# THE QUANTUM MECHANICAL MECHANISM BEHIND THE END RESULTS OF THE GTR: MATTER IS BUILT ON THE LORENTZ INVARIANT FRAMEWORK ENERGY x MASS x LENGTH$^2$ ~ h$^2$

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**ABSTRACT**

In a previous article, we have provided a whole new approach toward the end results of the General Theory of Relativity (GTR), based on just the energy conservation law, in the broader sense of the concept of "energy" embodying the mass & energy equivalence of the Special Theory of Relativity (STR). Thus, our approach was solely based on this latter theory (excluding the necessity of assuming the principle of equivalence of Einstein). According to our approach, the rest mass of an object embedded in a gravitational field (in fact in any field the object interacts with) decreases as much as the binding energy coming into play. Thereby, based on a general quantum mechanical theorem we prove, its internal energy, weakens as much; thus the classical red shift and time dilation.

This theorem (we did not have any room to provide a general proof of, previously), basically says that, if in a relativistic or non-relativistic quantum mechanical description, composed properly, the mass of the object in hand is multiplied by an arbitrary number $\gamma$, then the total energy of it, is multiplied by $\gamma$, and its size is divided by $\gamma$. This number however may very well not be arbitrary. For example, it would specify how much the rest mass of the object is altered when this is embedded in a gravitational field, leading via quantum mechanics, strikingly at once, to the end results of the GTR. This manipulation further yields the invariance of the quantity [energy x mass x size$^2$]. We conclude that, it is this quantum mechanical invariance, necessarily strapped to the square of the Planck Constant, which constitutes a given framework regarding the matter architecture, and insures the end results of the GTR.

Not only that our approach is incomparably simple as compared to the GTR, but it also avoids all incompatibilities (such as the breaking of the relativistic relationship $E=mc^2$), or inconsistencies (such as the breaking of the energy conservation law, as well as the breaking of momentum conservation law), or blockades (such as the impossibility of the quantization of the gravitational field), thus opens a whole clean avenue toward a unification of fields, and understanding of the matter and the universe at all levels, with just the same set of tools.

Since we do not have to use the principle of equivalence of the GTR, amongst others, we could show that, just like the gravitational field, the electric field too slows down the internal mechanism of a clock, had this interacted with the field. This result explains substantially, the retardation of the decay of the muon, bound to a nucleus.

**Keywords:** Mass Deficiency, Special Theory of Relativity, General of Theory of Relativity, Quantum Mechanics, Metric Change, Muon

## 1. INTRODUCTION

It was the author's idea that, owing to the law of conservation of energy, the internal energy of a particle bound to a field, should be weakened as much as the binding energy coming into play, no matter whether it is bound to an electric field in the atomic world, or a gravitational field in the celestial world, or any other field it can interact with.

All internal mechanisms, the particle of concern may embody, shall be affected accordingly, provided that the particle's inner articulations in relation to each other, are not degenerated via the binding process.

This is the essence of our approach, and we will show that, amongst other things, it can be successfully applied to predict the end results of the GTR, on the basis of merely quantum mechanics

We know that a gravitational binding does not alter the particle's inner articulations in relation to each other. That is, all parts of the bound particle, is affected in the same way, so that the particle holds up its original identity (unless perhaps the field is exceptionally strong).

Likewise, a charge particle will hold up its original identity when electrically bound, provided that the electric field affects equally all parts of the charged particle in consideration. This, at the first strike, requires that, all parts of the charged particle are charged, and uniformly charged.

In a nuclear binding however, the particle, such as the neutron, may not preserve its original identity, just like in a vegetable soup, the ingredients of concern cannot all the way, preserve their initial specific characteristics.

Below, for the sake of completeness, we will first briefly sketch the idea in question, and then give a general account of the quantum mechanical theorem we have discovered, underneath.

## 2. BRIEF SKETCH OF THE IDEA

Here, for simplicity, though without any loss of generality, we assume that the particle in question, is insignificant as compared to the host object binding it, so that we only have to worry about the changes this particle would undergo [1]; in other words, the host object binding the particle in consideration, will remain practically untouched through the binding process.

The binding in question, can be any type of binding provided that it does not degenerate the bound particle, though herein, we will mainly refer to gravitational binding. As we will see, the way we will develop our approach, does not at all restrict us to gravitational interaction; quite on the contrary it allows us to consider any field the particle in hand, interacts with (provided that the field dos not deteriorate the particle's identity).

Let us explain a bit further, why we would like to assume that the bound particle is insignificant as compared, to the binding host object.

Suppose an observer on Earth sets free from his elevated right hand, a stone to a free fall, and little after, he catches it with his lowered left hand. The overall energy of the closed system made of the stone in question and Earth (and just the two, i.e. disregarding, the air in between), must stay constant all along the free fall of the stone. The law of linear momentum conservation law requires that, because Earth is considerably more massive than the stone; in regards to a distant star, it will remain in place. Therefore throughout, it is only the stone, which gains kinetic energy. Once the observer interferes and catches the stone, with his lowered left hand, he retrieves from the closed system (made of the falling stone and Earth), an amount of energy equal to the kinetic energy, the stone would have acquired on the way, and this energy, evidently, is retrieved from the stone alone, once this is stopped.*

* Once the observer catches the stone, basically, his hand will be heated up. We may very well suppose that the related heat is released to the outer space as an infrared radiation. Thus indeed, the system made of Earth and the falling stone, looses energy, right after the observer catches the stone. The observer remains practically untouched, following the cooling down process.

Conversey, as the observer highers the stone, he will come to pile up an extra amount of energy equal to the energy he has to furnish to it, to elevate it to the given altitude.

Let us simplify things. Suppose one highers, just one atom of hydrogen. Then, what would it mean that, via highering the hydrogen atom, one piles up in it, an extra amount of energy equal to the energy he would have furnished to it?

**Owing to the Relativistic Equivalence of Mass & Energy, the Rest Mass of the Hydrogen Atom Will Get Altered. Wherever This Mass Intervenes, We Will Observe a Related Change.**

The answer to the question we just introduced, is the following.

Owing to the relativistic equivalence of mass & energy, the rest mass of the hydrogen atom will get increased as much .[†]

Thus, wherever this mass intervenes, we will observe a related change.

We can further analyze the situation in the following way.

Any entity must display an internal dynamics, based on a given "internal mechanism".

What do we mean by "internal mechanism"?

This is an "intrinsic periodic phenomenon". Already de Broglie has considered such a phenomenon in regards to a given particle at rest, were this, totally transformed into electromagnetic radiation [2].

We can be more specific than that:

A diatomic molecule for instance, vibrates. The motion in question delineates a particular "internal dynamics".

A diatomic molecule can as well rotate. The related motion delineates another internal dynamics.

One can associate a total energy with every specific internal dynamics, coming into play, provided that the "internal motions" in question can be envisaged to be independent from each other.

Thus, we can conceive any entity to embody an "internal dynamics, driven by a given internal mechanism. A given entity may embody many internal mechanisms, working simultaneously. To make things simple, let us assume that, there is only one internal mechanism of concern.

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[†] Note that within the frame of the general theory of relativity, a mass imbedded in a gravitational field dilates, and a mass carried away from a gravitational field, contracts. But then, the relativistic equivalence between mass & energy is broken. The present approach does not give rise to such annoyances.

**Clock Labor, Clock Mass, Clock Space, Clock Unit Period of Time**

Any such mechanism will delineate a clock's posture. It will then consist in a "clock labor", taking place in a given "clock space", and achieved by a "clock mass", displaying a "unit period of time". The resulting "internal dynamics" shall be founded on a "total energy", framing the "clock's mass motion".

Based on the Bohr Atom Model, the internal mechanism turns out to be the rotational motion of the electron around the proton; the clock mass is the reduced mass of the electron and the proton; the unit period of time is the period of time, a mass equal to the reduced mass in consideration, takes to rotate around the center of mass of the electron and the proton; since the reduced mass is practically the mass of the electron, then the unit period of time virtually becomes, the period of time the electron takes to rotate around the proton.

In more modern terms, the internal dynamics we refer to, can be characterized with the (no more than probabilistically predictable, yet still) measurable momentum of the electron, to be considered along with the usual quantum mechanical total energy.

Based on a non-relativistic approach, the total energy $E_{\infty n}$ of the hydrogen atom, at the $n^{th}$ principal level, in empty space, is (in CGS unit system), as usual, given by[‡]

$$E_{\infty n} = -\frac{2\pi^2 e^4 \mu_{0\infty}}{n^2 h^2} \; ; \tag{1}$$

*(total energy of the hydrogen atom)*

here e is the charge intensity of the electron or that of the proton, $\mu_{0\infty}$ is the reduced mass of the electron and the proton in empty space, and h is the Planck Constant; recall that $\mu_{0\infty}$ is practically equal to the electron's rest mass in empty space.

