ASTRONOMISCHE NACHRICHTEN.

Band 238. Nr. 5699. 11.

Relativity and the Law of Gravitation. By *Charles Lane Poor.*

The most important claim of ***Einstein*** is that of the discovery of a new law of gravitation; the claim that there is something radically wrong with ***Newton***'s law of inverse squares. This claim is based primarily upon a certain formula for planetary motion deduced by ***Einstein*** in 1915, and by the fact that this formula appears to explain the observed rotation of Mercury's orbit. But the way in which ***Einstein*** derived this formula for the supposed rotation of planetary orbits is not given in any standard work on relativity: even in ***Einstein***'s basic paper »Die Grundlage der allgemeinen Relativitätstheorie«[1]) one does not find a mathematical derivation of this fundamental formula. His original and only known derivation of this most important formula of relativity is, however, to be found in an obscure paper published in Berlin in 1915 under the title of »Erklärung der Perihelbewegung des Merkur«[2]). The present note is based upon the formulas and methods of this authentic and basic document of relativity.

In developing his formula for planetary motion ***Einstein*** starts with a very general mathematical expression, which, subject to special values of the factors involved, may represent the motions of a body under any and all conditions. For one specific set of values this general formula will represent the motion of a body at an infinite distance from all other bodies: for another specific set the formula represents the motion of a body when acted upon by a single gravitational mass, such as the sun. The successive introduction of these various factors, or specific values of factors, into the general expression is called by ***Einstein*** »approximations«. Thus, when all the factors vanish and the material point, or body is removed from all outside influence, or attraction, ***Einstein*** shows that the material point, or body will move uniformly and in a straight line. This is called the »zero$^{\text{th}}$ approximation«. When he introduces a single factor into his general formula, he calls the result the »first approximation«; when he introduces additional factors, or conditions he calls the new result his »second approximation«.

For his »first approximation« ***Einstein*** introduces into his general formula a factor representing the ***Newton***ian gravitational potential: a factor depending solely upon ***Newton***'s law of inverse squares. This fundamental tensor, thus introduced, is defined mathematically as being (p. 833):

$$g_{44} = 1 - \alpha/r$$

where »α is a constant determined by the mass of the sun«. And in tensor mathematics this corresponds to the negative value of the ordinary classical form, m/r, in which the ***Newton***ian potential is usually written. The derivative of this tensor gives the force due to the gravitational field; and the derivatives of the two factors, the ***Newton***ian and the ***Einstein***ian, are identically the same. Thus, the Einstein tensor assumes that the force at any point in the gravitational field is inversely proportional to the square of its distance from the central body. ***Einstein***, himself, states this fact in unmistakable words, when he says that this fundamental tensor »plays the part of the gravitational potential«.

With this »Ansatz«, or assumption as to the value of the fundamental tensor, ***Einstein*** writes the resultant equation for the component acceleration in the direction of the axis of X as (p. 835):

$$\frac{d^2x}{ds^2} = -\frac{\alpha}{2}\frac{x}{r^3}\,. \tag{1}$$

And there are two similar equations for the components in the directions of X_2 and X_3, or of Y and Z as usually written. If the axis of X be taken as coincident with the radius-vector of the planet, then this equation would be written:

$$\frac{d^2r}{ds^2} = -\frac{\alpha}{2}\frac{1}{r^2} \tag{1a}$$

and it is apparent that the law of force is the »inverse square« law of ***Newton***. Thus the ***Einstein*** equation is identical with the ordinary ***Newton***ian formula: the so-called »first approximation« is simply classical mechanics; uniform constant time, measuring-rods of constant lengths, and ***Newton***'s law of inverse squares.

