Internal Momentum Changes Manifesting as Clock Rate Changes in GPS Clocks.

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Is the consideration of mass of a body as the sole interactant correct?
It is important to take note that whereas the interactants in the Newtonian theory have to be vectors such as momenta and forces, and whereas Newton in *Principia* definition III recognizes that a body consists of ***vis*** *insita* or the innate **force** of matter (i.e., internal momentum), which has to be a vector or vector-like, hitherto neither Newtonian mechanics nor any other theory has figured out how this 'innate force' interacts with the externally applied momentum or force, or with gravitational constraints.

Further, whereas in definition II Newton recognizes that momentum consists of mass and velocity **conjointly** as two internal aspects of a single entity as a whole, when it concerns the interaction of external momentum with the internal momentum of a body, the reference to the velocity aspect of this **conjoint entity** has been ignored as if it did not exist, and the mass aspect has been considered as if it were the only aspect that matters in the interaction. To make the point in another way, there is the celebrated equation that the rest energy E of a body is given by $E = mc^2$. Newton's *vis insita* or *vis inertiae* (which was abbreviated to 'inertia') then turns out to be $E/c = m \times c$ where c is the velocity of atomic vibrations. Nevertheless, it appears that no attempt has been made to consider internal momentum as the entity that enters into interactions, but the tradition has continued to blindly consider the interactant as mass only.

The difference between Classical Vectors and the Omni-directional Vectors.
One of the difficulties that would have prompted the continuation of the above approach to be adopted in regard to the interaction of the two vectors is the fact that this internal momentum is the sum total of the momentum of random vibrations of atoms of the body, and therefore this sum total does not take the form of an ascertainable resultant vector with a definite magnitude and a direction. It appears that the problem of such a resultant vector of unknown magnitude and direction has therefore been circumvented by sweeping the velocity aspect of internal momentum under the carpet. This seems to be why in Newtonian mechanics and SRT, a body is considered as a point mass instead of considering it to consist of internal momentum. It is to be noted that it is this misrepresentation that a body consists of mass only (instead of internal momentum) that has obstructed the possibility of fathoming out the interaction between internal momentum of a body and the external momentum applied to the body to set it in motion. What this paper sets out to do is to discern the way this interaction happens. And this will be done by investigating how the time change in an atomic clock in a GPS satellite occurs in relationship to an atomic clock located on Earth.

A problem similar to the unknowable magnitude and direction of the resultant momentum of the random vibrations of atoms is the resultant momentum of the randomly moving molecules of a gas. And we know this problem had to be solved by a statistical approach

(of the root mean square speed) rather than looking for the empirical magnitude and direction of the resultant momentum of the molecules.

Therefore, it indicates that if we are to find a solution to this problem of how internal momentum interacts with external constraints, we need to look at it beyond the context of the division of physical quantities into vectors and scalars from the criterion of the possession of **a specific empirical direction**. We must make a distinction between 'speed' and 'root mean square speed' and take cognizance of the fact that whilst the first is a scalar, the second is an **omni-directional vector**. (And it must also be noted that an omni-directional vector can acquire the status of a universal constant such as the case of the magnitude of the root mean square velocity of a gas at the temperature of 0°K).

Maxwell's Insight towards how to overcome instances where Classical Mechanics Fail.

In the context of the cases of random motions of entities where the resultant vector cannot be ascertained, Maxwell wrote: "But even when the phenomena we are studying have not yet been explained dynamically (i.e. in terms of Newtonian mechanics), we are still able to make great use of the principle of the conservation of energy as a guide to our researches" (p. 71). Maxwell also wrote: (In comparison to the Newtonian method of action between bodies) ….:"The investigation of the mode in which the minute particles of bodies act on each other is rendered more difficult from the fact that both the bodies we consider and their distances are so small that we cannot perceive or measure them, and we are therefore unable to observe their motions as we do those of the planets, or of electrified or magnetized bodies. Hence the investigations of molecular science have proceeded for the most part by the method of hypothesis, **and comparison of the results of the hypothesis with the observed facts**"(p. 121-2).

The Hypothesis Involved in this Paper.

The hypothesis involved in this paper is that in the interactions between internal momentum of a body and external momentum, the sharp distinction between vectors and scalars disappear. Vectors and scalars commensurate with one another in so far as they are made qualitatively equivalent to one another **dimensionally**. Further in this paper, it is considered that a) the root mean square velocity of the atomic vibrations of a body remains at constant magnitude c independent of its state of motion or position, and b) that this velocity is potentially omni-directional and responds to external constraints as if it had acquired the direction perpendicular to the direction of the external constraint acting on the body. The above two premises are true for all states of the body, in regard to its position in a gravitational field as well as state of motion irrespective of whether the body is at rest or in motion with respect to its instantaneous position in the gravitational field.