**Decrease of the Mass, and Red Shift of Gravitationally Bound Hydrogen's Light**

A transition in the hydrogen atom, between an upper level n, to a lower level m, in empty space, yields an electromagnetic radiation of frequency $\nu_{\infty n \to m}$, so that

$$h\nu_{\infty n \to m} = \frac{2\pi^2 e^4 \mu_{0\infty}}{h^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) . \tag{2}$$

*(electromagnetic energy released from the hydrogen atom, through a* n to m *transition)*

‡ First notice that, Eq.(1), is a Lorentz invariant relationship, even though it is based on a non-relativistic approach (i.e. that of Bohr). Second, notice that Eq.(1) relates the total energy to the reduced mass, in terms of the universal constants e and h. A compact relationship, free of e, can be written, as usual, as

$$-8\pi^2 E_{\infty n} \mu_{0\infty} r_{\infty n}^2 = n^2 h^2 ,$$

where $r_{\infty n}$ is the Bohr radius, as referred to the center of mass of the proton and the electron. The reason we mention this, is that, it delineates (as we will soon show), a general matter architecture (specified along with the title of this article), well insuring the end results of the GTR.

When bound, at an elevation R, to a uniformly structured celestial body of mass M, the reduced mass $\mu_{0\infty}$ of the hydrogen atom (just like any mass taking place within the frame of this atom), is decreased as much as the gravitational binding energy $E_B(R)$ coming into play, to become $\mu_0(R)$:[§]

$$E_B(R) = [\mu_{0\infty} - \mu_0(R)]c^2 \ ; \tag{3}$$

*(gravitational binding energy, in terms of the mass variation the bound particle displays)*

here c, is the velocity of light, in empty space.

The gravitational binding energy $E_B(R)$, to a first approximation, can be expressed as

$$E_B(R) = G\frac{M\mu_{0\infty}}{R} \ ; \tag{4}$$

*(gravitational binding energy, derived from Newton's law of attraction)*

G is the universal gravitational constant, M the mass of the hosting celestial object, and R is the elevation at which the atom is bound.

Here we have tacitly adopted the Newton's law of gravitational attraction. We will though soon elucidate the fact that, the structure of this law is imposed by the STR.

Eq. (3) and (4) furnish $\mu_0(R)$:

$$\mu_0(R) = \mu_{0\infty}[1 - \alpha(R)] \ , \tag{5}$$

*(mass of the bound object)*

where $\alpha(R)$ is given by

$$\alpha(R) = \frac{GM}{Rc^2} \ . \tag{6}$$

Note that, to be rigorous one should consider the continuous change on the mass $\mu_0(r)$, through the binding process [1]. One should then, reconsider Eqs. (3) and (4):

§ The proton's mass $m_{P\infty}$ is decreased as much as the binding energy; so is the electron's mass $m_{e\infty}$. Thus the reduced mass $\mu_{0\infty} = m_{P\infty}m_{e\infty}/(m_{P\infty} + m_{e\infty})$ is also decreased as much.

$$dE_B(r) = -c^2 d\mu_0(r) \ , \tag{7}$$

$$dE_B(r) = -G\frac{M\mu_0(r)}{r}dr \ ; \tag{8}$$

$d\mu_0(r)$, here, is the infinitely small increase the rest mass $\mu_0(r)$ undergoes, if it is moved quasistatically through dr.

Thus at the elevation R, one arrives at

$$\mu_0(R) = \mu_{0\infty} e^{-\alpha(R)} \ . \tag{9}$$

*(rigorous expression for the gravitationally bound mass)*

In our approach, both the electron charge and the Planck Constant, are universal constants, and they remain untouched in either a gravitational field or an electric field, or seemingly, any other field. Note that they are as well Lorentz invariant quantities. Recall on the other hand that, the universal gravitational constant G is not Lorentz invariant. (So it is not as "universal", as one may think it is.)

Thus, in a gravitational field, a change in $E_{\infty n}$ of Eq.(1), and accordingly a change in $\nu_{\infty(n\to m)}$ of Eq.(2), must be based on a corresponding change, the reduced mass of the hydrogen atom undergoes within the frame of the suitable quantum mechanical description:

$$h\left|\Delta\nu_{\infty(n\to m)}\right| = \frac{2\pi^2 e^4 \left|\Delta\mu_0\right|}{h^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) = h\nu_{\infty(n\to m)}\frac{\left|\Delta\mu_0\right|}{\mu_{0\infty}} \ ; \tag{10}$$

*(change in the energy of the hydrogen atom's light, due to the change of mass)*

here the change $\Delta\mu_0$ in the reduced mass $\mu_{0\infty}$ of the hydrogen atom, in empty space, were this embedded in the gravitational field in consideration, is given by [cf. Eq. (9)]

$$\Delta\mu_0 = \mu_0(R) - \mu_{0\infty} = -\mu_{0\infty}(1 - e^{-\alpha}) \ . \tag{11}$$

*(change in the hydrogen atom's reduced mass due to gravitational binding)*

The negative sign that appears over here, points to the fact that, the hydrogen mass in the gravitational field, decreases.

Eq.(3), along with Eq.(10), leads to

$$\left|\Delta\nu_{\infty(n\to m)}\right| = \nu_{\infty(n\to m)}\frac{E_B(R)}{\mu_{o\infty}c^2} \quad . \tag{12}$$

*(red shift in the energy for the hydrogen atom's light, due to gravitational binding)*

This is nothing else, but a red shift in the energy of the hydrogen light, due to gravitational binding.

Note that, along with Eq.(11), the binding energy $E_B(R)$ of the particle bound at the elevation R, becomes

$$E_B(R) = [\mu_{0\infty} - \mu_0(R)]c^2 = \mu_{0\infty}c^2(1-e^{-\alpha}), \tag{13}$$

*(rigorous expression of the binding energy)*

which, for a small $\alpha$, and via Eq.(6), yields

$$E_B(R) \cong G\frac{M\mu_0}{R} \ , \tag{14}$$

which well turns out to be Eq.(4).

The frequency $\nu_{n\to m}(R)$, the hydrogen atom would produce at the altitude R, through a n to m transition, becomes

$$\nu_{n\to m}(R) = \nu_{\infty(n\to m)} - \nu_{\infty(n\to m)}\frac{E_B(R)}{\mu_{o\infty}c^2} = \nu_{\infty(n\to m)}\left[1-\frac{E_B(R)}{\mu_{o\infty}c^2}\right] \quad . \tag{15}$$

*(red shifted frequency of the bound hydrogen atom's light)*

It is weakened as much as $[1-E_B(R)/(\mu_{o\infty}c^2)]$.

**Decrease of the Mass and Stretching of the Unit Period of Time, as well as the Size of the Clock Space of a gravitationally Bound Clock**

As stated, de Broglie in his doctorate thesis, considered the electromagnetic energy amounting to the entire mass of the particle, i.e. the overall relativistic energy of it (at rest), and this, even long before the annihilation of the electron with a positron was discovered.

Thus, he wrote

$$h\nu_{0\infty} = m_{0\infty}c^2 \ , \tag{16}$$

*(the frequency de Broglie has associated with the presumed intrinsic periodic phenomenon an object of a given mass would delineate, or the same the frequency to be associated with the overall relativistic energy of the object in hand)*

for a particle of mass $m_{0\infty}$ in empty space; $\nu_{0\infty}$ is the frequency of the electromagnetic radiation, were the mass $m_{0\infty}$ somehow annihilated.

If the particle is embedded in a gravitational field created by the host celestial body of mass M, at the altitude R, then according to our approach, the electromagnetic energy $h\nu_{0\infty}$ will become $h\nu_0(R)$, which can be, via Eq.(9) written as

$$h\nu_0(R) = m_{0\infty}c^2 e^{-\alpha(R)} = h\nu_{0\infty}e^{-\alpha(R)} \ . \tag{17}$$

*(the overall relativistic energy of the given particle when this is embedded, at rest, in the gravitational field in consideration)*

This equation, via Eq.(13), but written for the mass $m_{0\infty}$, can be written as

$$h\nu_0(R) = h\nu_{0\infty}\left[1 - \frac{E_B(R)}{\mu_{o\infty}c^2}\right] \ , \tag{18}$$

*(the overall relativistic energy of the entity in hand, weakening as much as the binding energy coming into play)*

where $E_B(R)$, now becomes the binding energy of the particle of concern, at R.

Eq.(18) tells us that, when bound, the "overall relativistic energy" of the entity in hand, is weakened as much $\left[1 - E_B(R)/(\mu_{o\infty}c^2)\right]$.

