Quantum references: The determination of a zero point in quantum systems

Abstract.

Heisenberg’s microscope experiment for the determination of the position of an electron is fundamentally flawed because it does not define position in four dimensions, as an event in space-time. To rectify this the microscope is substituted for by an ideal radar system situated at the origin of a coordinate system, thereby defining a reference system. It is then demonstrated that the origin and resulting coordinate points have a minimum uncertainty due to the physical extension of the photon in space-time. Further analysis reveals that quantum mechanics may be characterized in general as the study of material processes for which the spatial extension of the photon must be taken into account.

Abstrait

L'expérience du microscope de Heisenberg pour la détermination de la position d'un électron est fondamentalement défectueuse parce qu'elle ne définit pas la position dans quatre dimensions, comme événement dans l'espace-temps Pour rectifier ceci le microscope est substitué près à un système idéal de radar situé à l'origine d'un système du même rang, définissant de ce fait un système de référence. On le démontre alors que l'origine et les points du même rang résultants ont une incertitude minimum due à l'extension physique du photon dans l'espace-temps. Davantage d'analyse indique que la mécanique quantique peut être caractérisée en général comme étude des processus matériels pour lesquels l'extension spatiale du photon doit être prise en considération.

Key words: reference system, coordinate system, configuration space, uncertainty, indeterminacy, photon, microscope experiment, zero point, spatial extension

1.0 Introduction

It has become increasingly popular to reexamine the early work of quantum mechanics[1-3]. When derivations made during the initial rapid advances are inspected carefully it is often revealed that they were, if not wrong, often hastily conceived of or incomplete. It is a healthy exercise which is in the interest of the advancement of physics and the understanding of physical processes to conduct these debates in order to establish not only what is known but also what is not known, questionable, or based on insecure conceptual foundations. One of the questions that has not been satisfactorily examined is the relationship of the abstract mathematical formalism of quantum mechanics to the real space and time of an observer.

In the case of classical mechanics it is the reference system that is used to combine the abstract with the real. Thus it includes both a mathematical part, the coordinate system; and a real part, a means of locating physical occurrences within that system. Because it requires actual methods for detecting events and relating them to an observer by means of the rules of geometry, it has a well-defined physical meaning; and in this sense differs from the coordinate system which has only an abstract or mathematical meaning. The reference system forms therefore a juncture between the equations of a mathematical theory and the physical implementation of the equations through measurement.

2.0 Classical systems

2.1 The heliocentric system

The problem of defining a reference system is at least as old as the question of how to formulate the laws of nature and beset with almost as many difficulties. A valid reference system is often hard to define because the observer is restricted physically to a single location in space and can only overcome this limitation by using intuitive powers. Thus the geocentric reference system was a product of the fact that we are all fixed in essentially the same position for the observation of the heavens and that it is impossible to observe the solar system from a fixed position relative to the sun. The correct physical interpretation was obtained by referring measurements performed geocentrically to the heliocentric system.

Even though serious errors in defining a reference system had been committed the appearances of the planetary motions could not be denied, consequently these errors were perpetuated for hundreds of years. Distortions caused by the physical limitation of observers also occurred due to the appearance of a "flat" earth and an earth-centered position in our galaxy. All of these theories proved to be inadequate due to the inaccurate determination of an origin from which to perform position measurements.

2.2 Configuration space

Reference systems employing Cartesian coordinates and a fixed origin proved too inflexible for the continued development of the laws of mechanics. If, for example, one has a collection of n point mass particles such that each particle moves independently of the others then the system has 3n spatial degrees of freedom. The generalized coordinates of the n particles together with their time derivatives the generalized velocities, may then be represented by a single point in a 3n dimensional hyperspace called "configuration space", where 3n+1 is now the number of independent variables necessary to define the state of the system. The mathematical description of a system of particles is thereby simplified.

2.3 The zero point

The general laws of mechanics tell us how some arbitrary system of particles behaves, but they do not describe a specific system. Thus they are formulated in such a way that the choice of origin is a matter of convenience. In order to apply equations of motion to a specific system of n point masses an origin must be chosen and 3n sets of coordinates specified. Therefore even though the choice of an origin is arbitrary, it is a necessary first step in applying the laws of mechanics to a particular system.

The spatial coordinate values, or positions, of the n point masses of a system are not physically significant. If they were it would require postulating the existence of an absolute space. In other words, one does not measure the coordinates of something, rather one measures the coordinates of something with respect to something else. Consequently the origin, like the reference system, has both a physical and an abstract significance. When the physical origin of a measurement is identifiable with the mathematical origin it will be referred to as a "zero point", signifying that its physical properties are to be regarded as both ideal and infinitesimal. In classical mechanics, for example, a point on the earth's surface is often used as the origin and zero point of position measurements. The sun acts as zero point when it is used to define both the physical origin of a gravitational field and the mathematical origin describing planetary motion.

