Dark Energy and its Possible Existence in Particulate Form in a Friedman dust Universe with Einstein's Lambda

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August 30, 2009

Abstract

It is shown that negatively gravitating *particles* can *consistently* be considered to exist and interact with normal positively gravitating particles in the contexts of general relativity and classical Newtonian gravitational theory. This issue arises from the discovery of *dark energy* which is considered to be causing an acceleration of the expansion of the universe. The issue is, can this dark *energy* occur in particulate form? A related issue was studied in the fifties by Herman Bondi, ([41]) in the context of the possible existence of negative mass in the general relativity context, long before dark energy appeared on the scene. He came to a paradoxical conclusion that seemed to rule out the actual physical existence of negatively gravitating particles. This paradox does not occur in this work because only positive mass particle are involved whatever their gravitational character may be. The structure of the differential equations that would apply in the case of a binary pair of opposite gravitational character components are used to show and explain how they can become consistent in general relativity or classical gravitation theory. This involves explaining a non-obvious relation between the principle of equivalence and Newton's *Action Equals Reaction* principle. A path structure for a mixed mass binary pair is set up which satisfies the equations of motion and does not have paradoxical properties. The force structure of the system is checked with a known classical dynamical test for the force per unit mass involved in the component particles motions. This test is used to demonstrates that the basic assumptions of this theory

are incorporated into the consequential orbital structure. An alternative to the *Action Equals Reaction* principle more appropriate to the astronomical situation is suggested. An animation using Mathematica has been derived, and is available, and shows how a mixed gravity binary pair move under their mutual gravitational action.

1 Introduction

The work to be described in this paper is an application of the cosmological model introduced in the papers *A Dust Universe Solution to the Dark Energy Problem* [23], *Existence of Negative Gravity Material. Identification of Dark Energy* [24] and *Thermodynamics of a Dust Universe* [32]. The negatively gravitationally characterised mass density involved from which such hypotetical might be formed has the value, $\rho_\lambda^\dagger$, which is twice the Einstein dark energy density, $\Lambda c^2/(8\pi G)$. The issue to be examined in this paper is the form that the *dark energy negatively gravitating material* is likely to take. I shall assume that it is in the form of material particles like the normal positively gravitating particles and consider what consequences this has for the dynamics of this exotic material in relation to the normal gravitating material. The previous work on this topic does lead towards making this particulate assumption about the form of dark energy by showing that with the new definition, $\rho_\lambda^\dagger$, for dark energy density it comes physically onto a par with positively gravitating material simply by its gravitating strength being denoted by -G times mass in place of +G time mass as is usually the case. All of this work and its applications has its origin in the studies of Einstein's general relativity in the Friedman equations context to be found in references ([16],[22],[21],[20],[19],[18],[4],[23]) and similarly motivated work in references ([10],[9],[8],[7],[5]) and ([12],[13],[14],[15],[7],[25],[3]). The applications can be found in ([23],[24],[32],[36],[34][40]). Other useful sources of information are ([17],[3],[30],[27],[29],[28]) with the measurement essentials coming from references ([1],[2],[11],[37]). Further references will be mentioned as necessary.

2 Positive, Negative Gravitating Particle Dynamics

The term *pure particle* will be used with an extended sense of meaning either astronomical accumulations of mass with order of the amount of mass that might be found within a planet, a star or a galaxy or quantum elementary particle and accumulated structures formed from elementary particles such as atoms or molecules. In other words, a pure particle can be mass-wise any physical object formed from a mass accumulation with some restrictions. I shall assume that particles can have the two possible states of either being gravitationally positive, M^+, and consequently causing acceleration towards itself on all other particles in its vicinity or being gravitationally negative, M^-, and causing an acceleration away from itself on all other particles in its vicinity. All the particles involved are assumed to have *positive mass* and the restriction *pure* means that all such particles, whatever mass size will each have a definite gravitational coupling value, $\pm G$, G being Newton's gravitational constant. That is to say, for a massive particle of mass $M^\pm$ gravitating strength equals $\pm GM^\pm$. This is a provisional restriction that will certainly need to be lifted that I introduce to avoid mixed character particles that would involve changed values for G. Further restrictions on this very wide definition for possible masses will be discussed later. I shall designate the gravitationally characterised positive particles as being of red colour and gravitationally characterised negative particles as being of green colour for ease of reference and diagrammatic use and further represent the red particles with a $+$ superscript and the green particles with a $-$ superscript. I emphasise that both types of particle have *positive* rest mass. Thus a red particle induces, at distant points, accelerations *towards* itself in *all* other particles in its vicinity. A green particle induces, at distant points, accelerations *away* from itself in *all* other particles in its vicinity. This assumption about gravitational characterisation is very different from the electric characterisation for charged particles with charges $\pm e$ where oppositely characterised particles mutually attract and same characterised charges mutually repel, the distinction emphasised in the *all* for the gravitational case. This may seem to the reader to invite a paradox but it will be explained why this paradox does not arise. The classical Newtonian equations that describes how a red particle behaves dynamically under the gravity field of another red particle are

$$
\begin{aligned}
\mathbf{f} &= m_1^+ d^2\mathbf{r}/dt^2 = -Gm_1^+ m_2^+ \hat{\mathbf{r}}/r^2 && (2.1)\\
d^2\mathbf{r}/dt^2 &= (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\mathbf{t}} && (2.2)\\
\alpha_{r,R} = \hat{\mathbf{r}} \cdot d^2\mathbf{r}/dt^2 &= (\ddot{r} - r\dot{\theta}^2) = -Gm_2^+/r^2. && (2.3)\\
\alpha_{\theta,R} = \hat{\mathbf{t}} \cdot d^2\mathbf{r}/dt^2 &= (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = r^{-1}d(r^2\dot{\theta})/dt) = 0, && (2.4)
\end{aligned}
$$

The force acting is the gravitational force originating from the positively gravitating particle m_2^+, regarded as fixed at its specific origin of coordinates, is directed towards it. That is to say, m_2^+ is at the tail of the vector $\mathbf{r}$ and m_1^+ is at the sharp end of $\mathbf{r}$ and usually in motion. These equations contain much information about classical Newtonian gravity. Very striking is the fact that the two components of acceleration $\alpha_{r,R}$ and $\alpha_{\theta,R}$ of the particle, m_1^+, do not involve the mass of this particle, in fact it could be red or green and in either case the equation would be unchanged. The mass of the red particle m_2^+ is involved in the $\hat{\mathbf{r}}$ component through the influence of the gravitational field. Thus the accelerating effect on any particle of any mass or any other characteristic under the influence of a red gravitating source is the same for all particles passing through the same position distant from m_2^+ at any time. Thus from classical dynamical theory, we obtain the essence of Einstein's *principle of equivalence*. The main conclusion, I wish to emphasise is that the acceleration, $\alpha_{r,R}$, is always negative with direction, $-\hat{\mathbf{r}}$ that is towards m_2^+, for any particle experiencing the effect of a positively characterised particle such as m_2^+ because G and m_2^+/r^2 are both always positive. That is the induced acceleration is towards a positively gravitating source, m_2^+ whatever the gravitational character the subjected particle, $m_1^\pm$, may have,

$$\alpha_{r,R} = \mathbf{f} \cdot \hat{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2) \leq 0. \quad (2.5)$$

Let us now consider the case of a green particle fixed at the origin of coordinates being the source of the gravitational force on a red particle. For this case equation 2.3 has to be replaced with the negative $-G$ becoming $+G$ as in the next equation.

$$\alpha_{r,G} = \mathbf{f} \cdot \hat{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2) = +Gm_2^-/r^2. \quad (2.6)$$

This then implies also from equation (2.3)

$$\alpha_{r,G} = \mathbf{f} \cdot \hat{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2) \geq 0, \quad (2.7)$$

as all rest masses and G/r^2 are always positive. Thus a negatively gravitating mass source particle induces acceleration away from itself for all particles in its vicinity regardless of any other characteristics they may have. Particle, m_2^-, in equation 2.6 differs from the positively gravitationally characterised particle m_2^+ in equation 2.3 in that it is negatively gravitationally characterised