For an electromagnetic radiation, one by definition, has

$$c = \frac{\text{wavelength}}{\text{period of time}} = \text{wavelength x frequency} . \tag{19}$$

In other words, the frequency and the corresponding period of time associated with the given electromagnetic radiation are inversely proportional to each other.

Thus, let $T_{0\infty}$ be the period of time associated with the frequency $\nu_{0\infty}$. When the particle is embedded in the gravitational field in consideration, its internal energy weakens, $T_{0\infty}$ stretches just as much, to become $T_0(R)$, i.e.

$$T_0(R) = \frac{T_{0\infty}}{1 - \dfrac{E_B(R)}{m_{o\infty}c^2}} \ . \tag{20}$$

*(the period of time associated with the overall relativistic energy of the entity in hand, stretching as much as the binding energy coming into play)*

The same should be expected to occur in relation to any package of energy driving a given internal dynamics, within the particle in hand.

Let us for instance consider the ground rotational period of time $T_{e\infty}$ of the electron of mass $m_{e\infty}$ around the proton in empty space, within the frame of Bohr Atom Model.

Assuming that the proton is infinitely more massive than the electron, $T_{e\infty}$ turns out to be[**]

$$T_{e\infty} = \frac{h^3}{4\pi^2 (e^2)^2 m_{e\infty}} \quad . \tag{21}$$

*(electron's rotational period of time around the proton, in empty space)*

Here for simplicity we have assumed that the proton is infinitely more massive than the electron.

When the hydrogen atom is embedded in the gravitational field in consideration, $m_{e\infty}$ will get decreased in accordance with Eq.(9); thus $T_{e\infty}$ stretches as much, to become $T_e(R)$, at the altitude R, so that

$$T_e(R) = \frac{h^3}{4\pi^2 (e^2)^2 m_{e\infty} e^{-\alpha(R)}} = T_{e\infty} e^{\alpha(R)} = \frac{T_{e\infty}}{1 - \dfrac{E_B(R)}{m_{e\infty} c^2}} \quad . \tag{22}$$

*(electron's rotational period of time around the proton, stretching as much the binding energy coming into play)*

This means that the related internal dynamics loosens as much as the internal energy loss, amounting to the binding energy (c.q.f.d.).

Any information coming from the atom, such as an electromagnetic radiation, based on the energy difference between two states, must as well weaken as much [cf. Eq.(15)]; thus, once again, the gravitational red shift.

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[**] The Bohr ground rotational velocity (assuming that the proton is infinitely more massive than the electron), is given by

$$v_n = \frac{2\pi e^2}{h} = \frac{2\pi r_{e\infty}}{T_{e\infty}} \ ,$$

where $r_{e\infty}$ is the Bohr ground orbit radius in empty space; it is expressed as

$$r_{e\infty} = \frac{h^2}{4\pi^2 e^2 m_{e\infty}} \ .$$

This then, well leads to Eq.(21).

Conversely, as we elevate the hydrogen atom, in a gravitational field, owing to the relativistic equivalence between mass & energy, we come to increase its mass. This in return strengthens just as much, the total energy of the atom, which concurrently shortens as much the period of time, one can associate with the electromagnetic radiation, one would obtain, if the entire mass of the hydrogen atom were transformed into electromagnetic energy.

Note that by the same token, the size of the clock space of the object embedded in the gravitational field, stretches. This can be right away seen from Eq.(19). The fact that the mass decrease [via Eq.(16)] yields, a decrease of the frequency corresponding to the electromagnetic energy, were the object annihilated, means that (since c remains constant), the corresponding wavelength must stretch just as much.[††]

Thus, let $\lambda_{0\infty}$ be the period associated with the frequency $\nu_{0\infty}$. When the particle of mass $m_{0\infty}$ is embedded in the gravitational field in consideration, $\lambda_{0\infty}$ stretches just as much, to become $\lambda_0(R)$, i.e.

$$\lambda_0(R) = \frac{\lambda_{0\infty}}{1 - \frac{E_B(R)}{m_{o\infty}c^2}} \quad . \tag{23}$$

*(clock space size stretching as much as the binding energy coming into play)*

Accordingly, any space size, in which a given internal dynamics takes place, must as well stretch, as much.

**We Have Elaborated on the Idea We Developed, to Predict All of the Measurable End Results of the General Theory of Relativity, Without Though Having to Assume the "Principle of Equivalence". As a Result, Our Approach Opens a Whole New Horizon.**

We have elaborated on this interesting idea to predict, all of the measurable end results of the general theory of relativity, without though any further assumption than the energy conservation law. It is evidently striking to obtain the same results as those of the general theory of relativity, through a completely different set up than that of this latter theory, up a second order Taylor Expansion [3, 4].

We would further like to emphasize that, it really must take quite a captivation, not to have considered for many decades that a gravitationally bound object would indeed exhibit straight, conventional changes, at the atomic level. The fact that Dirac, the monumental father of the relativistic quantum mechanics (who was deeply interested in the general theory of relativity, and who has published a book about it, just before he died), seems not to have given any thought, to quantum mechanical changes that a gravitationally bound object may exhibit, furnishes a firm sign about the strength of the captivation in question [5].

[††] Here again note that, what we draw in regards to masses and lengths, is the opposite of what the general theory of relativity establishes; in this latter theory indeed, when embedded in a gravitational, masses increase, and lengths contract.

Anyway, as we have demonstrated, given that along with our approach, one does not have to assume the "principle of equivalence" of the general theory of relativity, in order to arrive at the end results of this theory, he comes to discover a whole new horizon.

Generally speaking, the bound particle in consideration may just not be a mass (such as a stone) gravitationally bound to a celestial body; but it may also be a charged particle electrically bound to a charged body. The particle of concern, may even be a nuclear entity such as neutron, bound to a nuclear field, provided that, the binding process does not destroy the inner articulational characteristics of the original entity in hand.

At the first strike, our claim (that the internal energy of the bound particle must be decreased as much as the binding energy coming into play), may seem trivial, since this is nothing else but the energy conservation law. Yet, not only that our approach was overlooked for gravitationally bound objects, but also, even the "internal energy" of a charged particle, such as that of an electron, was not given a particular consideration, for say, the electron was always considered as a point-like particle; thereby an eventual change of its internal energy, was not considered at all.

**The Internal Dynamics of an Electron Bound to a Proton, Must Weaken As Much As the Binding Energy Coming Into Play.**

What we basically do here is to consider the internal dynamics of the entity in hand. According to our approach, the internal dynamics of the given entity does not only weaken when gravitationally bound to a celestial body, but it also weakens, say in the case of a charged particle, such as an electron or a muon, electrically bound to a charged object, such as a nucleus [6].

Then, the internal dynamics of an electron bound to a proton, must weaken as much as the binding energy coming into play.

Well, what is the internal dynamics of an electron? We do not know. So far, no one knows. But one way or the other, the electron must have an internal dynamics. The electron cannot be reduced to just a point. It has a mass, and a charge. These cannot be reduced to an imaginary point.

We may not know what the internal dynamics of an electron consists in. Nonetheless, we can well consider a muon, instead. This particle is unstable. It sure has a certain internal dynamics. Just like any other internal dynamics, the muon's internal dynamics too, constitutes a clock. It can be sensed via the muon's decay rate.
Thus, what we claim is that, when gravitationally or electrically bound, due to the energy conservation law, the internal dynamics of a muon must weaken, as much as the binding energy coming into play (assuming that the field of concern acts equally on all parts of the muon). Such a weakening must concurrently cause the retardation of muon's decay.

According to our approach, the muon's decay rate must slow down in an electric field, just like it is expected to slow down in a gravitational field.

That is the heart of the present approach.

**Alpha Decay of a Gravitationally Bound Nucleus**

Note that along our approach, the gravitational slowing down of a radioactive process, can be easily checked, for instance, on the basis of alpha disintegration.

The alpha disintegration half life $T_{\alpha\infty}$ in empty space, can be as usual expressed, as [7]

$$T_{\alpha\infty} = \frac{2\ln 2\{m_{\alpha\infty} R_\infty^2\} e^{\gamma}}{h} \quad ; \tag{24}$$

*(the alpha disintegration half life in empty space)*

here $m_{\alpha\infty}$ is the mass of alpha particle in empty space, $e^{\gamma}$ the barrier transmission coefficient, and $R_\infty$ the radius of the nucleus alpha disintegrating (still in empty space).

The mass of the alpha radioactive nucleus is decreased in a gravitational field, along with Eq.(9). Its size stretches as much, along with Eq.(23). One can show that, the barrier transmission coefficient is not altered.‡‡

The alpha disintegration half life $T_\alpha(R)$ at the altitude R, in the gravitational field, then becomes [cf. Eq.(20)]

$$T_\alpha(R) = \frac{T_{\alpha\infty}}{1 - \dfrac{E_B(R)}{m_{o\infty} c^2}} \quad . \tag{25}$$

*(the alpha disintegration half life stretching as much the binding energy coming into play)*

This result can be extended to other disintegration types, such as beta decay. But we will save the related effort for an other article, not only to show that, for instance the muon decay rate is retarded in exactly the same way as that presented above for an alpha particle, in a gravitational field, but also the muon decay rate retards, when the muon is bound to an electric field, such as that of a nucleus.