The **validity of the above hypothesis** is borne out by the fact that motion of particles conform to the following well established equation

$$E^2/c^2 = E_0^2/c^2 + p^2$$

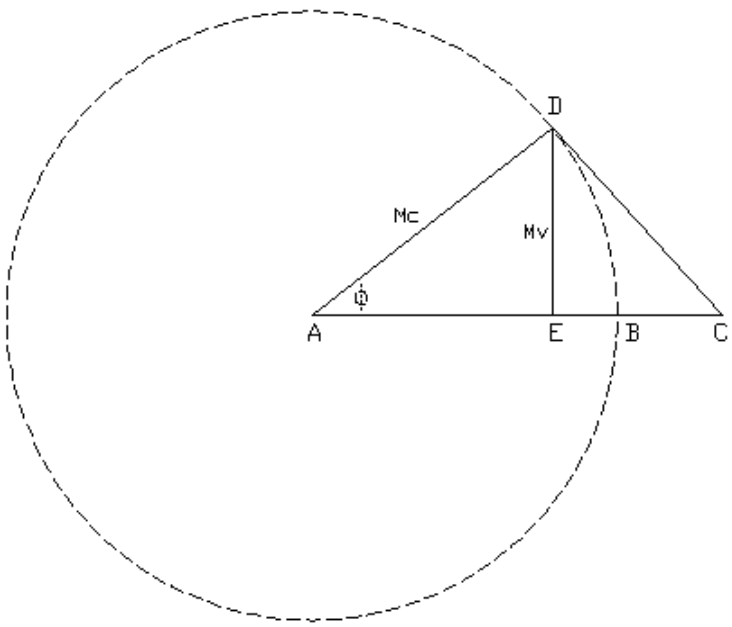


Fig 3

AC = E/c, AD = E_0/c, and DC = p

It will be found that the above equation is tenable By virtue of the fact that a vector (such as p) and a scalar (such as E_0) can commensurate with one another on the basis of converting one to the dimensional quality of the other. It shows that E_0/c is of the same dimensional quality as p, otherwise the equation would not have been tenable. Also the structure of the equation shows that E_0/c is an omni-directional vector such that no matter what the empirical direction of p is, E_0/c responds to it by acquiring a direction perpendicular to that of p.

It is therefore to be noted that the basic tenets of the hypothesis have been confirmed by observed facts as borne out by the validity of the equation. Further, the '**algorithm of motion**' (as shown in the above figure) that is developed in this paper has been discerned on the basis of the above equation.

The interactions between internal momentum and external momentum can interchangeably, and alternatively be considered as interactions that occur dimensionally in terms of energy, momentum, or even mass. This is possible because, with every change of state of the system, the system adjusts the omni-directional velocity of its internal momentum to maintain it at the constant value equal to c. By virtue of this re-adjustment of the velocity of the system to c, it means the same thing whether we refer to the change of state of the system in terms of energy change, momentum change or the change of mass. (Note that these changes occur due to transfers between parts of the system, while the energy, momentum and mass of the system as a whole remains conserved).

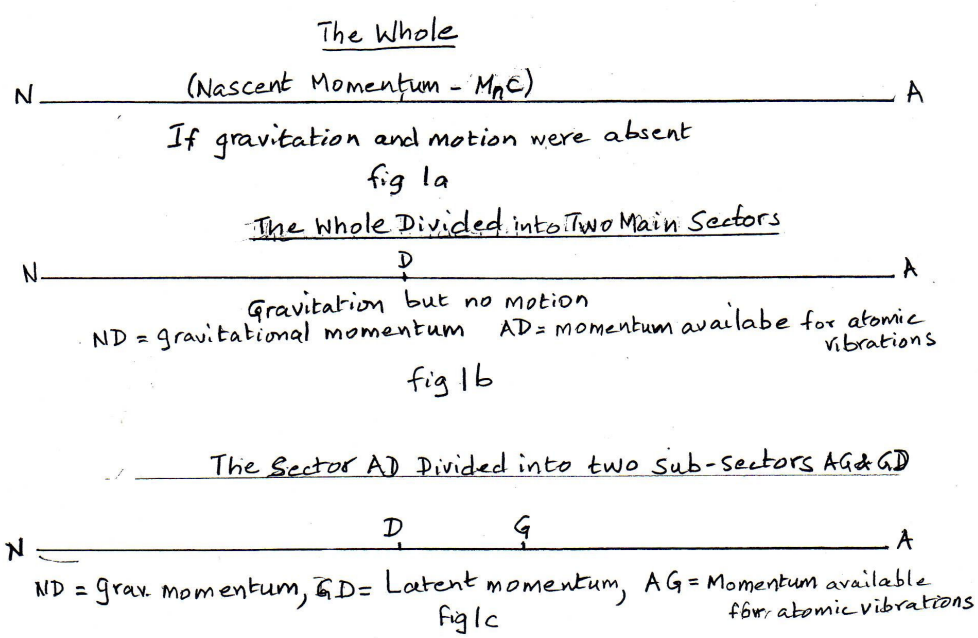
Re-writing the equation we have: $E^2 = E_0^2 + p^2c^2$

