SUPERLUMINAL INTERACTION, OR THE SAME, DE BROGLIE RELATIONSHIP, AS IMPOSED BY THE LAW OF ENERGY CONSERVATION

PART I: ELECTRICALLY BOUND PARTICLES

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ABSTRACT

Previously, based on the law of energy conservation, we figured out that, the steady state electronic motion around a given nucleus generally depicts a rest mass variation throughout, though the overall relativistic energy remains constant. Consider for instance, the simple case of the electron in a stationary *elliptic motion* around the proton. Thus, according to our approach, an *infinitesimal portion of the rest mass* of the electron is transformed into *extra kinetic energy,* as the electron moves toward the proton, and an *infinitesimal portion of the kinetic energy* of it is transformed into "extra rest mass" as it slows down away from the proton. Note that our approach is, in no way, conflicting with the usual quantum mechanical approach. Quite on the contrary it provides us with the possibility of elucidating the "quantum mechanical weirdness". It requires, though, the revision of the concept of *field,* which cannot in anyway be determined without the concept of *force.* Our approach is, solely based on the factual concept of *force* and the related concept of *energy.* This makes that it is more natural than the standard approach, based on the concept of *field.* Thus, regarding a bound electron for instance, it is not the *"field energy"* that we decrease, but the *"electron's internal energy",* or the same the *"electron's rest mass".*

We happened to develop our theory originally vis-à-vis gravitational bodies in motion with regards to each other; hence, it is comforting to have both the atomic scale and the celestial scale described on just the same conceptual basis.

One way to conceive the phenomenon we disclosed is to consider a *"jet effect".* Accordingly, a particle on a given orbit through its journey must eject a net mass from its back to accelerate, or must pile up a net mass from its front to decelerate, while its overall relativistic energy stays constant throughout. The speed U of the jet, strikingly, points to the de Broglie wavelength λ_B, coupled with the period of time T_0, inverse of the frequency ν_0, delineated by the electromagnetic energy content $h\nu_0$ of the object of concern; $h\nu_0$ is originally set by de Broglie equal to the total mass m_0 of the object *(were the speed of light taken to be unity).* This makes that, on the whole, the *jet speed* becomes a *superluminal speed* $U = \lambda_B / T_0 = c_0^2 \sqrt{1 - v_0^2 / c_0^2} \big/ v_0$, yet excluding any transport of energy. Recent measurements appear to back up our conjecture. Our result, in any case, seems to be important in many ways. Amongst other things, it means that, either gravitationally interacting macroscopic bodies, or electrically interacting microscopic objects, sense each other, with a speed greater than that of light, and this, in exactly the same manner. Here, furthermore may be a clue, for the *wave-particle duality.*

Our study will be presented in two distinct parts for convenience; the first part deals with generalities, and then with electrically bound particles. The second part deals with the gravitationally bound particles. A general conclusion is drawn at the end of the second part.

1. INTRODUCTION

Consider an object of mass m_0 at rest. In his doctorate thesis, de Broglie has anticipated that,[1] there should be a *periodic phenomenon,* inside m_0, depicting a frequency ν_0, such that

$$h\nu_0 = m_0 c_0^2 \quad . \tag{1}$$

(de Broglie's definition of the periodic phenomenon's frequency inside the object in hand, at rest)

Here, h is the Planck Constant, and c_0 the speed of light in "empty space". It is evident that, de Broglie has envisaged the "extreme case", where the entire mass m_0 would be transformed into electromagnetic energy. It is on the other hand remarkable that he considered Eq.(1), at a time even when, the "annihilation process" of an electron with a positron remained far away to be discovered.

Thus, let λ_0 be the wavelength, and T_0 the period of time, to be associated with the *electromagnetic wave* coming into play. Then, by definition,

$$c_0 = \frac{\lambda_0}{T_0} = \lambda_0 \nu_0 \quad . \tag{2}$$

Eqs. (1) and (2), as usual, lead to

$$\lambda_0 = \frac{h}{m_0 c_0} \quad . \tag{3}$$

(wavelength of the electromagnetic radiation associated with the mass m_0*, as originally assigned by de Broglie, to describe the periodic phenomenon inside the object in hand)*

The frequency ν_0 and the mass m_0 are transformed differently were the object brought to a uniform translational motion;[2] relativistically, the frequency decreases while the mass increases. This observation *(as he mentions it, himself)* intrigued de Broglie for a long time.[1] He ended up with the introduction of a new wavelength λ_B describing the manifestation of the *wave character* of the object; suppose that the object is moving with the velocity v_0; thus de Broglie framed λ_B, similarly to the RHS of Eq.(3), as

$$\lambda_B = \frac{h}{m v_0} \quad ; \tag{4}$$

(de Broglie relationship written for the object in hand, brought to a translational motion)

m is the *relativistic mass* of the moving object, i.e.

$$m = \frac{m_0}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} \quad . \tag{5}$$

Via Eqs. (3), (4) and (5), one can write, in a straightforward way, though unusual, the relationship

$$\lambda_B = \lambda_0 \frac{c_0}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} \quad , \quad v_0 \neq 0 \ , \tag{6-a}$$

[de Broglie wavelength written along with Eq.(1), *in terms of* λ_0, *the wavelength of the periodic phenomenon displayed by the object, at rest]*

between the two wavelengths λ_B and λ_0, in question.