‡‡ To a good approximation we have [6],

$$\gamma = \pi\left(\frac{2Zzc}{137V}\right) - \frac{4}{137}\left(2z \frac{M_\infty}{m_{\alpha\infty}} \frac{R_\infty}{r_{0\infty}}\right) .$$

Z is the proton number of the daughter nucleus (resulting from the decay of the parent, achieved via throwing the alpha particle) , z is 2, V is the alpha particle's velocity with respect to the daughter nucleus, $M_\infty$ is the mass of the daughter, $R_\infty$ is the radius of the daughter, and $r_{0\infty}$ is the classical electron radius, i.e. $e^2/(m_{e\infty}c^2)$, where $m_{e\infty}$ is the electron's mass in empty space. Since $M_\infty$ and $m_{\alpha\infty}$ on the one hand, and $R_\infty$ and $r_{0\infty}$ on the other hand, are altered in exactly the same manner in a gravitational field, the barrier transmission coefficient, remains untouched (c.q.f.d.). (Recall that because both lengths and periods of time stretch, the velocities remain the same.)

## 3. RECAPITULATION: EXPECTED OCCURRENCES ABOUT A PARTICLE BOUND TO A FIELD

Let us state briefly, what we have so far established, no matter whether we may have, time to time, lacked generality. We will anyway soon introduce a general quantum mechanical theorem, encompassing all of our foregoing derivations.

**Theorem 1:** The energy conservation law requires that, a particle at rest, when embedded in a field, it interacts with, must discharge an amount of energy equal to the binding energy coming into play. Likewise, as the bound particle is carried out of the field in consideration, it will pile up, an amount of energy equal to its binding energy, which is in fact, the energy one has to furnish to the particle, in order to remove it out of the field. Here, for simplicity, though without any loss of generality, we assumed that the bound particle is insignificant as compared to the host object binding it.

**Theorem 2:** The energy conservation law, in the broader sense, drawn by the relativistic equivalence between mass & energy, requires that the "rest mass of a particle", when embedded in a field, the particle interacts with, in its entirety, decreases as much as the binding energy coming into play.

**Theorem 3:** Any internal dynamics the particle may embody, along with a given mass $m_{0\infty}$, which we call "clock mass", interacting with the field of concern, in its entirety, must accordingly, slow down. Thus, the frequency associated with a given internal phenomenon is red shifted as much as $\left[1-E_B/(m_{0\infty}c^2)\right]$, where $E_B$ is the binding energy coming into play. This result is the same as the red shift predicted by the general theory of relativity, were the particle embedded in a gravitational field, though it is obtained through a totally different set up than that of this latter theory. The corresponding period of time is, accordingly, stretched as much as $1/\left[1-E_B/(m_{0\infty}c^2)\right]$. This result is the same as that related to the clock retardation, predicted by the general theory of relativity, were the particle, still embedded in a gravitational field. The present approach furnishes the end results of the general theory of relativity, though through a totally different set up.

**Theorem 4:** Concurrently to the decrease of mass, and the stretching of unit period of time, the clock space size, the clock motion takes place in, stretches as much as $1/\left[1-E_B/(m_{0\infty}c^2)\right]$.

The above theorems are derived based on plain insights and simple checks. We will improve our approach by providing a mathematically sound and general quantum mechanical theorem, embodying all of the foregoing theorems, at once. But before this, it is worth to review the way we conceive the notion of "field".

## 4. DISCUSSION ABOUT THE CONCEPT OF FIELD: THE CLASSICAL COULOMB'S LAW, OR NEWTON'S LAW REIGNS IN BETWEEN, EXCLUSIVELY, STATIC CHARGES AND STATIC MASSES, RESPECTIVELY, WHILE THEIR $1/r_0^2$ DEPENDENCY IS A REQUIREMENT IMPOSED BY THE SPECIAL THEORY OF RELATIVITY (STR)

Sure, according to our approach the concept of field, has to be revised, and we are to clarify our stand point. The concept of force is the fundamental concept, to be experimentally relied on; the concept of field, though useful, is only, an extended concept. It cannot be measured; only force can be measured. Two interacting masses exert upon each other a gravitational force, just like two interacting charges exert upon each other an electric force.

We should stress that, the "total relativistic energy" delineated by two masses or two electric charges, according to our approach, is not anyway materialized by the surrounding space, but only by the "internal dynamics" of the charges of concern.

What is essential is the "conventional Coulomb's Force reigning in between two static charges, only", or the "conventional Newton's Force reigning in between two static masses, only", or any similar force, say the weak force, but expressed in similar terms.

Let us elaborate on this; let us first consider the Coulomb's Force.

The frame of Coulomb's Force is essential in the following way: The electric charges are Lorentz invariant; owing to this fact, the $1/\text{distance}^2$ dependency of the Coulomb's Force between two static charges, can be shown to be imposed by the STR, if this dependency is assumed to be in the form $1/\text{distance}^n$. A derivation of this fundamental result is provided in Appendix A.

Thus Coulomb's Force, reigning between only two static electric charges, as adopted, is thoroughly compatible with the STR. Recall however that, here we consider static charges, exclusively.

Our reasoning regarding Coulomb's Force, also holds for Newton's Force, with the difference that, in the latter case the masses are obviously not Lorentz invariant, but the product [(universal gravitational constant) x (mass one) x (mass two)] appearing on the denominator of the Newton's Force expression, is well Lorentz invariant. Here again, the $1/\text{distance}^2$ dependency of the Newton's Force between two static masses, is imposed by the STR. Thus, Newton's Force, as it is, but reigning between only two static masses, is also thoroughly compatible with the STR [8].

It seems that any other force law, must be built on similar characteristics. The Yukawa mesonic force law, constitutes a proof of this claim. To simplify our reasoning, let us continue on the basis of Coulomb's Law. What is believed so far, is that Coulomb's Force holds, if the source charge is static, regardless whether the test charge is at rest or in motion. However, we discover that, this is not so; if the test charge is in motion, then Coulomb's force is decreased by the factor $\sqrt{1-v_0^2/c_0^2}$ [8]. This occurrence drives us (contrary to what has been so far done), to consider the electron not in the accustomed simplistic way; we sympathize by the fact that, the electron is generally considered as a "point-like particle". It must be obvious though, as tiny as it may be, the electron cannot be reduced to a point, given that a "point" cannot be a "material being".

Thus, it is pointless to consider the electron as a point-like particle. The electron must embody an "internal dynamics", just like any other particle. Perhaps its "mass" is simply the "internal energy" of the "electric property", which we call "electric charge". This internal energy, is thus to be associated with (how ever it may be), the internal dynamics delineated by the electric charge.

When the electron is bound, say, to a proton, its internal dynamics is then (as a requirement of the energy conservation law), slown down, as much as the binding energy coming into play, assuming for simplicity that the proton (being much more massive than the electron), is not affected by the process of binding.

Our claim regarding the weakening of the internal dynamics of the bound electron can be rechecked, right away, through a backwards process (just the way we proceeded with the elevated hydrogen atom vis-à-vis a gravitational field). Suppose then we propose to bring back to infinity, the bound electron. Accordingly, we have to furnish to it, an amount of energy equal to its binding energy (still supposing that, moving away the electron, would not disturb, the supposedly infinitely more massive proton). The two particles, forming a "closed system"; furnishing energy to the electron, owing to the energy conservation law, will increase the internal energy, thus the rest mass of the latter. In other words, when entirely detached from the interaction domain, with the proton, the electron's rest mass would then get increased as much as the energy we would have furnished to it, i.e. by an amount equal to its original binding energy.

Hence, the free electron is not anymore the previous bound electron, or vice versa, the bound electron is not anymore the same as the free electron. It is indeed hard to accept that it would be, given that one cannot make an omelette, and keep the eggs as they are, prior to cooking!

The bound muon decay rate retardation, that we will consider herein seems to be an experimental proof of our assertion.

One still would question, "How the interaction between the proton and the electron occurs, if their respective energy is not spread in the surrounding space"; we have worked that out elsewhere [8]. In fact the same occurs between two celestial bodies, in exactly the same manner.