3.0 Quantum systems

3.1 The uncertainty principle

Heisenberg's uncertainty principle states that the product of the uncertainties of position and momentum of an electron will be equal to or greater than Planck's constant h.

$$\Delta x \; \Delta p \geq h \qquad 1)$$

Heisenberg's intent when he formulated the uncertainty relations was to describe microscopic events in a reference system by using definite physical methods of measurement. This may be seen from the reason he gave for conducting his microscope thought experiment[4]. "When one wants to be clear about what is to be understood by the words 'position of the object', for example of the electron (relative to a given frame of reference), then one must specify definite experiments with whose help one plans to measure the 'position of the electron'; otherwise this word has no meaning." Heisenberg's use of **uncertainty** to describe 1) places emphasis upon the observer's lack of knowledge of the particle. In other words, the observer is uncertain of the actual parameters of the electron.

The uncertainty principle is a classical relation in the sense that particles are assumed to have well-defined parameters, but due to 1) it is impossible to determine them precisely. If a particle's position is measured to a high accuracy, its momentum will be correspondingly less accurate. We conclude from 1) that particle trajectories cannot be precisely described. Since the purpose of a reference system is to map trajectories uncertainty places limits upon how it functions.

3.2 Indeterminacy

More recent discussions define uncertainty as a spread in the possibilities defined by a wave function. Previous to a measurement position refers to a potentiality because it does not even exist. The maximum accuracy of the product of position and momentum is then given by h/2π, or ħ, and 1) must be rewritten as follows:

$$\Delta x \ \Delta p \geq \hbar \qquad 2)$$

Because 2) is believed to describe a property of matter ħ is used to indicate the exchange of angular momentum, and the term **indeterminacy** is preferred. A position measurement localizes a particle so that it approximates an exact location in space, but the momentum is then indeterminate. Indeterminacy is more restrictive than uncertainty since it questions the very legitimacy of the reference system to serve as a map of microscopic phenomena.

Indeterminacy may also be expressed in terms of the parameters of energy and time.

$$\Delta E \, \Delta t \geq \hbar \qquad 3)$$

3.3 Conceptual difficulties

The quantum mechanical interpretation of a "reference system" leaves us with a certain dissatisfaction. Instead of a completely redefined method for locating particles we use classical techniques such as measuring rods for recording events on the macroscopic level while indeterminacy is applied to microscopic events. In order to bridge the gap between classical and quantum systems it is necessary to apply the correspondence principle. This proves unsatisfactory because the point of transition between them is not precisely defined.

Although Heisenberg does not describe the physical characteristics of the reference system used in his thought experiment, it is assumed that the electron is localized relative to a system of orthogonal rods and clocks. This is what is meant when it is stated that quantum theory is formulated in a special relativistic, or "flat", space-time. However, in order to verify that a system of rods and clocks designed to register macroscopic events can perform this same function for microscopic events it will be necessary to detail the physical procedure for accomplishing this.

If the observation of an electron by means of a microscope is compared to photography then the background of the photograph represents the coordinate space and localization refers to the extent of blurring that occurs in the image. However, position coordinates may only be measured incompletely with a microscope. No matter how sharply defined an object's image, it does not assist in specifying that object's four-coordinate position. In order to measure the position of the object relative to the camera it is necessary to determine when the shutter was tripped and the distance of the object from the focal plane (film) of the camera. A similar procedure would be necessary to specify the position in space and time of a particle. Otherwise the microscope simply extrapolates arbitrarily small spatial and temporal differences to macroscopic size. Therefore the microscope cannot act as physical intermediary between four-coordinate microscopic events and a macroscopic system of rods and clocks.

4.0 Redefining the reference system

4.1 A reinterpretation of uncertainty

A properly designed reference system must be capable of representing either macroscopic or microscopic phenomena. To do this the Heisenberg thought experiment will be modified so that it can locate particles in the laboratory frame. A simple method for analyzing all aspects of the proposed measurement process is to replace the microscope with an ideally constructed radar system. It will then be possible to choose a coordinate system whose origin coincides with the radar, thereby defining a zero point. Because a specific measurement process has been specified a reference system of this type has a well-defined physical content, in contrast to most classically defined reference systems.

Photons are emitted by the radar at exact instants in time and at known angles. When they encounter a particle they bounce off and are then detected at the point of emission. The angle and time delay is registered and may be used to determine the location of the particle relative to an imaginary coordinate system. Both emitter and detector are assumed to function without error, and physical processes are assumed to occur without loss. This method may be used to locate and track objects of any size by simply choosing photons of an appropriate wavelength.

