

SR Theory of Electrodynamics for Relatively Moving Charges

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Abstract

This paper summarizes the derivation of SRT electrodynamics laws for relatively moving charges. These laws are based on Coulomb's Law and SRT, both of which have been basic in conventional physics for over 100 years. Relativists have determined an equation for the transform of the electric field of a relatively moving electric charge. However, that transform is usually based on a transform of the Lorentz Force Law. Part of the Lorentz Force law was found to be unnecessary to derive the magnetic force on a stationary charge due to a moving charge. Several electrodynamics laws emerge from this transform: The "cardinal law" as described by Maxwell, the law for the force between slow relatively moving charges, the law between a stationary charge and a current element, similar to Gauss's Law, and the old law for force between current elements as determined by Ampère in 1822. An experiment was performed to check on the validity of the old Ampère's Law.

Introduction

The purpose of this paper is to collect work the author has done on this subject and to present explanations of the author's treatment of important aspects of the theory. Another purpose is to show that the old Ampère's Law can be derived from SRT, thus giving credence to this law over Grassmann's Law.

Postulates and Assumptions

1. SRT Postulates
 - a. Relativity Principle (RP): All inertial frames are totally equivalent for the performance of all physical experiments. (gravity is not accounted for here and is negligible).
 - b. The speed of light is constant with respect to its source and its receiver. An alternate postulate $E=mc^2$ or relativistic mass increase may be employed here to arrive at the same results for SRT, the Lorentz Transform.
2. The charge of an electron or proton is mathematically considered herein to be a point charge and does not have a finite size.
3. An uniform electric field exists about a stationary electric point charge, pervading the space equally in all directions and falling off in intensity at $1/r^2$.
4. Velocities are *relative* between interacting particles.
5. The electric field of a moving charge pervades all the space of its inertial frame, thus having instant reaction with a charge in contact with it. Acceleration of a charge creates a new velocity that changes the electric field that spreads out over the new inertial frame at the speed of light.
6. The charge value, q , is invariant from one inertial frame to another.
7. A positive sign on the overall magnetic force represents repulsion while a negative sign represents attraction.

8. A negative sign must be entered into the equations for negative charges such as electrons. A positive sign must be entered into the equations for positive charges such as protons. This makes the direction of the overall force appear correctly as in 7 above.

The Basic Electrodynamics Law of SRT for a Moving Charge

This law is expressed by Eq. (1) [1]. It expresses the electric field intensity at a stationary point, emanating from a relatively moving charge q_1 at a 3-vector distance of \mathbf{r}_1 .

$$\mathbf{e}_1 = \frac{kq_1\mathbf{r}_1}{\gamma^2 r^3 [1 - (v^2/c^2)\sin^2\theta]^{\frac{3}{2}}} \quad (1)$$

where $v =$ magnitude of \mathbf{v} , the relative velocity between q_1 and a stationary point, $\gamma = 1/\sqrt{1 - v^2/c^2}$, $\theta =$ angle between \mathbf{r}_1 and \mathbf{v} . The constants are $k = 1/4\pi\epsilon_0$ ($\epsilon_0 =$ permittivity of free space) and $c =$ speed of light. Place a test charge q_2 at the stationary point and you can create an expression that represents the total electrodynamic force between the stationary charge and the moving charge. This force consists of the electric Coulomb force and the magnetic force. This expression is good for relative velocity from zero up to c . This is why this author calls this formula “The Basic Electromagnetic Law”. Relativists might say Eq. (1) determines only the electric field and the magnetic field is something else. But the magnetic force emerges as the difference in the electric field of a stationary charge and the electric field of the moving charge as observed by a stationary charge or observer.

The SRT derivation of Eq. (1)

Regrettably, Eq. (1) has to be defended and this section is written for that purpose. Eq. (1) was derived using the Lorentz force law (three-force) as the starting point [1]:

$$\mathbf{f} = q \left(\mathbf{e} + \frac{\mathbf{u} \times \mathbf{h}}{c} \right) \quad (2)$$

where \mathbf{u} is the three vector relative velocity between q and the three vector magnetic field \mathbf{h} . The electric field \mathbf{e} which interacts with the charge q produces part of the force \mathbf{f} .