## 5. GENERAL QUANTUM MECHANICAL THEOREM: THE QUANTITY {(TOTAL ENERGY) X (CLOCK MASS) X (CLOCK SPACE SIZE)$^2$}, COMPOSED WITH RESPECT TO AN OBJECT, TURNS OUT TO BE A UNIVERSAL INVARIANT STRAPPED TO THE SQUARE OF THE PLANCK CONSTANT

Let us now give a rigorous prove of the above theorems we have drawn above, based on rather simple considerations. In order to do that, we demonstrate a general quantum mechanical theorem, in Appendix B. Thus, for an object existing in nature, we have shown elsewhere [9] the following theorem, first, on the basis of the Schrodinger Equation, as complex as this may be, then on the basis of the Dirac Equation, whichever may be appropriate, in relation to the object in hand. An atomistic or molecular object existing in nature, involves a potential energy, whose appearance is imposed by the STR, just like a "Coulomb Potential energy" or, a "Newton Potential energy", or anything as such. Thence, even a relativistic Dirac description, embodying potential energy terms made of potential energies other than the mentioned potential energies (compatible with the STR), may not represent a valid description, for the object in consideration.

**Theorem 5:** Consider a relativistic or non-relativistic quantum mechanical description of a given object, depending on whichever, may be appropriate. This description points to an internal dynamics which consists in a "clock motion", achieved in a "clock space", along with a "unit period of time". The description excludes "synthetic potential energies" (which may otherwise lead to incompatibilities with the STR). It is supposed to be based on $\mathcal{K}$ particles, altogether. If then different masses $m_{k0}$, $k = 1, \ldots, \mathcal{K}$, involved by this description of the object at rest, are all multiplied by the arbitrary number $\gamma$, the following two general results are conjointly obtained:

a) The total energy $E_0$ associated with the given clock's motion of the object is increased as much, or the same, the unit period of time $T_0$, of the motion associated with this energy, is decreased as much.

b) The characteristic length, or the size $\mathsf{R}_0$ to be associated with the given clock's motion of concern, contracts as much.

In mathematical words this is:

$$[(m_{k0},\ k = 1, \ldots, \mathcal{K}) \rightarrow (\gamma\, m_{k0},\ k = 1, \ldots, \mathcal{K})]$$

$$\Rightarrow [(E_0 \rightarrow \gamma\, E_0) \text{ or } (T_0 \rightarrow \frac{T_0}{\gamma}), \text{ and } (\mathsf{R}_0 \rightarrow \frac{\mathsf{R}_0}{\gamma})]. \tag{31}$$

What this theorem fundamentally says, is that, if an object ever experiences, for instance a mass decrease, then its total energy weakens as much, yielding the stretching of the period of its internal motion framed by the total energy in question (which should be considered quite understandable).

Next, we define a quantity $M_0$, which we call the "clock mass"; it is a compound mass whose motion constitutes the internal dynamics of the object; it is manufactured based on different masses embodied by the object in hand; thus multiplying these masses by $\gamma$, alters $M_0$ just as much. The clock mass, for instance is, as mentioned, the reduced mass of the proton and the electron, in the case of the hydrogen atom.

Eq.(31) immediately yields the invariance of the quantity $E_0M_0\mathsf{R}_0^2$. We call this invariance the "quantum mechanical invariance" of $E_0M_0\mathsf{R}_0^2$. It is remarkable, since $E_0M_0\mathsf{R}_0^2$, turns out to be Lorentz invariant as well (were the object brought into a uniform translational motion).

It is sure striking that the quantum mechanical invariance of $E_0M_0\mathsf{R}_0^2$ versus a mass change within the frame of the quantum mechanical description in question, and the relativistic invariance of it, are identical. This makes that, as uncomplicated as it may seem, the gravitational field, or any other field interacting with the object in hand, affects it, just like the uniform translational motion affects it. Indeed, our original claim was that, the rest mass of a given object embedded in a field, interacting with it, is decreased as much as the binding energy coming into play [cf. Eqs. (9), (13), and (18)], to be then input to the object's quantum mechanical description at rest; this happens to be, as we have demonstrated, the basis of the quantum mechanical invariance of $E_0M_0\mathsf{R}_0^2$, which in return, is nothing else, but a relativistic invariance yeld by the STR.

Note that the quantum mechanical invariance of $E_0M_0R_0^2$, is obtained no matter what the description in hand is relativistic or non-relativistic. The only requirement, as mentioned, is that the potential energy terms input to the description in hand, are compatible with the STR. Coulomb Potential is; so is Yukawa potential. These are the mere two forms, yeld by the Klein Gordon equation built on the fundamental relativistic relationship between momentum and energy (see Appendix B). Thus, any other potential representing any other possible field, must be made similarly.

We can further show that, the quantity $E_0M_0R_0^2$ is necessarily strapped to the square of the Planck Constant, $h^2$ (being proportional to it, through generally a complex, anyway dimensionless, and relativistically invariant quantity, which is somewhat a characteristic of the bond configuration of the elements making up the object in hand). This can be grasped as follows: $E_0M_0R_0^2$ is Lorentz invariant; the three quantities $E_0$, $M_0$, and $R_0$ are somehow related to each other, through the quantum mechanical description in hand; thus the product $E_0M_0R_0^2$ must be strapped to a Lorentz invariant universal constant involved by this description; this is nothing else but $h^2$. Hence $E_0M_0R_0^2$ is necessarily strapped to the square of the Planck Constant, $h^2$.

We call the relationship

$$E_0M_0R_0^2 \sim h^2, \tag{26}$$

*(quantum mechanical invariance yeld by the change of mass input to the quantum mechanical description in consideration, strapped to the square of the Planck Constant*

the UMA (Universal Matter Architecture) Cast.

It discloses already many structural properties, otherwise left obscure since very many decades [10,11,12].

Note that primarily what we state along with Eq.(26), is not the result of a "dimension analysis"; indeed $E_0M_0R_0^2$ would not be invariant in regards to a mass change, if the description of the object in consideration were not made of potential energy terms compatible with the STR, though of course, dimension-wise there would still be no problem. It is that the STR, stringently imposes an interrelation in between $E_0$, $M_0$ and $R_0$ (and this, already at rest), which is precisely the proportionality of $E_0M_0R_0^2$, to a Lorentz invariant universal constant, i.e. $h^2$. The spatial dependency of the forces bringing the elements of the object in hand, already constitutes a major ingredient of it.

In other terms, in order to insure the end results of the STR, when brought to a uniform, translational motion, or the end results of the GTR when embedded in a gravitational field, or any other field interacting with, the object in hand, already at rest, must be structured in just a given way. Or, the other way around, because the object, already at rest is structured in just a given way, it well yields the end results of the STR, when brought to a uniform, translational motion, or the end results of the general theory of relativity, when embedded in a gravitational field, or possibly any other field apt to interact with the object.

Figure 1, amongst others, along this line shows, how beautifully, the *lowest classical vibrational period of time* ($T_0$) is architectured versus the *clock mass* $(M_0 m_e)^{(1/2)}$ (composed of the *produc*t of, the *nuclei reduced* mass $M_0$, by the *electron mass* $m_e$), and the *internuclear distance* ($r_0$), to provide us with the Lorentz invariant interrelation for all diatomic molecules, thus, for the case we consider herein, especially for alkali molecules ($H_2$, $Li_2$, LiNa, $Na_2$, NaK, $K_2$, KRb, $Rb_2$, $Cs_2$),[12.13]

$$T_0 \sim \sqrt{M_0} r_0^2 \; ; \tag{27}$$

here, we kept just the nuclei reduced mass, since the *electron mass* $m_e$ is a constant throughout; the proportionality constant $4\pi^2 \sqrt{g} / h\sqrt{r_0 / r_{00}}$, where $r_{00}$ is the internuclear distance in $H_2$ molecule, and g a constant, we have associated with the alkali family, we specified right above, is Lorentz invariant; note further that the ratio $r_0 / r_{00}$ corresponds to the product of the quantum numbers, each associated with the electrons *(of the diatomic molecule at hand),* taking place in the bond of the molecule.

It is in fact owing to the architecture sketched by Eq.(27) that, one can test the validity of the STR, already at rest. *(Along this line, note that, even the solution of a non-relativistic quantum mechanical description, happens to be Lorentz invariant.)*

In short, the mass increase we introduced, to arrive at Theorem 5, may very well not be arbitrary, and this is indeed what one experiences for instance, when a clock is removed out of a gravitational field; its rest mass, following our claim, as required by the law of conservation of energy and the mass & energy equivalence of the STR [13], should be increased as much as the binding energy the object displays vis-à-vis the host celestial body of concern (just like the mass of the hydrogen atom is increased, as the electron is removed away, from its orbit around the proton). The unit time displayed by the internal dynamics of the object in hand, according to our Theorem 5, should then be altered as much. This is exactly what happens in the scope of the General Theory of Relativity.

According to our approach, the same phenomenon would occur, in exactly the same way, for ionized quantum mechanical clocks in an electric field, or for quantum mechanical clocks bearing an electric dipole, still in an electric field, or quantum mechanical clocks bearing a magnetic dipole in a magnetic field [14].