It follows that for a binary system composed of a positive gravitating mass and a negative gravitating mass we need two equations of motion such as,

$$\mathbf{f}_1 = m_1^+ d^2\mathbf{r}_1/dt^2 = +Gm_1^+ m_2^- \hat{\mathbf{r}}_1/r_1^2 \tag{2.8}$$
$$d^2\mathbf{r}_1/dt^2 = (\ddot{r}_1 - r_1\dot{\theta}_1^2)\hat{\mathbf{r}}_1 + (r_1\ddot{\theta}_1 - 2\dot{r}_1\dot{\theta}_1)\hat{\mathbf{t}}_1 \tag{2.9}$$
$$\alpha_{r_1,G} = \mathbf{f}_1 \cdot \hat{\mathbf{r}}_1 = (\ddot{r}_1 - r_1\dot{\theta}_1^2) = +Gm_2^-/r_1^2 \geq 0. \tag{2.10}$$
$$\alpha_{\theta_1,G} = \mathbf{f}_1 \cdot \hat{\mathbf{t}}_1 = (r_1\ddot{\theta}_1 + 2\dot{r}_1\dot{\theta}_1) = 0, \tag{2.11}$$

$$\mathbf{f}_2 = m_2^- d^2\mathbf{r}_2/dt^2 = -Gm_1^+ m_2^- \hat{\mathbf{r}}_2/r_2^2. \tag{2.12}$$
$$d^2\mathbf{r}_2/dt^2 = (\ddot{r}_2 - r_2\dot{\theta}_2^2)\hat{\mathbf{r}}_2 + (r_2\ddot{\theta}_2 - 2\dot{r}_2\dot{\theta}_2)\hat{\mathbf{t}}_2 \tag{2.13}$$
$$\alpha_{r_2,R} = \mathbf{f}_2 \cdot \hat{\mathbf{r}}_2 = (\ddot{r}_2 - r_2\dot{\theta}_2^2) = -Gm_1^+/r_2^2 \leq 0. \tag{2.14}$$
$$\alpha_{\theta_2,R} = \mathbf{f}_2 \cdot \hat{\mathbf{t}}_2 = (r_2\ddot{\theta}_2 + 2\dot{r}_2\dot{\theta}_2) = 0, \tag{2.15}$$

The two sets of equations above refer to a system of two particles one, m_1^+, with a positive gravitating characteristic and one, m_2^-, with a negative gravitating characteristic, both have positive rest mass. In the first set m_2^- is fixed at the origin of coordinates and is the source of the gravitational field that determines the dynamics of m_1^+ which is found at vector position $\mathbf{r_1}$ at time t relative to the position of m_2^+. In the second set m_1^+ is fixed at the origin of coordinates and is the source of the gravitational field that determines the dynamics of m_2^- which is found at vector position $\mathbf{r_2}$ at time t relative to the position of m_1^+. Thus there are two *space* frames of reference involved with their origins separated by the relative position vector, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, of the particles. The key quantities that determine the kinematic and dynamic behaviour of the total system are the four component accelerations $\alpha_{r_2,R}, \alpha_{r_1,R}, \alpha_{r_2,G}, \alpha_{r_1,G}$ but being measured in different frames of reference it is difficult to see how their effects are to be combined. This is necessary if we are to understand how the particles interact. However, Inspection of the two basic equations of motion (2.8) and (2.12), repeated below, which somehow or other are to hold as a simultaneous pair for the two particle system under examination,

$$\mathbf{f}_1 = m_1^+ d^2\mathbf{r}_1/dt^2 = +Gm_1^+ m_2^- \hat{\mathbf{r}}_1/r_1^2 \quad (2.16)$$

$$\mathbf{f}_2 = m_2^- d^2\mathbf{r}_2/dt^2 = -Gm_1^+ m_2^- \hat{\mathbf{r}}_2/r_2^2 \quad (2.17)$$

$$\hat{\mathbf{r}}_1 = - \hat{\mathbf{r}}_2 \quad (2.18)$$

the first impression is that they are incompatible as they seem to defy the Newtonian law that action and interaction should be equal. The first equation, gives the force from m_2 acting on m_1 . The second equation, gives the force from m_1 acting on m_2. Usually, if the action is taken to be $\mathbf{f}_1$, the reaction is regarded as being -$\mathbf{f}_2$, then, $\mathbf{f}_1 = -\mathbf{f}_2$, but here because of (2.18) it follows that $\mathbf{f}_1 = \mathbf{f}_2$. This is a well know paradox arising when trying to work with negatively gravitating particles. Subtle, you may say but nevertheless a source of great consternation in this area of work. It is clear from these equations that two negatively gravitating particles in interaction would also give the result $\mathbf{f}_1 = -\mathbf{f}_2$, implying that their interaction satisfies Newton's third law so that we only have to consider the two situations of positively gravitating pairs and oppositely gravitating pairs to analyse an explain this situation. Thus we now have to consider the way this difficulty can be circumvented. We can get some direction in this investigation by considering how the similar well known problem in the study of the orbiting positive gravitating particles of a binary system in astronomy is handled. There are also two equations of motion as in (2.16, 2.17),

$$\mathbf{f}_1 = m_1^+ d^2\mathbf{r}_1/dt^2 = -Gm_1^+ m_2^+ \hat{\mathbf{r}}_1/r_1^2 \quad (2.19)$$

$$\mathbf{f}_2 = m_2^+ d^2\mathbf{r}_2/dt^2 = -Gm_1^+ m_2^+ \hat{\mathbf{r}}_2/r_2^2 \quad (2.20)$$

$$(2.21)$$

Clearly there are no negatively gravitating particles involved here and here we do have action and reaction equal because $\hat{\mathbf{r}}_1 = -\hat{\mathbf{r}}_2$ which implies $\mathbf{f}_1 = -\mathbf{f}_2$. The usual procedure is to get the two equations combined by defining the *centre of mass*, $\mathbf{r}_{cm}$, frame in place of the two obviously different frames used above as follows:

$$\mathbf{r}_{cm} = \frac{m_1^+\mathbf{r}_1 + m_2^+\mathbf{r}_2}{m_1^+ + m_2^+} \quad (2.22)$$

In the absence of any *external* forces acting on a binary system, it is assumed that the center of mass can acquire no acceleration. Thus we can, by differentiating twice with respect to t the centre of mass vector, deduce that

$$0 = d^2\mathbf{r}_{cm}/dt^2 = m_1^+ d^2\mathbf{r}_1/dt^2 + m_2^+ d^2\mathbf{r}_2/dt^2, \tag{2.23}$$

as the centre of mass denominator factors out. It follows that

$$0 = d^2\mathbf{r}_{cm}/dt^2 = m_1^+ d^2\mathbf{r}_1/dt^2 + m_2^+ d^2\mathbf{r}_2/dt^2 \equiv \mathbf{f}_1 + \mathbf{f}_2 = 0. \tag{2.24}$$

Thus the relations above become identities because the usual action equals reaction condition holds for the classical binary particle pair case. Clearly also, this condition as it stands does not hold for the mixed character particle pair case and it is fairly obvious why this is so. The centre of gravity has to replace the centre of mass in the mixed mass case. However, notably, in the classical situation the centre of mass and the centre of gravity of the systems coincide. The centre of gravity in the mixed mass case can be defined as

$$\mathbf{r}_{cg} = \frac{G_+ m_1^+ \mathbf{r}_1 + G_- m_2^- \mathbf{r}_2}{G_+ m_1^+ + G_- m_2^-} = \frac{m_1^+ \mathbf{r}_1 - m_2^- \mathbf{r}_2}{m_1^+ - m_2^-} \tag{2.25}$$

$$G_+ = +G \tag{2.26}$$

$$G_- = -G. \tag{2.27}$$

In the mixed mass case, differentiating $\mathbf{r}_{cg}$ twice with respect to t and assuming that the centre of gravity cannot acquire any acceleration in the absence of external forces gives

$$0 = d^2\mathbf{r}_{cg}/dt^2 = m_1^+ d^2\mathbf{r}_1/dt^2 - m_2^- d^2\mathbf{r}_2/dt^2 \equiv \mathbf{f}_1 - \mathbf{f}_2 = 0, \tag{2.28}$$

because in this case, the condition, $\mathbf{f}_1 - \mathbf{f}_2 = 0$, contradicting action and reaction are equal holds. In the light of the form these structures take, it is perhaps tempting to conclude that all that need to be done to get a consistent theory is to redefine the equality of action and reaction as the *magnitude* of action and reaction forces are always equal. I am reluctant to do this for reasons to be discussed later.

Using the basic two types of structure either centre of mass or centre of gravity orientated together with the separation vector of the two particles concerned $\mathbf{r}(t) = \mathbf{r}_1(t) - \mathbf{r}_2(t)$ the position vectors for the two cases the cases (2.24) and (2.28) can respectively be each expressed in terms of $\mathbf{r}(t)$ in two equations,

$$\mathbf{r}_1 = \mathbf{r}_{cm} - \frac{m_2^+}{m_1^+ + m_2^+}\mathbf{r}(t) \qquad (2.29)$$

$$\mathbf{r}_2 = \mathbf{r}_{cm} + \frac{m_1^+}{m_1^+ + m_2^+}\mathbf{r}(t) \qquad (2.30)$$

$$\mathbf{r}_1 = \mathbf{r}_{cg} + \frac{m_2^-}{m_1^+ - m_2^-}\mathbf{r}(t) \qquad (2.31)$$

$$\mathbf{r}_2 = \mathbf{r}_{cg} + \frac{m_1^+}{m_1^+ - m_2^-}\mathbf{r}(t). \qquad (2.32)$$