When we divide the equation by c^2 it reveals to us that energy/c has the same dimensionality as momentum, and when we divide it by c^4 it reveals that momentum/c has the same dimensionality as mass (i.e. E/c^2). It shows that the same equation can be transformed to be written in terms of energy, momentum or mass. Accordingly in this paper the terms 'mass', 'velocity', 'momentum', 'energy' etc., are assigned to quantities in so far as a given quantity has a dimension that corresponds to a given one of these terms.

Physics and Mathematics of the Paper.

The theory presented in this paper has two aspects (1) physics and (2) mathematics which are grounded in the two most fundamental of the laws of nature. Physics is based on the principle of conservation of energy, and mathematics is based on the law of proportions.

Physics of the theory concerns the physical basis that underlies relativistic phenomena which Maxwell outlined as follows: "In fact the ***special work*** which lies before the physical inquirer in the present state of science is the determination of the quantity of energy which **enters or leaves** a material system during the **passage of the system** from its standard state to any other definite state"(p. 74). From what we have discussed above, it would mean just the same thing if we were to consider the "quantity of energy"/c (i.e. momentum) which enters and leaves a material system. Hence the present theory is developed on the basis of the above prescription of how to account for the changes of momentum when a body changes its state in terms of its position and velocity with respect to another body with which it is in two-body gravitation.



The theory considers the internal momentum of a body as a system consisting of three sectors. The internal momentum of the system as a whole is represented by NA (fig.1a). The system consists of two main sectors ND and AD (fig. 1b). The sector AD is in turn divided into two sub-sectors AG and GD (fig. 1c). If there were to be no motion, then the two sub-sectors AG and GD merge and only the two main sectors ND and AD remain. If the gravitational influence disappears for some reason then the two main sectors ND and AD also become merged, and the whole of internal momentum will be dedicated to the atomic vibration, hence in this ideal situation of no gravitation and no motion the whole of NA (which is referred to as "Nascent momentum" – M_nc) acts as clock momentum. When the body is in motion in a gravitational field we find that ND = Gravitational momentum (i.e. gravitational energy/c), DG = the latent momentum (i.e. the fraction of internal momentum that becomes latent when the system is in motion), AG = the clock momentum (the internal momentum that remains available for atomic vibrations on which the atomic frequency depends).

When the system changes its state due to a change of position in a gravitational field or due to change of state of motion, NA the **total momentum of the system remains conserved**, while fractions of that momentum pass from one part to the other within the system as enforced by the **law of proportions** in relation to the gravitational and the motive constraints encountered by the system (NA).

The mathematics of the theory concerns the manner in which the law of proportions enforces the transfer of momentum between the two main sectors, and between the two sub-sectors by way of formation of two geometric algorithms whose function it is to determine various fractions of momentum to be transferred or induced, in the form of trigonometric proportions of the total momentum. If the proportions were to be determined arithmetically, relationships within the system would have been linear but they are found to be otherwise. The fact that the relationships within the system are non-linear reveals that this non-linearity arises from the trigonometric determination of these fractions by the formation of these algorithms. The algorithm of gravitation is shown fig 2. (And the algorithm of motion is shown in fig. 3)

The Algorithm of Gravitation:

