

Most elementarily-intuitive, tentative approach to evaluate the bending of a light-ray passing a black hole.

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Assuming the hypothesis of a $1/r^2$ law of ether absorption-speed in the vicinity of massive celestial bodies and using an elementary, intuitive, calculus procedure, the bending of a light-ray passing near a black hole is evaluated. The theoretical results so obtained are, in some aspects, stunning because they suggest that some strong astronomic anomalies may, or should be observable by adequate means. And also because some accepted cosmologic fundamentals may become questionable. The so arisen problems are just suggested without any pretense to be rigid statements.

“The universe is wilder than we
 ordinarily dare to imagine”
 R.P.Kirschner, *The
 extravagant universe* [1].

I. The problem dealt-with in the present paper is to evaluate, by theoretical means, the bending of light rays when passing near a black hole. The chosen approach is elementarily intuitive. It is based on a hypothesis suggested by H.E.Ives in his last years [2], and more explicitly configured in “On this, till now, so shy universal ether” [3], scheduled to appear in Galilean Electrodynamics. In fact one assumes that a kind of “material” ether, everywhere present, exists under the form of a universal flow-field determined by local absorption in celestial bodies, the speed distribution being the classic one of a sink in a non compressible fluid.

The chosen solving method is a step by step evaluation of the light’s trajectory by dividing it in small segments of finite elementary length, the light propagation proceeding in a classic way into the frame of the above assumed flow-field. By the so chosen approach one assumes that: **a)** the phenomenon of light-propagation *at the macroscopic level* is 100% of undulatory nature, **b)** light propagates perpendicularly to the element of wave-front which is, itself, carried by the ether’s flow assumed configured as one of an uncompressible fluid drained into a sink located in the black-hole’s center.

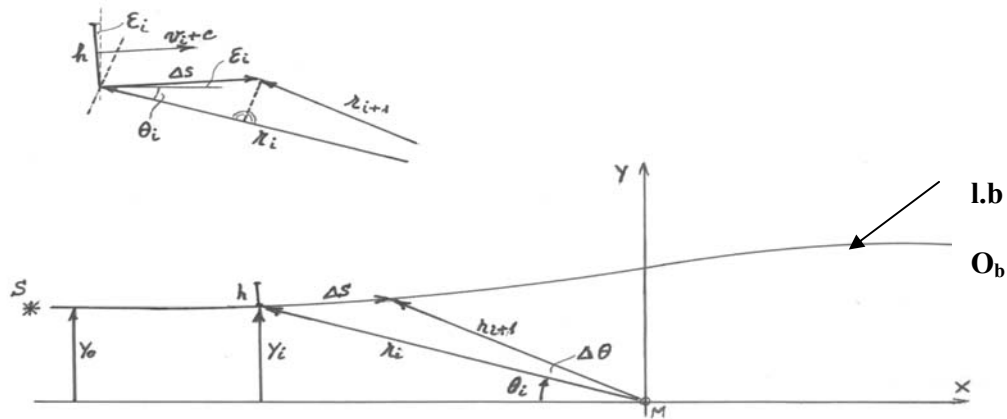


Fig.1.

S = light source infinitely remote; M= black hole ; l.b= light beam ; O_b= observer infinitely remote

The calculus procedure is an Excel program based on conventions and symbols indicated in fig. 1. The chosen independent variable is θ_i sequentially incremented by equal steps of $\Delta\theta$ value.

Expressing the Excel program by columns, the first two columns comes out as:

<u>$\Delta\theta$ (radians)</u>	<u>θ_i (radians)</u>
A1= $\Delta\theta_0$	B1=A1
A2= $\Delta\theta$	B2=B1+A2
A3=A2	B3=B2+A3
.....

In view to be able to run the program with different values of the free parameters – i.e. the parameters *arbitrarily* chosen - one allots specific columns to: y_0 (Fig.1); h , which means the diameter of the light beam taken into consideration, assumed of the order of a big telescope’s aperture, i.e. 8m in a first approximation; M , the total mass proper to the black-hole. As initial values for θ and ε , one considers $\theta_0 = \Delta\theta$ and, for a first round of investigations, $\varepsilon_0 = 0$, different values for ε_0 being eventually investigated in a more advanced phase of the study. So, the first steps in the three following Excel columns are:

ε_0 (radians)	H (meters)	M (kilograms)
C1= ε_0	D1= h	E1= M
C2=C1	D2=D1	E2=E1
C3=C2	D3=D2	E3=E2
.....

Undertaking the task of determining the general evolution of r , one must be aware that r depends essentially on “ c ”, the light’s propagation speed - referred to the local ether – but also on the transport phenomenon due to the ether flow-field, an influence accounted for from an *absolute referential point of view*. Meanwhile, having chosen to unfold the calculus by incrementing θ in steps of $\Delta\theta$ value, in essence a purely geometric procedure, the dependence of r_{i+1} on r_i and $\Delta\theta$, results clearly from figure’s 1 meaning. In concordance with the figure’s details, one assumes the following system of two equations:

$$r_i = r_{i+1} \cos \Delta\theta + \Delta s \cos(\theta_i + \varepsilon)$$

$$r_{i+1} \sin \Delta\theta = \Delta s \sin(\theta_i + \varepsilon)$$

Eliminating Δs , one finds:

$$r_{i+1} = \frac{r_i \sin(\theta_i + \varepsilon)}{\sin(\theta_{i+1} + \varepsilon)}$$

valid till $(\theta_{i+1} + \varepsilon_i) = \pi_{-0}$, point at which the asymptotic branch begins.