If we now differentiate these four equations through twice with respect to time assuming that the acceleration of the centre of mass or the centre of gravity is zero and then multiply through by the appropriate gravitationally subjected particle we get

$$m_1^+ d^2\mathbf{r}_1/dt^2 = -\frac{m_1^+ m_2^+}{m_1^+ + m_2^+} d^2\mathbf{r}(t)/dt^2 = \mathbf{f}_1 = -\mathbf{f}_2 \qquad (2.33)$$

$$m_2^+ d^2\mathbf{r}_2/dt^2 = +\frac{m_2^+ m_1^+}{m_1^+ + m_2^+} d^2\mathbf{r}(t)/dt^2 = \mathbf{f}_2 \qquad (2.34)$$

$$m_1^+ d^2\mathbf{r}_1/dt^2 = +\frac{m_1^+ m_2^-}{m_1^+ - m_2^-} d^2\mathbf{r}(t)/dt^2 = \mathbf{f}_1 = +\mathbf{f}_2 \qquad (2.35)$$

$$m_2^- d^2\mathbf{r}_2/dt^2 = +\frac{m_2^- m_1^+}{m_1^+ - m_2^-} d^2\mathbf{r}(t)/dt^2 = \mathbf{f}_2. \qquad (2.36)$$

The last four equations can be used to summarise results so far. The first two equations above are well known results from Newtonian gravitational theory and are used to find the astronomical orbits of binary star systems. The last two of the four are new results that describe the dynamics of binary mass systems composed of one positively gravitating mass together with one negatively gravitating mass and it has been shown that the new system seems to defy Newton's action and reaction law. This being the puzzling situation that the normally gravitating mass attracts the negatively gravitating mass whilst the negatively gravitating mass repels the normally gravitating mass. It has long been thought that this is a non reconcilable paradox that excludes the existence of negatively gravitating particles from the physical arena. An important point I wish to emphasise is that the *negatively gravitating* mass in this theory does not have negative mass. The gravitational negativity is an intrinsic property of its positive mass structure just as the negative charge of an electron is an intrinsic property of its positive mass structure. From the equations above the two component systems reduce to a single component equation in $\mathbf{r}(t)$ for the normal component pair and the mixed component pair. We note also that the usual *reduced* mass that arises in the binary system, $m_1 G_+ m_2/(G_+ m_1 + G_+ m_2)$, in the mixed mass system is replace by $m_1 G_- m_2/(G_+ m_1 + G_- m_2)$ and this will change sign if the mass character of the masses are interchanged. The form of the reduced mass for the mixed mass system also exposes a difficulty with the single equation of motion that arises for the mixed mass system. If the two mass components m_1^+ and m_2^- in such a system are equal their reduced mass becomes infinite. Thus rendering the reduced mass equation of state unusable. Consequently a binary mixed gravitating pair with *equal* masses must be excluded from discussion. I am sure that this difficulty has some fundamental significance but I do not know what it is. In the next section, I find a full solution for the paths of a mixed mass pair under their mutual gravitational interaction.

3 Mixed Mass Pair Paths

In this section, I shall derive a complete solution to the dynamical problem of a positively gravitationally characterised mass and a negatively characterised mass moving in conjunction under their mutual gravitational interaction. In the previous section, it was shown that such a system seems to violate the usual version of Newton's law of action and reaction being equal. I shall work with a special case and show the solution does not involve bizarre features such a one particle chasing another to infinity which arose in the Bondi analysis. The production of this solution, I regard as something like a mathematics *existence* theorem, here showing that negatively gravitation particles can occur in nature in interaction with positively gravitating particles with unambiguous orbits, with a rational explanation and not contradicting general relativity or Newtonian dynamic. Repulsive inverse square law force is well known in the electromagnetic theory in the context of the electron and positron interaction for example. The hyperbola is particularly interesting in the inverse square law context as it has two branches each with it own focus and with reference to one or other of its two foci, the two branches can represent an attracted particle path and a repelled particle path. In gravitation, theory the repelled particle path has hitherto been regarded as of no interest because gravity has always been thought to be only attractive. To give these remarks some mathematical basis let us first consider the pedal, (r, p), equations of the two hyperbolic branches for a particle moving under inverse square law gravity,

$$(b/p)^2 = 2a/r + 1 \tag{3.1}$$

$$(b/p')^2 = 1 - 2a/r' \tag{3.2}$$

$$1 = (x/a)^2 - (y/b)^2. \tag{3.3}$$

The first equation above represents the orbit or branch of an hyperbola occupied by a particle being attracted to the focus within the orbit. This orbit is concave to the active focus. The second equation above represents the orbit or branch of the hyperbola occupied by a particle being repelled from a focus outside the orbit. This orbit is convex to the active focus. The second equation, (3.2) would usually be rejected in the gravitational context. The third equation involves both the branches and is represented in a frame of reference with the centre of the hyperbola at the centre of coordinates. Even though the receptor particle m_2^- is negatively gravitationally characterised, the numerical value of the attractive gravitational force from the particle m_1^+, fixed at the local focus, on a particle m_2^- on the first branch above is given by,

$$f_2 = \frac{m_2^- m_1^+ G}{r^2} = \frac{m_2^- h^2}{p^3} dp/dr, \tag{3.4}$$

where $h = r^2\dot{\theta}$ is the constant arrived at by integrating equation (2.11) or (2.15). Integrating equating (3.4)with respect to r, we get

$$\left(\frac{h}{p}\right)^2 = \frac{2m_1^+ G}{r} + C. \tag{3.5}$$

If we multiply equation (3.5) through with $(b/h)^2$, we get

$$\left(\frac{b}{p}\right)^2 = \left(\frac{b}{h}\right)^2 \frac{2m_1^+ G}{r} + C\left(\frac{b}{h}\right)^2 \tag{3.6}$$

and comparing (3.6) with (3.1), using the relation, $b^2 = a^2(e^2-1)$, between b, a and e, the eccentricity, we find,

$$C = \left(\frac{h}{b}\right)^2 = \frac{h^2}{a^2(e^2-1)} = \frac{(e^2-1)(m_1^+G)^2}{h^2} = m_1^+G/a \tag{3.7}$$

$$a = \left(\frac{b}{h}\right)^2 m_1^+G = \frac{a^2(e^2-1)m_1^+G}{h^2} \tag{3.8}$$

$$a = \frac{h^2}{(e^2-1)m_1^+G}. \tag{3.9}$$

Let us now consider the second branch of the hyperbola given by (3.2). Firstly Suppose that we are not sure about the force that would have to be at the focus of the first branch to control the motion followed by a particle m_1^+ on the second path. Thus let us call this unknown or uncertain force force $G_?$. We now have to integrate the formula,

$$f_1 = \frac{m_1^+m_2^-G_?}{r^2} = \frac{m_1^+h^2}{p^3}dp/dr, \tag{3.10}$$

where $h = r^2\dot{\theta}$ is the constant arrived at by integrating equation (2.11) or (2.15). Integrating the equation (3.10) with respect to r, we get

$$\left(\frac{h}{p}\right)^2 = \frac{2m_2^-G_?}{r} + C. \tag{3.11}$$

If we multiply equation (3.11) through with $(b/h)^2$, we get

$$\left(\frac{b}{p}\right)^2 = \left(\frac{b}{h}\right)^2 \frac{2m_2^-G_?}{r} + C\left(\frac{b}{h}\right)^2 \tag{3.12}$$

and comparing (3.12) with (3.2), using the relation, $b^2 = a^2(e^2-1)$, between b, a and e, the eccentricity, we find,

$$C = \left(\frac{h}{b}\right)^2 = \frac{h^2}{a^2(e^2-1)} = \frac{(e^2-1)(m_2^-G_?)^2}{h^2} = m_2^-G_?/a \tag{3.13}$$

$$a = -\left(\frac{b}{h}\right)^2 m_2^-G_? = -\frac{a^2(e^2-1)m_2^-G_?}{h^2} \tag{3.14}$$

$$a = -\frac{h^2}{(e^2-1)m_2^-G_?}. \tag{3.15}$$

The two branches refer to the same hypperbola, (3.13), so that the value for a obtained by the two different routes followed above should yield the same result. Thus from (3.9) and (3.15) it follows that

$$\frac{a_{first\ route}}{a_{second\ route}} = 1 = -\frac{m_2^- G_?}{m_1^+ G_+}. \tag{3.16}$$

Here we seem to have a big problem, because we have been assuming that the force induced by a negative gravitating particle at a distant point involves $G_- = -G$ rather than $G_+ = G$ and here we see that substituting G_- for $G_?$ gives the result

$$\frac{a_{first\ route}}{a_{second\ route}} = 1 = \frac{m_2^-}{m_1^+}. \tag{3.17}$$

This seems to be disastrous because it implies that the two masses have to be equal, a situation excluded earlier. However, it can also be taken to imply that we require two different sized hyperbolae each with its own a value to get a consistent binary system. Thus we conclude that $G_?$ can be taken to equal to $G_- = -G$ provided we work with one hyperbola with a major axis a_R and a second hyperbola with a major axis a_G and such that

$$\frac{a_R}{a_G} = \frac{m_2^-}{m_1^+} \neq 1. \tag{3.18}$$

However, the conclusion that two hyperbolae are necessary for the construction of a binary gravitational system is otherwise obvious. It would seem impossible to set up two particles moving on the branches of one definite hyperbola and at the same time have the particles fixed at the two available foci.