Here, we have taken the precaution to write the de Broglie wavelength for a non-zero velocity, since ordinarily one would think that de Broglie relationship could only be defined, along with a motion. But as will be elaborated later, it seems that, it can be defined for a zero velocity, as well. In this case, it becomes infinitely long. As we will soon detail, it appears then to constitute, without however involving, any *mass or energy exchange,* the basis of an immediate action at a distance.

For this reason, in what follows, we will drop the restriction $v_0 \neq 0$. It becomes interesting to recall that our conjecture is well compatible with the *quantum mechanical uncertainty principle,* since for $v_0 = 0$, the momentum is zero too, which implies that, as strange as it may look at the first strike, the *uncertainty* about the location, is infinite. From our standpoint, this result alone can be considered as a clue for an *"immediate action at a distance",* which we better call a *"wave mechanical interaction".*

Note that, λ_B decreases as v_0 increases, and for $v_0 = c_0$, λ_B becomes null [cf. Eq.(6-a)]. This would mean that, the *wave mechanical interaction* ceases at the level of the ceiling c_0 of the speed of light. But, according to our approach, only a photon of infinite energy would bear the ceiling speed. For instance, a photon falling in a gravitational field gains energy, though its velocity does not significantly increase, no matter how strong is the field, and the photon velocity would never attain c_0. This will be detailed in the next part. At any rate the *wave mechanical interaction* does not involve any mass or energy exchange; any interaction involving *mass or energy exchange,* as usual, cannot occur with a speed above the speed of light. Note further that λ_B becomes λ_0, for $v_0 = c_0\sqrt{2}/2$.

Let us dig a little deeper in Eq.(6-a): $\lambda_1 = \lambda_0\sqrt{1 - v_0^2/c_0^2}$ is *(as assessed by the outside observer)* the "contracted wavelength" along the direction of the translational motion.

The original frequency ν_0, is concurrently reduced into $\nu_1 = \nu_0\sqrt{1 - v_0^2/c_0^2}$ *(pointing to the usual relativistic time dilation).*

The ratio c_0 / v_0, on the other hand, is the *"phase constant"* introduced by de Broglie,[1] lying between the *"phase of the periodic phenomenon of frequency* ν_1*"*, and the *"phase of the periodic phenomenon of frequency* $\nu = \nu_0 / \sqrt{1 - v_0^2 / c_0^2}$ *"*; this latter quantity is induced by Eq.(1), to match the *relativistic mass,* i.e. $m_0 / \sqrt{1 - v_0^2 / c_0^2}$.

Thereby, de Broglie postulated that, the periodic phenomenon of frequency ν_1 propagates with the velocity v_0, and that the periodic phenomenon of frequency ν propagates with the velocity c_0^2 / v_0.[1] Then, the two waves, are constantly in *harmony* with each other.*

It is in fact, the generation of these two *different periodic phenomena* through the uniform translational motion that (according to his own statement), confused de Broglie for a long time, and led him to the formulation of Eq.(4), based on the assumptions he introduced.

Thus, based on Eq.(2) (i.e. wave velocity = wavelength x frequency), via writing c_0^2 / v_0 instead of c_0, i.e.

$$\frac{c_0^2}{v_0} = \lambda_B \nu = \lambda_B \frac{\nu_0}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} , \tag{6-b}$$

(set up for the de Broglie relationship, in terms of the inflated frequency ν*, propagating with the velocity* c_0^2 / v_0*, and pointing to the wave character of the moving object)*

he could indeed very well end up with Eq.(4) [or the same, Eq.(6-a)].

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* Suppose that the two periodic functions, respectively describing the two waves, point to the same *"phase"* ϖt, at t=0 *(as assessed by the outside observer).* Recall that, $\varpi = 2\pi \times$ *(frequency in consideration)*.

At time t, the periodic function related to the first wave of frequency $\nu_1 = \nu_0 \sqrt{1 - v_0^2 / c_0^2}$ propagating with the velocity v_0, will point to the argument $2\pi\nu_1 t$, which at the location x, can be, via Eq.(1) written as

$$2\pi\nu_1 t = 2\pi \frac{m_0 c_0^2}{h} \sqrt{1 - \frac{v_0^2}{c_0^2}} \frac{x}{v_0} . \tag{i}$$