Accordingly, column **F** becomes:

$$\underline{r}_i \text{ (m.)}$$

$$F1 = Y_0 / \sin B1$$

$$F2 = F1 * \sin(B1 + \varepsilon_1) / \sin(B2 + \varepsilon_1)$$

$$F3 = F2 * \sin(B2 + \varepsilon_2) / \sin(B3 + \varepsilon_2)$$

.....

till $(\theta_i + \Delta\theta + \varepsilon_i) = \pi_{-0}$, limit where the asymptotic regime is practically attained.

It is important to observe here that, because of the in this work adopted algorithm, ε_i remains along a few further stages of calculus purely theoretical, its value waiting to be explicitly determined by the yield of the following stages. This shortcoming is due to the fact that ε_i depends on Δs , so far not yet explicitly expressed. Its values result from the same - by geometry motivated - system previously used, so that:

$$(\Delta s)_i = \frac{r_i \Delta \theta}{\sin(\theta_i + \Delta \theta + \varepsilon)}$$

also valid until the asymptotic regime is attained.

Observing now that $(\Delta s)_i$ is linked by a propagation phenomenon to a correspondent time interval Δt_i - asked for r_i to evolve towards r_{i+1} - and knowing that h is nearly everywhere insignificantly small if compared to r_i and r_{i+1} , the following relations resulting from figure's 2 meaning are valid for all θ_i :

$$(\Delta s)_x = (c \cos \varepsilon_i + ((v_r + v_h)/2) \cos \theta_i) \Delta t$$

$$(\Delta s)_y = (c \sin \varepsilon_i - ((v_r + v_h)/2) \sin \theta_i) \Delta t$$

the symbols v_r and v_{r+h} representing the ether's-flow transport velocities at the two ends of the linear element h , and "c" maintaining its above stated meaning.

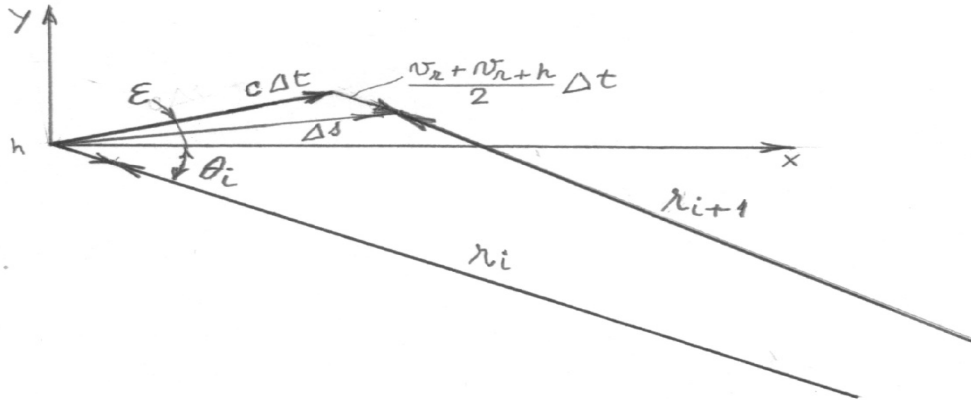


Fig.2.

Knowing that $\Delta s = v_a \Delta t$, v_a meaning the average absolute transport velocity on the transverse section of the light beam - actually a surface but, in the present case, adequately represented by the linear element $h - \Delta t$ will be correctly expressed by the formula:

$$\Delta t = \frac{\Delta s}{\sqrt{(C \cos \varepsilon_i + \frac{V_r + V_{r+h}}{2} \cos \theta_i)^2 + (C \sin \varepsilon_i + \frac{V_r + V_{r+h}}{2} \sin \theta_i)^2}}$$

This last relation being of a strong "kinematical" nature, it is convenient to evaluate first the entity $(r+h)_i$ which, by simple geometry, comes out from fig.3 as:

$$(r \tilde{h})_i = r_i + h \sin(\theta_i + \varepsilon_i),$$

valid for all values of θ_i till here found significant. The “ \tilde{r} ” symbol in $(\tilde{r}+h)_i$ is to be understood as meaning a vector summation. Consequently, the dependent variables v_r and v_{r+h} are expressible as:

$$v_{r+h} = \sqrt{\frac{2kM}{(r+h)_i}} \quad v_r = \sqrt{\frac{2kM}{r_i}}$$

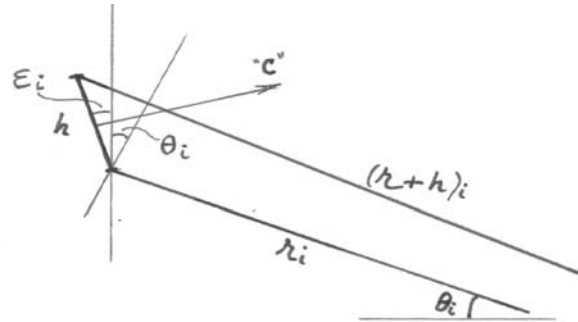


Fig.3.