There are three *simple* ways to view a binary system. Two of them are from the rest mass frame of one or other of the pair and the third is from the centre of gravity frame for the pair. This observation is valid for all combinations of gravitationally characterised components. I have chosen to carry through the calculation here within the rest mass frame of the positively gravitating mass m_1^+. As earlier, I shall call m_1^+ the red component of the binary pair. In the rest frame of the red particle, the negatively gravitating green particle m_2^- will be assumed to move on an hyperbola which as a whole is at rest, The path on which the green particle moves will be called the green path. This path will be an hyperbolic branch containing the stationary red particle fixed at the focus of that branch. The second fixed branch of this hyperbola does not play an active role so its focus will be displayed as an empty green circle in the diagrams 1 on page 24 and 3 on pages 25. It will be helpful to refer to these diagrams for the following discussion. Working in the rest frame of the red particle, which is by definition fixed in this frame, will have the consequence that the *path* of the red particle, the red path, will have to be in motion, rotating and translating while changing points on the red path remain attached to the definite fixed position of the red particle. Thus for the frame we are working in, the red path moves through the fixed red particle position rather than the red particle moves on the red path. The green particle is on the green path but if it is to exert a repulsive force on the red particle it must also be at a remote focus of the red path. Thus mathematically we have to set up the two conditions:

1 Red particle on local focus of green path
2 Green particle on remote focus of red path

The hyperbolic parameterisations for various hyperbolic branches with respect to the two foci posibilities are given below,

$$x_{LL}(\theta, a, e) = a(e - \cosh(\theta)) \tag{3.19}$$
$$x_{RL}(\theta, a, e) = -a(e + \cosh(\theta)) \tag{3.20}$$
$$x_{LR}(\theta, a, e) = a(e + \cosh(\theta)) \tag{3.21}$$
$$x_{RR}(\theta, a, e) = -a(e - \cosh(\theta)) \tag{3.22}$$
$$y_{LL}(\theta, a, e) = a(e^2 - 1)^{1/2} \sinh(\theta) \tag{3.23}$$
$$= y_{LR}(\theta, a, e) \tag{3.24}$$
$$= y_{RL}(\theta, a, e) \tag{3.25}$$
$$= y_{RR}(\theta, a, e). \tag{3.26}$$

The two letter subscripts LL etc refer to the focus and branch involved respectively. For example LR means left focus for origin of coordinates and right branch for path. This terminology is OK provided we rethink when a rotating hyperbola turns through more than $\pm\pi/2$. Thus to impose the conditions (1) and (2) into the mathematics of our binary system, we note that the separation distance r_s, between our red and our green particle can be expressed in two ways, in terms of the red particle parameters or in terms of the green particle parameters, each way giving the same numerical value for r_s We are working in the red rest frame for the red particle and this is also the rest frame for the whole hyperbolic path of the green particle. This path will also have a fixed axis. However the red particle's hyperbolic path will be rotating in this frame so that we have to introduce its axial rotation away from the fixed axis of the green particle path. This rotation I shall call the angle β. β is the angle between the green path's axis and the red path's axis. The positions of the various parameter components are given by the blue lines in diagram 3 on page 25 together with identification positions A, B, C, D, E, F defined below

$$FA = x_{RL} \tag{3.27}$$
$$FD = y_{RL} \tag{3.28}$$
$$\angle BFA = \beta \tag{3.29}$$
$$\angle EFD = \beta \tag{3.30}$$
$$FB = x_{RL}\cos(\beta) \tag{3.31}$$
$$ED = y_{RL}\sin(\beta) \tag{3.32}$$
$$BA = x_{RL}\sin(\beta) \tag{3.33}$$
$$CB = y_{RL}\cos(\beta) \tag{3.34}$$
$$DC = x_{LL} \tag{3.35}$$
$$CA = y_{LL} \tag{3.36}$$
$$FB - ED = DC \tag{3.37}$$
$$CB + BA = CA. \tag{3.38}$$

Thus using (3.37) and (3.38) and equating components for r_s in the two parameterisations, we obtain

$$x_{RL}(\theta_R, a_R, e_R)\sin(\beta) + y_{RL}(\theta_R, a_R, e_R)\cos(\beta) = y_{LL}(\theta_G, a_G, e_G) \tag{3.39}$$
$$-y_{RL}(\theta_R, a_R, e_R)\sin(\beta) + x_{RL}(\theta_R, a_R, e_R)\cos(\beta) = x_{LL}(\theta_G, a_G, e_G). \tag{3.40}$$

My objective is to produce a single nontrivial case of a mixed gravity binary system to establish that such systems can theoretically exist without internal contradiction or violation of physical laws. This can be most easily achieved by taking the simplest case. We have seen that the masses of the two components need be different and this translates into the $a's$ being not equal. The other parameter e can be taken equal for the two orbits just to reduce notation and the arithmetic. Thus from now on I shall take $a_G = a$ and $a_R = 0.3a_G = 0.3a$. Then the basic equations become

$$\begin{aligned} x_{RL}(\theta_R, 0.3a, e)\sin(\beta) &+ y_{RL}(\theta_R, 0.3a, e)\cos(\beta) \\ &= y_{LL}(\theta_G, a, e) \quad (3.41) \\ -y_{RL}(\theta_R, 0.3a, e)\sin(\beta) &+ x_{RL}(\theta_R, 0.3a, e)\cos(\beta) \\ &= x_{LL}(\theta_G, a, e). \quad (3.42) \end{aligned}$$

These last two simultaneous equations can be solved for the pair of variables $\sin(\beta), \cos(\beta)$ to give

$$\sin(\beta) = \frac{x_{RL}(\theta_R, 0.3a, e)y_{LL}(\theta_G, a, e) - y_{RL}(\theta_R, 0.3a, e)x_{LL}(\theta_G, a, e)}{x_{RL}(\theta_R, 0.3a, e)^2 + y_{RL}(\theta_R, 0.3a, e)^2} \quad (3.43)$$

$$\cos(\beta) = \frac{y_{RL}(\theta_R, 0.3a, e)y_{LL}(\theta_G, a, e) + x_{RL}(\theta_R, 0.3a, e)x_{LL}(\theta_G, a, e)}{x_{RL}(\theta_R, 0.3a, e)^2 + y_{RL}(\theta_R, 0.3a, e)^2}. \quad (3.44)$$

The expanded versions for $\sin(\beta) = S(\theta_R, \theta_G)$ and $\cos(\beta) = C(\theta_R, \theta_G)$ are given next in terms of the parameters, θ_R and θ_G, for position on the two hyperbolae,

$$\begin{aligned} \sin(\beta) &= \left(\frac{(e^2-1)^{1/2}}{0.3}\right)\left(\frac{\cosh(\theta_G)\sinh(\theta_R) - \sinh(\theta_G)\cosh(\theta_R)}{(e\cosh(\theta_R)+1)^2} - \frac{e(\sinh(\theta_G)+\sinh(\theta_R))}{(e\cosh(\theta_R)+1)^2}\right) \\ &= S(\theta_R, \theta_G). \quad (3.45) \end{aligned}$$

$$\begin{aligned} \cos(\beta) &= \frac{3}{10}\left(\frac{e^2(\sinh(\theta_R)\sinh(\theta_G) - 1) - e(\cosh(\theta_R) - \cosh(\theta_G))}{(e\cosh(\theta_R)+1)^2} - \frac{\sinh(\theta_R)\sinh(\theta_G) + \cosh(\theta_R)\cosh(\theta_G)}{(e\cosh(\theta_R)+1)^2}\right) \\ &= C(\theta_R, \theta_G). \quad (3.46) \end{aligned}$$

Thus it is possible to find the relation between θ_R and θ_G for the binary system to hold together consistently by using the identity,

$$\sin^2(\beta) + \cos^2(\beta) = 1 \tag{3.47}$$

and employing the functions above at (3.45) and (3.46). However, the formula that emerges by that route is intrinsic and complicated in the variables θ_R and θ_G and it is difficult to see how to extract an explicit relation giving for example a function of one in terms of the other such as, $\theta_R(\theta_G)$. A useful result that can easily be obtained from the S and C functions is the angle between the two orbit axes for any pair of positions. This is given by the inverse tangent

$$\beta(\theta_R, \theta_G) = \tan^{-1}\left(\frac{S(\theta_R, \theta_G)}{C(\theta_R, \theta_G)}\right) \tag{3.48}$$

and can be used in finding the moving rotating path of the red particle in the green frame or vice versa. If we return to the original pair of equations, we can find a the relation between the two position on curve parameters, θ_R and θ_G, simply by squaring the two equations and adding the results together to obtain,

$$\begin{aligned} r^2_{s,G}(\theta_R, e) &= x_{RL}(\theta_R, a_R, e_R)^2 + y_{RL}(\theta_R, a_R, e_R)^2 && (3.49)\\ &= x_{LL}(\theta_G, a_G, e_G)^2 + y_{LL}(\theta_G, a_G, e_G)^2 && (3.50)\\ &= r^2_{s,R}(\theta_G, e), && (3.51)\end{aligned}$$

whilst eliminating β at the same time. This result simultaneously defines the square of the separation distance, $r^2_{s,G}(\theta_R)$, between the two particles, when viewed from the green particle or as, $r^2_{s,R}(\theta_G)$, when viewed from the red particle, θ_R varying from the green point of view and vice versa. Expanding the expressions in (3.49) and (3.50) we find,

$$\begin{aligned} r^2_{s,G}(\theta_R, e) &= 0.3^2(e\cosh(\theta_R) + 1)^2 && (3.52)\\ r^2_{s,R}(\theta_G, e) &= (e\cosh(\theta_G) - 1)^2 && (3.53)\\ r_{s,G}(\theta_R, e) &= \pm 0.3(e\cosh(\theta_R) + 1) && (3.54)\\ r_{s,R}(\theta_G, e) &= \pm(e\cosh(\theta_G) - 1) && (3.55)\\ \theta_{R,\pm,\tilde{\pm}}(\theta_G) &= \tilde{\pm}\cosh^{-1}(\pm\cosh(\theta_G)/0.3 \mp 1/(0.3e) - 1/e) && (3.56)\\ \theta_{G,\pm,\tilde{\pm}}(\theta_R) &= \tilde{\pm}\cosh^{-1}(\pm 0.3\cosh(\theta_R) + 1/e \pm 0.3/e). && (3.57)\end{aligned}$$