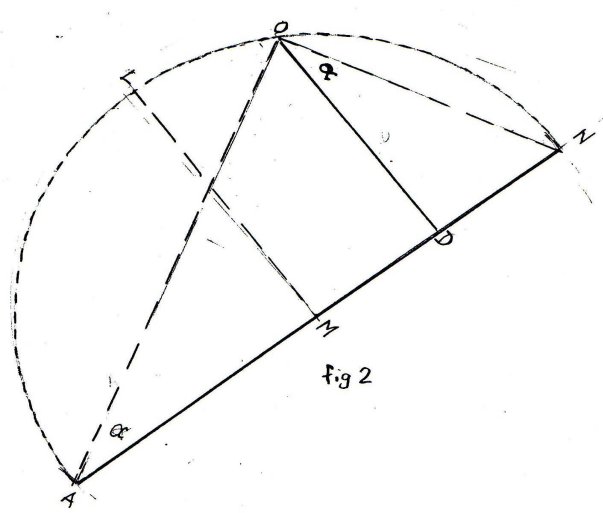


fig 2

This algorithm considers the system to be at rest at a distance R from the centre of the gravitational field. Let the total momentum be $M_n c$. (M_n denotes 'nascent mass' meaning the **true mass** of the body that corresponds to the whole of NA). The gravitational constraint OD is the square root of the gravitational potential at the position R times the apparent mass of the system. (This is the same as what would be the momentum of orbit of the system, if it were to orbit at a distance R from the centre of the gravitational field). The point O is always located on the arc LN where ALN is a semi-circle of which L is it mid-point. (Note: LM is the gravitational potential at the Schwarzschild radius). The law of proportions is enforced in the following manner. OD is placed perpendicular to NA, and the point D is determined such that, , $AD = NA \cos^2\alpha$, $ND = NA \sin^2\alpha$ and $OD = NA\sin\alpha\cos\alpha$. When the division of NA into the two sectors has taken place, the velocity component of AD still retains the value c (as discussed above). Therefore $AD = (M_n\cos^2\alpha).c$. Hence $\alpha = \tan^{-1} (GM/Rc^2)^{1/2}$.

This would make it **appear** as if the mass of the system has reduced to $M_R = M_n\cos^2\alpha$. M_R is only the **apparent mass** of the system at the position R in the gravitational field. Hence the **rest mass** of a body is its **apparent mass** when at a given position R in a gravitational field.

$AD = M_R c$ and $OD = M_R.(GM/R)^{1/2}$. Then $\alpha = \tan^{-1} OD/AD = (GM/Rc^2)^{1/2}$.

$ND = AD.\tan^2\alpha. = M_R c.\ GM/Rc^2 = M_R(GM/Rc)$.

This algorithm just developed above, can be applied to solve problems of two-body gravitation such as the time changes of an atomic clock within a GPS satellite.

Clock Rate Increase Due to altitude increase in a GPS Clock.
Let us consider that the atomic clock in the GPS satellite to be at rest (momentarily) at the orbit radius R_o = 26.600 km (just before it is imparted with tangential momentum to set it in orbit).

OD_o at $R_o = M_{Ro}(GM/Ro)$, $\tan\alpha_o = (GM/R_oc^2)^{1/2}$.

Clock momentum AD_o at $R_o = M_n\cos^2\alpha_o.c = M_nc/(1+\tan^2\alpha_o) = M_nc/(1 + GM/R_oc^2)$

When the atomic clock is at the observatory at Earth's equator, at radius R_e = 6378.km

OD_e at the equator $R_e = M_{Re}\ GM/R_e$), $\tan\alpha_e = (GM/R_ec^2)^{1/2}$.

Clock momentum AD_e at $R_e = M_n\cos^2\alpha_e.c = M_nc/(1+\tan^2\alpha_e) = M_nc/(1 + GM/R_ec^2)$

Gain in clock momentum Δp at $R_o = AD_o - AD_e$

$\Delta p = M_nc[\{1/(1 + GM/R_oc^2)\} - \{1/(1 + GM/R_ec^2)\}]$ ----------(1)

(Note: The above is the accurate formula for Δp)

This equation simplifies into $\Delta p = M_n c(R_o - R_e)/ [(R_o R_e c^2/GM + (R_o + R_e) + GM/c^2]$

Since $R_o R_e c^2/GM >> [(R_o + R_e) + GM/c^2]$

Gain in clock momentum $\Delta p \approx M_n c.GM(R_o - R_e)/(R_o R_e c^2)$ ----------(2)

(Note: Equation (2) is only an approximate formula for Δp and therefore there will bound to be long term corrections to be made when Δp is calculated using (2)).

Gain in clock momentum as a fraction of total momentum $M_n c$ =

$\Delta p/M_n c = GM(R_o - R_e)/(R_o R_e c^2)$

Consequently the atomic frequency of the clock in the satellite becomes greater by the fraction $\Delta p/M_n c$

Time is determined by the number of cycles of atomic vibrations. Since the number of cycles increase with the increase of momentum by Δp, the gain in time Δt is in one to one proportionality to the fractional gain $\Delta p/M_n c$ in clock momentum.

$\Delta t = \Delta p/M_n c$ of a sec per sec.

Therefore the time increase per day = $86400 x GM(R_o - R_e)/(R_o R_e c^2)$

(R_o = 26,600 km, R_e = 6378 km, GM = 3.986×10^5 km^3/sec^2, c = 2.99792×10^5 km/sec)

Gain in time due to increase of altitude = $4.567398579 \times 10^{-5}$ sec per day

= 45,674 nanoseconds per day.