The second wave of frequency $\nu = \nu_0 / \sqrt{1 - v_0^2 / c_0^2}$, propagates with the velocity c_0^2 / v_0; let us thus calculate the phase ϖt, of the periodic function describing it, at time $(t - x c_0^2 / v_0)$:

$$2\pi\nu\left(t - \frac{x v_0}{c_0^2}\right) = 2\pi \frac{m_0 c_0^2}{h\sqrt{1 - \frac{v_0^2}{c_0^2}}}\left(\frac{x}{v_0} - \frac{x v_0}{c_0^2}\right) = 2\pi \frac{m_0 c_0^2}{h} \sqrt{1 - \frac{v_0^2}{c_0^2}} \frac{x}{v_0} , \tag{ii}$$

which is indeed the same as the previous result; this is what de Broglie called the *"Theorem of Harmony of Phases"* [i.e. the equality of $2\pi\nu_1 t$ and $2\pi\nu\left(t - x v_0 / c_0^2\right)$], easily proven under the assumptions in consideration. Note that it is this latter frequency, i.e. $\nu = \nu_0 / \sqrt{1 - v_0^2 / c_0^2}$, which is associated with de Broglie's wavelength *(necessarily propagating with the speed* c_0^2 / v_0*, without though carrying any energy)*.

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This whole idea seems to have been forgotten; de Broglie relationship takes place in all related textbooks, but how de Broglie had arrived at this idea, is practically nowhere around.

Hence, it is the velocity c_0^2 / v_0, which is associated with the de Broglie wave. Regardless the fact that energy cannot be carried by a wave of such velocity, de Broglie wavelength should still be considered as a *fundamental physical quantity,* no doubt carrying a given information, and this is the *very wave-like information.* Electron diffraction observed already in 1927, thus long ago had been a concrete proof of this statement.[3] The perplexing results about neutron diffraction, later on was observed.[4] The results are perplexing, because they clearly display both the "wave character" and the "particle character" of diffracting neutrons, conjointly *(and not just one, at a time).* In other terms, neutrons lead to diffraction, although the beam is known to consist in scarce neutrons traveling in a raw, one after the other, and this is how presumably, they should have gone through the diffraction slits; yet they still display a wave character; neutrons are finally detected on the screen, as particles, hence here again they display their particle character!

This outcome evidently seems to be profound, and should be considered along with the propagation velocity c_0^2 / v_0 of the *wave* framed by de Broglie. He, himself, does not seem to have considered the case where $v_0 = 0$, for which the *de Broglie wavelength* becomes infinite. In this case, the corresponding propagation velocity too, becomes infinite. This is nothing else, but *(as will be elaborated on, throughout),* the propagation velocity of the *"information"* associated with the wave of frequency ν_0, i.e. the original frequency of the *"periodic phenomenon of the internal dynamics of the object at rest",* as introduced by de Broglie [cf. Eq.(1)]. In other words, the internal beats of the objects, already at rest, are *"felt"* instantaneously, everywhere in space, without though any transfer of energy.

How this can be? We do not know! Perhaps it is a *property of space* that comes into play. Nevertheless we are inclined to think that the internal beats of the object at rest, are somehow right away *"felt"* everywhere in space, and conversely the object itself, very probably *"feels",* practically at once all *(atomistic, nuclear, gravitational, or other) internal beats,* existing at all possible scales, in its surrounding, i.e. the entire space. So one way or another, it must be question of a *"universal network of interaction"* in between all existing substances. Thus, though free of any energy exchange, the interaction in question occurs instantaneously between objects, at rest. Recent measurements seem to back up this arresting deduction.[5, 6]

On the other hand, here may be a clue for the mysterious *"duality".* The *wave character* is clearly due to the internal dynamics, the object in hand periodically delineates. This, most likely, causes a *given disturbance in space,* which is transmitted all over. An object at rest, or in motion, thus continuously emanates a *given wave-like information (again, without however, any energy transfer).* This information would obviously go through any diffraction slits, erected on the way, and eventually, yields an interference. The de Broglie wave becomes though finite, only if the object moves, which makes a corresponding diffraction measurement possible.

Thus perhaps, it is not, the *"diffracting single neutron"* itself, which goes through both slits, but the *"beating internal information"* that it emanates. It may be this information, which somehow digs out beforehand, the *"space channels"* hosting neutrons, on the way.

At any rate, the propagation velocity c_0^2 / v_0 associated with de Broglie wave [cf. Eq.(6-b)], is much higher than the object's speed. That is, in the first place, the *"de Broglie wave"*, and the *"particle"* do not presumably, move at the same speed (though the wave of frequency $\nu_1 = \nu_0 \sqrt{1 - v_0^2 / c_0^2}$, corresponding to the dilated period of time displayed by the internal dynamics of the object, moves with the same speed as that of the object).

It is further interesting to note that, the *wave character* would be destroyed through an *energy and momentum exchange* process; indeed it seems clear that a *momentum chock* received from the exterior, perturbs the original beating systematic of the object in hand, thus most likely wiping out the anterior wave-like information together with the space channels that would have been originally framed.