The next formula is SRT’s formula for the four-vector force \mathbf{F} (the four-force):

$$\mathbf{F} = \gamma(u) \left(\mathbf{f}, \frac{\mathbf{f} \cdot \mathbf{u}}{c^2} \right) \quad (3)$$

Substituting (2) into (3), we get

$$\mathbf{F} = q\gamma(u) \left(\mathbf{e} + \frac{\mathbf{u} \times \mathbf{h}}{c}, \frac{\mathbf{e} \cdot \mathbf{u}}{c^2} \right) \quad (4)$$

The components of a four-vector transform just as the four components of a Lorentz Transformation: (x, y, z, t) and (x', y', z, t') . Following the path described in (2), (3), and (4) above and setting $\mathbf{e}' = (q/r'^2)(x', y', z')$ and $\mathbf{h}' = 0$, and using length contraction on r 's, we arrive at Eq. (1) above and a formula for the magnetic field of the moving charge:

$$\mathbf{h}_1 = \frac{1}{c} \mathbf{v} \times \mathbf{e}_1 \quad (5)$$

The details of (1) to (5) are many. It is not the author's intention to discuss these in detail, but only to point out certain features of these equations. First, it is observed that the Lorentz force law Eq. (2) is the starting point of the derivation. The relativists who derive Eq. (1) take that formula as truth. Their derivation of Eq. (1) and Eq. (5) is correct based on SRT and the correctness of Eq. (2). Also, it shows how e-fields and h-fields transform. The present author questions the appropriateness of using the right side of Eq. (2) relating to the h-field when we are just trying to determine the force between a stationary charge and a uniformly moving charge. That h-field part of the Lorentz force comes from a separate source other than the e-field part that comes from the moving charge. Also, that part of the Lorentz force law and the Biot-Savart Law combines to create the Grassmann's Law for force between current elements. This puts the "chicken" before the "egg". The "egg" in (2) is the electric field of the moving charge from which we will ultimately derive the old Ampère's Law. In addition, the force on the stationary charge due to the magnetic field \mathbf{h} of Eq. (5) would be zero (using the same part of the Lorentz force law) since the stationary charge is not moving. Therefore, we must conclude the total force on the stationary charge due to the moving charge is totally due to the oval shaped electric field of the moving charge the stationary charge sees. Considering this, we work with just Eq. (1). A contribution to the field of electrodynamics this author has made is in just recognizing the magnetic field is just the difference between the e-field of a moving charge and the e-field of a stationary charge.

Jan Olo Jonson has criticized the derivation of Eq. (1), stating that one had only perform length contraction on the r term of the e-field of a stationary charge $\mathbf{e} = \frac{kq\mathbf{r}}{r^3}$. One must note that the Lorentz transformation of a four-vector-force involves a gamma factor in the y and z terms of the transformed e-field that would not otherwise be there if only the length contraction formula was applied to r term to arrive at Eq. (1). The gamma factor in the y and z terms is necessary to derive Eq. (1).

The Magnetic Force Law Between Slow Relative Moving Charges

This law is derived from Eq. (1) by using the Binomial Series, eliminating the higher order terms of v^2/c^2 and subtracting out the stationary Coulomb force [2].

$$\mathbf{f}_{12} = \frac{kq_1q_2}{r_{12}^3} \frac{v^2}{c^2} \mathbf{r}_{12} (0.5 - 1.5\cos^2\theta) \quad (6)$$

This formula looks familiar. It shows the angle dependence of the magnetic force. This magnetic force law obeys Newton's Third Law and depends on the relative velocity. The force is in the direction of a line between the two charges (no torque is generated), and it does not depend on an ether.