The fifth equation above is the sought relation between θ_R and θ_G resulting from the equality of the two separation views from the first two equations. The leading $\tilde{\pm}$ at (3.56)and (3.57) results from the evenness of the cosh function with the $\sim$ indicating that this $\pm$ is independent of the others, so implying four possibilities. The sixth expression above is the inverse transformation of the fifth. The polar forms for the separation distances can be obtained by replacing the parameters θ_R and θ_G by functions of their angular equivalents ϕ_R and ϕ_G, say,

$$\phi_R(\theta_R) = \tan^{-1}\left(\frac{-y_{RL}(\theta_R, 0.3a, e)}{x_{RL}(\theta_R, 0.3a, e)}\right) \tag{3.58}$$

$$\phi_G(\theta_G) = \tan^{-1}\left(\frac{y_{RL}(\theta_G, a, e)}{x_{RL}(\theta_G, a, e)}\right) \tag{3.59}$$

$$\phi(\theta_G) = \tan^{-1}\left(\frac{y_{LL}(\theta_G, a, e)}{x_{LL}(\theta_G, a, e)}\right). \tag{3.60}$$

Both expressions at (3.54) and (3.55) after the $\pm$ signs above are always positive because $e > 1$ and $\cosh(x) > 1$. Consequently, one might infer that the negative options can be ignored on the grounds of being unphysical, if the $r_{s,R/G}$ quantities represent physical distance. This turns out to be not the correct inference so that the final definitions of these quantities have to be such that they, being scalar distance, are carefully defined to come out positive. This will be explained. The inversions of the three equations above, the θs in terms of the ϕs are

$$\theta_R(\phi_R) = \cosh^{-1}\left(\frac{e\tan^2(\phi_R) - \sec(\phi_R)(e^2-1)}{e^2 - \sec^2(\phi_R)}\right) \tag{3.61}$$

$$\theta_G(\phi_G) = \cosh^{-1}\left(\frac{e\tan^2(\phi_G) + \sec(\phi_G)(e^2-1)}{e^2 - \sec^2(\phi_G)}\right) \tag{3.62}$$

$$\theta(\phi) = \cosh^{-1}\left(\frac{e\tan^2(\phi) - \sec(\phi)(e^2-1)}{e^2 - \sec^2(\phi)}\right)). \tag{3.63}$$

As a result of these equations, we can re-express the formulae for the distance between red and green particles, the $r_{s,R}(\theta_G)$ and $r_{s,G}(\theta_R)$ functions at (3.54) and (3.55), in terms of the angles ϕ_G and ϕ_R respectively.

$$
\begin{aligned}
r'_{s,G}(\phi_R, e) &= \pm 0.3(e\cosh(\theta_R(\phi_R)) + 1) &(3.64)\\
&\rightarrow \left|0.3\left(\left(\frac{e^2\tan^2(\phi_R) - \sec(\phi_R)e(e^2-1)}{e^2 - \sec^2(\phi_R)}\right) + 1\right)\right| &(3.65)\\
&= \left|\frac{0.3(e^2-1)}{\cos(\phi_R)e + 1}\right| &(3.66)\\
r'_{s,R}(\phi_G, e) &= \pm(e\cosh(\theta_G) - 1) &(3.67)\\
&\rightarrow \left|\left(\left(\frac{e^2\tan^2(\phi_G) + \sec(\phi_G)e(e^2-1)}{e^2 - \sec^2(\phi_G)}\right) - 1\right)\right| &(3.68)\\
&= \left|\frac{(e^2-1)}{\cos(\phi_G)e - 1}\right| &(3.69)
\end{aligned}
$$

the primes being introduced to differentiate these new functions of ϕ from the original functions of θ. The $\pm$ options have been assigned so that both these length functions are positive as indicated by the modulus sign, $||$ enclosing their final definitions. This is the point I suggested earlier should be treated with care as the introduction of the ϕ versions with the inverse coshes seems to open up the possibility for negative values. The relations between the ϕ_Rs and ϕ_Gs corresponding to the relations between the θ_Rs and θ_Gs at (3.56) and (3.57) which make the two lengths equal are obtained easily by taking the two lengths *as equal* and then solving for either ϕ_G or ϕ_R and are

$$
\begin{aligned}
\phi_G(\phi_R) &= \cos^{-1}((\cos(\phi_R)/0.3 + 1.3/(0.3e))) &(3.70)\\
\phi_R(\phi_G) &= \cos^{-1}((0.3\cos(\phi_G) - 1.3/e)) &(3.71)\\
& &(3.72)
\end{aligned}
$$

This completes most of the technicalities and leaves us with the two distances, $r'_{s,G}(\phi_R)$ and $r'_{s,R}(\phi_G)$, between the red and green particles as would be seen by an observer on the green particle in terms of the red particle's angular parameter ϕ_R and an observer on the red particle in terms of the green particle's angular parameter ϕ_G respectively. The green observer sees the red particle and its path and the red observer sees the green particle and its path. However, these distances are not of equal magnitude unless the basic θ_R and the basic θ_G are related by the formula (3.56) or the formula (3.57) or the equivalent ϕ_R and ϕ_G are equivalently related.

4 Assembling the Paths

Most of the mathematics for this problem has now been completed. We have seen that in the rest frame of the red particle, the path of the green particle is also at rest, the red particle being at the focus of this hyperbolic path. It is also established that the axis of the path of the red particle will have to have rotated relative to the green path axis by the angle β given at (3.48). Concentrating on the view of the situation from the rest frame of the red particle in which the path of the green particle is a simple hyperbolic branch also at rest, it is convenient to introduce a rotated frame, relative to this at the angle β to it, in which the green particle is at rest and the path of the red particle is also at rest. To do this, I introduce the rotation-translation transformation of coordinates based at the empty green focus which is at (0,0),

$$X_{rot}(x, y, d_1, \beta) = (x\cos(\beta) - y\sin(\beta)) + d_1 \quad (4.1)$$

$$Y_{rot}(x, y, d_2, \beta) = (x\sin(\beta) + y\cos(\beta)) + d_2 \quad (4.2)$$

$$d_1 = x_{RL}(\theta_G, a, e) \quad (4.3)$$

$$d_2 = y_{RL}(\theta_G, a, e), \quad (4.4)$$

where d_1 and d_1 are displacements following the rotation that take the empty green focus to the momentary position of the green particle assumed to be at parameter value θ_G. Thus this transformation takes us into a reference frame in which the green particle is at the origin of the transformed x-axis which is at the angle β relative to the axis of the green parabolic path. In other words, the x-axis of this frame is on the x-axis of the red hyperbola with the green particle on its active focus. Thus to find the parametric equation for the red particle path all we have to do is place its parametric representation into the rotations (x, y) coordinate position and replace the *rot* subscripts with *red* to indicate we now have the two parametric components for the path of one of the branches of the moving red hyperbola as functions of ϕ_R and θ_G.

$$\begin{aligned}
X_{red}(\phi_R, \theta_G) &= x_{RL}(\theta_R(\phi_R), 0.3a, e)\cos(\beta(\theta_R(\phi_R), \theta_G)) \\
&- y_{RL}(\theta_R(\phi_R), 0.3a, e)\sin(\beta(\theta_R(\phi_R), \theta_G))) \\
&+ x_{RL}(\theta_G, a, e) \qquad (4.5) \\
Y_{red}(\phi_R, \theta_G) &= x_{RL}(\theta_R(\phi_R), 0.3a, e)\sin(\beta(\theta_R(\phi_R), \theta_G)) \\
&+ y_{RL}(\theta_R(\phi_R), 0.3a, e)\cos(\beta(\theta_R(\phi_R), \theta_G))) \\
&+ y_{RL}(\theta_G, a, e), \qquad (4.6)
\end{aligned}$$

where θ_R has been related to its angular equivalent ϕ_R and β has been replaced by the function of θ_R and θ_G at (3.48) repeated below.

$$\beta(\theta_R, \theta_G) = \tan^{-1}\left(\frac{S(\theta_R, \theta_G)}{C(\theta_R, \theta_G)}\right). \qquad (4.7)$$

The other co-moving branch is given by

$$\begin{aligned}
X'_{red}(\phi_R, \theta_G) &= x_{RR}(\theta_R(\phi_R), 0.3a, e)\cos(\beta(\theta_R(\phi_R), \theta_G)) \\
&- y_{RR}(\theta_R(\phi_R), 0.3a, e)\sin(\beta(\theta_R(\phi_R), \theta_G))) \\
&+ x_{RL}(\theta_G, a, e) \qquad (4.8) \\
Y'_{red}(\phi_R, \theta_G) &= x_{RR}(\theta_R(\phi_R), 0.3a, e)\sin(\beta(\theta_R(\phi_R), \theta_G)) \\
&+ y_{RR}(\theta_R(\phi_R), 0.3a, e)\cos(\beta(\theta_R(\phi_R), \theta_G))) \\
&+ y_{RL}(\theta_G, a, e), \qquad (4.9)
\end{aligned}$$

with the same adaptations as for the first branch.