This may indeed constitute a clue to the *classical wave-particle duality.*

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Now, dividing the two sides of Eq.(6-a), by T_0 [cf. Eq.(2)], or the same, rearranging Eq.(6-b), yields

$$\frac{\lambda_B}{T_0} = \frac{\lambda_0}{T_0} \frac{c_0}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} = \frac{c_0^2}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} \quad . \tag{7-a}$$

We define the LHS as U_B :

$$U_B = \frac{\lambda_B}{T_0} \quad . \tag{7-b}$$

(definition)

Note that, both λ_B and T_0 are in fact, just like λ_0, defined in relation to the outside fixed observer. (It is true that T_0 and λ_0 are transformed, with the motion, though the above definition stays perfectly valid.)

U_B, via Eq.(2), becomes

$$U_B = \frac{c_0^2}{v_0} \sqrt{1 - \frac{v_0^2}{c_0^2}} \quad . \tag{7-c}$$

(velocity defined based on de Broglie relationship and the period of the periodic phenomenon of the object at rest)

Eq.(6-b), or more specifically Eq.(7-a), or the same Eq.(7-c), tells us that, through a motion such as the Bohr rotational motion of the electron around the proton in the hydrogen atom, each time *(as assessed by the outside observer)* the inherent periodic phenomenon of the electron assumed by de Broglie, and described *(at rest)* by λ_0, beats; a *"wave echo"* beats, all the way through the stationary orbit of perimeter λ_B.[*]

He could, in effect, show in his doctorate thesis that, in order to display a stationary motion on a given *circular orbit,* the wavelength λ_B to be associated with the electron's motion *(depicting a constant velocity)* along Eq.(4), should come to be equal to the orbit's perimeter.

On the other hand, once a wave is confined, the quantization of it follows from classical physics;[†] hence, de Broglie, landed at Bohr's angular momentum quantization assumption.[7]

It is a pity that this achievement is not either cited in very many textbooks.

The thing is that, the velocity U_B induced by the LHS of Eq.(7) turns out to be greater than the speed of light; at the slowest, it is the speed of light; it can be infinite for a zero v_0. For this reason, de Broglie considered it very rationally, as a velocity not carrying any energy *(and we do well stick to this interpretation).*

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[*] Recall that λ_0 for the electron, based on Eq.(3), turns out to be $2.45\text{x}10^{-10}$ cm. The de Broglie wavelength λ_B to be associated with the electron at the ground state of the hydrogen atom, is about $3.33\text{x}10^{-8}$ cm. Thus , $\lambda_B/\lambda_0 = 135$. T_0, on the other hand, is $0.82\text{x}10^{-20}$ seconds.

[†] Classically, a wave displaying a resonant standing motion of wavelength λ_n, through say, a string of length L, obeys the relationship

$$L = \frac{n\lambda_n}{2} \text{ , } n = 1, 2, 3, ... ; \tag{i}$$

if visualized for a particle in the box, along with the de Broglie relationship [cf. Eq.(4)], Eq.(i), thus already classically, yields well the Schrödinger's equation's solution, i.e. the quantization of energy levels.

The resonance condition, for a circular orbit, is

$$2\pi r_n = n\lambda_{Bn} = \frac{nh}{mv_{0n}} \text{ ,} \tag{ii}$$

for the de Broglie wavelength λ_{Bn}, to be associated with the nth energy level of the Bohr hydrogen atom, for which the orbital velocity of the electron is v_{0n}.

This makes that, for the case in hand, Eq.(4), or the same Eq.(6-a), should in general, be written as

$$\lambda_{Bn} = \frac{nh}{mv_{0n}} = n\lambda_0 \frac{c_0}{v_{0n}}\sqrt{1 - \frac{v_{0n}^2}{c_0^2}} \text{ ,} \tag{iii}$$

which is in fact, nothing else, but [based on Eq.(i)]

$$2\pi r_{0n} v_{0n} m = nh \text{ ,} \tag{iv}$$

i.e. the Bohr postulate, written for the nth energy level, were the orbit circular, with a radius of r_{0n}.

Hence, once we have de Broglie relationship, the quantization displayed within the frame of Eq.(iv), follows from classical physics. Eq.(iii) will be used below.

We will see that, not only one can derive Eq.(6-a), i.e. the *de Broglie wavelength,* based on a mere energy conservation approach, but also that, the velocity depicted by the LHS of Eq.(7-c) turns out to be quite physical, if strikingly things are not considered *particle-wise,* but just *wave-wise,* not involving any *energy exchange with the exterior,* as will be implied by the electric and gravitational interactions that we will deal with, in this work. Thus, in fact, de Broglie wavelength does not indeed carry any energy, but it certainly carries a given *information* in regards to the interaction in question.

Amazingly, our approach can be applied to both gravitational and electric interactions, underlining the fact that both interactions work exactly the same way.
It really seems silly, that the *"instant interaction",* clearly evoked by different aspects of quantum mechanics *(that will be discussed in this work),* was not only thought to be against the Special Theory of Relativity (STR), but also it was not a bit correlated with the de Broglie relationship, the basis of the wave theory of matter *(or the same, quantum mechanics),* and derived, following considerations remaining *(as will be elaborated on),* within the mere frame of the STR.

