A relativistic wave-particle based on Maxwell's equations: part II

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At PIRT 2006, a speculative concept of a photon-like solution to Maxwell's classical equations was presented [1] in which helically rotating solutions appeared to have properties that are typically associated with photons. Since that time, our understanding of these solutions has increased and the previous work has to be modified. This paper will present a digest of the latest results that enables us to clarify past work and review new experimental evidence for the theory.

There is already sufficient experimental evidence to show that single photons can have a measurable group velocity [2], measurable phase velocity [3] and can be localised [4]. Various Maxwellian models that provide a confined packet of classical energy have been discussed theoretically by several researchers [5,6] but these packets all rely on specific field profiles. Here the task is to find a packet that applies to *all* classical modes, is invariant to Lorentz transformations, and has a property that is recognisable as spin that increments in energy in appropriate units. At present the work is confined to beams in free space.

As a starting point, general solutions of Maxwell's equations are conveniently labelled as TE (Transverse Electric fields) or TM (Transverse Magnetic fields) solutions (Figure 1). Here **E, B** and the direction of propagation form a right handed set of vectors [7]. TE and TM modes have a group velocity $v_g < c$ with a phase velocity $= c^2/v_g > c$. It is therefore always possible to travel in a frame of reference moving with the group velocity of these electromagnetic waves. In such a frame of reference, and using light cone coordinates [8] , there are wave vectors k_{f} and k^r (associated with light on the forward and reverse branches of the light cone) which are equal and opposite : $k_{\mathrm{f}} = -k^r$ as is found in most resonators.

With these concepts it is found possible to invent a Lorentz invariant wave-packet with a definite frequency, definite duration, definite phase velocity $v_p > c$ and definite group velocity $v_g < c$:

$$F_z = E_z + \mathrm{i}\, cB_z = F_{zo} \exp[\mathrm{i}(k_o z - \omega_o t)] \cos[(\omega_o/c)(z - \underline{v}_g t)\, \delta].$$

Here δ is an arbitrary Lorentz invariant number that defines the phase-length of the packet and also defines the relative spread of the frequencies composing the wave-packet. This packet produces a Lorentz invariant envelope for the axial fields but fails to envelope properly the transverse fields $\mathbf{F_T} = \mathbf{E_T} + \mathrm{i}\, c\mathbf{B_T}$.

Different mechanisms are used to ensure that the transverse fields are localised. These mechanisms are called here *distributed spin-rotations*. They are localised rotations of the transverse fields of any mode and can be observed mathematically only in the vector formulation of Maxwell's equations. They provide helical modulation moving at the group velocity and form an enveloping packet for the transverse fields. Spin-rotations should not be confused with the helical phase fronts observed by other workers [9]. In principle distributed spin-rotations can have arbitrary frequencies. However to ensure that the packet enveloping the transverse fields has the same duration as the packet enveloping the axial fields, two fields with equal and oppposite spin rotations are required with magnitudes that are quantised to be proportional to $(2N+1)(\omega_o/c)$ where N is integer. Figure 1 shows the idea schematically (N = 1). The lateral extent of the packet can be controlled, not just by the classical field profile, but by additional imaginary distributed spin rotations. Thus distributed spin rotations in principle allow for a flexible range of packets that might either emerge from confined sources or be detected by compact detectors. The duration of this photon-like packet is also flexible (determined by the value $1/\delta$) but, when this duration is taken into account, the helical distributed spin rotations appear to contribute to a classical energy proportionally to (2N+1) (ω/c) H where N is some Lorentz invariant integer. At present the theory is unable to evaluate H.

The paper will end with a brief review of experiments that could support this theory as a model for a photon.

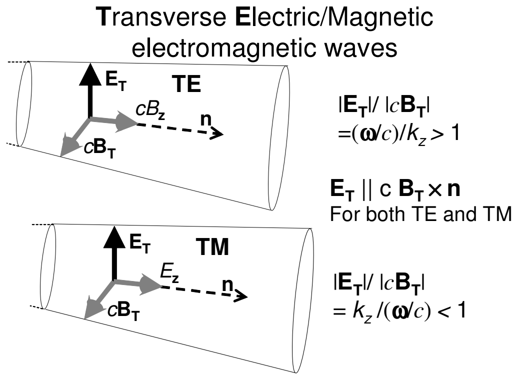

Figure 1. TE and TM waves
TE waves are 'driven' by cB_z
TM waves are 'driven' by cE_z
Phase velocity = $\omega/k = v_p > c$
Group velocity $v_g = c^2/v_p < c$
Can therefore always find a real frame of reference moving with the velocity of the electromagnetic waves.

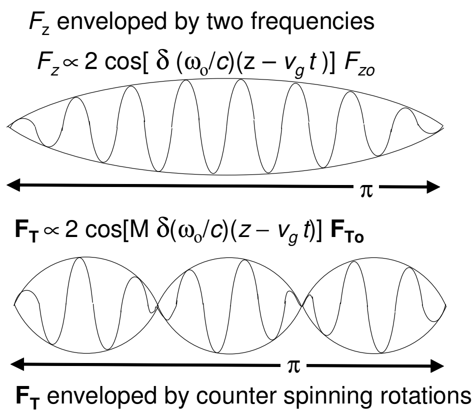

Figure 2. Lorentz Invariant Envelopes
Every modal field with axial fields $F_z = E_z + icB_z$ and transverse fields $\mathbf{F_T} = \mathbf{E_T} + ic\mathbf{B_T}$ can be enveloped about a central frequency ω_o with a Lorentz invariant parameter δ controlling the phase duration of the envelope. The transverse fields are enveloped by counter rotating spins. M=2N+1 is always an odd integer to ensure the axial and transverse fields to have the same duration of packet. The classical energy added by this spin is proportional to M (ω/c) H where H is some Lorentz invariant number.

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