Empirical Gap in Newtonian Gravity

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For practical and historical reasons, most of what we know about gravity is based on observations made or experiments conducted *beyond the surfaces* of dominant massive bodies (*exterior solution*). Here we consider one particular type of *interior solution* experiment that would not be too difficult to do, but has never been done.

1 Extrapolation

A problem concerning Newtonian gravity often found in undergraduate physics texts [1] [2] [3] [4] is as follows: A test object is dropped into an evacuated hole spanning a diameter of an otherwise uniformly dense spherical mass. One of the reasons this problem is so common is that the answer, the predicted equation of motion of the test object, is yet another instance of *simple harmonic motion*.

In no case that I know of, however, is it suggested that we might like to have some empirical evidence to verify the theoretical prediction. Confidence in the prediction is primarily based on the success of Newton's theory for phenomena that test the *exterior* solution. It is of course understandable that one would then extrapolate Newton's law to the interior. Essentially the same prediction follows from Einstein's theory of gravity. [5] [6] [7] [8] Since there is no obvious reason to doubt the predicted simple harmonic motion, the impression is sometimes given that it is a physical fact. A perfect example is found in John A. Wheeler's book, *A Journey Into Gravity and Spacetime*, in which he refers to the phenomenon as "boomeranging." Wheeler devotes a whole (10-page) chapter to the subject because, as he writes, "Few examples of gravity at work are easier to understand in Newtonian terms than boomeranging. Nor do I know any easier doorway to Einstein's concept of gravity as manifestation of spacetime curvature." [9] Later we'll come back to the connection with spacetime curvature, i.e., General Relativity. Presently, the point is that nowhere in Wheeler's book is there any discussion of empirical evidence for "boomeranging." This is typical, but it is certainly not necessary.

The primary purpose of the present paper is not to raise doubts about what would happen if the experiment were actually performed. We might well think of it as a *demonstration* rather than as a *test* of Newton's theory. I'll argue that even if that's all it would be, carrying out the experiment would still be worthwhile.

2 Feasibility

What kind of apparatus do we need? In the 1960's–1970's a few proposals to measure Newton's constant, G, involved through-the-center oscillations having the potential to confirm Wheeler's boomeranging. Y. T. Chen discusses these ideas in his 1988 review paper on G-measurements. [10] Each example in this particular group of proposals was intended for space-borne *satellite* "laboratories." The original motivation of these ideas was to devise ways to improve the accuracy of our knowledge of G by timing the oscillation period of the simple harmonic motion. Though having some advantages over Earth-based experiments, they also had drawbacks which, ultimately prohibited them from ever being carried out.

What distinguishes these proposals from experiments that have actually been carried out in Earth-based laboratories is that the test objects were to be allowed to *fall freely* the whole time. Whereas G-measurements conducted on Earth typically involve *restricting* the test mass's movement and measuring the *force* needed to do so. The most common, and historically original, method for doing this is to use a torsion fiber. Another distinguishing characteristic of Earth-based G-measurements is that the test masses are always *outside* the larger "source" masses. Since the movement of the test masses is restricted to a small range of motion, these tests can be characterized as *static* measurements. Torsion balance experiments in which the test mass is *inside* the source mass have also been performed. (For example, Spero, et al [11] and Hoskins, et al. [12]) These were not G-measurements, but tests of the inverse square law. Again, however, these were *static* measurements; the test masses were not free to move beyond a small distance compared to the size of the source mass.

These considerations all point to a possible modification of the torsion balance so as to demonstrate – at least as a first approximation – the simple harmonic motion of New-

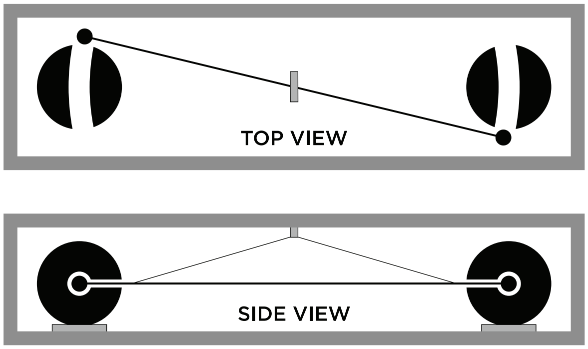

Fig. 1: Schematic of modified Cavendish balance.

ton's interior solution in an Earth-based laboratory. As implied above, the key is to design a suspension system which, instead of providing a restoring force that prevents the test masses from moving very far, allows unrestricted or nearly unrestricted movement. Two available possibilities are fluid suspensions and magnetic suspensions (or a combination of these). In 1976 Faller and Koldewyn succeeded in using a magnetic suspension system to get a G-measurement. [13] [14] The experiment's accuracy was not an improvement over that gotten by other methods, but was within 1.5% of the standard value.