For convenience herein we will handle just the *"electric interaction"*. In the subsequent part, we will handle the *"gravitational interaction".*

Below, we will first summarize our previous work, which led to a novel equation of motion; accordingly, we provide a discussion about the notion of *"field"* (Section 2). Then, we describe our *"jet model",* simulating both the electric interaction and the gravitational interaction (Section 3). Next, based on our approach, and essentially the law of energy conservation, we derive the de Broglie relationship (Section 4). A short conclusion is drawn (Section 5). A general conclusion will be provided in Part II.

Consider now, two electrically interacting objects such as the proton and the electron. We will in general call the proton, assumed to be at rest, the *"source charge",* and the electron, either at rest or in motion, the *"test charge".*

2. PREVIOUS WORK: A NOVEL APPROACH TO THE EQUATION OF ELECTRIC MOTION, AND DISCUSSION ABOUT THE NOTION OF FIELD

In a previous work,[8] a completely new approach to the derivation of the celestial equation of motion was achieved; this led to all crucial end results of the General Theory of Relativity .[9]

The same approach was applied to the atomic scale, as well. This led to the derivation of a new relativistic quantum mechanical description well equivalent to that established by Dirac, if geared alike.[10,11]

Thus, we had started with the following postulate, essentially, in perfect match with the *"relativistic law of conservation of energy",* thus embodying, in the broader sense, the concept of *"mass",* though we will have to specify, accordingly, the notion of *"field".*

Postulate: The rest mass of an object bound either gravitationally or electrically, amounts to less than its rest mass measured in empty space, the difference being, as much as the mass, equivalent to the binding energy vis-à-vis the field of concern.

A mass deficiency, conversely, via quantum mechanics, yields the stretching of the size of the object in hand, as well as the weakening of its internal energy, on the basis of quantum mechanical theorems proven elsewhere[12] still in full conformity with the STR.*

Such an occurrence can be experimentally checked, if say a muon is considered to be bound to a nucleus instead of the electron; the decay rate of the bound muon is indeed retarded as compared to the decay rate of a free muon;[13,14,15,16,17,18,19,20] our prediction about this, remains better than any other available predictions.

Not to attract conservative reactions, let us precise what we mean by *binding energy*.

Binding Energy

Take for instance a piece of stone on Earth. We can assume that Earth is infinitely more massive than the stone. Then the binding energy of the stone to Earth is the energy we have to furnish to the stone in order to bring it to infinity. The calculation of the gravitational binding energy is peculiar though, since the rest mass of the stone is increased as much as the energy furnished to it, on the way. A detailed study of this problem is furnished in Reference 6 (http://www.ensmp.fr/aflb/AFLB-293/aflb293m137.htm).† Briefly speaking, as the stone falls from a practically infinite distance onto Earth, the energy it would acquire at the moment it strikes Earth is equal to its binding energy to Earth.

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* An easy way to grasp this is to consider Eq.(1). If the rest mass is decreased due to binding, so will be the frequency. Thus the gravitational red shift. Eq.(2) makes that, the size is accordingly stretched.

† As a first approximation, the binding energy E_{BStone} of the stone of mass m_{0Stone}, measured at infinity, bound to Earth of mass $\mathcal{M}$ and radius R, can be calculated as usual, to be,

$$E_{BStone} = \int_R^\infty G\frac{\mathcal{M}\, m_{0Stone}}{r^2}\,dr = G\frac{\mathcal{M}\, m_{0Stone}}{R}\,, \tag{i}$$

G being the universal gravitational constant.

Otherwise one should write for the binding energy $E_{BStone}(r)$, the stone delineates at a distance r to the center of Earth (see Reference 6),

$$E_{BStone}(r) = G\mathcal{M}\int_r^\infty \frac{m_{0Stone} - \dfrac{E_B(r')}{c_0^2}}{r'^2}\,dr'\,, \tag{ii}$$

which on Earth (at R from the center of Earth), yields

$$E_{BStone}(R) = m_{0Stone}\, c_0^2\left[1-\exp\left(-\frac{G\mathcal{M}}{c_0^2 R}\right)\right]. \tag{iii}$$

The binding energy of the electron to the proton in the hydrogen atom in its ground state, for instance, is *(supposing first, for simplicity, that the proton is infinitely more massive as compared to the electron),* is the energy one has to furnish to the electron in order to bring it, from its ground state, to infinity. The proton will remain at rest. Hence, under the given circumstances, it is the electron, which will pile up the amount of energy, in consideration. *(A detailed analysis on this, will be presented below.)* In other words, the relativistic rest energy *(i.e. the relativistic equivalent of the rest mass)* of the free electron, weigh more than the relativistic energy of the bound electron, and this as much as the electron's binding energy *(in the hydrogen atom).* Or the same, when bound, still under the given circumstances, this much of energy ought to be retrieved from the relativistic rest energy of the electron. As trivial as this may sound to most readers, this is important with regards to conservative reactions, directed to the present approach; therefore we should insist a bit further on it. Thus let us go back in more details.