The above result is very close to the observed result of 45,900 ns/day as recorded in Van Flandern's paper. Hence this result will validate the algorithm of gravitation as indicated in fig 2.

THE ALGORITHM OF MOTION:

Hitherto, in Newtonian mechanics and in SRT, motions of bodies in general have been considered as occurring in spaces devoid of a gravitational field, as inertial motions. On the other hand, Etovos, in his well know experiment demonstrated that the 'gravitational mass' is equal to the 'inertial mass' of a body. Ref. fig.2 let us identify, the extensive component of the momentum (i.e. mass) represented by the sector AD as 'gravitational mass' (See Appendix below for the reason for the use of the terms 'extensive' and 'intensive' component respectively for mass and velocity and its explanation).

Since we have already identified that the intensive component of momentum of the sector AD (i.e. the velocity of AD) adjusts itself to retain the value c, AD = $(M_n\cos^2\alpha).c$. Hence the apparent mass M_R of the body at R is equal to $M_n\cos^2\alpha$. It is this apparent mass that has been identified as the 'gravitational mass'. Therefore what the equivalence of 'gravitational mass' and 'inertial mass' means is that the momentum to be applied to set a body in motion at rest at a distance R in a gravitational field has to be proportional to its 'apparent mass' M_R at R, which is its clock momentum AD divided by c.

Thus if we are to have a unified theory, where we incorporate the inevitable fact that **a body without exception will be located in a gravitational field** and as a consequence it has always got to be considered to be potentially or actually at rest in a gravitational field as the starting point of an investigation, then we need to have the algorithm of gravitation (fig.2) as the basis, and that the sector AD in it as the effective internal momentum of the body to which momentum p is applied externally to set it in motion. (Note: As we discussed, ref. fig. 1 total momentum of the body is NA, which is divided into two main sectors ND and AD. ND turns out to be the self-acting gravitational momentum exerting on the body. The momentum that remains at this stage for atomic vibrations is AD. It is AD/c = M_R which is the apparent mass of the body, that comes to be considered as the 'gravitational mass').

The Algorithm of Motion

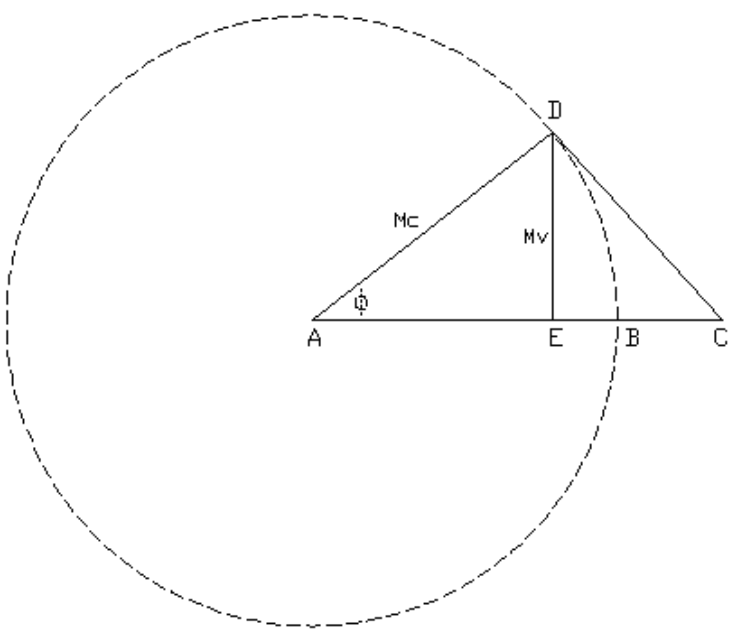


Fig 3

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As indicated above the outline of the algorithm has been discerned logically from the structure suggested by the following equation

$E^2/c^2 = E_0^2/c^2 + p^2$

AC = E/c, AD = E_0/c, and DC = p

Fig. 3 indicates the final form of the algorithm of motion. But it can be discerned that there is a prehistory to it from which it evolves in its formation.

The Precursive Process of the formation of the Algorithm of Motion.
Note that the semi-circular part AON in fig. 3a.is the algorithm gravitation, and AD in it is the internal momentum of the body that determines its atomic frequency when the body is at rest at the distance R in gravitational field. And it is to this AD that the momentum p is applied to set the body in motion. As discussed momentum AD is omni-directional.

No matter whatever the direction in which the momentum p = DC' is applied, the omni-directional momentum AD of the algorithm of gravitation aligns itself perpendicular to DC'. Then the two quantities of momenta rotate about D through angle ϕ, where $\phi = \tan^{-1}$ DC'/A'D and attain the positions AD and DC. (It may be recalled that Minkowski and Einstein both have referred to the 'rotation of axes', without explaining how it happens or for what reason this happens).