Before conducting an experiment to locate the position of an electron it will be helpful to calibrate the radar. Let a photon be emitted directly towards a reflecting surface fixed on the x axis at x_o. The energy of the photon is exactly known if it is produced by the decay of an electron between highly stable orbitals. Then from 3) there is an uncertainty in the time of emission of the photon,

$$\Delta t \;\geq\; h/E \;=\; h/h\nu = \;\tau \qquad 4)$$

where τ is the period of the photon and h replaces $\hbar$ due to the exchange of only linear momentum. After the photon is reflected at x_o, it returns to the microscope and triggers the detection process by ejecting an electron. The same uncertainty must be applied to the time of detection.

From 1) there is also a minimum uncertainty Δx in the distance of the reflector from the origin given by,

$$\Delta x \;\geq\; h/p \;=\; hc/h\nu \;=\; \lambda \qquad 5)$$

where $p=E/c$. Because the distance is expressed in terms of the elapsed time as $x_o = ct_o/2$, the uncertainties of emission and detection need not be summed.

The calibration of the radar in our system of reference has determined that events are uncertain by the amounts $\Delta x \geq \lambda$ and $\Delta t \geq \tau$, where these values represent the minimum error of a position measurement and change according to the energy of the photon. Therefore the coordinate points of a reference system cannot be endlessly subdivided. The error stems from the fundamental requirement that a reference system include a real means for performing position measurements, and from the observed physical extension of photons. It may now be hypothesized that the uncertainty principle 1) is a direct result of the spatial extension of photons. The generality of the derivation suggests that it can be applied to all material systems for which field effects are ignorable.

4.2 Heisenberg's derivation of uncertainty

The above thought experiment demonstrates that the accuracy of a microscopic position measurement is inherently limited by the physical extension of photons in space-time an amount λ and τ. To see why Heisenberg did not correctly interpret the meaning of uncertainty it will be helpful to review the sequence of events which led up to its derivation[5]. Shortly after he discovered a set of complex mathematical rules interrelating the frequencies and intensities of the spectral lines of hydrogen, Born recognized them as matrices leading to the commutation relation:

$$\mathbf{pq} - \mathbf{qp} = i\hbar \qquad 6)$$

where **p** and **q** are matrix operators in Hilbert space, an abstract complex linear vector space, and they represent the observables, momentum and position respectively.

Equation 6) was derived by comparing the amplitudes of the spectral lines of hydrogen atoms. In quantum mechanics these are equivalent to the transition probabilities of electrons, so it was obtained by studying the excited states of atoms. The structural characteristics of the hydrogen atom that lead to 6) are revealed therefore by adding energy quanta to electron shells. The resulting excited states are products of both the electron and the energy quantum. However, Heisenberg's derivation of uncertainty presumes the presence of the excitation energy without taking into account its existence as a photon before and after the excited state. In other words, the photon-electron resonance is considered significant, but the photon's prior and subsequent interactions with a macroscopic system are not.

A reference system is not properly defined by an ideal microscope since there exist aspects of the measurement process which are not fully described by their mathematical representation. The physical agent of the position measurement, the photon, does not interface with the macroscopic system of the observer. Furthermore because the origin is not well-defined the meaning of position is unclear. These shortcomings only become apparent after a suitable reference system has been chosen as in 4.1. Because the concepts of position and measurement have undefined meanings quantum theory should not be used to interpret related classical concepts such as observer, trajectory, and determinism.

4.3 A "real" microscope experiment

Heisenberg derived 1) by imagining the experimental determination of an electron's future position, but he did not do so in a real setting. Suppose the experiment is actually attempted by directing x-ray photons at an atom until one of the electrons is ejected by an inelastic collision. If a heavy atom is chosen so that the atom is fixed relative to the laboratory frame it will be uncertain which of the atom's electrons was ejected. Therefore it is impossible to construct the probability distribution of an electron in, say, the 1s orbital of a particular element by repeatedly directing photons at it. If, on the other hand, hydrogen is chosen so that a specific electron is targeted the position of the atom is indeterminate. Once again the probability distribution of the electron cannot be mapped. The question arises then whether Heisenberg's thought experiment for determining the position of an electron is possible even as an idealization.

If in fact an electron is localized by a photon of very high energy such $\lambda = \tau \approx 0$ the experiment will indicate not that the electron's position has been located in a reference system, i.e. fixed relative to the origin of a coordinate system. Rather it indicates that the "diameter" of the electron has been reduced to nearly zero[6]. The thought experiment demonstrates that the electron, or any particle for that matter, cannot be conceived of physically as a singularity. Consequently the indeterminacy relation 2) may be interpreted as evidence that all particles have structure and are extended in space. In other words, if Planck's constant were equal to zero particles could not exist because they would have no physical dimension.