With the parameterisation of the moving hyperbolic path for the red particle obtained at (4.5) and (4.6), it is possible to plot that path which has the moving green particle at its active focus and so give a clear diagrammatic picture for any chosen position, the value of θ_G, of the green particle on it own path which you will recall is fixed, together with the red particle at its left focus, in the basic reference frame on which we are concentrating. The fixed path branches for the green particle is given by the parameterisations,

$$\begin{aligned}
X_{green}(\phi_G) &= x_{RL}(\theta_G(\phi_G), a, e) \qquad (4.10) \\
Y_{green}(\phi_G) &= y_{RL}(\theta_G(\phi_G), a, e) \qquad (4.11) \\
X'_{green}(\phi_G) &= x_{RR}(\theta_G(\phi_G), a, e) \qquad (4.12) \\
Y'_{green}(\phi_G) &= y_{RR}(\theta_G(\phi_G), a, e). \qquad (4.13)
\end{aligned}$$

We now have all the basic mathematical structure to plot and analyse the path structure and the relative motions of the particles in terms of the angular changes of ϕ_R and or ϕ_G. Most of the emphasis here has been the view of the binary pair of an observer fixed to the rest frame of the red particle and the accompanying fixed hyperbolic path of the green particle. From that frame of reference the observes sees only the motion of the green particle around its orbit. It is not difficult to recast the whole structure in terms of the view of an observer on the green particle and his accompanying fixed hyperbolic orbit of the red particle. Further these two basic views can then be put together using the definitions of the centre of mass or centre of gravity systems to give an observers view from either the centre of mass or the centre of gravity frame. However, only the last of these reference frame possibilities could be taken to be truly inertial. Using the *Mathematica* animation process, I have produced a simulation of the movement of the mixed character binary pair with 301 frames. One of these frames has been used to produce the diagram on page 24. The full simulation in mathematica note book language can be downloaded in the file *mixmass.nb* from my website at, QMUL Maths. It can now be shown that the structure of the force configurations for this binary pair composed of oppositely gravitationally characterised particles does conform to the correct conditions. They are that the red particle exerts a gravitational attraction on the green particle and the green particle exerts a gravitational repulsion on the red particle. This can be done by using a well know result from classical dynamics and inverse square law forces which recovers the force involved from just knowing the distance in detail of a possibly influenced particle from the force source as a function of the angle ϕ. The formula required is derivable from (2.3)and is

$$\begin{aligned}\alpha_{r,R} &= \hat{\mathbf{r}} \cdot d^2\mathbf{r}/dt^2 = (\ddot{r} - r\dot{\phi}^2) = -Gm_2^+/r^2(\phi) \\ &= h^2u^2(\phi)(d^2u(\phi)/d\phi^2 + u(\phi)) = -Gm_2^+u^2(\phi) \qquad (4.14)\\ u(\phi) &= 1/r(\phi) \qquad (4.15)\\ h &= r^2(\phi)\dot{\phi}^2, \qquad (4.16)\end{aligned}$$

where normally m_2^+, as indicated, the gravitating source would be positively gravitating. h is the constant arising from integrating the transverse component of acceleration $\alpha_{\phi,R}$ on the assumption that the mutual interaction of the particle pair only acts along their line of separation. As we are working with the inverse square law of gravitation $u^2(\phi)$ appears twice in the formula (4.14) and so can be cancelled from both sides of the second equality to define $G_{type}(\phi)$ and to give,

$$G_{type}(\phi) = h^2(d^2u(\phi)/d\phi^2 + u(\phi)) = -Gm_2^+, \tag{4.17}$$

where $u(\phi)$ is the inverse of the magnitude of a position vector for distant points originating at the position of the mass m_2^+. This shows immediately, because of the $-G$, that we are dealing with an attraction towards the source under gravity which in the past has been thought to always hold. The detailed steps in deriving the formula (4.17) outlined above can be found in A. S. Ramsey's book ([42]). They are elementary but I think this formula needs to be treated with care because it is rather profound. In particular, it should be noted that the force involved on the right hand side above is the force originating from the particle m_2 which is at the origin of coordinate from which the radial length, $r(\phi) = 1/u(\phi)$, is measured and which is also the magnitude of a vector, $\mathbf{r}(\phi)$, which has its tail at the same origin. However, the angular parameter ϕ on which it depends refers to the angular component of acceleration at the sharp end of the same vector and is involved with the path where the induced distant acceleration field may be influencing other particles but even so this ϕ is an angular parameter measuring an angle as perceived by an observer at the source of the force. According to general relativity all such other particles at the sharp end of the vector $\mathbf{r}(\phi)$ will experience the same acceleration due to the gravitational field of the particle m_2. So there is some sense that general relativity is at least a part feature of the formula. Thus when using this formula to find the nature of the force involved we must not lose sight that it refers exclusively to the source location of the force. The formula determines how the source observer sees the process at a distance but it contains no feedback from the distant events. This has the consequence that the function $u(\phi)$ which is used in the formula should be specific to the source as origin which one is trying to interpret. That is to say, it should not be in a form which makes it equal to the distance between the binary pair as observed from the other member of the pair. If the physically equal distances are made mathematically equal before using the formula, The two $u(\phi)$ functions become *identically* equal and so the differential form in the expression (4.17) gives the same result for the two binary members. The significance of this formula is that it gives information about the local source force, whether attractive or repulsive, valid for all receptor particles at whatever distance from the source they may be and whatever gravitational character they may have. This independence on receptor distance is indicated in the diagrams 1 and 2 on page 25 by plotting the formula value with corresponding value of $u(\phi)$. Thus we can use the raw functions of ϕ_R and ϕ_G at (3.66) and (3.69) to determine the type of gravity originating from the green particle or the red particle respectively. These expressions are repeated below

$$r'_{s,G}(\phi_R, e) = \left|\frac{0.3(e^2-1)}{\cos(\phi_R)e+1}\right| = 1/u_{Green}(\phi_R, e) \quad (4.18)$$

$$r'_{s,R}(\phi_G, e) = \left|\frac{(e^2-1)}{\cos(\phi_G)e-1}\right| = 1/u_{Red}(\phi_G, e), \quad (4.19)$$

with the second equalities giving the appropriate $u(\phi, e)$ function.