Commuting to Excel, the next stage of calculus will be:

$$\begin{aligned} & \mathbf{(r_i + h)} \text{ (m.)} \\ & G1 = F1 + D1 * \sin(B1 + C1) \\ & G2 = G1 + D2 * \sin(B2 + \epsilon_1) \\ & G3 = G2 + D3 * \sin(B3 + \epsilon_2) \\ & \dots \end{aligned}$$

the following entities resulting as consequences:

$$\begin{aligned} & \mathbf{v_r} \text{ (m/s)} \\ & H1 = \text{SQR}(2k * E1 / F1) \\ & H2 = \text{SQR}(2k * E2 / F2) \\ & H3 = \text{SQR}(2k * E3 / F3) \\ & \dots \end{aligned}$$

$$\begin{aligned} & \mathbf{v_{r+h}} \text{ (m/s)} \\ & I1 = \text{SQR}(2k * E1 / G1) \\ & I2 = \text{SQR}(2k * E2 / G2) \\ & I3 = \text{SQR}(2k * E3 / G3) \\ & \dots \end{aligned}$$

Looking now for h 's elementary rotation during its propagation travel along path Δs_i , by means of fig.4 and in a first order of approximation assessment the following relation comes out:

$$\text{tg} \Delta \epsilon_i = ((v_r - v_{r+h})_i \cos \theta_i) \Delta t / h$$

valid for all $\theta_i < \pi$.

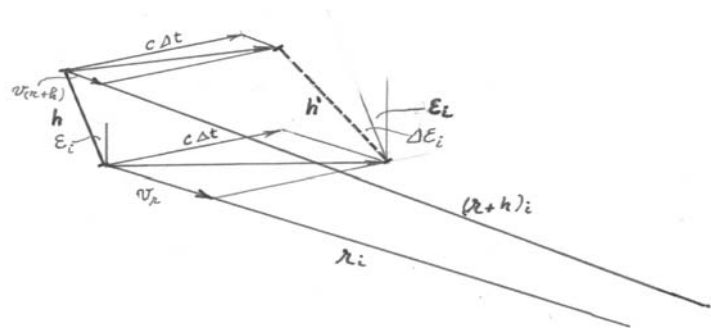


Fig.4.

The phenomenon being in this way understood and the functional dependence of Δs being assumed from the previously determined equations, the next Excel steps are:

Δs (m.)
 $J1 = F1 * \sin(B1+C1) / (\sin(B1 + A1 + C1))$
 $J2 = F1 * \sin(B2+\varepsilon_1) / (\sin(B2 + A2 + \varepsilon_1))$
 $J3 = F1 * \sin(B3+ \varepsilon_2) / (\sin(B3 + A3 + \varepsilon_2))$

.....
 relations valid for all steps before the asymptotic regime is reached.

For reasons of calculus fluency it is convenient to insert here the intermediary K_i computing steps

K_i :
 $K1 = (300000000 * \cos C1 + ((H1+I1)/2) * \cos B1)^2 +$
 $\quad + (300000000 * \sin C1 - (H1+I1)/2) * \sin B1)^2$
 $K2 = (300000000 * \cos \varepsilon_1 + ((H2+I2)/2) * \cos B2)^2 +$
 $\quad + (300000000 * \sin \varepsilon_1 - (H2+I2)/2) * \sin B2)^2$
 $K3 = (300000000 * \cos \varepsilon_2 + ((H3+I3)/2) * \cos B3)^2 +$
 $\quad + (300000000 * \sin \varepsilon_2 - (H3+I3)/2) * \sin B3)^2$

.....
 Consequently to that and knowing that $(\Delta t)_i = (\Delta s)_i / v_a$ in which v_a means *the absolute transport velocity* due to the ether flow, the necessary time-interval for Δs_i to be built comes out from:

Δt_i (s.)
 $L1 = J1 / \text{SQR}(K1)$
 $L2 = J2 / \text{SQR}(K2)$
 $L3 = J3 / \text{SQR}(K3)$

.....
 Assuming figure's 4 meaning together with the previously deduced relation:

$$\text{tg} \Delta \varepsilon_i = ((v_r - v_{r+h})_i \cos \theta_i) \Delta t / h$$

the values $\Delta \varepsilon_i$ of the light-beam's elementary bending-increments will result from the sequences:

$\text{tg} \Delta \varepsilon_i$ (rad.)
 $M1 = (H1 - I1) * \cos(B1) * L1 / D1$
 $M2 = (H2 - I2) * \cos(B2) * L2 / D2$
 $M3 = (H3 - I3) * \cos(B3) * L3 / D3$

Summing the so determined elementary light-front rotations, one obtains the progressive beam's bending:

$$\begin{aligned} \underline{\epsilon}_i \text{ (rad.)} \\ N1 &= C1 \\ N2 &= N1 + \text{arctg}M2 \\ N3 &= N2 + \text{arctg}M3 \\ &\dots\dots\dots \end{aligned}$$

It seems important to observe here that the so assumed procedure determines ϵ_i with a *delay of one calculus-step*. Yet it is easy to find out that this fact *does not break the procedure's coherence*.