Had we not assumed that the proton is infinitely more massive than the electron, then the binding energy is the energy one has to furnish in order to dissociate the hydrogen atom into the electron and the proton *(i.e. while almost all of this energy is to be delivered to the electron, a minimal part of it is to be delivered to the proton).*[*] This energy, in other words, is the *"ionization energy"* of the hydrogen atom, i.e. about 13.6 ev.

Thus, the mass & energy equivalence driven by the STR, together with the law of conservation of energy, requires that the total rest mass of the proton and the electron considered separately in a space free of field, shall weigh 13.6 ev less, when bound in a hydrogen atom. In other words, the hydrogen atom, weighs 13.6 ev less than the sum of the rest masses of the proton and the electron considered in a space free of field. And, how this mass deficiency will be accounted for, by the original mass of the proton and that the electron? As explained, as a first approximation, it is the electron relativistic energy at rest, considered in free space, which will undergo, practically all of the mass deficiency in question. But, still to avoid conservative reactions, let us simplify things and work out before everything else, the *"static binding energy"* of a nucleus of charge +Ze (composed of Z protons) and an electron of charge intensity e, thus altogether at rest, and situated at a given distance from each other. We will need this energy soon, anyway.

When a Massive Charge +Ze and an Electron Are Bound Altogether at Rest, the Electron's Rest Mass Measured at Infinity, is Decreased as much as the Binding Energy Coming Into Play

Supposing, again for simplicity, that the nucleus in consideration is infinitely more massive than the electron, the binding energy of the nucleus and the electron, situated at rest, at a given distance from each other, is the energy one has to furnish to the electron in order to bring it, from its bound location to infinity.

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[*] This energy which we call E_{BH}, can be calculated from Bohr Atom Model, as well as the solution of the related Schrodinger Equation, as usual, to be (in CGS unit system)

$$E_{BH} = \frac{2\pi^2 e^4 \mu_0}{h^2} \ ;$$

(dissociation energy of the hydrogen atom)

here e is the charge of the electron or the proton, μ_0 is the reduced mass of the proton and the electron, and h the Planck Constant; E_{BH} is about 13.6 ev.

Had we not assumed that the nucleus is infinitely more massive than the electron, then the binding energy in question, is the energy one has to furnish in order to dissociate the pair of nucleus and electron bound at rest (situated at the given distance from each other), into the free nucleus (of charge +Ze) and the free electron (i.e. while once more, almost all of this energy is to be delivered to the electron, a minimal part of it only, is then, to be delivered to the nucleus).*

Thus, just like in the case of the hydrogen atom, the nucleus of charge +Ze and the electron situated at a distance R from each other, and altogether at rest *(owing to the mass & energy equivalence drawn by the STR, along with the law of conservation of energy)* shall weigh $E_{BZe^2} = Ze^2/R$ ergs less than the sum of the rest masses, of the free nucleus and the free electron (see the footnote, at the bottom of the previous page). *(Note that the electric charges are not affected, throughout, only the rest masses are.)*

This energy in question, ought to be retrieved, practically from the electron, alone. The reason is simple. Suppose the electron falls from a sufficiently large distance, onto the nucleus in consideration, and originally at rest. If this nucleus is infinitely more massive than the electron, then the law of linear momentum conservation requires that the nucleus stays, practically in place, while the electron keeps on falling. An outsider can intervene somehow and stop the electron at a given distance to the nucleus. Then, the only energy he would tap, would be the kinetic energy, the electron would have piled up on the way. Thus, the system originally composed of the nucleus of charge +Ze and the electron, when bound at the given distance R, from each other *(and originally at rest altogether),* will loose the kinetic energy, the electron would have acquired on the way, i.e. the energy $E_{BZe^2} = Ze^2/R$ ergs, and since the nucleus would virtually not move throughout, this energy ought to be extracted from the electron's mass alone.†

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* The static binding energy E_{BZe^2} of the nucleus of charge +Ze and the electron of charge -e, altogether at rest, at the distance R from each other (assuming that the nucleus is infinitely more massive than the electron), is (in CGS unit system)

$$E_{BZe^2} = \int_R^\infty \frac{(Ze)e}{r^2} dr = \frac{Ze^2}{R} ;$$

(static binding energy of the nucleus of charge Ze *and the electron, situated at rest, at a distance* R *from each other)*

recall that the charges, unlike the gravitational charges, are not affected by the energy piled up, as they are carried away from each other.

† Suppose the electron's mass in a space free of field, is m_0; thus when bound, at rest to a nucleus of charge +Ze, still at rest, at a given distance R, its (rest) mass m(R) becomes

$$m(R) = m_0 - \frac{Ze^2}{Rc_0^2} , \qquad \text{(i)}$$

(rest mass of the electron bound at a distance r *from a nucleus of charge* +Ze, *also at rest, and infinitely more massive than the electron)*

or the same, the total relativistic energy of the bound electron at rest is

$$m(R)c_0^2 = m_0c_0^2 - \frac{Ze^2}{R} . \qquad \text{(ii)}$$

(rest total relativistic energy of the electron bound at a distance r *from the nucleus of charge* +Ze, *also at rest, and infinitely more massive than the electron)*

We can still have conservative reactions, as to what, we mean by "an electron and a nucleus held at rest, at a given distance from each other". How can one achieve such a pair? Is it realistic to talk about it?