The Magnetic Force Law Between a Stationary Charge and a Stationary Current Element

This law is derived from Eq. (1) in the same manner as Eq. (6) with the exception that q_1 in Eq. (1) is replaced with $-\gamma\sigma_1 ds_1$, where σ_1 is charge line density and $\sigma_1 ds_1$ is the current element charge [2]:

$$d\mathbf{f}_{12} = -\frac{kq_2\sigma_1 ds_1}{r^3} \frac{v^2}{c^2} \mathbf{r}_{12} (1-1.5\cos^2\theta) \quad (7)$$

The γ in $-\gamma\sigma_1 ds_1$ expresses length contraction of the distance between the moving charges in the current element as seen by the test charge q_2 thereby increasing the charge density, and the minus sign recognizes the charge is negative due to moving electrons. Notice the subtle differences between Eq. (6) and Eq. (7). The “0.5” in Eq. (6) is replaced by “1.0” in Eq. (7) and the moving charge q_1 is replaced by the current element charge $\sigma_1 ds_1$. Also, notice that Eq. (7) describes a force between a stationary charge and a current element “magnetic field”. This force has been reported in the works of Cooper [3] and Spencer [4].

Eq. (7) is very similar to the Gauss Law (with the Coulomb term removed) derived in about 1835 [4]. Gauss’s Law is

$$d\mathbf{f}_{12} = -\frac{kq_1 q_2}{r_{12}^3} \frac{v^2}{c^2} \mathbf{r}_{12} (1-1.5\cos^2\theta) \quad (8)$$

The q_1 in Gauss’s Law is misleading. It should be a current element charge instead of an isolated charge. Gauss Law denotes a force between relatively moving charges, which is different. In SRT, Eq. (6) is the formula for the force between relatively moving charges. It is important to recognize that Eq. (7) and Eq. (8) describes the magnetic force between a stationary charge q_2 or a stationary $\sigma_2 ds_2$, and a current element charge, $\sigma_1 ds_1$. In Eq. (7) the velocity term is the relative velocity between the positive ion lattice and the moving electron lattice creating the current. When the equation is applied to the moving electron lattice of another current element then velocity term in Eq. (7) is the relative velocity between that lattice and the moving electron lattice of the other current element.

Eq. (7) is employed to derive Ampère’s Law [2]. Three force relationships of Eq. (7) are added together to form Ampère’s Law as describes below.

The Magnetic Force Law Between Stationary Current Elements (Ampere’s Law)

The test charge q_2 in Eq. (7) may be replaced by $\sigma_2 ds_2$ of another wire current element with charge line density σ_2 . Then, the two σds ’s are replaced by two $I ds/v$ ’s, representing the moving charges in the two current elements. The positive ion lattices of the two stationary current elements are stationary with respect to each other. Their Coulomb repulsion has already been subtracted out of Eq. (7) and this force does not need to be added. Then applying Eq. (7) three times to the cross combination of charges in the two current elements and adding the forces, one arrives at Ampere’s Law [2] [5]:

$$d^2 \mathbf{f}_{12} = -\hat{\mathbf{r}}_{12} \frac{kI_1 I_2 ds_1 ds_2}{c^2 r_{12}^2} (2\sin\theta_1 \sin\theta_2 \cos\eta - \cos\theta_1 \cos\theta_2) \quad (8)$$

where $\hat{\mathbf{r}}_{12}$ = unit vector in the direction of \mathbf{r}_{12} and r_{12} = magnitude of the vector \mathbf{r}_{12} joining the two current elements. The constants are $k = 1/4\pi\epsilon_0$ (ϵ_0 = permittivity of free space) and c = speed of light. The I_1 and I_2 are current magnitudes and ds_1 and ds_2 are current element lengths. The angles are: θ_1 = angle between ds_1 and \mathbf{r}_{12} ; θ_2 = angle

between ds_2 and \mathbf{r}_{12} ; $\eta =$ angle between the plane of ds_2 with \mathbf{r}_{12} and the plane of ds_{12} with \mathbf{r}_{12} .

Weber has derived an identical magnetic force law to Eq. (8). Eq. (8) is sometimes referred to as Weber's magnetic force law.