To obtain his »second approximation« ***Einstein*** transforms the formulas of the »first approximation« by introducing additional tensors representing the relativity tenets of varying units of time and of space. This transformation is accomplished in successive steps by use of the ordinary formulas of calculus for a change in the independent variable. In all this work, however, the ***Newton***ian value of the fundamental tensor is left unchanged. The resultant typical equation, which represents the component acceleration of his »second approximation«, is (p. 837):

$$\frac{d^2x}{ds^2} = -\frac{\alpha}{2}\frac{x}{r^3}\left(1 + \frac{3B^2}{r^2}\right) \tag{2}$$

where B is the constant of areas. And this equation differs in form from that of the »first approximation« solely by the presence of the factor $(1+3B^2/r^2)$ in the second member. It is the small term, $3B^2/r^2$, of this factor which, in the integrated form of the equation, becomes the so-called »perihelial term« of the relativists, and which gives rise to the celebrated 43″ per century in the supposed relativity motion of Mercury.

In these two equations (1) and (2) the letter s represents the »time variable« (p. 837), and ds thus represents an infinitely

[1]) *Einstein*, »Die Grundlage der allgemeinen Relativitätstheorie«, Ann. d. Phys. **49** (1916).

[2]) *Einstein*, »Erklärung der Perihelbewegung des Merkur«. Sitzungsber. d. Preuß. Akad. d. Wiss. 47.831 (1915). The references to this Paper are noted in the text by the number of the page on which the quotation is found.

small interval of time. Although designated in the formulas by the same letter, or symbol, these time variables are not the same. This fact of non-identity is shown not only by the mathematical processes used by *Einstein*, but it is stated by him in clear and unequivocal words. In the section, in which are introduced the necessary factors to change the »first« into the »second« approximation, is to be found the direct statement (p. 837):

»Wählt man endlich $s\sqrt{(1-2A)}$ als Zeitvariable, und nennt man letztere wieder s, · · ·«.

In this transformation *Einstein* introduces other »time factors« in addition to the one here specifically admitted. The entire transformation factor, as used by *Einstein*, is:

$$1+3B^2/r^2\,.$$

And this factor represents the relation between the respective $(\mathrm{d}s)^2$ of the two formulas: represents the ratio of the squares of the time intervals, or of the »time variables«.

Einstein, however, failed to complete the transformation. In his equation (2) one factor is still expressed in terms of the original time variable. This is α, the factor, or constant »determined by the mass of the sun«. In the formulas and methods of celestial dynamics this factor or symbol (whether designated by m, k^2, KM, or α) is not an absolute constant. In the ordinary formulas of physics and dynamics there are three fundamental, arbitrary, and independent units of measurement: those of length, of time, and of mass. In celestial dynamics, however, there are only two such independent units: those of length, and of time. In celestial dynamics there is no way of directly measuring mass: there is no way of determining how much matter there is in the earth, or in the sun. This can only be determined indirectly through the motions caused by these bodies. The mass of the sun is measured by the acceleration it will give to a particle of matter at a definite distance and in a definite interval of time. Mass in celestial mechanics, therefore, is a derived unit, and depends upon and is expressed in terms of the adopted units for time and space. This is shown by the very equations under discussion. In order that the various terms shall be homogeneous as to units, the constant α must be of the dimensions L^3/T^2. Hence, while, of course, the mass of the sun itself is constant, the expression for, or the measure of such mass will change with the units employed for space and time. The ordinary astronomical units are the mean solar day and the mean distance (150000000 kilometers) of the earth from the sun. In these astronomical units, the mass of the sun is expressed by the fraction 1/3379, or more accurately by the figure:

$$0.000295969.$$

But, if either, or both of these units of length and of time be changed, then the numerical figure, or constant representing the mass of the sun will also be changed.

This variation of the »constant determined by the mass of the sun« with the time unit employed is well shown by the fundamental formula of planetary motion:

$$\mu=(\sqrt{m})/a^{3/2}$$

where m, or α in *Einstein*'s notation, is the mass of the sun, μ the mean or average motion of the planet in unit time, and a the semi-axis major of the orbit. Thus the mean motion (μ) and the measure of the mass ($\sqrt{m}$) vary, the one with the other: if one be constant the other is constant; if one changes, the other must change. In one solar day, as may be easily verified, the earth moves over 3548″.193 and the constant for the mass of the sun (m) has the value 1/3379, as heretofore given. But in the shorter sidereal day the earth travels only 3538″.505, and the above unquestioned formula of celestial mechanics shows that, for sidereal time, the value of mass constant (m) must be reduced to 1/3398.