Similarly, if a muon is bound to a proton, its half life should quantum mechanically stretch, as much as its binding energy. This happens, to our knowledge, something totally overlooked. And it is worth to state Eq.(32), as our next theorem.

**Theorem 6:** The quantity [total energy x clock mass x (clock space size)$^2$], composed with respect to a quantum mechanical object, turns out to be a universal invariant strapped to $h^2$, the Square of the Planck Constant, and this, vis-a-vis any field the object interacts with.

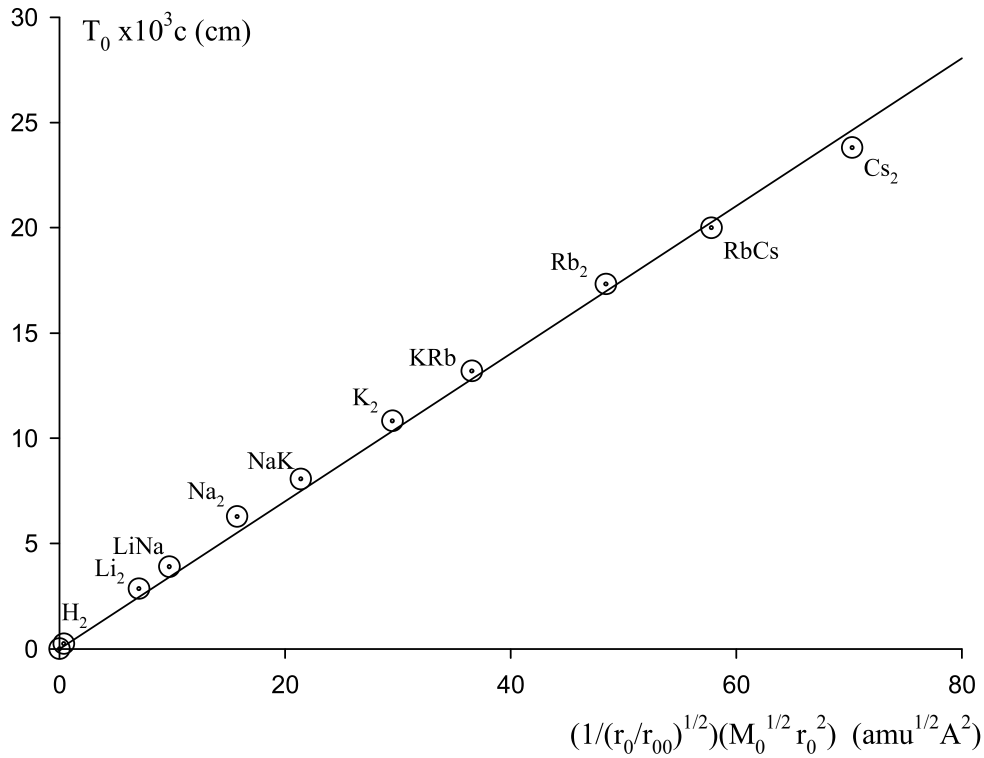


**Figure 1** Period of alkali molecules versus
$(1/(r_0/r_{00})^{1/2})(M_0^{1/2}\, r_0^2)$
$r_{00}$ : internuclear distance of $H_2$

## 6. CONCLUSION

It was the author's idea that, owing to the law of conservation of energy, in the broader sense of the concept of "energy" embodying the mass & energy equivalence of the STR, the rest mass of a particle bound to a given field, should be weakened as much as the binding energy coming into play, whether it is question of the atomic world or the celestial world. When this result is inserted into the quantum mechanical description of the object in hand (i.e. the rest masses taking place in such a description, are decreased as much), it is (as can be expected through even a plain insight), found that the total energy of a particle bound to a given field, decreases as much as the binding energy it delineates.

Thus, as dreadfully simple as this may be, one ends up with the red shift and the time dilation of the GTR, if the particle were bound to a gravitational field, through though, purely quantum mechanical means. One can further right a general equation of motion, capable to predict the well known results of the GTR, such as the precession of the perihelion of a planet, and the light deflection nearby a celestial body, matching perfectly well the data, yet through a completely different set up than that of Einstein [3].

Thus our approach summarized in Table 1, is solely based on the STR.

This makes that our theory is a more general theory than the GTR. Furthermore, it excludes the necessity of assuming the principle of equivalence of Einstein, with regards to the end results of the GTR.

In our previous work (where we have established a simple general equation of motion, alternative to the cumbersome set up of GTR), we did not have any room to detail mathematically, how quantum mechanics played a role in our approach. We simply stated our main theorem [3]. Herein we worked out this theorem. Yet, not to attract conservative reactions, we took the time to develop our idea on simple considerations first, and only afterwards undertook a general quantum mechanical proof of it.

We would like to emphasize that, it really must take quite a captivation, not to have considered for very many decades, that a gravitationally bound object, would indeed exhibit straight, conventional quantum mechanical changes, bringing into play, on its own, metric changes. It is understandable that Einstein himself, who never believed in quantum mechanics, did not consider such an avenue; but it seems less understandable that, seemingly no one (including fathers of quantum mechanics) did, after him.

Briefly it is that, the effect of gravitation (and this through quantum mechanics), and that of the uniform translational motion are alike, given that both point to (amongst others), the same invariance of the quantity [energy x mass x lenght$^2$].

We conclude that, it is this quantum mechanical invariance, necessarily strapped to the square of the Planck Constant, which constitutes a given framework regarding the matter architecture, and insures the end results of the GTR. (We will in a subsequent article show, how this works as the internal machinery of the STR, were the object in hand brought to a uniform translational motion.)

In regards to the end results of the GTR, not only that our approach is exceptionally simple as compared to the GTR, but it also avoids all incompatibilities (such as the breaking of the relationship $E=mc^2$), or inconsistencies (such as the breaking of the energy conservation law, as well as the momentum conservation law), or blockades (such as the impossibility of the quantization of the gravitational field), thus opens a whole clean horizon toward a unification of fields, and the understanding of the matter and the universe at all levels, with just the same tools.

Our approach moreover allows quantization of the gravitational field. Out there, it is all one nature, and one should not really have to conceive different packages of conception in order to predict different scales of it.

Along our approach, we could as well show that, just like the gravitational field, the electric field too slows down the internal mechanism of a clock, which interacts with the field. This result explains substantially, the retardation of the decay of the muon, bound to a nucleus.

Thus, when a particle is bound to any field, based on our approach, a given internal mechanism, the particle may embody, shall be affected accordingly, provided that the particle's inner articulations in relation to each other, are not degenerated via the binding process, i.e. the particle preserves its original characteristics. This seems quite trivial, but very much against the general wisdom, since neither Dirac nor anyone else after him, has seemingly, given a thought to alter the rest mass of the bound electron.

**Table 1** Flow Chart of the Quantum Mechanical Mechanism Presented Herein

The object of rest mass $m_0$ in empty space, is embedded, quasistatically, in the field of concern (gravitational, electric, or else). It is assumed that the object does not loose its identity in the field.

$\Downarrow$

Owing to the special theory of relativity, the rest mass of the object decreases as much as the binding energy $E_B$ coming into play, so that

$$m_0 \to \gamma m_0,$$

$$\Delta m_0 = m_0 - \gamma m_0,$$

$$\gamma = \frac{m_0 - \Delta m_0}{m_0} = 1 - \frac{\Delta m_0}{m_0},$$

$$c^2 \Delta m = E_B,$$

$$\gamma = 1 - \frac{\Delta m_0}{m_0} = 1 - \frac{E_B}{m_0 c^2}.$$

$\Downarrow$

Compose the Non-Relativistic or the Relativistic Quantum Mechanical Description of the object in hand, whichever is appropriate.

$\Downarrow$

Achieve the transformation [ $m_0 \to \gamma m_0$ ], in there.
If the object is made of different particles, each of mass $m_{0i}$, then apply the transformation [ $m_{0i} \to \gamma m_{0i}$ , i = 1, …, I], to all of the I particles of concern, as if the rest masses are decreased as much , were the object embedded in a field.

$\Downarrow$

As a result, the total energy of the object is decreased just as much, thus the gravitational red shift, if the object were embedded in a gravitational field, and its size is stretched as much. Though, the quantity [total energy x mass x size] remains invariant. This quantity is further a Lorentz invariant quantity.

## APPENDIX A

## PROOF OF THE FACT THAT "COULOMB'S FORCE OR NEWTON'S FORCE REIGNING BETWEEN RESPECTIVELY STATIC CHARGES AND STATIC MASSES - ONLY, MUST ACT AS $1/r^2$", IS IMPOSED BY THE SPECIAL THEORY OF RELATIVITY (STR): HENCE, BOTH LAWS AS SUCH, MUST BE UNIVERSAL AS MUCH AS THE STR IS

Our claim can be achieved easily by noting that the quantity,

$$H = \text{force x mass x length}^3 \ , \tag{A-1}$$

is Lorentz invariant.