In intention to show the light-beam's shape, the resulting values of y_i are also computed:

$$\begin{aligned} \underline{Y}_i \text{ (m.)} \\ O1 &= F1 * \sin B1 \\ O2 &= F2 * \sin B2 \\ O3 &= F3 * \sin B3 \\ &\dots\dots\dots \end{aligned}$$

Finally, replacing in Excel's columns - wherever asked for - ϵ_i with N_i or, correspondingly, with $C1$, a consistent theoretical tool fit for modeling optic phenomena in an as here assumed ethereal cosmos is obtained.

II. The theory as well as the computational program so assumed were tested by running a first course of appliances aimed towards finding the bent shape of a light beam coming from cosmic minus-infinity and so oriented as to pass in the vicinity of a black hole.

The program was run with fixed values for some parameters – more specifically for: $M = 10^{31}$ kg. i.e 5 times the sun's mass, $\epsilon_0 = 0$ and $\Delta\theta = 0,001$ rad., linked with the ones of r_0 which results from those values when associated with the ones ascribed to y_0 , which is the only *assumed-as-variable* parameter.

Table 1 synthesizes the results so obtained.

Table nr.1.

General parameters: $M=10^{31}$ kg, $\Delta\theta=0,001$ rad., $\epsilon_0=0$

Variable parameter: Y_0 .

Y_0 (m)	0,1*	10*	700	1000	500000	1000000	17695000	40000000	1E+11
r_0 (m)	1000000	100000000	700000	1000000	500000000	1000000000	17695000000	40000000000	1E+14
$r_{\pi/2} = y_{\pi/2}$ (m)	3,704	109,3	2434	3225,6	567157	1300246	18114585	40642500	1E+11
$r_{y=0}$ (m)	158,3	4029	14334	17718	107638208	545370000	□	□	□
r_{as} (m)	5554780	10262000	17723	7842000	6,188E+09	1179000000	10176000000	58493047658	1,687E+14
θ_{as} (m)	3,616	4,209	6,37	5,789	3,148	3,143	3,14	3,14	3,141
$\epsilon_{\pi/2}$ (rad.)	0,3424	0,4455	0,33946	0,323	0,05072	0,0344	0,00942	0,00631	0,000128
$\epsilon_{y=0}$ (rad)	-0,1055	-0,18652	-2,08516	-1,3	-0,006448	-0,002617	□	□	□
ϵ_{as} (rad)	-0,4759	-1,069	-3,229	-2,648	-0,0064547	-0,002629	-0,0002328	-0,000114	-6,69E-07
y_{as} (m)	-2537000	-8989191	1,537	-3719472	-3,965E+07	-1659392	16207579	34666113	9,995E+10
d (m)	8292	16340	1,05	4648	293500	1440000	18580000	99830000	1,001E+11

* means: with 10000 steps a at $\Delta\theta=10^{-7}$ rad. foruner program

As a procedural detail, consequence of the fact that for small values of y_0 the distance r_0 – meaning, in fact, the distance at which the phenomenon's evaluation starts - becomes inconclusively small, a preliminary phase of 10000 steps of sequential calculus at a reduced rate of 10^{-7} rad./step was introduced and linked, as a fore-runner phase, to the main sequence.

No such refining is needed on the side of the observer (Fig.5), because, beyond $r(\theta_{as})$ the light's trajectory is very nearly straight, the evolution of $r(\theta)$ and ϵ_{as} shown by the charts clearly proving that.

As a first observation, the so obtained results - if compared to the ones by intuition foreseen - may seem stunning because one finds that, on approaching the black hole, the beam is *always* bent in the *away* direction, i.e. passing at an $y_{\pi/2} = r_{\pi/2} > y_0$, with $\epsilon_{\pi/2} > 0$, as revealed by fig.5 and in the table nr.1 included figures.