In the apparatus Cavendish used for his original G-measurement the torsion arm and test masses were isolated from the source masses by a wooden box. In Faller and Koldewyn's experiment the arm was isolated from the source masses by a vacuum chamber. The modified design requires that there be no such isolation, as the arm needs to swing freely through the center of the source masses. Since we are interested not so much in the precision measurement of a number, but in simply demonstrating the low order correctness of the simple harmonic oscillation prediction, it is reasonable to expect that the technology used by Faller and Koldewyn could be adapted to this purpose. In fact, it is not unreasonable to expect that advances in technology (e.g., better magnets, better electronics, etc.) since 1976 would make the experiment quite doable for any "institution grade" physics laboratory. Figure 1 shows a schematic diagram of the modified Cavendish balance.

3 Completeness and Aesthetics

One hardly needs to mention the many successes of Newtonian gravity. By success we mean, of course, that empirical observations match the theoretical predictions. Einsteinian gravity is even more successful. A significant fraction of contemporary gravity experiments are designed to measure the typically small differences between Newton's and Einstein's theories. In every case Einstein's theory has proven to be more accurate. This is impressive. But consider that Newton's and Einstein's theories *both remain untested* with regard to the problem discussed above. The simple harmonic motion prediction is so common and so obvious that we have come to take it for granted. Wouldn't it be more satisfactory if, when discussing the *prediction*, we could at the same time cite the *physical evidence*?

If R represents the surface of a uniformly dense spherical mass, our empirical knowledge of how things move because of the mass within R is essentially confined to the region, $R \leq r < \infty$. The region $0 \leq r \leq R$ is a rather basic and a rather large gap. For the sake of *completeness*, shouldn't we resolve to fill the gap?

One of the distinctive features of the kind of experiment proposed above is that its result is, in principle, independent of *size*. The satellite versions mentioned by Chen were thus referred to as "clock mode" experiments. The determining factor in the oscillation period is the *density* of the source mass. If the source mass is made of lead (density, $\rho = 11,340$ kg/m^3) the oscillation period is about one hour. I'd guess that many students and physicists have enough patience to observe for an hour and would be fascinated to watch the oscillation take place, knowing that the *mass* of the larger body is the essential thing making it happen. In my opinion this would be a *beautiful* sight. Beautiful for completing the range from $0 \leq r \leq R$ and beautiful simply to see what has never before been seen.

4 Possible Bearing on Astrophysical Observations

Star clusters in general and globular clusters in particular, contain the closest natural circumstances to the idealized laboratory experiment described above. Stars bound to such systems have, at one extreme, perfectly *radial* orbits (through the center) or, at the other extreme, perfectly *circular* orbits (around the center). Of course in most every real case, the trajectories are more complicated. Nevertheless, since these systems are also extremely *old*, the statistical distribution of orbit types is supposed to be "isotropic," i.e., preferentially neither radial nor circular. In this section we'll consider one method used by astronomers to get a clearer picture of the pattern of motion taking place in these magnificent objects.

There are two distinctly different ways of measuring the velocities of individual stars within clusters. The first method is to measure the Doppler shifts with respect to the average for the cluster. This gives the speed of the star along the *line of sight*; it is known as a *radial* velocity. (This is not to be confused with radial motion with respect to the center of the *cluster*. Rather, it's radial motion with respect to Earthian observers.) The second method is to measure the change in position of individual stars at widely different *times* so as to deduce an angular velocity across the plane of the sky. These are known as *proper motions*. The latter type of measurement