The Combined Algorithm of Gravitation and Motion
(Note in the diagram below A' is at the end of the horizontal line starting at D).

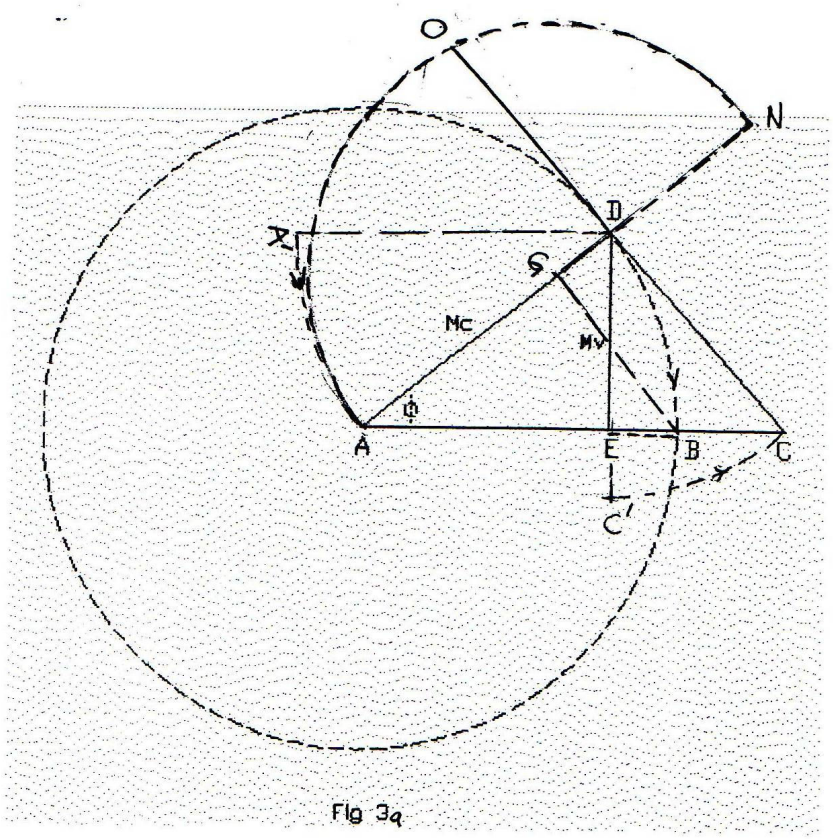

Fig 3a

What appears as a 'rotation' of the external momentum p is in fact, that DC' renders EC' to be a latent component and makes DE to be the effective momentum of motion, at the same time by this 'rotation' it **induces** EC (centrifugal momentum).In the case of internal momentum there is a 'double rotation' back to back. First A'D rotates about D to AD, and this immediately followed by a rotation of AD about A to AB (AB = AD = AD'). And in the course of this latter rotation AB renders a part of it, namely EB to be latent and causes the effective internal momentum (i.e. clock momentum) to be AE, and at the same time **induces** ED to be equal and opposite to DE.

ED being equal and opposite to DE, forms the dynamic equilibrium as required by Newton's third law. (It may be noted that although Newton stated the third law as a **law of motion,** how this law operates in the motion of bodies has hitherto not been demonstrated. It is seen only as law that applies in statics. Although it seems obvious as a law, it should apply to all cases of motions of bodies, how it applies to all cases of motions of bodies has not been discerned).

Ref. fig. 3a, consider BG as perpendicular dropped from B to AD. It will be seen that EB = GD. From the gravitational algorithm (fig. 2), ND is the gravitational momentum and AD is the clock momentum if the body were at rest at R. When the body is in motion G divides the clock momentum and renders GD to be latent. The momentum that is left over for the activation of atomic vibrations is AG =AE = ADcosϕ.

From the above algorithm, considering GD to have been rendered latent and therefore considering the quantity of internal momentum that remains for activation of atomic vibrations as AG, we can determine the Clock rate decrease of a GPS clock due to orbital motion.

Clock rate decrease due to orbital motion of the GPS Clock.
If the clock were at rest at R_o the atomic frequency would have been proportional to AD. When the clock is in orbit the atomic frequency is proportional to AE = ADcosϕ. Therefore the atomic frequency would slow down when in orbit by the fraction AD(1-cosϕ)/AD = 1-cosϕ

Since in an atomic clock the clock rate is directly connected to the frequency, therefore when the frequency decreases the clock rate diminishes by the same proportion.

From the algorithm it will be seen that
sinϕ = orbital velocity/velocity of atomic vibrations = $(GM/R_o)^{1/2}/c$

Therefore the fraction $\Delta t'$ by which the clock rate diminishes is given by:

$\Delta t' = 1 - \cos\phi = 1 - (1 - GM/R_o c^2)^{1/2}$ per sec.

