A Dialogue on Position[1]

Student: Would you please answer a question for me?

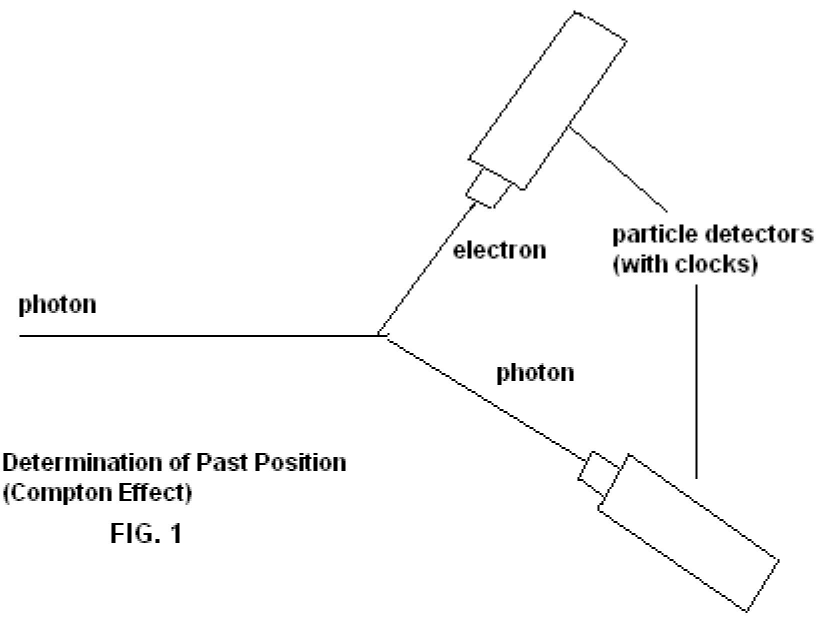
Professor: Can you express it simply?

S: Yes. I want to find out how to determine the position of a particle.

P: Is that all you want to know? I was expecting something more difficult, like determining the magnetic moment of an electron.

S: Please be patient. You see I want to have the concept of position clear in my mind before proceeding to more complicated questions.

P: Very well. In order to determine the position of an electron, you need to collide a photon of high energy with it, resulting in what is known as the Compton effect (figure 1). If particle detectors register the time of arrival of the photon and electron some distance from the impact, the data may be used to determine the time of impact and the point of intersection of the particles. This will give you the electron's past position in three spatial coordinates and time[2].



Determination of Past Position
(Compton Effect)

FIG. 1

S: What about the future position of an electron?

P: Determinations of future position are governed by the uncertainty principle, or as it is more properly termed, indeterminacy. This is clearly illustrated (figure 2) in the diffraction of electrons by a slit of known width[3].

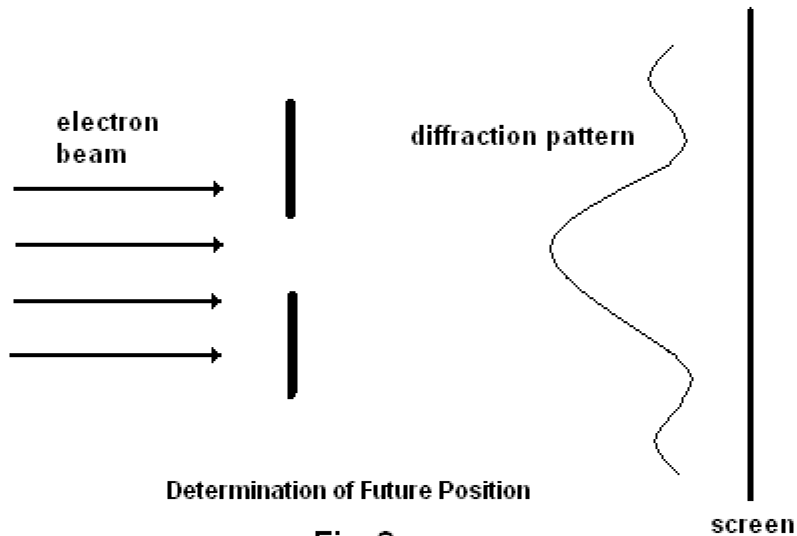


Fig. 2

The uncertainties in vertical position Δy and vertical momentum Δp_y depend upon the slit width, but their product is a constant and is approximately equal to h.

$$\Delta y \cdot \Delta p_y \approx h \qquad 1)$$

S: In the example you gave me of the determination of an electron's past position, the four position coordinates are given relative to the laboratory frame. A particle's future position, however, is given in terms of a probability distribution. Both experiments claim to measure position, but they are represented in completely different forms. How do we know they refer to the same thing?

P: Both experiments were carefully prepared to perform a specific measurement. What is important is that each experiment is appropriate for its intended purpose: the determination of past position on one hand and of future position on the other.