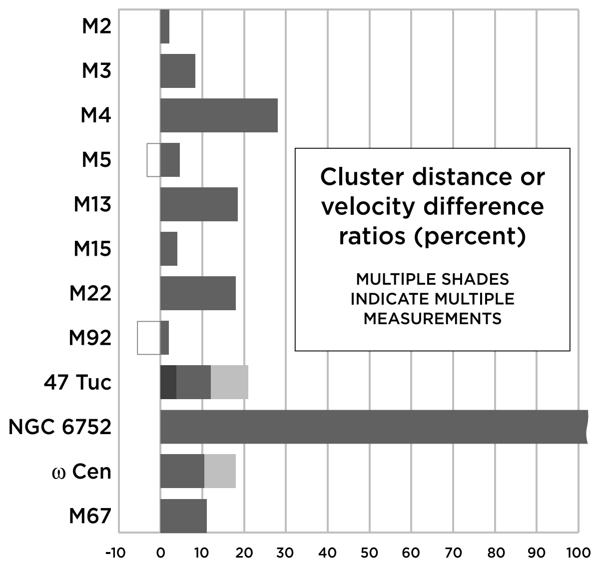

Fig. 2: Proper motion velocity dispersions tend to be greater than radial velocity dispersions. The discrepancies are commonly interpreted as implying the need to shorten the distance scale; this would bring the velocity dispersions into agreement. Bar length indicates the needed adjustment. Sources are: Harris, [15] Rees, [16] Popowski, [17] Jimenez, [18] van den Bosch, [19] McNamara, [20] Monaco, [21] Del Principe, [22] Rees, [23] Zoccali, [24] McLaughlin, [25] Gratton, [26] Drukier, [27] van de Ven, [28] van Leeuwen, [29] Girard, [30].

has only relatively recently become possible due to availability of data from the requisite different times.

Since the velocity distributions are supposed to be iso tropic, the so-called velocity *dispersion* from the radial measurements of a given cluster is supposed to match the velocity dispersion from the proper motion measurements of the same cluster. Various complications make the comparison a little less direct than one might like. For example, the radial measurements are independent of *distance*. The Doppler effect by itself gives a direct enough measure of the velocity. But this is not so for proper motions. Having only a measure of angular position change on the sky, one needs an independent measurement of distance to convert this into a linear velocity.

Fortunately, astronomers have devised independent means for measuring distances to the clusters. Given these independent distance measurements, when the proper motions (angular velocities) for stars in a given cluster are converted to linear velocities, the resulting velocity dispersion is supposed to match the radial velocity dispersions for the same cluster. Curiously, they rarely match. The proper motion velocities tend to be greater than expected. The most common interpretation of this result is to regard the independent distance measurements as being too large. In other words, the proper motion velocities can be made to agree with the radial motion velocities if the clusters were closer than the previous distance measurements would indicate. In fact, this kind of comparison is regarded as a distance measurement in it own right (called an "astrometric" or "dynamical" distance). The graph in Figure 2 reveals a general pattern suggesting one of two possible interpretations: Either the proper motion velocities are too large or the independent distance measurements to the clusters are too large. (The data are sometimes presented according to one interpretation, and sometimes according to the other.)

The most conspicuous discrepancy, involving globular cluster NGC 6752, warrants further comment. After discus sing in detail the difficulties involved in making such measurements and the care thus taken, the authors of the research, Drukier, et al concluded:

> Our HST (Hubble Space Telescope) proper motions suggest that the velocity dispersion in the center of NGC 6752 is surprisingly large. At 12.5 km s^{-1} it is much larger than the measured dispersion along the line of sight. While there is some uncertainty in the distance to NGC 6752 it is certainly known to better than the factor of roughly two which would be required to bring the two measurements into agreement. . . a most peculiar situation. [27] [31]

With a little reflection on the geometry and statistics involved here, one can deduce that, if the distance to the cluster is accurately known and the proper motions still indicate higher velocities than the radial velocities, this indicates *anisotropy* in the sense of a tendency toward *circular* orbits. Such a tendency is "peculiar" because it does not conform to what we otherwise think we know about star clusters. The challenge to resolve the puzzle is exacerbated by the fact that the clusters are very far away; their densities are far from uniform and it is a complicated task to analyze the velocity data. It might therefore be a safe guess that the "peculiar situation" concerning NGC 6752 or the less dramatic trend of the other clusters has no bearing on our proposed modified Cavendish experiment (nor vice versa). Perhaps the trend pointed out above (Figure 2) will go away with the accumulation of more data. Of course, in assessing this situation it certainly *wouldn't hurt* to know that Newton's interior solution has been verified by experiment. On the other hand, if verification fails, perhaps the astrophysical anomalies would be easier to understand.