Analyzing more in depth the theoretical meaning of the data gathered in table nr.1 it is worth observing that the bend-away phase is always followed - whatever the value of y_0 - by a "bend-towards" phase characterized by a gradual decrease of ϵ until significant negatives

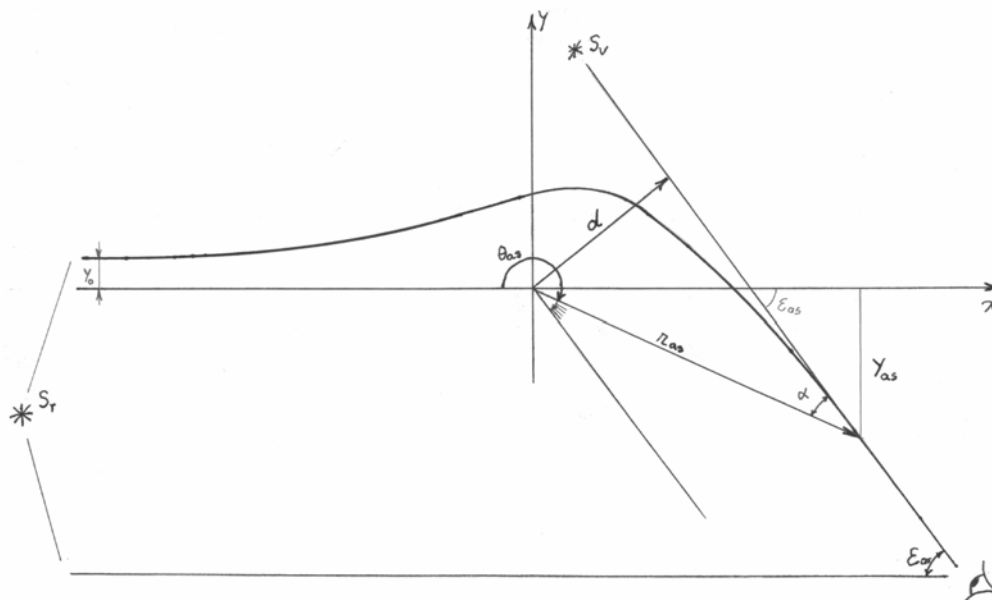


Fig. 5.

values are reached and, by this, settling the beam on a final asymptotic straight line with the eventual observer at its end, as shown in fig.5. The so foreseen physical behavior would make the observer see the source not in its real position, but in a virtual one, in essence in S_v .

In the frame of the above adopted values for the *set of fixed parameters* one may distinguish between some specifically differing phenomenological cases:

a) If the light-source, the black hole and the observer are situated almost on the same straight line then all beams passing the black hole *at relatively large distances* - in essence of the order of 17 to 100 millions meters - should be deflected so as to build a virtual image shaped as a fuzzy ring with an angular diameter in the limits of some $4,656 \cdot 10^{-4}$ to $1,338 \cdot 10^{-6}$ radians. This fact is suggested by the by calculus delivered figures - reproduced in table nr.1 and illustrated by charts nr. 1, 2 and 3 - yet without these data being a really conclusive proof on that mater. More elaborate theoretical/computational means could deliver a more refined answer.

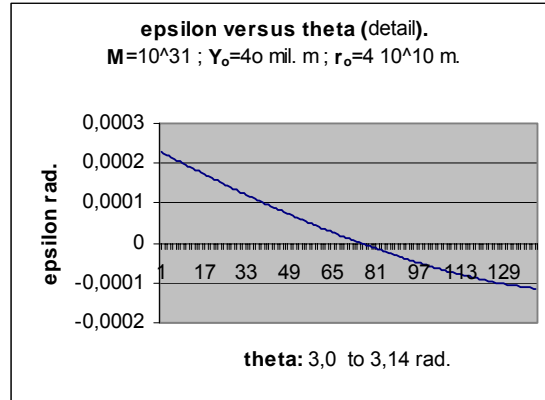
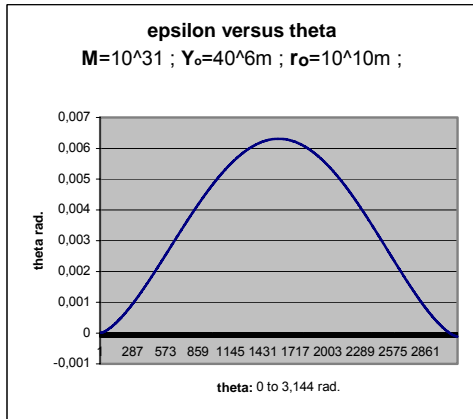


Chart nr.1, and 2.

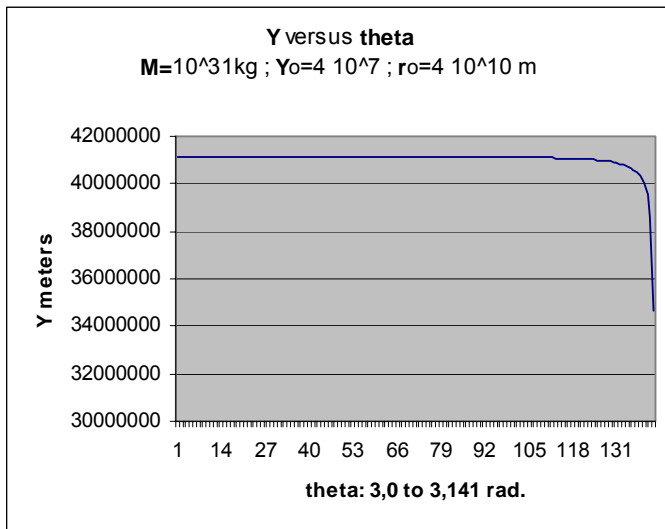


Chart nr.3.