Well, one can conceive many ways. Here is one. Just consider a dipole, such as a *water molecule,* in which, the oxygen atom (O) attracts, respectively the two binding electrons of the hydrogen (H) atoms, delineating an angle HOH of about 105º. This makes that, the hydrogen atoms get charged positively, and the oxygen atom, negatively. Thus, water molecule can indeed be described by a dipole, made of -2e situated nearby the oxygen atom, and +2e situated on the median of the triangle HOH, in between the hydrogen atoms (e is again, the electron's charge intensity). We call r_{Dipole}, the distance between the two representative charges +2e and -2e. So we have the charges +2e and -2e held still at a the distance r_{Dipole}, from each other.

Thus we can answer the question we have just introduced: Yes, indeed, we can well conceive a dipole composed of +Ze and –e, at a given distance from each other, and at rest, since this is not any different than the dipole (composed of +2e and -2e), a water molecule delineates.

The binding energy of the water molecule *(assumed at rest),* or the same, that of the dipole made of +2e and -2e, is the energy one has to furnish to it, in order to carry these two charges, far away from each other; in other words, this is the energy one has to furnish to the water molecule, in order to dissociate it, into its oxygen atom and the two hydrogen atoms. This energy, which we call E_{BH_2O}, is about 9.5 ev. Knowing the angle HOH and the distance between the pair O and H, one can easily calculate it, in terms of the hydrogen atom's ionization energy, since each arm OH of water molecule, consists in the bond of the pair of +e and –e bound with each other. Thus, that much of energy, should be extracted from the sum of the rest masses, of respectively the hydrogen atoms, and the oxygen atom, weighed separately from each other. Noting that the oxygen atom is much more massive than the hydrogen atom, roughly speaking, 9.5 ev *(more specifically, the mass equivalent of this much energy)* must be extracted, from the hydrogen atoms. Hence, the bound hydrogen atoms, in water molecule, shall each weigh less, and this, about half of the dissociation energy E_{BH_2O} of the molecule, as referred to the hydrogen atom's rest mass $m_{H\infty}$, measured at infinity,* weighed separately. Thus, the mass of the hydrogen atom bound to O, in a H_2O molecule, shall nearly weigh, $m_{H\infty} - E_{BH_2O}/(2c_0^2)$.

This is indeed an approximation, since the mass ratio of the hydrogen atom to the oxygen atom, is about 1/16. We can do much better than that. The mass ratio of the hydrogen atom to the tellurium (Te) atom, indeed, is about 1/128. Thus, when two H atoms are bound to a Te atom, in a H_2Te molecule, bearing the dissociation energy E_{BH_2Te}, one can with confidence affirm that, each of the H atoms will, to an acceptable precision, weigh $E_{BH_2Te}/2$ ev less, as compared to the H atom weighed at infinity, while the Te atom (owing to the law of linear momentum conservation, as explained above), practically remains untouched.

* For quantities defined at infinity, normally we use the subscript "$_0$"; but because here, this symbol can be confused with "O" of the oxygen molecule, we prefer to use "$_\infty$", instead.

Thus, the mass of the hydrogen atom bound to Te, in a H_2Te molecule, shall practically weigh, $m_{H\infty}$ - $E_{BH_2Te}/(2c_0^2)$.

Furthermore, suppose that the molecule H_2Te undertakes a routine rotational motion. Since Te is much too heavy as compared to H, the rotational motion shall take place around Te atom.

Let V_{Rot} be the tangential velocity of the H atoms, rotating around Te. The overall relativistic energy of such an H atom thus becomes $[m_{H\infty} - E_{BH_2Te}/(2c_0^2)]/\sqrt{1-V_{Rot}^2/c_0^2}$.

What we do here may seem boring to many readers, and we should apologize for that. Yet such readers should be reminded that the author has had a great deal of difficulty to convey to many colleagues, as well as reviewers, the controversial results, we are soon going to base on the foregoing discussion.

Thus, following our discussion we conclude that, when a charge +Ze and an electron are bound altogether at rest (supposing that Ze is very much more massive that the electron), at a distance R from each other, the electron's mass m_0 measured at infinity, is decreased as much as the static binding energy $E_{BZe^2} = Ze^2/R$ ergs, coming into play, to become $m_0 - Ze^2/(Rc_0^2)$; the mass of the heavy nucleus (owing to the law of linear momentum conservation), is not virtually touched.