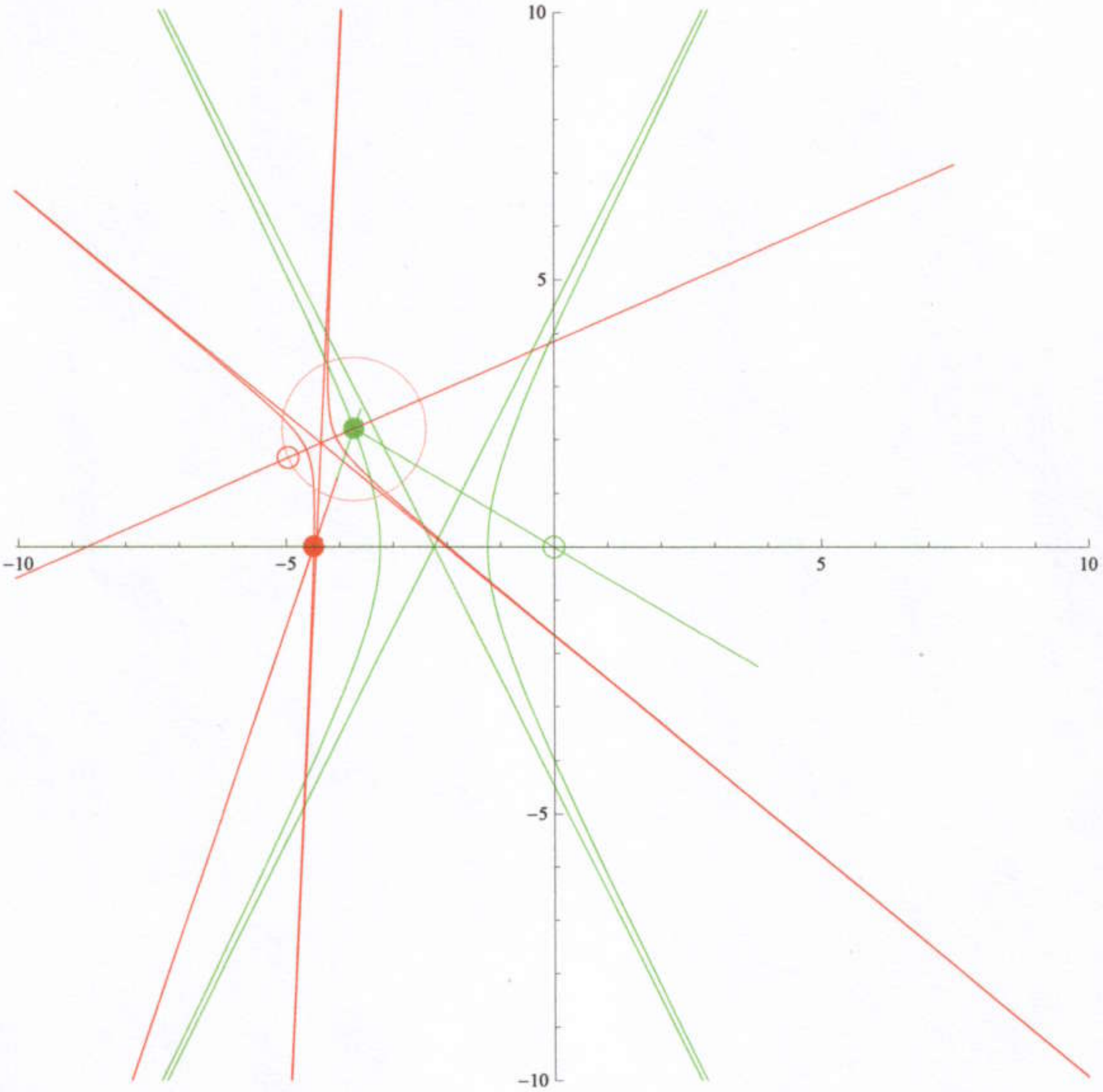
If we evaluate the first and second derivatives of $u_{Green}(\phi_R, e)$ and $u_{Red}(\phi_G, e)$, with respect to ϕ_R and ϕ_G respectively and then use them to find the value given by G_{type} for the two cases, we find

$$G_{type}(\phi_R) = h^2(d^2u_{Green}(\phi_R, e)/d\phi_R^2 + u_{Green}(\phi_R, e)) > 0 \quad (4.20)$$

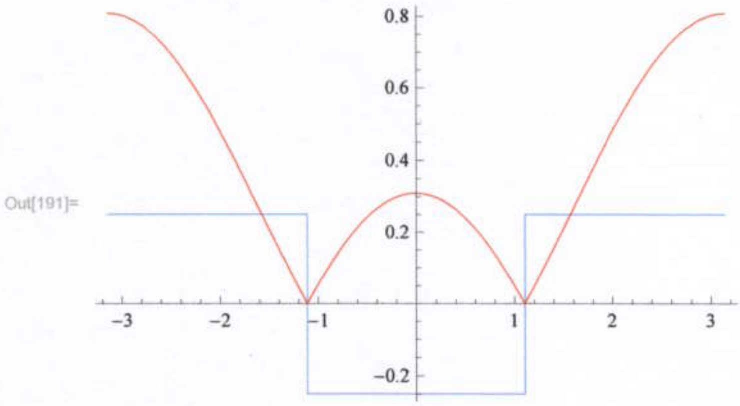
$$G_{type}(\phi_G) = h^2(d^2u_{Red}(\phi_G, e)/d\phi_G^2 + u_{Red}(\phi_G, e)) < 0, \quad (4.21)$$

implying that $G_{type}(\phi_R) = Gm^+$ and $G_{type}(\phi_G) = -Gm^-$ and thus showing, as expected, that the red particle will attract all other particles in its neighbourhood and the green particle will repel all other particles in its neighbourhood. The situation can be clearly seen from plots of $G_{type}(\phi_R)$ for the two cases that are given at diagrams 1 and 2 on page 25. The corresponding $u(\phi)$ for the two cases is plotted with the $G_{type}(\phi)$ block function and this shows clearly how the character of any source does not depend on $r(\phi) = 1/u(\phi)$, the distance of any other particle in its vicinity whatever character that particle may have.

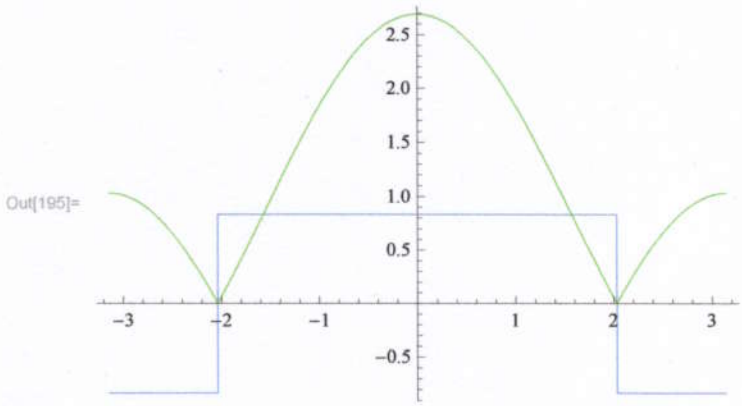
Diagram 1
Frame from Binary Pair Animation



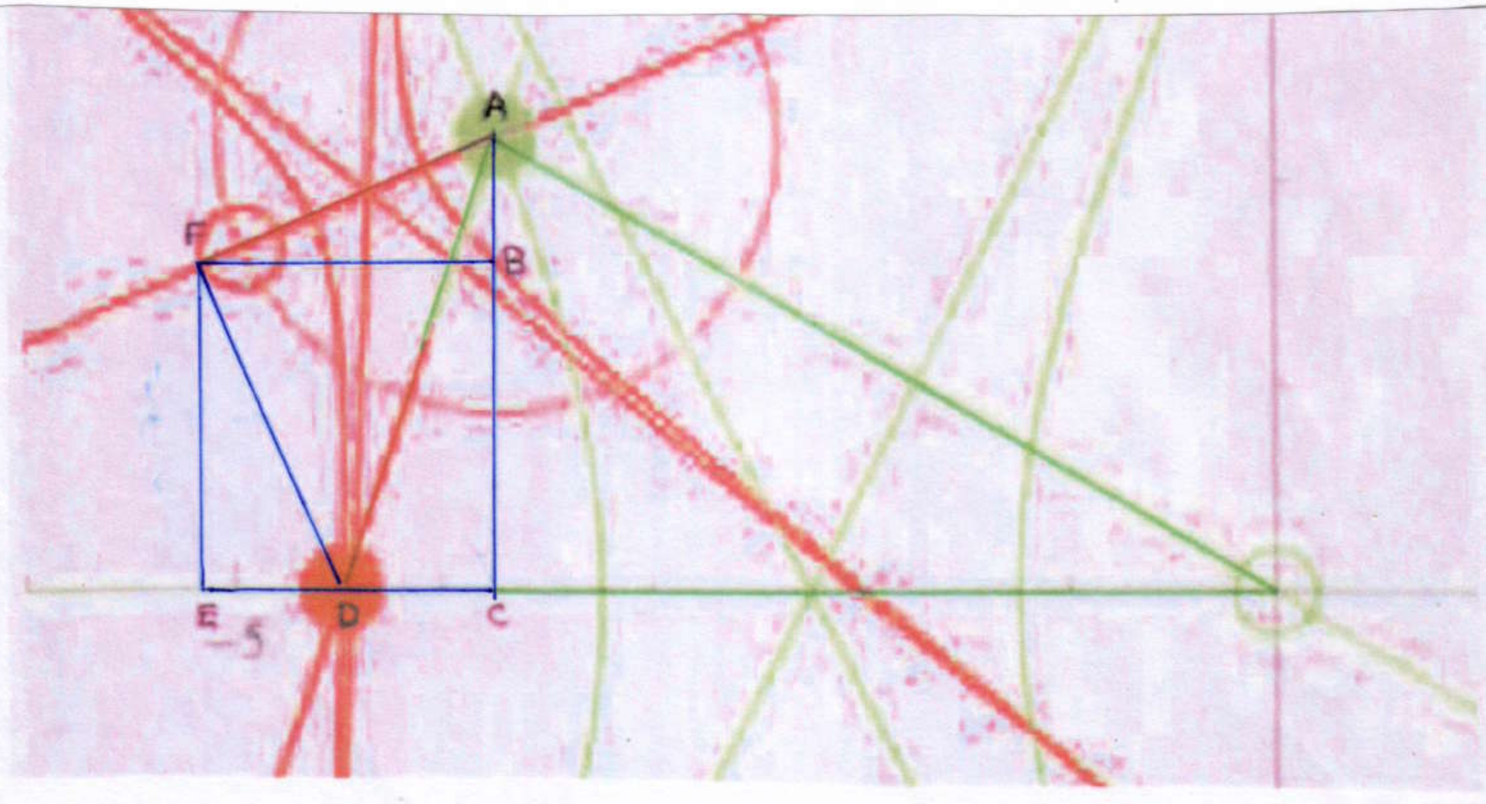
1 Character test Red particle as Source



2 Character test Green particle as Source



3 Parameterisations Connections



5 Conclusions

The work in this paper makes strong *theoretical* evidence for the possible existence of *particulate negative gravitating* positive mass particles. This has been achieved by showing that negatively gravitating particles can be involved in composite systems and follow orbits in conjunction with positive gravitating particles in much the same way as the later type of usual particles can form systems and gravitationally interact. In the specific case examined here, this new orbital system type does not have the bizarre features that were noted in the fifties studies by Herman Bondi. In the light of this special but typical case, it seems reasonable to assume that if such particles do exist their interaction generally with the normal type of particles would be theoretically predictable and not remarkably odd. Of course, I have not given any physical evidence that such particle do exist.