It thus appears very simply that, if one changes the unit of time, one must correspondingly change the »constant« for the mass of the sun. But *Einstein* was apparently unfamiliar with this ordinary fact of celestial mechanics, and he applied the methods of the mathematician and of laboratory physics to his problem of planetary motions, and kept his mass symbol, α, an absolute constant, instead of changing it to conform to his new unit of theoretical, relativity time. And thus the equation (2) of his »second approximation« is not homogeneous in units: the constant, α, is expressed in terms of the uniform time of classical astronomy, and the »time variable« (s) in terms of relativity units.

This non-homogeneity as to units employed in the various terms of the *Einstein* equation may be corrected by noting that, as the expression for mass in celestial mechanics varies with the square of the time unit employed, the value of the constant becomes, when expressed in relativity units:

$$\alpha_r=\alpha(1+3B^2/r^2).$$

With this relativity value for the »constant determined by the mass of the sun«, *Einstein*'s equation (2) becomes:

$$\frac{\mathrm{d}^2x}{\mathrm{d}s^2}=-\frac{\alpha_r}{2}\frac{x}{r^3}\,. \tag{3}$$

The equation is now homogeneous as to units of time, and it is of identically the same form as that of *Newton*. Thus a complete transformation in time units leaves the form of the equation unchanged; and the geometrical solution is identically the same as that of *Newton*: a fixed ellipse.

All of this is in accord with the fundamental relativity concepts of varying time. In the »second approximation« time is measured in terms of the local, relativity time of the theoretical »clock« attached to the planet. And this local time varies with the speed of the planet through space, and changes with its varying distance from the sun. Thus, *Einstein*, in his fundamental paper, states:

»Thus the clock goes more slowly if set up in the neighborhood of ponderable masses.«

and gives formulas for the variation in clock rates with the distance of the clock from the central gravitational body (the sun), and for differing speeds of the planet and clock through space. Hence as the relativist observer on the planet approaches nearer and nearer the sun, his speed will increase, his clock will run more and more slowly, his unit of time will become greater, and his expression for, or his measure of the mass of the sun (α) will also become greater and greater. Now, according to the formulas, this variation in α is just exactly what it should be to adjust this measure of the mass of the sun to the changing time unit of the travelling »relativity clock«. The relation between the time unit ($\mathrm{d}s$) and the measure of the mass of the sun (α) is thus constant and the same in all parts of the orbit, and the geometrical solution of the equation is identically the same as though ds and α were constant and equal to the respective *Newton*ian values.

Another way of showing that these two formulas, the *Newton*ian and the *Einstein*ian, represent the same general law of gravitation is by considering the specific meanings of the formulas themselves. The first members of the equations (1) and (2) each represent component accelerations of the planet caused by the gravitational field of the sun: the first in terms of units of classical mechanics; the second in terms of the special units of general relativity. The second members show that these accelerations are not equal. This inequality of the two accelerations may be caused either by different laws of gravitation, or by different intervals of time during which the accelerations were measured. *Einstein* apparently accepts the first alternative, and claims that the second formula represents a different law of gravitation from that of *Newton*. This without any investigation, or adequate explanation, and in spite of the admitted fact (p. 837) that the time intervals, or »time variables« are different in the two formulas.

Now the measure of the acceleration of a body varies directly with the square of the time unit. The acceleration at the surface of the earth, due to the earth's gravitational action, is some 9.806 meters per second: in two seconds it is four times this, or 39.22 meters: and when measured in terms of minutes, this same acceleration is some 35.2 kilometers. The two accelerations under consideration, that given by the *Newton*ian formula (1) and that given by the *Einstein*ian formula (2), are accelerations for, and measured in, different units of time (p. 837). And these accelerations, if due to the same gravitational action, should be directly proportional to the squares of the respective »time variables«, or units of time. From *Einstein*'s own formulas and statements, one has seen that these time variables are different, and one can find that the ratio of the squares of these time variables is given by the formula:

$$1+3B^2/r^2.$$

But this is the factor which appears in the second member of *Einstein*'s equation, and which alone represents the difference between the two formulas. Thus the increased acceleration of *Einstein*'s »second approximation« is due, solely and entirely, to the increased time interval in which the acceleration is measured.

