartheta\ldots}^{M}W_s\right) +$$

$$+B_{KLM}^{qrs\lambda\ldots\mu\ldots\vartheta\ldots}\left(\partial_{\lambda\ldots}^{K}Wq\right)\left(\partial_{\mu\ldots}^{L}W_r\right)\left(\partial_{\vartheta\ldots}^{M}W_s\right) + \tag{7.4}$$

$$+\overline{C}_{KLMN}^{pqrs\eta\lambda\ldots\mu\ldots\vartheta\ldots\xi\ldots}\left(\partial_{\eta\lambda\ldots}^{K+1}W_p\right)\left(\partial_{\mu\ldots}^{L}W_q\right)\left(\partial_{\vartheta\ldots}^{M}W_r\right)\left(\partial_{\xi\ldots}^{N}W_s\right) +$$

$$+C_{KLMN}^{pqrs\lambda\ldots\mu\ldots\vartheta\ldots\xi\ldots}\left(\partial_{\lambda\ldots}^{K}W_p\right)\left(\partial_{\mu\ldots}^{L}W_q\right)\left(\partial_{\vartheta\ldots}^{M}W_r\right)\left(\partial_{\xi\ldots}^{N}W_s\right)\Big]$$

(K, L, M, N=0,1, 2, 3, …Z) .

We from the magnitudes

$$W_r^{\Delta} = \sum_{\rho} \chi_{\rho}^{\Delta}\left(\Lambda^{\Delta}W_r\right)_{\rho} \tag{7.5}$$

where Λ^{Δ} conttains the symbol Δ. Thus W_r^{Δ} are the generalized coordinates in the Lagrangean.

Consequently this Lagrangean is now written

$$\mathsf{L}_{A} = \sum_{\substack{KKrs \\ \lambda\ldots\mu\ldots\eta}} \Big[\overline{A}_{KL}^{rs\eta\lambda\ldots\mu\ldots}\left(\partial_{\eta\lambda\ldots}^{K+1}W_r^{\Delta}\right)\left(\partial_{\mu\ldots}^{L}W_s^{\Delta}\right) + A_{KL}^{rs\eta\lambda\ldots\mu\ldots}\left(\partial_{\lambda\ldots}^{K}W_r^{\Delta}\right)\left(\partial_{\mu\ldots}^{L}W_s^{\Delta}\right)\Big] + N\left(W_r^{\Delta}\right) \tag{7.6}$$

where the nonlinear part is

$$N = N_1 + N_2 \tag{7.7}$$

$$N_1 = \sum_{\substack{pqrKLM \\ \lambda\ldots\mu\ldots\vartheta\ldots\eta}} \Big[\overline{B}_{KLM}\left(\partial_{\eta\lambda\ldots}^{K+1}W_p^{\Delta}\right)\left(\partial_{\vartheta\ldots}^{M}W_r^{\Delta}\right) + B_{KLM}\left(\partial_{\lambda\ldots}^{K}W_p^{\Delta}\right)\left(\partial_{\vartheta\ldots}^{M}W_r^{\Delta}\right)\Big] \tag{7.8}$$

$$N_2 = \sum_{\substack{pqrKLMN \\ \lambda\ldots\mu\ldots\vartheta\ldots\xi\ldots\eta}} \Big[\overline{b}_{KLMN}\left(\partial_{\eta\lambda\ldots}^{K+1}W_p^{\Delta}\right)\left(\partial_{\mu\ldots}^{L}W_r^{\Delta}\right)\left(\partial_{\vartheta\ldots}^{M}W_q^{\Delta}\right)\left(\partial_{\xi\ldots}^{N}W_s^{\Delta}\right) +$$

$$+b_{KLMN}\left(\partial_{\lambda\ldots}^{K}W_p^{\Delta}\right)\left(\partial_{\mu\ldots}^{L}W_q^{\Delta}\right)\left(\partial_{\vartheta\ldots}^{M}W_q^{\Delta}\right)\left(\partial_{\xi\ldots}^{N}W_s^{\Delta}\right)\Big] \tag{7.9}$$

We saw that the Lagrangean L_I led to the electromagnetic and gravitational fields,which are long range, macroscopic fields. It is them suggestive to presume that the Lagrangean L_A is liable for the desctiption of the elements of matter and microscopic fields. Thus the universal field, through its from (7.5) gives rise to all specific observable microscopic fields.

8. Elementary equations.

In order to obtain first and second order equations resulting from the Lagrangean (7.6), we make the following decomposition:

$$L_{\Delta} = \sum_{j=o}^{\Gamma} L_j^v + \sum_{k=o}^{\Omega} L_k^u \tag{8.1}$$

where

$$L_j^v = \sum_{\lambda} E_j^{\lambda} \left(\partial_{\lambda} v_j^+\right)\left(\partial_{\lambda} v_j\right) - F_j v_j^+ v_j + f_{vj}^2 \frac{1}{2\Gamma} N \tag{8.2}$$

$$L_k^u = \sum_{\lambda} G_k^{\lambda} u_k^+ \left(\partial_{\lambda} u_k\right) - H_k u_k u_k + f_{uk}^2 \frac{1}{2\Omega} N \tag{8.3}$$

(the cross indicates the conjugate)

The initial Lagrangean (7.6) is obtained if the fields v_j and u_k are given by

$$v_j = f_{vj} i \sum_{r\lambda\ldots} \sum_{a=0}^{z} V_a^j \partial_{\lambda\ldots}^a W_r^{\Delta} \tag{8.4}$$

$$u_k = f_{uk} i \sum_{s\mu\ldots} \sum_{b=0}^{z} U_b^k \partial_{\mu\ldots}^{ba} W_s^{\Delta} \tag{8.5}$$

with necessary relations between the coefficients.

The Euler-Lagrange equations written with the Lagrangeans (8.2) and (8.3) allows to obtain the equations of the fields u_k and v_j :

$$\sum_{\lambda} E_j^{\lambda} \partial_{\lambda\lambda}^2 v_j + F_j v_j + f_{vj}^2 \frac{1}{2\Gamma}\left(\sum_{\lambda} \partial_{\lambda} \frac{\partial N}{\partial\left(\partial_{\lambda} v_j^+\right)} - \frac{\partial N}{\partial v_j^+} \right) = 0 \tag{8.6}$$

$$\sum_{\lambda} G_k^{\lambda} \partial_{\lambda} u_k - H_k u_k + f_{uk}^2 \frac{1}{2\Omega}\left(\frac{\partial N}{\partial u_k^+} - \sum_{\lambda} \partial_{\lambda} \frac{\partial N}{\partial\left(\partial_{\lambda} u_k^+\right)} \right) = 0\,. \tag{8.7}$$

The equations of these fields are nonlinear, N being given by (7.7), (7.8), (7.9). N is very complicated; parts of the nonlinear terms can serve to several aims as: interaction with other fields, selfinteraction, definition of some characteristic magnitudes, etc. Discussions about nonlinear equations previous papers, e. g. [11,12].

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"Politehnica" University of Timişoara
Physics Department
P-ța Regina Maria, Nr.1
1900, Timişoara-ROMÂNIA
Telephone: 056-220370, Fax 40-56-19.03.21