S: But you define future position in terms of statistical averages applied to a large number of electrons. That is nothing more than an extension of the methods of thermodynamics to the electron. It predicts the behavior of a system, not of a particle. How can that be interpreted as a determination of the future position of a particle?

P: If the screen is examined closely the electrons may be seen as individual flashes of light which are realizations of the predicted positions.

S: The results of thermodynamics can also be derived by analyzing the properties of individual particles in the study of statistical mechanics. How is quantum mechanics different?

P: The probabilities of gas molecules are obtained by averaging over the microscopic coordinates of many particles, whereas in quantum mechanics the probabilities are intrinsic to each particle.

S: A simple position measurement in one coordinate system can appear to be very complicated when it is transformed to another. For example, a fixed point on the earth's surface (geocentric coordinates) describes a complex waveform as it moves around the sun (heliocentric coordinates)[4]. Has anyone ever tried to explain quantum probabilities in terms of the transformation of a particle's internal coordinates, which are inaccessible to direct observation?

P: Speculations should not enter into a theory of nature. The wave function together with the observables includes all that can be known about a particle. Quantum mechanics is founded exclusively upon relationships between quantities which are in principle observable[5].

S: Then future position should not be included in quantum theory because it cannot be observed.

P: As can be seen in the electron diffraction experiment the wave function does give the future position of individual electrons.

S: But because electrons are indistinguishable the wave function defines an "averaged" electron, not a particular electron. For example, if I collide a photon with one of the electrons in a multi-electron atom, will the uncertainty principle together with the probability distributions of the various electron shells tell me which electron has been localized?

P: No. You can only calculate the probability of striking one of the electrons.

S: Probability distributions seem to describe the relative density of the electron cloud, or atomic structure, rather than future position. Can't the future position of a particular electron be calculated?

P: Yes, if a hydrogen atom is used.

S: Does the position measurement identify an electron as belonging to a particular atom?

P: No. The physical properties of all hydrogen atoms are the same so it is impossible to distinguish among them.

S: Then when you use "future" it refers to the experiment or the system, not to the particle. Apparently the uncertainty principle cannot be applied to a particular electron.

P: Yes it can. Heisenberg used a thought experiment to derive the uncertainty principle for single electrons[6]. He imagined a high energy photon colliding with an electron constrained to move along the x-axis. If the photon is detected by an ideal microscope, then the electron's indeterminacy is given by,

$$\Delta x \cdot \Delta p \geq h \qquad 2)$$

All matter is subject to this equation. Even Einstein, who firmly believed that exact solutions to the natural laws existed, could not find a way to avoid indeterminacy.

S: Indeterminacy is tested by averaging the imprecision of measurement of many electrons in carefully constructed experiments. Because this does not occur in Heisenberg's thought experiment it cannot in principle be performed. How can it be of any significance if it is impossible to perform in practice?

P: It is an extrapolation of the averaged behavior of many particles down to the level of single particles so we know it must be true.

S: Statistical mechanics is also expressed in terms of the average behavior of many particles, but it is not necessary to extrapolate that to the level of single particles. Why is it important for quantum mechanics?

P: By describing a single particle interaction Heisenberg's experiment allows us to define the space in which the interaction occurs[7].

S: The localization of one particle defines an entire space?

P: Yes. The particle's position is defined in an absolute space of infinite extent.

S: Is the position of anything else defined in that space?

P: No. Each observation or measurement process localizes at most one particle.

S: Do you mean to say that the electron is the only thing of known position in an absolute space of infinite extent?

P: Yes.

S: Then quantum theory is formulated in the same manner as the Ptolemaic theory, except that an electron is designated to be the center of the universe instead of the earth. Isn't that rather impractical?

P: Actually it works quite well. The position measurement of the electron occurs in, and can be imagined as being projected, or superimposed on, the ordinary space of experience. For example, the most probable position of a hydrogen atom's electron may be calculated quantum mechanically and it compares favorably with the measured value of .5 angstrom.

S: The reference point from which the measurement was taken, the nucleus, is of unknown position in the laboratory frame, so that doesn't really measure position does it?

P: It doesn't matter because the same value is obtained no matter where the nucleus is located in the laboratory.

S: The numerical value, yes, but the representation in space is not the same. For example, the three spatial coordinates that are used to locate positions on the earth's surface; longitude, latitude, and altitude; cannot be used to locate the earth's position relative to the sun. Two coordinate systems are required: a three-dimensional geocentric and a four-dimensional heliocentric. Does quantum theory make a similar distinction between atomic coordinates and laboratory coordinates?

P: That would be way too complicated. It would require an independent coordinate system for every particle in the universe and a space in which to represent them. Quantum mechanics simplifies things by using one space, to represent all observable phenomena. Because particles are treated as singularities their structures and motions can be described in the same space. You are aware of course that mathematical points have no intrinsic structure.