Using vector analysis Ampère's Law is mathematically equivalent to and is more conveniently expressed as:

$$d^2\mathbf{f}_{12} = -\hat{\mathbf{r}}_{12} \frac{kI_1I_2 ds_1 ds_2}{c^2 r_{12}^2} (2d\hat{\mathbf{s}}_1 \bullet d\hat{\mathbf{s}}_2 - 3(d\hat{\mathbf{s}}_1 \bullet \hat{\mathbf{r}}_{12})(d\hat{\mathbf{s}}_2 \bullet \hat{\mathbf{r}}_{12})) \quad (9)$$

where \mathbf{f}_{12} is the force on current element 1 caused by current element 2, and \mathbf{r}_{12} is the relative vector displacement from 1 to 2, i.e., $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$. I_1 and I_2 are current magnitudes in current elements 1 and 2 respectively. The constants are $k = 1/4\pi\epsilon_0$ ($\epsilon_0 =$ permittivity of free space) and $c =$ speed of light.

A study of Ampère's Law reveals that successive current elements with current in the same direction repel each. This fact was illustrated by Peter Graneau's experiments of exploding wires [6] [7] by conducting huge currents through them. Parallel current elements with current in the same direction attract each other.

Eq. (9) is an instantaneous action at a distance formula. It does not need a term to describe contact action that would involve retardation of the field between the current elements. It is postulated the electric field of a moving charge has already pervaded all the space of its inertial frame. Also, since SRT effects involve only instantaneous velocity and not acceleration, no term is needed for acceleration of the moving charges. Only as acceleration affects the relative velocity is the magnetic force affected.

Other magnetic force laws between current elements are discussed briefly below.

Grassmann's force law

Grassmann's force law is the combination of part of the Lorentz force law and the Biot-Sarvart law [8]. The part of the Lorentz force law is as follows:

$$\mathbf{f}_{12} = (k/c)I_1 d\mathbf{s}_1 \times d\mathbf{B}_2 \quad (10)$$

The Biot-Sarvart law:

$$d\mathbf{B}_2 = I_2 d\mathbf{s}_2 \times \hat{\mathbf{r}}_{12} / (cr_{12}^3) \quad (11)$$

Combining (10) and (11) results in the Grassmann's force law:

$$\mathbf{f}_{12} = \frac{kI_1I_2}{c^2 r_{12}^2} [d\mathbf{s}_2 (d\mathbf{s}_1 \bullet \hat{\mathbf{r}}_{12}) - \hat{\mathbf{r}}_{12} (d\mathbf{s}_1 \bullet d\mathbf{s}_2)] \quad (12)$$

Eq. (12) with an acceleration term in it is the currently used, *classical force formula*. It works fairly well and gives the same results as Eq. (9) when integrated around a closed loop. One can see from (12) that the $d\mathbf{s}_2$ vector is not necessarily in line with $\hat{\mathbf{r}}_{12}$, the vector joining the two current elements. Therefore, this law violates Newton's third law. This fact alone severely discredits this law and is physically and mathematically unacceptable. Eq. (9) does not violate Newton's third law.

New Gaussian force Equation

Domina Eberle Spencer [4] developed an equation to express the force between relatively moving charges to replace the Gauss Eq. (8). When that equation is applied to current elements the resulting equation is:

$$d^2 \mathbf{F}_G = \frac{1}{8\pi\epsilon_0 c^2} \left[\begin{array}{l} \frac{I_1 I_2}{r^2} ((d\mathbf{s}_1 \cdot \mathbf{a}_r) d\mathbf{s}_2 + (d\mathbf{s}_2 \cdot \mathbf{a}_r) d\mathbf{s}_1 - 2(d\mathbf{s}_1 \cdot d\mathbf{s}_2) \mathbf{a}_r) \\ + \frac{\left(\frac{dI_1}{dt}\right) I_2}{cr} ((d\mathbf{s}_2 \cdot \mathbf{a}_r) d\mathbf{s}_1 - (d\mathbf{s}_1 \cdot d\mathbf{s}_2) \mathbf{a}_r) \end{array} \right] \quad (13)$$