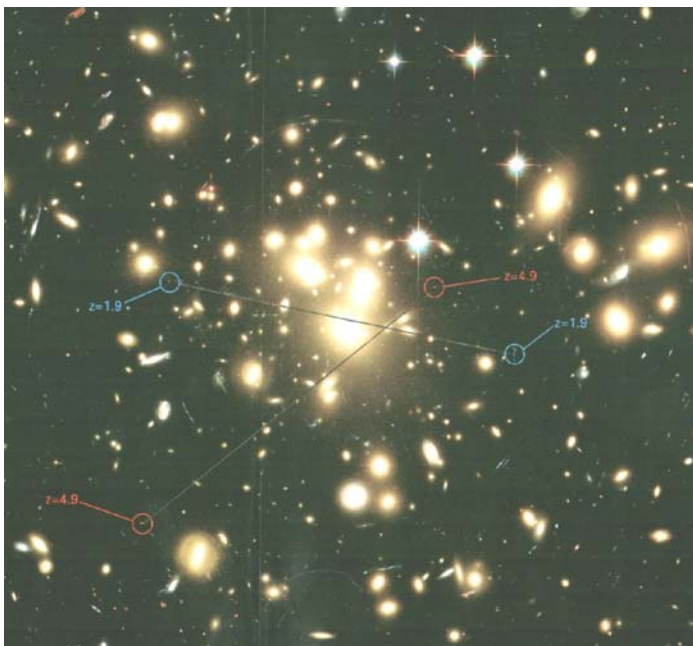
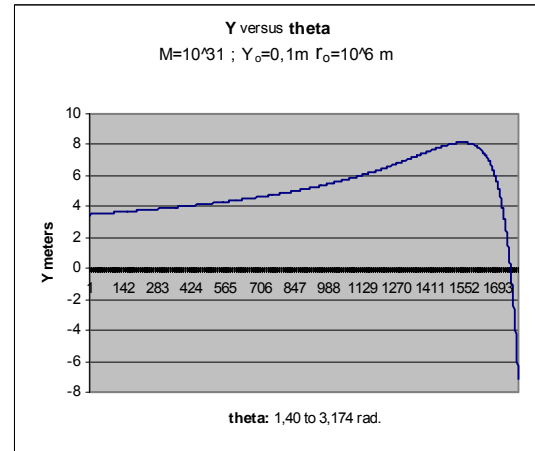
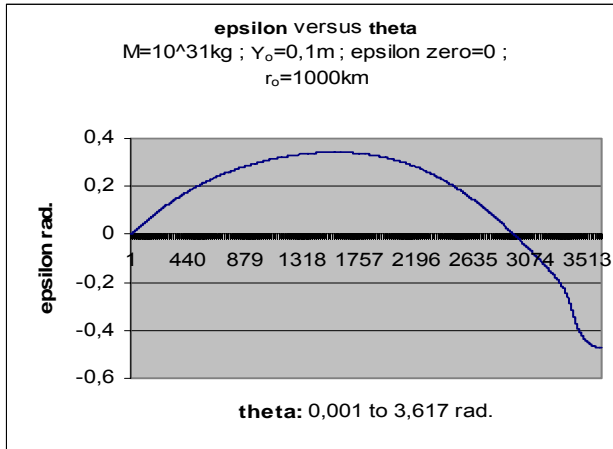


Fig. 6

b) A slight off-alignment of the source relative to the black-hole-observer line could cause the observer to see two virtual images of the source, one on each side of the black hole's direction. The apparent linear distance d of the virtual image, eventually so borne, to the black hole's position is evaluated by means of $d = r_{as} \sin \alpha_{as}$ with $\alpha_{as} = \theta_{as} - \pi - \epsilon_{as}$ (fig.5), the *angular distance* between the two images being of the same order as the one before mentioned. And if so, could this not be accepted as a clue for a new kind of explanation of the phenomenon evidenced in the image caught by the Hubble Space Telescope (fig.6, reproduced and adapted from a National Geographic issue [4]).

c) On the other extreme, the one of the smallest possible values of y_0 – in the range of 0,1 to 100m – the phenomenon may be very similar to the one above described, the foreseeable difference being that ϵ_{as} goes farther in negative values together with y growing in an asymptotic way towards some negative quasi infinite dimension. This means that an observer sited in a position corresponding to values of ϵ_{as} in the range of -0,47587 to -1,0691 rad. may be able to see a

real image S_r as well as a *single* virtual one, the angular distance between the two images being approximately the corresponding ϵ_{as} .



Charts nr. 4 and 5.