In fact, dimensionally speaking, it amounts to the square of the Planck Constant [mass x length$^2$ x (period of time)$^{-1}$] , which in return is Lorentz invariant.

On the other hand, it is known that the electric charges are Lorentz invariant. If not, say in excited atoms, energetic electrons would exhibit electric charge intensities different than the electric charge intensity of the electrons on the ground level, which is not the case. (A further insight regarding the Lorentz invariance of electric charges will be presented in Appendix B.)

Now suppose we have a dipole, such as a water molecule,[§§] which can be represented by charges q and Q, situated at rest, at a distance $r_0$, from each other; it has the rest mass $m_0$.

Coulomb's Force $F_{C0}$, reigns between q and Q. Suppose we assume that this force is as usual expressed as proportional to the electric charges coming into consideration, also to $1/r_0^n$, where though we do not know, a priori, the exponent n, i.e.

$$F_{C0} = \frac{qQ}{r_0^n} \ . \tag{A-2}$$

Suppose now, we bring the dipole to a uniform translational motion of velocity v, along the direction of the line connecting the electric poles.

[§§] In water molecule, the oxygen atom, attracts the binding electrons of the hydrogen atoms. This makes that the hydrogen atoms get charged positively and the oxygen atom negatively. Thus the water molecule can be represented by a dipole, made of -2e situated nearby the oxygen atom, and +2e situated on the median of the triangle HOH, between the hydrogen atoms; e is the charge intensity of the electron.

Through the motion, the quantity bearing the dimension

$$[I] = [\text{mass}] \text{ x } [\text{length}] \quad , \tag{A-3}$$

remains invariant, if the length in consideration, lies on the direction of motion.

Thus,

$$I = m_0 r_0 = (\beta m_0)\left(\frac{r_0}{\beta}\right) = \text{Constar} \; ; \tag{A-4}$$

$\beta$ is the usual Lorentz dilation factor, i.e.

$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad . \tag{A-5}$$

It becomes evident that the invariance of

$$\begin{aligned} H &= \text{force x mass x length}^3 \\ &= \frac{qQ}{r_0^n} \text{ x } m_0 \text{ x } r_0^3 = \frac{qQ}{r_0^n} \text{ x } I \text{ x } r_0^2 = \frac{qQ}{r_0^n \gamma^{-n}} \text{ x } I \text{ x } r_0^2 \, \gamma^{-2} = \text{Constar}, \end{aligned} \tag{A-6}$$

holds, if and only if n=2, i.e. if Coulomb's Force, behaves as

$$F_{C0} = \frac{qQ}{r_0^2} \quad \text{(c.q.f.d.)} \quad . \tag{A-7}$$

Note that the same holds, if "charges", in question, are "gravitational charges"; in this case however, the product of charges should be considered, together with the universal gravitational constant.

In other words:

i) The $1/r_0^n$ dependency of Newton's Force $F_{N0}$, reigning between two static masses m and M, and expressed as

$$F_{N0} = G\frac{mM}{r_0^n} \quad (n \equiv 2) \quad , \tag{A-8}$$

with n being exclusively 2, is imposed by the STR.

ii) The quantity $GmM$ is Lorentz invariant, thus G, the universal gravitational constant G, alone, is not Lorentz invariant. This result makes that G is not as universal as one may think it is.***

By the same token, the quantities $\sqrt{G}m$, or $\sqrt{G}M$ (bearing the dimension of an electric charge), are well Lorentz invariant.

The fact that the $1/r_0^2$ dependency of Coulomb's Force, is imposed by the STR, is certainly correlated with the fact that, this dependency is well furnished as a result of the Klein Gordon Equation, built via replacing the energy and momentum quantities in the relativistic equation

$$p^2c^2 + m_{0\infty}^2 c^4 = E^2 , \tag{A-9}$$

by the corresponding quantum mechanical symbols; here p is the momentum of a moving particle of rest mass $m_{0\infty}$, and E its total energy. With regards to Coulomb's Force, the rest mass $m_{0\infty}$ is taken to be zero.

Evidently, the cast of Newton's Force is the same as that of Coulomb's Force, and we show elsewhere that (see primarily, Section 2 of the text), through our approach, based on Newton's Force, but written exclusively for static masses, one is able to arrive at the end results of the general theory of relativity, up to a measurable sensitivity.

We can conclude that both Eqs. (A-2) (i.e. Coulomb's Force Law), and (A-8) (i.e. Newton's Force Law), but once again, written for respectively, static charges, and static masses, delineate universal force laws, provided that the STR is valid (and it seems it is).

This tells that, the weak force law, governing beta decays too, must be somehow built in a similar manner. This result is important to generalize the theorem, we will demonstrate in the Appendix B.

*** Consider for instance the sun's rotational motion around the center of the Milky Way. Neglect for simplicity the rotation of the sun around its pole-to-pole axis. The relatively slow rotational motion of the sun around the center of the galaxy can be considered as a uniform translational motion. Take the attraction force between the sun and a proton bound at rest to the sun, as assessed by a distant observer, in a reference frame where both the sun and the observer can be assumed at rest. This force, reigning at rest, is not the same force when assessed relative to the center of the galaxy (because of the motion of the solar system around the center of the galaxy), as implied by the special theory of relativity. Let M be the mass of the sun, and m the mass of the proton in consideration as assessed by the mentioned distant observer. Suppose we define G too, in the frame of this observer. The quantity $GmM$ [bearing the dimension of Lorentz invariant quantity (electric charge)$^2$], is the same, whether we consider it relative to the center of the galaxy, or relative to the distant observer. (Recall that electric charges are Lorentz invariant.) But the masses of concern, are not the same when assessed relative to the center of the galaxy. Thus, the universal gravitational constant does not remain the same when one switches from the first frame of reference, to the second.

## APPENDIX B

## PROOF OF THEOREM 5 OF THE TEXT

Herein we will prove the Theorem 5 of the text.

**Theorem 5:** Consider a relativistic or non-relativistic quantum mechanical description of a given object, depending on whichever, may be appropriate. This description points to an internal dynamics which consists in a “clock motion”, achieved in a “clock space”, along with a “unit period of time”. The description excludes “synthetic potential energies” (which may otherwise lead to incompatibilities with the STR). It is supposed to be based on $K$ particles, altogether. If then different masses $m_{k0}$, $k = 1, \ldots, K$, involved by this description of the object at rest, are all multiplied by the arbitrary number $\gamma$, the following two general results are conjointly obtained:

a) The total energy $E_0$ associated with the given clock’s motion of the object is increased as much, or the same, the unit period of time $T_0$, of the motion associated with this energy, is decreased as much.

b) The characteristic length, or the size $R_0$ to be associated with the given clock’s motion of concern, contracts as much.

In mathematical words this is:

$$[(m_{k0}, k = 1, \ldots, K) \to (\gamma\, m_{k0}, k = 1, \ldots, K)] \Rightarrow [(E_0 \to \gamma\, E_0) \text{ or } (T_0 \to \frac{T_0}{\gamma}), \text{ and } (R_0 \to \frac{R_0}{\gamma})].$$

Let us accentuate that, if the object is, say an atom, then $R_0$ is (no matter how this is defined) the radius of it; if the object is a diatomic molecule, $R_0$ is the internuclear distance, etc; $R_0$, in fact, may be just any length one may pick, within the framework of the object in hand, and Theorem 5, as can be shown, shall still be valid.

**Proof of the First Part of Theorem 5:**
$[(m_{k0}, k = 1, \ldots, K) \to (\gamma\, m_{k0}, k = 1, \ldots, K)] \Rightarrow [E_0 \to \gamma\, E_0]$

For our purpose, for simplicity, without though any loss of generality, we consider the (time independent) Schrödinger Equation, i.e. with the familiar notation, written for an atomistic or a molecular object composed of J nuclei, of respective masses $m_{j0}$, $j = 1, \ldots, J$, and I electrons (altogether), of (the same) mass $m_{i0}$, $i = 1, ..., I$:

$$\left(-\sum_j \frac{h^2}{8\pi^2 m_{j0}}\nabla_j^2 - \sum_i \frac{h^2}{8\pi^2 m_{i0}}\nabla_i^2 - \sum_{i,j}\frac{Z_{j0}e^2}{r_{ij0}} + \sum_{i,i'}\frac{e^2}{r_{ii'0}} + \sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{r_{jj'0}}\right)\psi_0(\underline{r}_0)$$

$$= E_0\psi_0(\underline{r}_0)\,. \tag{B-1}$$

$E_0$ is the eigenvalue, and $\psi_0(\underline{r}_0)$ the related eigenfunction; $Z_{j0}$ is the atomic number of the $j^{th}$ nucleus; $r_{ij0}$ is the distance between the $i^{th}$ and the $j^{th}$ particles.