Spencer states this equation satisfies many experiments and is most viable of all the magnetic force equations for current elements. She also states that it satisfies experiments and is most viable of all magnetic force equations for current elements. She also states that it satisfies the repulsion attributes of successive current elements as observed by Peter Graneau. Doing all that, the equation deserves much credit. However, by noting that the ds element vectors in Eq. (13) may point in directions not in line with the position vector between the two current elements, one can conclude that it suffers the same defect as the Grassmann's force law as having violated Newton's third law. She based the derivation of this equations on a couple of Gauss's Criteria [4]. This author's approach to acceleration of moving charges is presented below in the Section on wave propagation.

Retarded force law

This law, based on retarded action at a distance and Coulomb's Law, was derived by Jan Olof Jonson [9].

$$d^2 \mathbf{F}_{12} = \frac{(\mu_0 I_1 I_2 \cos\theta \cos\phi ds_1 ds_2) \mathbf{a}_r}{4\pi r_{12}^2} \quad (14)$$

Jonson claims this force equation has the dominant magnetic force effect and SRT has negligible effect. This equation has at least two problems: One is that SRT does not support action at a distance for steady currents. The next is that this author took the integral of Eq. (14) to calculate the inductance this law presented for a one-turn solenoid (radius = 10 in.) with a measured inductance of 2.71 micro henries. The computer-calculated-inductance using Eq. (14) was 1.76 micro henries. The computer-calculated-inductance using the integral of Ampere's Law (8) was 2.71 micro henries.

Application of SRT Force Law to Wave Propagation

The SRT Force Law Eq. (7) appears to be a 'natural' for radio wave propagation: Taking the q_2 out of Eq. (7) we have an equation for the e-field at a distance \mathbf{r}_1 .

$$\mathbf{e}_1 = -\frac{k\sigma_1 ds_1 v^2}{r^3 c^2} \mathbf{r}_1 (1-1.5\cos^2\theta) \quad (15)$$

While this Eq. (7) describes a force between a charge and a current element, it can also describe an electric field \mathbf{e}_1 at a distance from the current element. An antenna is a wire that carries current and emits radiation. Therefore, implicit in Eq. (15) is radio wave propagation. By varying the velocity of the electrons (varying current flow in the wire or antenna) a varying electric field ("magnetic" field) should appear outside the antenna.

And this should be the varying electric field that propagates throughout space at the speed of light. The following is a possible derivation of the propagation formula. It assumes a transmitting antenna being a vertical wire protruding from the earth's surface.

The line charge density σ_l is related to the current I_l in the antenna and the electron current velocity v by:

$$\sigma_l ds_1 = \frac{I_l ds_1}{v} \quad \text{and} \quad v = \frac{I_l}{\sigma_l} \quad (16)$$

The current can be made to vary sinusoid:

$$I_l = I_{l\max} \sin(\omega t) \quad (17)$$

Insert time retardation due to wave propagation traveling as speed c :

$$t_R = \frac{r}{c} \quad (18)$$

Substitute (16), (17), and (18) into (15), replace ds_s by L (the antenna length) and we have an equation for the varying electric field at distance r from the antenna:

$$\mathbf{e}_1(t) = -\frac{kL}{\sigma_s r_1^3} \frac{I_{l\max}^2 \sin^2[\omega(t - r/c)]}{c^2} \mathbf{r}_1 (1 - 1.5 \cos^2 \theta) \quad (19)$$

This equation illustrates the basic relativistic nature of radio wave propagation and field patterns. This is due to length contraction of the space surrounding the basic electron charge when it's movement is seen by a non-moving point. This equation has not been tested nor fully developed by the author and should not be considered as exact. However, based on successful experiments with Ampère's Law, it should have some validity. Also, it is an example of retarded action at a distance versus instantaneous action at a distance.