Charts 4 and 5 together with fig.5 and the data in Table 1 illustrate these conclusions. Yet one has to be aware that figure 5 represents reality in a strongly deformed way because of its incapacity to *show infinity*, its ability being limited to only suggest it.

d) The big and, in a certain way, *terrible* surprise comes out with the computed figures corresponding to values of y_0 in the hundred meters range. For such values the beam's

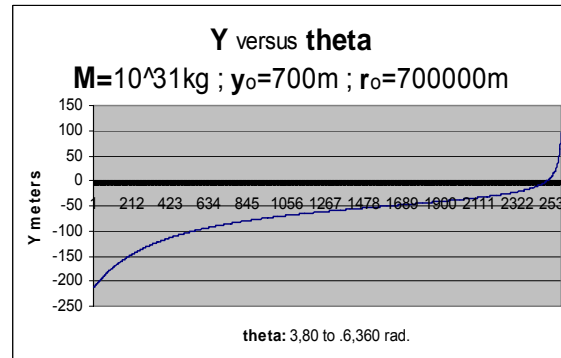
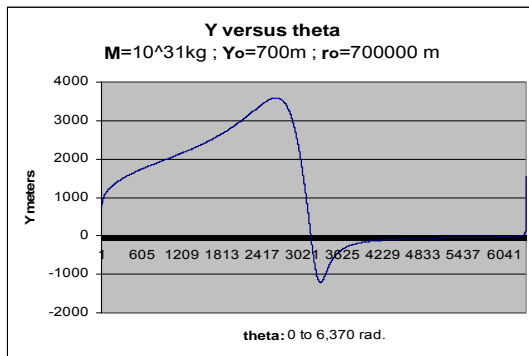


Chart 6, a and detail b.

bending becomes extremely large - quite "monstrous" one may say - in the vicinity of $y_0=700m$, as made evident by in table 1 figured data and by the variable's evolution shown in charts 6 and 7. The mentioned data imply that the bending-angle may grow towards $\theta_{as} > 2\pi$, this meaning that the bending may possibly exceed one turn around the black hole.

This fact suggests that the by the source emitted light beams in an extremely narrow domain of y_0 's values may be eventually captured on a closed, most probably wavy, nearly circular trajectory around the black hole's nucleus. Much more rigorous and detailed abstract simulations are needed to elucidate this point of the problem.

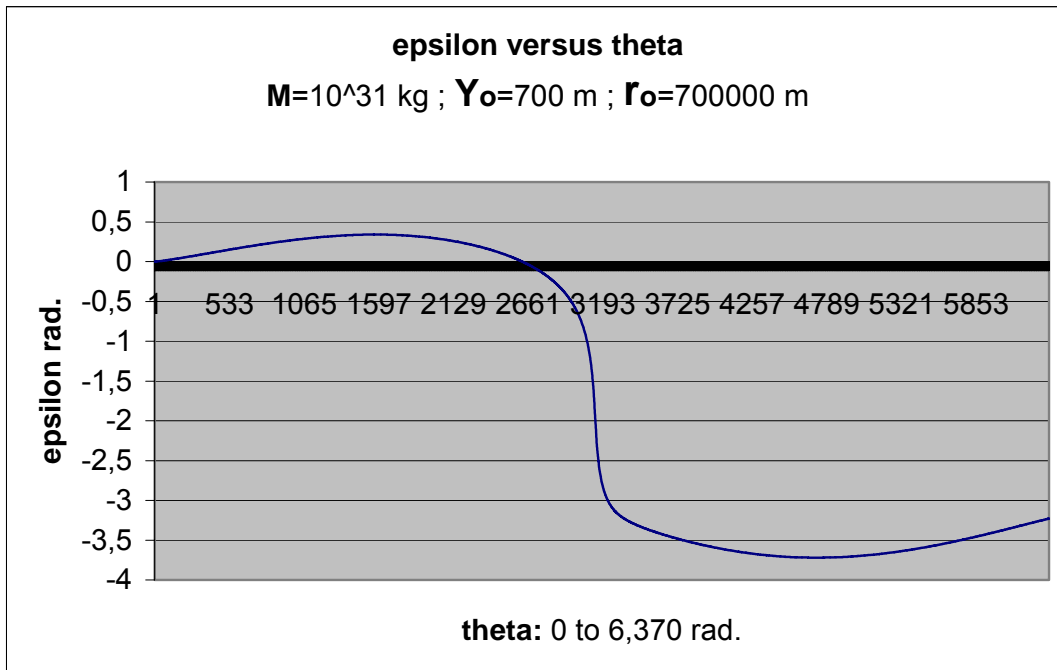


Chart nr.7.

The surprise is, indeed, “terrible” from more than one point of view. The most striking one would be the possibility that a far away galaxy situated, let’s say, somewhere at the back of the observer - and, if looked at that way, seen as a *real*, direct, image - could eventually be seen also, as a virtual image, in a somewhat diametrically opposed direction, in essence somewhere in front of the observer, perhaps showing itself as a more remote and much older *virtual object* than the direct one.