Eq.(B-1) already represents a given complexity. Instead, we could have considered straight a much more general quantum mechanical description with respect to a given number of unspecified particles, without making any distinction between these, even those which would be identical. But then the picture we would have to paint would have remained too abstract, and the derivations that would follow, too cumbersome to follow.

This is why we have avoided a more general quantum mechanical description, to start with.

On the other hand, in any case, any potential energy term to be input to such a description must be compatible with the STR. Otherwise, even a relativistic Dirac description would furnish results which do not consist in Lorentz invariant forms. In Appendix A, above, we have come to show that, in order to be compatible with the STR, a potential energy term's spatial dependency must bear the $1/r_0$ dependency, on the background. An extra (Lorentz invariant) exponential term may be added to the numerator, such as that coming into play along with the Yukawa potential. But what is primordial, is that the $1/r_0$ dependency, must appear on the background. (And in Appendix A, we have shown that, this is imposed by the STR.)

Thus, vis-à-vis our goal, an atomic or molecular quantum mechanical description made of Coulomb Potential energy terms, is a fine basis to work on; furthermore, it well lands itself to further generalization.

*

Thus multiply all, electron masses $m_{i0}$ (i = 1, ..., I), as well as nuclei masses $m_{j0}$ (j = 1, …, J), appearing in Eq.(B-1), by $\gamma$; the eigenfunction and the related eigenvalue will accordingly be altered:

$$\left(-\sum_j \frac{h^2}{8\pi^2 \gamma m_{j0}}\nabla_j^2 - \sum_i \frac{h^2}{8\pi^2 \gamma m_{i0}}\nabla_i^2 - \sum_{i,j}\frac{Z_{j0}e^2}{r_{ij0}} + \sum_{i,i'}\frac{e^2}{r_{ii'0}} + \sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{r_{jj'0}}\right)\psi_{new}(\underline{r}_0)$$

$$=E\,\psi_{new}(\underline{r}_0)\,. \tag{B-2}$$

This is the same as

$$\left(-\sum_{j}\frac{h^2}{8\pi^2 m_{j0}}\nabla_j^2-\sum_{i}\frac{h^2}{8\pi^2 m_{i0}}\nabla_i^2-\sum_{i,j}\frac{Z_{j0}e^2}{\frac{r_{ij0}}{\gamma}}+\sum_{i,i'}\frac{e^2}{\frac{r_{ii'0}}{\gamma}}+\sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{\frac{r_{jj'0}}{\gamma}}\right)\psi_{new}(\underline{r}_0)$$

$$=\gamma E\,\psi_{new}(\underline{r}_0)\,. \tag{B-3}$$

Let now

$$\underline{r}_0\to\underline{r}=\gamma\underline{r}_0\quad , \tag{B-4}$$

together with

$$\psi(\underline{r})\equiv\psi_{new}(\underline{r}_0)\,. \tag{B-5}$$

Since

$$\frac{\partial\psi(r_0)}{\partial u_0}=\frac{\partial\psi(r)}{\partial u}\frac{\partial u}{\partial u_0}\ ;\ u_0=x_0,y_0,z_0;\ u=x,y,z; \tag{B-6}$$

we have

$$\frac{\partial\psi(r_0)}{\partial u_0}=\gamma\frac{\partial\psi(r)}{\partial u}\,. \tag{B-7}$$

Eq.(B-3) thus becomes

$$\left(-\sum_{j}\frac{h^2}{8\pi^2 m_{j0}}\gamma^2\nabla_j^2-\sum_{i}\frac{h^2}{8\pi^2 m_{i0}}\gamma^2\nabla_i^2-\sum_{i,j}\frac{Z_{j0}e^2}{\frac{r_{ij0}}{\gamma}}+\sum_{i,i'}\frac{e^2}{\frac{r_{ii'0}}{\gamma}}+\sum_{j,j'}\frac{Z_{j0}Z_{j'0}e^2}{\frac{r_{jj'0}}{\gamma}}\right)\psi(\underline{r})$$

$$=\gamma\ E\psi(\underline{r})\,. \tag{B-8}$$

Dividing by $\gamma^2$, and using Eq. (B-4), this yields

$$\left( -\sum_j \frac{h^2}{8\pi^2 m_{j0}} \nabla_j^2 - \sum_i \frac{h^2}{8\pi^2 m_{i0}} \nabla_i^2 - \sum_{i,j} \frac{Z_{j0} e^2}{r_{ij}} + \sum_{i,i'} \frac{e^2}{r_{ii'}} + \sum_{j,j'} \frac{Z_{j0} Z_{j'0} e^2}{r_{jj'}} \right) \psi(\underline{r})$$
$$= \frac{E}{\gamma} \psi(\underline{r}) . \qquad \text{(B-9)}$$

In comparison with Eq.(B-1), we can deduce at once that

$$\frac{E}{\gamma} = E_0 \Rightarrow E = \gamma E_0 \quad \text{(c.q.f.d.)} . \qquad \text{(B-10)}$$

Thus, we have come to achieve the demonstration of the first part of Theorem 5.

It would be interesting to notice that, no $\gamma$ is left out next to the electric charges, through the foregoing, mathematical operations. (It is evident that, otherwise, we could not land at the above relationship. But, this is not the point.) Recall that our derivation, will anyway take us to the quantum invariance of the quantity $E_0 M_0 R_0^2$ with regards to a mass change introduced into the quantum mechanical description in hand (cf. Theorem 6 of the text). Yet, this quantity is as well Lorentz invariant. The fact that "what we do, is ultimately well coherent with the STR", in conjunction with the fact that "no $\gamma$ is left out next to the electric charges, through our derivation above", can be seen as a sign that electric charges are indeed Lorentz invariant. Thus, so is $[\text{charge}]^2$. Accordingly, as discussed in Appendix A, the same should hold for the product of gravitational charges along with the universal gravitational constant.

**Proof of the Second Part of Theorem 5:**

$[(m_{k0}, k = 1, \ldots, K) \to (\gamma\, m_{k0}, k = 1, \ldots, K)] \Rightarrow [R_0 \to \frac{R_0}{\gamma}]$

Next we focus on a size of interest $R_0$ (i.e. as we just pointed out, the "size of an atom", anyway we would like to define it, or the "internuclear distance" in a diatomic molecule of concern, or whatever), to be associated with the wave like object in hand. $R_0$ shall be determined based on the solution of Eq.(B-1). Following the mass perturbation, $R_0$ becomes $R_{0new}$, and this latter shall be found based on the solution of Eq.(B-2). According to Eq.(B-4), $R_{0new}$ is transformed into R, so that $R = \gamma R_{0new}$. (Note that according to this equation, any distance, say $r_0$ we would consider, becoming $r_{onew}$ due to the mass change, is transformed into r, so that $r = \gamma\, r_{onew}$. Thus the derivation presented herein, in fact holds for any distance, thence also for a given specific distance $R_0$ we would pick up.)

R is to be determined as the solution of Eq.(B-9). But since this equation is identical with Eq.(B-1) [along with Eq.(10)], the solution of Eq.(B-9) in regards to R, is the "original size" of interest, i.e. $R_0$.

Hence

$$\gamma R_{0new} = R_0 , \qquad \text{(B-11)}$$

or the same

$$R_{0new} = \frac{R_0}{\gamma} \quad \text{(c.q.f.d.)} \; . \tag{B-12}$$

This ends the demonstration of Theorem 5 of the text.

**Proof of** $[\,E_0 \to \gamma\, E_0\,] \Rightarrow [\,T_0 \to \frac{T_0}{\gamma}\,]$

Following the multiplication of masses by the factor $\gamma$, the total energy $E_0$ has increased by $\gamma$ [cf. Eq.(B-10)].

On the other hand, $E_0$ can well be written as

$$|E_0| = ah\nu_0 = \frac{ah}{T_0} \; , \tag{B-13}$$

where $T_0$ is the unit period of time associated with the dynamics in consideration, and the coefficient a, is just a constant insuring the equality [for a quick check one can multiply the magnitude of the right hand side (RHS) of Eq.(1) by the RHS of Eq.(21) of the text, to discover that a, in the case of the hydrogen atom, becomes 1/2].

This makes that

$$(\,E_0 \to \gamma\, E_0\,) \Rightarrow (\,T_0 \to \frac{T_0}{\gamma}\,) \quad \text{(c.q.f.d.)} \; . \tag{B-14}$$

*

One can easily check that the foregoing results can be obtained with respect to a relativistic quantum mechanical description, as well as nuclear potentials input to the description, instead of Coulomb potentials.


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