There are a number of interesting issues that arise from this work. One such issue is the meaning and significance of the term *particle*. We talk about particle in quantum theory and at the other end of the scale, in cosmology, galaxies are often regarded as particles. In the definition of particle that has been used in this work the whole of this vast range has been included because it is generally believed that all such *particles* whether they have rest mass or not are influenced by and can cause gravity fields. However we think we know that negative gravity particles, if they exist at all, will be very rare entities. It is a fact that antiparticles in general are very rare, a fact that is regarded as a great mystery. The question is, where are the missing antiparticle? Some scientists believe that normal and anti particle where created together in equal numbers at an early stage of the evolution of the universe but now most of the antiparticle seem to have disappeared. This prompts me to make the speculation that *dark energy* is composed of those missing antiparticles and further those antiparticles that we do detect are manifestations of the local sea of dark energy particles and like them are also anti gravitational. This view is also supported by the accepted cosmological fact that at some time t_c in the evolution of the universe there was equal quantities of normal mass and dark energy mass within the boundaries of the universe. This time could be the moment of creation of the equal anti and normal particle densities.

Staying with the particle concept problem, there is second interesting issue. How is it that accumulations of mass arise? It a convincing idea in cosmology that large accumulations of mass such as a in a galaxy could have arisen by gravitational accretion or by a dispersed distribution of particulate mass falling together as a result of the mutual gravitational attraction of its component particles. Stars could be formed in the same way. This may be what happens in some or many cases but it is possible that such structure are formed and spill out from some unexplained minute but greatly massive singularity. Whatever the true situation is, it would seem that a dispersed distribution of negative gravitating particles would not accrete into a single mass because the elements of the distribution would be mutually repulsive. This suggests that negative gravity galaxies of the usual type are unlikely to exist. However, there is another possible twist to this issue and that is how do the mass accumulations within elementary particles form. Some researchers think that gravity is involved in the structure of elementary particle. They could have been formed by some sort of gravitational collapse and it could be that gravity holds them together. The snag is that gravity is a very week force and such ideas are not at all convincing. In fact, we have no idea what holds elementary particle together or how they are first formed from *energy*. In quantum, mechanics however, we do have good ideas about how particle transform between themselves. This does have a bearing on the question of possible galactic size *negative* mass particles. Dense matter that is found in atoms, molecules, rocks, planets, stars and galaxies is held together by atomic forces of one sort or another. The smaller accumulated parts then held together in the large by positive gravity. This suggests that large accumulations of negative mass particles could be possible if constructed by atomic processss without the intervening spaces found in normal galaxies. I am suggesting that galactic sized molecules could be formed from negatively gravitating particles. Enough speculation, I return to the main issue of the orbiting binary pair and its lessons on fundamentals in the next paragraph.

Another issue of interest is the question of Newton's *Action equals Reaction* principle, his third Law, and how that squares with *Action at a distance* in the case of the gravitational interaction between separated positive and negatively characterised particles. Other than mentioning this issue, I have carried through the work in this paper as though there is no problem. In fact, that is the case, there is no problem this being an important consequence that follows through from Einstein's principle of equivalence in general relativity.

To explain this issue, I first quote one popular version of Newton's third law:

If a first body exerts a force **f** *on a second body,*
the second body exerts a force -**f** *on the first body.*
f *and* -**f** *are equal in size and opposite in direction.*

Alternatively, in a form nearer to Newton's original presentation:

All forces occur in pairs, and these two forces are
equal in magnitude and opposite in direction.

Newton's original version of this law is not very specific in respect of what a force is and what it is doing but it does seem to be talking about one basic force and its somehow reflected mate such as we all experience when we push an object with our hand and feel the resisting response, presumably the reaction to our action. Such a situation is decidedly local and is essentially all about a single basic force and is very intuitively convincing. The first definition talks about bodies, says essentially the same as does Newton's law but although it mentions bodies it does not define a body or explain how these two bodies are orientated in space. Of course, fundamental definitions have to be minimal. Neither of these definition obviously apply to bodies separated in space in the way that a binary pair of particle, whatever their gravitational characteristics may be, are separated in space. However, it is usual practice to invoke the action and reaction principle in the standard binary pair case to justify saying the force on one from the other must be of opposite sign to the force from the other to the one. This practice does give a correct result in the usual binary pair case but as we have seen it seems not to work for the mixed mass pair. This situation prompts me to suggest that calling on Newton's third law to support the proposition that the two possible gravitational forces between distant particle from one or the other must be equal is philosophically incorrect, in spite of it giving a correct result. Before I make this case, let us consider how general relativity theory describes the properties of and incorporates a local gravitational field arising from a distant source. Local *acceleration* fields generated by distant gravitating particles take precedence over the gravitational forces experienced by any local particle. This is essentially *Einstein's Principle of equivalence* which manifests itself by the fact that all particles in a given location move with the same acceleration due to local gravity, regardless of their individual masses and, indeed this is also part of the Newtonian theory structure as described by equations (2.8) and (2.12) from which it is clear that the mass of a gravitationally influenced particle cancels out from both side of the equation of motion under gravity. Essentially this means that two spatially separated gravitating particles produce *independent* acceleration fields in which the other particle moves. Each of the separated particles causes an action at a distance on the other particle but the two particles are acting independently. This is certainly not reaction of one to the action of the other as implied by Newton's third law from whichever of the two possible directions it may be viewed. However, I have shown above that in the classical case the forces involved do satisfy the condition

$$\mathbf{f}_1 = -\mathbf{f}_2, \tag{5.1}$$

which is the condition for action and reaction to be equal in this classical case of a positively characterised pair. Thus if the equal action reaction property holds, how is it reconcilable with my claim that this is not an example of Newton's third law? The answer to this question is that there is a more fundamental simple and kinematic reason of greater generality than Newton's third law. We can see this clearly by examining the equations at (2.33) and (2.35) rewritten below in a rearranged form.

$$\begin{aligned} -\, d^2\mathbf{r}(t)/dt^2 &= \frac{\mathbf{f}_1}{M^+} = -\frac{\mathbf{f}_2}{M^+} && (5.2) \\ M^+ &= \frac{m_1^+ m_2^+}{m_1^+ + m_2^+} && (5.3) \\ d^2\mathbf{r}(t)/dt^2 &= \frac{\mathbf{f}_1}{M^-} = +\frac{\mathbf{f}_2}{M^-} && (5.4) \\ M^- &= +\frac{m_1^+ m_2^-}{m_1^+ - m_2^-}. && (5.5) \end{aligned}$$

The relative separation acceleration vectors for a binary system as seen from each particle of the pair for the classical case, the first two, and for the mixed mass case, the second two, can be defined as,

$$\mathbf{a}_1^+ = \frac{\mathbf{f}_1}{M^+} \tag{5.6}$$

$$\mathbf{a}_2^+ = \frac{\mathbf{f}_2}{M^+} \tag{5.7}$$

$$\mathbf{a}_1^- = \frac{\mathbf{f}_1}{M^-} \tag{5.8}$$

$$\mathbf{a}_2^- = \frac{\mathbf{f}_2}{M^-}. \tag{5.9}$$

If negatively gravitating particles do exist then, according to (5.2) and (5.4) all the situations that can occur with a same type gravitating pair or a mixed pair can be summarised as,

$$\mathbf{a}_1^\pm = \mp\mathbf{a}_2^\pm. \tag{5.10}$$

I suggest that in astronomical physics in the context of separated bodies Newton's law of action and reaction could usefully and more realistically be replaced with the new law:-

The relative gravitational separation acceleration vectors $\mathbf{a_1}$ *and* $\mathbf{a_2}$ *for two bodies satisfies one or other of the two case,* $\pm$*, relation*

$$\mathbf{a}_1^\pm = \mp\mathbf{a}_2^\pm. \tag{5.11}$$

Whether negatively gravitating particles exist or not Newton's law of action and reaction in the astrophysics of separated bodies could usefully be replaced as above with only the + sign in the index and the minus sign multiplying.

I have carried through the work in this paper using only Newton's equations of motion but with much guiding influence from general relativity ideas. I think a fully relativistic treatment of this topic would be very complicated and not reveal any substantial deviations from the conclusions that I have arrived at here.

Acknowledgements

I am greatly indebted to Professors Clive Kilmister and Wolfgang Rindler for help, encouragement and inspiration and additionally to Clive and professor Steven Barnett for for drawing my attention to the Bondi negative mass paradox.

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