S: Yes, I am. If the electron were actually singular though it could not be imagined as having a spin or a magnetic moment, nor would it have a measurable diameter. Why are you so sure that electrons are singular? Can it be proved experimentally?

P: No. We use singularities because it permits complicated physical relationships to be expressed mathematically. For example, the equations of general relativity theory would have been impossible to solve without using mass points as approximations for the heavenly bodies. The singular model of the electron in quantum electrodynamics (QED) is also highly successful. That's why nobody has objected very strongly to the use of a sophisticated fudging process called "renormalization" to eliminate the infinities that arise due to a singular electron[8].

S: But aren't you worried that by placing too much emphasis on a mathematical model the ability to describe some of the electron's physical characteristics will be lost?

P: No. The only object of theoretical physics is to calculate results that can be compared with experiment[9]. Since quantum theory has never been known to fail in its predictions it is unlikely that a competing theory will ever improve upon it.

S: But doesn't it matter to physicists that real electrons have measurable diameters so they can't possibly be represented by a singular model?

P: No, not at all.

S: Why not?

P: Because no one understands quantum mechanics anyway[10].

S: What about the photon? It is easy to show that photons cannot be localized more accurately than λ and τ[11]. This implies that a photon occupies a volume of space. Because these calculations reflect the results of real experiments, don't they mean that the photon is not singular?

P: No, it means only that the photon can be found at some point within the interval λ and τ, but due to the photon's indeterminacy we don't know precisely at what point.

S: If the photon is indeterminate then why didn't Heisenberg include that in his microscope experiment? By not including the indeterminacy of the photon he seems to have ignored the participation of the observer.

P: Of course there is an observer. A microscope can't function without one.

S: The eye cannot detect a photon unless at least one electron is freed from an atomic orbital. This introduces a minimum uncertainty in the position of the electron relative to the observer of λ and τ. To assume that photon detection is performed without error, as Heisenberg has done, is an impossibility.

P: The observer's position relative to the electron is of no physical significance. It would not change the result of any experiment.

S: Then quantum mechanical "position" measurements are defined in center-of-mass coordinates, not in the laboratory frame as Heisenberg intended.

P: No matter what coordinates are used it will be impossible to determine the trajectory of the electron. This means that classical determinism is denied, which is the idea that the future position of a particle may be determined exactly if all aspects of its past history are known.

S: You are using equation 2), a result derived in atomic coordinates, to describe what occurs in the ordinary coordinates of experience. That's like comparing apples to oranges, isn't it?

P: Not at all. The laws of motion are all formulated in the same space. Atomic coordinates are just more detailed.

S: In the lengthy calculations that lead up to and result in the uncertainty relations, which includes all of matrix mechanics and the derivation of the commutation relations, the electron is conceived of as a singularity in the space of an atom. That method results in an exact solution given by equation 2). In QED, however, serious problems result when a singular electron is described in ordinary space and they have yet to be satisfactorily resolved. How can a singular electron cause such extreme difficulty in QED, yet it yields an exact solution when found in the atom?

P: The problem you are speaking of, renormalization, must be solved mathematically. I expect a solution to be found soon. Over half the work in physics is being done by mathematicians, you know.

S: What if the two spaces have physically distinct origins so that the two results can't be compared. Atomic space describes matter's structure, which is exact, while ordinary space is used to describe our perception of matter.

P: It is impossible to describe more than what indeterminacy allows. Conjectures about what occurs behind the natural phenomena are pointless because only observables may be included in a theory of nature.

S: You mean except for singularities, right?

P: Yes, of course. But it's OK to use singularities in a theory because everybody does it.

S: Let's leave questions of position for a moment and look at motion. How exactly does quantum mechanics reconcile the concept of motion with indeterminacy?

P: Quantum mechanics describes motion in terms of the emission of particles at one location and their detection at another. This is expressed formally through the use of Dirac's bra-ket notation.

S: Then quantum mechanics does not describe what happens to a particle in the interval between emission and detection?

P: That's right. The particle occupies all possible paths.

S: What about bound electrons?

P: We know that bound electrons move in orbits around the nucleus because they have a well-defined angular momentum; but it is impossible to describe that motion in terms of distinct trajectories.

S: There is a finite probability of finding a bound electron anywhere in the universe; however, Schroedinger's wave equation is derived non-relativistically. How is it possible that a bound electron may be found so far away yet it does not exceed the speed of light or increase its mass, in other words, it gets there without moving?

P: The wave function gives the possibilities, or potential positions of the electron; so the electron is in all possible places of the abstract space, all of the time.

S: Quantum mechanics refers to the "motion" of an electron when it is convenient, but when it's not the abstract nature of the space is cited. Either motion is a valid concept all of the time, or it should not be used at all.