Knowing the values of GM, R_o and c, the clock rate diminution per day due to orbit

$= [1 - (1 - GM/R_o c^2)^{1/2}] \times 86400 = = - 7.202766 \times 10^{-6}$ secs

The combined Clock Rate change due to change of Positon and Motion of the GPS Clock.

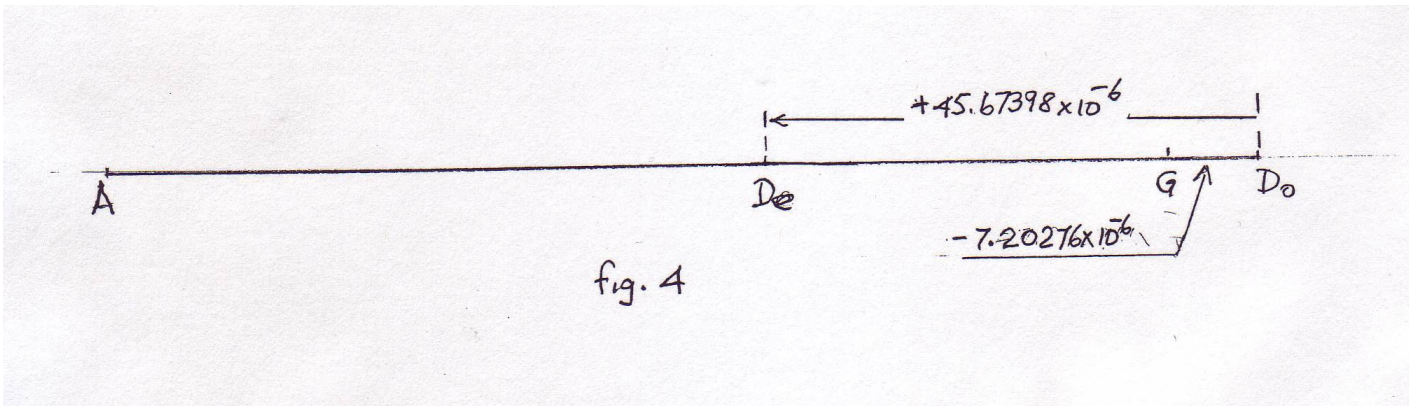

fig. 4

When the clock is on earth, the clock momentum is AD_e, When the clock is at the altitude R_o and when it is considered to be stationary, the clock momentum increases to AD_o. The consequent increase in clock rate $D_oD_e = 45.67398579 \times 10^{-6}$ sec per day. When the clock is in orbit, the clock momentum decreases from AD_o to D_oG. Therefore the net increase in clock rate when the clock is in orbit is $D_oG = D_oD_e - D_oG$ ($D_oD_e = 45.67398579 \times 10^{-6}$ sec per day, $D_oG = 7.202766 \times 10^{-6}$ sec/sec).

The net increase in clock rate when the clock is in orbit = 38.471219×10^{-6} sec/day.

If the clock were at rest at R_o the atomic frequency would have been proportional to AD. When the clock is in orbit the atomic frequency is proportional to $AE = AD\cos\phi$. If there were to be a stationary clock at R_o its time unit t would be proportional to AD which corresponds to a given number of cycles completed. When the clock is in motion the time taken to complete the same number of cycles $t' = t.AD/AE = t.\sec\phi$.

This result ($t' = t \sec\phi$) can be verified with the known result of the delay in decay time of a fast moving muon.

The Delay of Decay Time of Fast moving Muons.

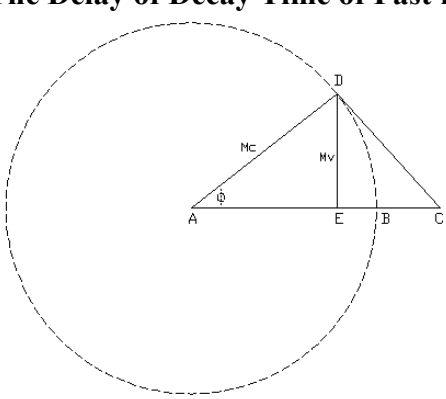

Fig 3

In a laboratory on Earth, the disintegration of a muon occurs in a time $t = 2.2 \times 10^{-6}$ sec. By the estimated distances traveled by muons in cosmic rays moving at velocities near 0.9c the time $t' = t.\sec\phi$ taken for these muons to disintegrate can be calculated upon knowing that $v = 0.9c$ and $t = 2.2 \times 10^{-6}$ sec and $\sin\phi = v/c$.

$$t' = t.\sec\phi = t(1-v^2/c^2)^{-1/2}$$

$$= 5.047 \times 10^{-6} \text{ sec.}$$

This is the time that corresponds to the observed displacement of a muon moving at velocity 0.9c in a cosmic ray before disintegration as recorded by Feynman.