5 Analogy

> *To the same natural effects we must, as far as possible, assign the same causes.*
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> *– Isaac Newton's Rule of Reasoning in Philosophy II* [32]

As is well known, Einstein's theory of gravity, General Relativity (GR) differs from Newton's theory in that freely falling bodies are not made to move by a *force*, but by the *curvature of spacetime*. How does this relate to the interior solution and the simple harmonic motion prediction? According to Newton's theory, the prediction follows from the fact that the center of the sphere represents the bottom of a "potential well." In GR, this corresponds to the fact that the rate of a clock at the center is supposed to be a minimum. Differences in the rates of clocks from one place to the next are supposed to make things "fall."

It is also well known that, both prior to and after completing GR, Einstein often appealed to *uniform rotation* as an analogy for understanding curved spacetime. The speed of rods and clocks located on a rotating body varies with distance from the axis; and this speed affects the rods' lengths and the clocks' rates. Einstein deemed that the best way to describe these relationships was in terms of non-Euclidean geometry. Einstein saw this as being analogous to what is found in a gravitational field.

A key fact here is that, though moving, and *moving with a range of different speeds*, a uniformly rotating body is nevertheless *stationary*. In the case of rotation it is fairly obvious that the variations in length and clock rate are caused by the speed of rotation. We could thus say that the cause of rotation-induced spacetime curvature is motion, *stationary motion*. In the case of gravity we evidently have the opposite circumstance. The gravitational field (e.g., a Schwarzschild field) is regarded as being *static*. So Einstein's analogy is really more like an *anti*-analogy. According to GR, it's not motion that causes spacetime curvature; spacetime curvature is supposed to be the cause of motion.

The "inverted" character of Einstein's analogy comes out especially by considering again the centrally located clock. What causes it to run slow? What is the physical mechanism? The analogy with rotation doesn't seem to help here. For a clock on Earth's *surface* you could perhaps say its reduced rate is due to the asymmetric distribution of matter with respect to the clock. At the center, however, there is no asymmetry, so the cause would have to be something else.

Now suppose we try abiding by Newton's second Rule of Reasoning. What would it mean in this case to "assign the same causes to the same effects."? Turning the anti-analogy into a direct analogy implies the idea that the stationary motion of *rotation* corresponds to some kind of "stationary motion of gravitation." Newton's Rule thus evokes the question: Is there a reasonable way to conceive that, if motion is what causes *rotating* rods and clocks to contract and slow down, it may also be the case for gravitation? If this were possible, would we still expect the central clock to have a minimum rate? Or, by analogy with a clock on the rotation axis, should the central clock instead have a *maximum* rate?

These questions clearly do not fit within the context of Newton's theory of gravity nor within the context of GR. Perhaps, therefore, they are nonsense. Of course, we could establish *for a fact* that they are nonsense if we knew *for a fact* that the simple harmonic motion prediction were correct. For this prediction *requires* the central clock to have the slowest rate.

One might like to conduct an experimental test of the prediction for the sake of *completeness*. One might like to test the prediction for the sake of *aesthetics*. Or, by confirming the prediction one might gain a modicum of security in one's assessment of astrophysical anomalies or Einstein's theory of gravity. In any case, even if the above arguments and ideas have failed to kindle the desire to experiment, it is beneficial to at least acknowledge what we do not know.

> *It is absolutely necessary that we should learn to doubt the conditions we assume, and acknowledge we are uncertain ... In the pursuit of physical science, the imagination should be taught to present the subject investigated in all possible and even in impossible views; to search for analogies of likeness and (if I may say so) of opposition – inverse or contrasted analogies; to present the fundamental idea in every form, proportion, and condition; to clothe it with suppositions and probabilities – that all cases may pass in review, and be touched, if needful by the Ithuriel spear of experiment.*
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> – *Michael Faraday* [33]

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