The »first approximation«, or *Newton*ian formula (1), measures the component acceleration, due to the sun's gravitational field, in terms of units of standard, uniform, astronomical time: the »second approximation«, or *Einstein*ian formula (2), measures the same acceleration, due to the same gravitational action, in terms of units of the local, planetary time of general relativity. The two formulas represent the same law of gravitation and the same effect of gravitation; but they measure that effect in terms of different units of time.

And thus the »perihelial term« of *Einstein* is merely a factor of transformation from one system of units to another, analogous to the factors which change feet into meters, or dollars into marks. The unit of time, used by *Einstein* in his form of the planetary equation, is greater than a unit of ordinary astronomical time; and in a definite number of such relativity days, therefore, a planet will travel farther than it would in the same number of astronomical days. In one hundred years of *Einstein*'s theoretical relativity time Mercury will travel farther than in a century of astronomical time. In the planetary formula the factor of reduction for this difference appears as 43″; and it is this 43″ that *Einstein* erroneously interprets as a »perihelial motion« of the planet's orbit.

Thus the original, but now unquoted and apparently forgotten, paper of *Einstein* shows, directly and without the possibility of doubt, that his formula of planetary motion is based upon and involves the *Newton*ian law of inverse squares; shows that he derived his formula from that of *Newton* by a direct transformation in time units. The so-called relativity rotation of planetary orbits is a mathematical illusion: an illusion due to an incomplete mathematical transformation and to an illogical interpretation of the resulting formula. The *Newton*ian law has not been abolished: there is no *Einstein*ian law of gravitation.

Columbia University, New York, 1930 Jan. 18. *Ch. Lane Poor.*

Weitere Bemerkungen zum Nova-Problem. Von *H. Gehne.*

Die Ausführungen, die ich in AN **235**.93 zu dem Thema »Weiße Zwerge und Neue Sterne« machte, bedürfen in manchen Punkten der Ergänzung. Es kommt mir dabei auch darauf an, an Hand der »*Fermi-Dirac*-Statistik« zu zeigen, wie sich die großen Energiebeträge ergeben, die der dichten Materie in den Weißen Zwergen zukommen. Diese Untersuchung setze ich an den Schluß dieses Berichtes. Es sei aber gleich bemerkt, daß ich mich darauf beschränken werde, nur die durch Ionisation freigewordenen Elektronen als »*Fermi*-Gas« starker Entartung zu behandeln. Zu meinem früheren Artikel sind ein paar Ergänzungen zu machen, die erstens die Anbringung der bolometrischen Korrektion an die visuellen Helligkeiten betreffen und zweitens hinsichtlich der Frage der Polytropenklasse die erforderliche Unbeschränktheit herstellen. Weiter werde ich kurz zeigen, daß sich der absteigende Ast der Lichtkurve Neuer Sterne, den ich in AN 5619 als Kontraktion verbunden mit Pulsationen deutete, als eine Lösung der *Vogt-Jeans*schen Differentialgleichung der Stabilität der Sterne ergibt.

Pike[1]) hat in MN **89** das Nova-Problem von einem ganz ähnlichen Standpunkt behandelt wie ich und hat dabei die bolometrischen Korrektionen berücksichtigt. Ausgehend von den bei *Eddington* gegebenen bolometrischen Korrektionen für die effektiven Temperaturen von 7000–20000° rechnet *Pike* dieselben bis 60000° und extrapoliert weiter bis 100000°. Für die letztere Temperatur wird die bolometrische Korrektion $\varDelta m = +6^{m}3$. *Pike* erhält diesen Wert durch graphische Extrapolation, indem er $\log T$ gegen $\log \varDelta m$ aufträgt; benutzt man dagegen T und $\varDelta m$ selber, so kann man ohne Schwierigkeiten bolometrische Korrektionen erhalten, die für $T > 60000°$

[1]) *S. R. Pike*, MN **89**.538 (The physical conditions in New Stars).