5.0 Zero point determination

5.1 Independent systems

Past experience has shown the importance of correctly specifying the origin of a reference system. A universal law of gravity cannot be formulated in the geocentric system because planets revolve in epicycles about imaginary origins (see figure 1). As a result a property of the earth's motion, its period of revolution, was for several centuries believed to characterize planetary motions as well. These errors can be eliminated if a zero point, the sun, is chosen as origin. In this way the reference system may be used to identify error due to an improperly defined origin.

When the zero point of a microscopic system is properly defined, as in the thought experiment of 4.1, the size of the photon is seen to define the lower limit of discrimination. Since this result holds for all electromagnetically coupled systems for which field effects are ignorable, it may be generalized to include any arbitrary number of linear oscillators in configuration space. Thus the indistinguishability of photons and the use of boson statistics to describe a photon gas in the derivation of Planck's law (e.g. see reference 5) may be interpreted as requirements imposed by the reference system, or representation of the system; rather than as a property of the photon. In other words, a physical condition has been placed upon the reference system and in turn on the configurational coordinates.

5.2 The harmonic oscillator, a one-dimensional system

The one-dimensional harmonic oscillator can be used as a theoretical model for the vibration of atoms in a molecule. Solving the Schroedinger wave equation indicates that the minimum energy state is equal to $h\nu/2$. In other words, the minimum energy of a quantum oscillator is greater than zero, and consequently is referred to as the "zero point energy". This rather unexpected result arises due to the uncertainty principle, and leads to an improved agreement with experiment[7].

In the case of electromagnetically coupled systems the origin was shown to have an uncertainty of at least one quantum. For an internally coupled system such as the harmonic oscillator, the origin may be placed within the system thereby yielding a minimum uncertainty in the origin equal to $\Delta x \geq \lambda/2$ and $\Delta t \geq \tau/2$. The zero point energy which is one-half that of a quantum may then be interpreted as arising due to the resulting uncertainty in the configurational coordinates. It is required because energy quanta have non-zero dimension. These examples show that properties of the reference system, or representation, must be included in the description of a material system even if that system is not directly observed.

5.3 The hydrogen atom, a system in three dimensions

It makes no difference classically whether position or momentum is measured first; in short, physical variables commute. Quantum mechanics, on the other hand, distinguishes itself from classical mechanics due to the noncommutativity of observables, as may be seen in equation 6). To see why they differ we recall two previous findings: when an atom becomes excited the resonant state that is formed is a product of both photon/energy quantum and electron; and secondly, the photon is physically extended in space-time.

The physical variables of a bound electron will only commute if the excitation energy is applied to the electron singularly, as occurs in all classical interactions. Thus variables should not commute if the energy of excitation is spatially extended, so that a three-dimensional quantum is applied to a singular electron. The surprising thing about the commutation relations, however, is that they do not take on general values as might be expected for a wide variety of applications and energies, but only the definite value $\hbar$. If the assumption is now made that the energy quantum and the photon are physically proportionate in structure, then it may be concluded that $\hbar$ is the characteristic trace of a photon that has been captured. It expresses the proportionality between a photon's local properties, energy and momentum, and its physical extension in space-time.

6.0 Uncertainty and indeterminacy

In quantum mechanics indeterminacy is defined relative to exact coordinate values in absolute space. However, a properly performed position measurement is defined relative to something other than space, something physical; so coordinates cannot be exact. All position measurements performed in a reference system have a minimum uncertainty of representation aside from instrument error that depends upon the extension in space-time of photons. Indeterminacy, on the other hand, refers to the mediation of energy quanta within material systems. Thus uncertainty and indeterminacy concern the way photons mediate between point masses in space-time.

To avoid confusion in the study of mechanics uncertainty should be used to refer to the space and time of an observer, and indeterminacy to the structure of matter. They are independent concepts because they refer to the measurement of different physical attributes, distance and size; and they are necessary because representations in space and time are mediated by the photon, which embodies space-time both in its properties by the speed c and in its structure by λ and τ. Quantum mechanics may be characterized therefore as the study of material processes for which the spatial extension of photons/energy quanta must be taken into account.

I wish to acknowledge the assistance of an unknown referee who encouraged me to interpret my findings in terms of configuration space.

References

1 F. Selleri, *Wave-Particle Duality*, (Plenum, 1992).

2 F. Selleri, van der Merwe (Ed.), *Quantum Paradoxes and Physical Reality*, (Kluwer, 1990).

3 J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, (Cambridge, 1987).

4 W. Heisenberg, (1927) in J.A. Wheeler & W.H. Zurek (Eds.), *Quantum Theory and Measurement*, (Princeton, 1983), p. 62.

5 M. Jammer, *The Conceptual Development of Quantum Theory*, (Tomash, 1989).

6 The collision is referred to here in an idealized sense. An actual experiment of this type would produce positron-electron pairs rather than localizing the electron.

7 M. Born, *Atomic* Physics, (Dover, NY, 1969), p. 392.