APPENDIX:
Extensive and Intensive Components of Energy.
Both Newton and Maxwell have pointed out the necessity to develop a theory from a general point of view, rather than basing theory for a specific kind (such as electromagnetic, gravitational and the like).

Newton wrote: "To tell us that every Species of Things is endow'd with an occult specifick Quality (of Gravity, and of magnetick and electrick Attractions, and of Fermentations) by which it acts and produces manifest Effects, is to tell us nothing: But to derive two or three **general Principles of Motion** from Phaenomena, and afterwards to tell us how the **Properties and Actions of *all corporeal Things*** follow from those manifest Principles would be **a very great step** in Philosophy, though the Causes of those Principles were not yet discover'd: And therefore I scruple not to propose the Principles of Motion above mention'd, they being of **very general Extent**, and leave their Causes to be found out".(Query 31, p. 53).

Maxwell, being the progenitor of the theory of relativity, has had insights in regard to the direction to be taken in the development of a new theory to account for the **differences** the electromagnetic phenomena that he discerned, had with classical mechanics; and some of these ideas and insights are to be found in his book "Matter and Motion". Despite the fact that his findings **specifically** pertained to electromagnetic phenomena, his **main insight** is that in order to arrive at an **indisputable theory** which would account for the observed deviation from classical mechanics, all the specific forms of energy (viz., electromagnetic, thermal, gravitational, mechanical, chemical, etc.,) are to be considered in two **general categories** as generic kinetic energy and generic potential energy according as whether these are in **motion** or in **configurations** of material systems. Maxwell provides the rationale for the necessity of adaptation of this approach of considering specific types of energy under the general categories of potential energy and kinetic energy. He says: "The success of this approach depends on the generality of the hypothesis we begin with. If our hypothesis is the **extremely general one** that the phenomena to be investigated depend on the **configuration and motion** of a material system, then if we are able to deduce any available results from such an hypothesis, we may safely apply them to the phenomena before us. …. If, on the other hand, we frame the hypothesis that the configuration, motion, or action of the material system is of a

certain definite kind, and if the results of this hypothesis agree with the phenomena, then, unless we can prove that no other hypothesis would account for the phenomena, we must still admit the possibility of our hypothesis being the wrong one. ….. It is therefore of greatest importance that we should be thoroughly acquainted with the **most general properties** of material systems, and it is for this reason that in this book I have rather dwelt on these general properties than entered on the more varied and interesting field of the special properties of particular forms of matter"(p.122).

Therefore, in this paper, not only that we consider all types of energy as generic energy of motion and generic potential energy, we take one step further, we consider generic energy of motion to exist as the product of extensive component – (mass in mechanical motions and gravitation or charge in electrical motions) and intensive component - celerity (velocity) where these two components vary conjugately against each other as we discuss below. The above recognition that heat energy interacts by way of having of two opposing and conjugately varying internal components, namely the 'intensive' and 'extensive' components has been known in thermodynamics. In fact these interactions of involving these internal components forms the basis of thermodynamics. These internal components take the form of volume (extensive component0 and pressure (intensive component) or entropy (extensive component) and temperature (intensive component). In Boyle's law the same quantity of energy change its state by way of conjugate variation of the extensive (volume) and intensive (pressure) components. In Dalton's law of partial pressures two quantities of energy add by having the extensive component invariant common factor and the intensive component of the two quantities of energy adds $VP_1 + VP_2 = V(P_1 + P_2)$. The converse of this theorem also holds.

In mechanics, what is analogous to Dalton's Law has been recognized as the theorem of addition of velocities where the extensive component (mass) is considered constant and the intensive components of the two quantities of momentum add.
$Mv_1 + Mv_2 = M(v_1 + v_2)$

However, the converse theorem which applies in the case of division of the total internal momentum NA into ND, ED and AE has not been recognized in physics.

$ND = GM/Rc^2 \times c$, $DE = M_R(1-\cos\phi) \times c$, $EA = M_R\cos\phi \times c$

$NA = ND + ED + EA = c\,(GM/Rc^2 + M_R(1-\cos\phi) + M_R\cos\phi) = c(GM/Rc^2 + M_R)$

$= M_n c$ (since $GM/Rc^2 = M_n\sin^2\alpha$ and $M_R = M_n\cos^2\alpha$) – see gravitational algorithm

It will be clear that the recognition of the above theorem is important to represent the **principle of constancy of the velocity of atomic vibrations** under all states of motion and position of a body.