Experiment With Ampère's Law and the Current Element

The author performed an experiment using the integral of Ampère's Law to calculate the energy stored between current elements and in one-turn coils of various shapes, flat in one plane and also in three dimensions. Using a capacitor of known value, applying a resonant voltage, measuring the frequency and calculating its inductance, the measured value of the one-turn inductor was determined. The computer calculated inductance using the integral of Ampère's Law was then compared to the measured inductance. It was discovered that if the current element was of a certain length, a match occurred with the measured value equaling the calculated value. Varying the current element length, ds , above or below the desired length caused the computed value to be above or below the measured value. The main factor that the current element length depended on was that its self-inductance value be zero. Varying the wire diameter size changed the length ds required required for a match. Ampère's law can be successfully applied to just a current element by subdividing the current element into smaller current elements. Doing this, it was discovered that the current element had zero self-inductance. This experiment from the author's perspective verified Ampère's Law that was derived from SRT. The author presented a paper on this experiment at the NPA conference in Tulsa ('06) with the title: "Experiment with Ampère's Law and the Current Element".

Discussion

The author has shown that the total force between relatively moving charges is entirely due to the transformed electric field of the moving charge. The “magnetic field” has been eliminated. Eq. (1) along with SRT’s velocity transformation formulas may be used to describe forces relating to orbiting electrons at nuclear distances and velocities near the speed of light.

The author calls Eq.(1) the “cardinal” law of electrodynamics as sought for by Maxwell. However, it has been criticized as not having a term in it to describe a force due to acceleration so it cannot be the “cardinal law”. The answer to this is that acceleration only effects Eq. (1) as it changes its velocity. This force in SRT is only velocity dependent and not acceleration dependent. Because it has been shown how the old Ampère’s Law is derived from SRT, the Ampère’s Law should be the standard for physics instead of the Grassman’s law as generally accepted now. Both laws work for closed loop applications, but the Ampère’s Law works for all applications and obeys Newton’s third law while the Grassmann’s law does not. Peter Graneau [6] has used the Ampère’s Law successfully in experiments. He claims, however, the law is not of SRT origin, but of Newtonian origin and he therefore questions the validity of SRT. This author has shown the law is of SRT origin.

Other authors [4] [9] of electrodynamics formulas believe that retardation or propagation time of the force is an important factor that must be taken into account. However, the present author believes that retardation is not an important factor. 1. Because the e-field of a moving charge pervades all the space of its inertial frame and its interaction with another charge is instantaneous. 2. Acceleration of a charge would change its velocity and that would represent a change in its field that would propagate out at the speed of light. This is how radio wave transmission is accomplished.

With regard to the experiment with single turn inductors, it was discovered that the current element length had to be a certain length for the inductance measured with resonance to match the inductance calculated from the integral of Ampère’s Law. This length was found to depend on the wire size, mainly diameter. This dependency has not been entirely investigated in the experiment and could be a project for further work, like in the Bureau of Standards. An idea that the experiment points to is (as in arc welding) that when the current element is a size less than required by Ampère’s Law, excess energy may be liberated when the conducting medium is a plasma, then the energy released in that process may be greater than that expended from the original power source. That is because the moving electrons are attracted to the positive ions in the lattice with greater force than they normally are. In the movement toward each other, they may release excess energy. Research probably has already been performed in this area. Also, Ampère’s Law may be used to calculate inductances in microcircuits.

Dr. Wolfgang Rindler wrote in his book [1]: “In a laboratory current of a few amperes, v (electron velocity) is only about one millimeter per second. As C. W. Sherwin has said that it is hard to believe that this magnetic force, which has to suffer a denominator c^2 , is the “work force” of electricity, responsible for the operations of motors and generators.

And again considering that this force arises from transforming a purely electric field to another frame having a small velocity relative to the first. A. P. French has remarked: who says that relativity is important only for velocities comparable to that of light? The reason is that an ordinary current moves a very big charge: there are something like 10^{23} free electrons per cubic centimeter of wire. Their electric force, if it were not neutralized, would be enormous—of the order of two million tons of weight on an equal cubic centimeter at a distance of 10 km.”

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