In more detail commented: the observer, if imagined associated with each black hole and with every existing light source in the wide universe, defines a family rich of a hardly commensurable number of planes. In each of those planes the observer, whatever its position relative to the source and black-hole pair, would eventually be able to see the light-source’s real image as well as the virtual image of *the same source*, both images being situated in the particular plane considered. That may imply that every black-hole in the universe offers to the observer at least one supplementary image, a virtual one - visible or not depending on some particular conditionings - for every existing light-source in the cosmos. By this the number of really existing incandescent objects in our universe would be *apparently* increased, in a delusive way, by a number possibly of an order as big as $(2n - 1)N$ of virtual images, “n” meaning the number of black-holes and “N” the number of bright-enough objects to be observable in our universe.

Even if one considers that the number of images so built is – either due to the much longer travel imposed by the light-bending phenomenon, either because of some intervening gigantic red-shifts or because the bent-beam did not *yet* had enough time to reach the observer – significantly less, the number of cosmic objects actually seen would, under the here assumed hypotheses, still largely exceed the number of the really existing ones.

The *inevitable, terrible*, consequence of the as above configured hypotheses and theory, *if accepted as possible facts*, would be that the total mass contained into the known universe could be much less than the till now accepted as being the real one. That would imply the need for big conceptual changes in astronomy and, especially, in cosmology.

In astronomy the observers would be confronted with the obviously difficult task of finding the *virtual twin* of at least one of the *really* existing galaxies; a virtual image most probably situated at a quite large angular distance from the real one. To recognize that two images seen under different azimuths and, seemingly at different distances and of apparent different age, are optical products of one and the same source certainly won't be an easy astronomic task.

From the philosophy of the cosmos point of view the very grounds of the cosmological constant evaluation may be questioned. First, because in the here adopted frame of inquisitive philosophy the total gravitational mass in our universe may be found significantly smaller than the now assumed one. Secondly, because to *assume* on a preconceived basis the absolute constancy of **both** the *speed of light* and the *gravity's factor* values is, if framed in a cosmic ether hypothesis, definitely controversial. And, by that, it seems it would be prudent to consider, inquiringly, a possible model of universe more general than the one yet imagined and assumed on basis of *signals with phenomenological inherent limitative properties*.

Before closing the subject it may perhaps be of some interest to look at a supplementary – actually also somehow “wild” – hypothesis. In essence, to wonder if the part of electromagnetic radiation caught and continuously accumulated on circular orbits around the black hole, as above suggested, may not send out electromagnetic waves of some specific kind.

Sticking to the methodical philosophy of the most intuitive possible approach, previously adopted, one may resort to an incoming electromagnetic ray assumed, for simplicity reasons, polarized so that its electric component \mathbf{E} evolves in the flat surface determined by the ray itself and the black hole's center. Let us suppose that on the last part of the approach and just before being trapped on a permanent circular trajectory the incoming waves generate an emission of waves of the same kind only perpendicular to the absorbing direction. This supposition can be sustained observing that the incident beam's bending give birth into Maxwell's equations to a component as $(\delta\mathbf{E}/\delta\mathbf{t})_{\text{bend}} \neq 0$ and, consequently, $(\delta\mathbf{H}/\delta\mathbf{t})_{\text{bend}} \neq 0$, imagining the frequency of that emission being of the order of that of the captured waves circularly orbiting around the black hole, i.e. $v_{\text{bh}} = c/2\pi R_{\text{bh}}$, R_{bh} representing the trapped-waves radius.

Obviously the physical process so imagined is provocative, also disputable because it presupposes either that Maxwell's laws keep their sense in an absolute manifold independent of the local ether flow, either that the $(\delta/\delta\mathbf{t})_{\text{bend}} \neq 0$ effect is intrinsic to the bending of the electromagnetic waves. If one accepts one of those suppositions and observes that the so captured rays are incoming in cylindrical symmetry, one have to conclude that, after being captured, they should grow in a continuous accumulation of radiant energy confined in a spherical shell of predictable small thickness.

However, processes of continuous radiant energy accumulation in limited spaces *without* related compensatory emission seem not to be in the nature's habits. That makes one may presume that the captured waves in the above mentioned shell must build a coherent mode of oscillations capable to give birth to the here suggested kind of specific emission.

If in fact things would run so, one may foresee that every black hole in the sky would “glow” with an individual-specific, quasi-monochromatic radiation situated in the radio frequencies range.

If the hypothetical stuff above presented – somehow a kind of brain-storming exercise – is accepted as *potentially plausible*, one may conclude that the suggested phenomenon would be easily verifiable by simply looking into the till now by radio-telescope recorded data.

No further comments. If the subject is of some interest, the ground is surrendered to astronomers and astrophysicists.

In a kind of high spirited frame of mind, sometimes perhaps useful, one may, add the quotation: “Theorist are valuable as long as they are stimulating. It is not so important for them to be correct. Observations, on the other hand, are useful only when they are right” ([1], p.195). Quotation added in hope of eventual dedicated observations.

Acknowledgment: The author is indebted towards Dr. Cinthya Kolb Whitney for observing and warning me that my hypothesis on “*spiral-capturing* of electromagnetic waves by black holes” does not respect a fundamental principle of optics.

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- [4] Ron Cowen, “Galaxy Hunters, in National Geographic, vol.203, no.2, pp.26-27.