P: Don't get hung up on classical terminology. You are trying to describe reality as predicted by a mathematical model, but human languages simply don't have the means to express quantum concepts.

S: Well what about the motion of more massive particles? How is that described?

P: Either in theory with an ideal microscope or in practice by some sort of particle detector like a bubble chamber.

S: Those instruments are unable to measure the time parameter of a particle by comparing the occurrence of an event with a continuous time standard, such as Greenwich Mean Time.

P: Only time difference measurements are physically meaningful. The clock time that is registered when particles are detected does not enter into the dynamics of the interaction.

S: Yes, but the representation used to record the data differs greatly. Measuring the half-life of a radioactive atom does not reflect the internal processes that produce its eventual decay. Two physically distinct time parameters are required, one to describe structural events that produce the decay and another to record the decay as an event.

P: The use of two types of time would only complicate the mathematics. The same standard, the second, is used to measure all time phenomena so all time measurements may be represented in the same manner.

S: Although time difference measurements may be performed to any degree of accuracy, due to the indivisibility of a clock period measurements of the time parameter cannot be. For example, the period of an atomic clock, τ, is known with nearly exact precision, yet a minimum error $\Delta t \geq \tau$ is introduced when that same clock is used to measure the time coordinate of an event. How does quantum theory differentiate between the measurement of time differences and time as a parameter?

P: If you have a proper understanding of the mathematics you won't become confused about which time is being used.

S: The time parameter is one of the four coordinate axes used to define position. If position is included in quantum theory as a viable concept then it should define the measurement of time operationally to include a clock period's indeterminacy. Is the measurement of the time parameter assumed to be errorless?

P: You have to talk to a specialist on relativity theory about that. His office is at the end of the hall.

S: Unless we can agree on a precise meaning for concepts such as position we will continue forever to talk at, rather than to, each other about quantum phenomena. In order to place classical and quantum physics on an equal footing in the study of mechanics a common basis must be established. Over 70 years ago Einstein requested a clarification of the meaning of past and future position and was ignored[12]. The problem remains unresolved to this day.

P: In any case I hope I have cleared up some of your confusion about quantum measurement theory.

S: My question was about position.

P: Yes of course. Good-bye.

[1] Presented at the NPA/AAAS meeting held in Santa Fe, NM of April 1999. (Several members of the NPA were of assistance in improving this dialogue.)
[2] The accuracy of past position measurements is not limited by the uncertainty principle.
[3] From R. Feynman, *Lectures on Physics*, Vol. III, (Addison-Wesley, 1965), p. 2-2, 2-3.
[4] Due to rotation a fixed point on the earth's surface traces an irregular helix as it revolves around the sun.
[5] Quoted from the first ever paper on quantum mechanics by Heisenberg (1925).
[6] Richard Feynman has referred to renormalization as a "dippy" process.
[7] W. Heisenberg, Zeitschr. Phys., 43, 172, 1927, in J. Wheeler & W. Zurek (Eds.), *Quantum Theory and Measurement*, (Princeton, 1983), p. 62.
[8] In classical and relativity theory arbitrary positions may be located in an empty space so space has meaning independently of the objects it contains. This is not true in quantum theory because positions are specified relative to particles. When particles are absent from a quantum space nothing at all remains.
[9] This sentence is a quotation from Dirac.
[10] This is paraphrased from a well-known statement by Richard Feynman.
[11]Let an electron decay occur between known energy levels such that

$$\Delta E = E_2 - E_1$$

Then from the uncertainty relation for energy and time given by,

$$\Delta E \cdot \Delta t \geq h \qquad 3)$$

the time of emission of the photon is uncertain and is calculated as follows:

$$\Delta t \geq \frac{h}{E} = \frac{h}{h\nu} = \tau \qquad 4)$$

where τ is the period. Therefore it is impossible to specify the precise time of either the emission or the detection of a photon more accurately than the period. Similarly from 2), there is a minimum uncertainty in the precise location of emission and detection given by,

$$\Delta x \geq \frac{h}{\Delta p} = \frac{h\lambda}{h} = \lambda \qquad 5)$$

The relations 4) and 5) tell us that the photon cannot be localized more precisely than the amounts λ and τ.
[12] Fifth Solvay conference (1927). See, for example, A. Pais, *Niels Bohr's Times*, (Oxford, 1991), p. 318-19.

Abstract

A student who wants to "understand" quantum mechanics asks a physics professor how to determine the four-coordinate position of a particle. It soon becomes evident that Heisenberg's microscope experiment is totally inadequate as a model since the observer does not actually participate in the measurement process, and a procedure for measuring the time coordinate microscopically has never been defined. In fact, in a strict sense, quantum mechanics does not have a logically coherent method for determining position in even a single dimension. Their attempts to resolve these differences are an exercise in futility until the student finally realizes that before they can agree on anything they have to be able to communicate.