The energy $E_{BZe^2} = Ze^2/R$, is nothing else, but the classical *"potential energy"*. But we avoid this denomination, for reasons that will become clear soon. That is within the peculiarities we have introduced, as we will see, chiefly the total energy cannot be set equal to the sum of *"kinetic energy"* and *"potential energy"*, were they classically defined.

Indeed let us try to answer the following question:

What is, the *overall relativistic energy* of the electron quasistatically brought nearby the nucleus of charge Ze, if it is further set to a rotational motion of velocity v_0 around Ze? Is it $(m_0c_0^2 - Ze^2/R)/\sqrt{1-v_0^2/c_0^2}$, or $m_0c^2/\sqrt{1-v_0^2/c_0^2} - Ze^2/R$?

Based on the *"classical potential energy concept"*, all text books we know of, would answer, *"the second"*. Our answer though, is *"the first"*. We will soon discuss this question in detail. Nonetheless, now that we happened to have presented the foregoing discussion, our answer (we hope) should look a sound one. [Just think of the total relativistic energy of a H atom in the molecule of H_2Te, set to a rotational motion around Te. As discussed right above, it is $(m_{H\infty}c_0^2 - E_{BH_2Te}/2)/\sqrt{1-V_{Rot}^2/c_0^2}$, and not $m_{H\infty}c_0^2/\sqrt{1-V_{Rot}^2/c_0^2} - E_{BH_2Te}/2$. If so then the overall relativistic energy of the electron quasistatically brought nearby the nucleus of charge Ze, if it is further set to a rotational motion of velocity v_0 around this nucleus, must be $(m_0c_0^2 - Ze^2/R)/\sqrt{1-v_0^2/c_0^2}$, and not $m_0c_0^2/\sqrt{1-v_0^2/c_0^2} - Ze^2/R$. Soon, we will further dig in this question.]

In any case, henceforth, we will solely operate on the concept of *"relativistic energy" (and nothing else),* which we can minutiously define, and work out, with regards a given particle, either at rest *(based on the mass & energy equivalence drawn by the STR),* or in motion.

Let us recall that, in order to calculate the binding energy coming into play, for electrically bound particles, we make use of the Coulomb Force, yet with the restriction that, it can only be considered for static charges.

We do this, simply because (as will be elaborated on), we can relativistically assert that, Coulomb Force reigning between two static charges is a requirement imposed by the STR, and beyond this, a priori, we have strictly no idea, whether it will still hold or not, were one of the charges is in motion with regards the other, and as we will soon derive, it does not.

Why, Coulomb Force reigning between two static charges is a requirement imposed by the STR?

It is that we are able to derive the $1/\text{distance}^2$ dependency of the Coulomb Force between two static charges, still based on the STR.[4,5]

The reason for such a dependency, is merely that the quantity $(\text{force})\text{x}(\text{mass})\text{x}(\text{distance})^3$ is Lorentz invariant. Thus, suppose we take a dipole into a uniform translational motion. Consider for simplicity, the case where the motion takes place along the direction of the line joining the two poles. Let the mass in question, be the mass of the dipole in question.

Then, the quantity (mass)x(distance) is Lorentz invariant; for this case, accordingly, the quantity $(\text{force})\text{x}(\text{distance})^2$ is Lorentz invariant. The electric charges on the other hand, are following observations, Lorentz invariant. Otherwise the Galilean Principle of Relativity would be broken.

Thus the force reigning between the two poles, expressed as the [product of static electric charges coming into play] / distance^n, can only allow the exponent $n=2$. Therefore, the structure of the classic Coulomb Force reigning between two static charges, is solely implied by the STR.

Hence, the framework we set up herein, fundamentally lies on the STR.

*

Note that below, just like we did above, we consider solely the *closed system* made of two charges *(of opposite signs).* This means we will continue to tackle, all the way through, with these two charges, somehow engaged with each other everlastingly.

Thus we exclude the possibility of having to deal with one charge only up to a given point of a possible process, suddenly allowing the pop out of the second charge, right next to the first one *(which can, for instance, be achieved via charging at a given moment the plates of a capacitor, while the first charge is lying in the inside of it).*

Processes taking place in accelerators also fall in this category, which is that of an electric charge experiencing in its frame of reference the creation of an electric field on its way. .

The Equation of Motion

We defined m_0 the mass of the electron, at infinity. When this is bound at rest, to a nucleus of charge +Ze, assumed for simplicity infinitely more massive as compared to m_0, this latter mass, following the discussion we have just presented, will be diminished as much as the *static binding energy* coming into play, through the binding process, to become $m(r_0)$, at a distance r_0 to the nucleus, so that[4]

$$m(r_0) = m_0 c_0^2\, \kappa(r_0)\,, \tag{8}$$

(mass of the bound electron, at rest)

where $\kappa(r_0)$ is

$$\kappa(r_0) = 1 - \frac{Ze^2}{r_0\, m_0 c_0^2}\ . \tag{9}$$

Note that the distance between the electron and the nucleus, when measured by an observer bound to the electron, and when measured by the distant observer, does not point to the same quantity [simply because the change of the mass, via Eqs. (1) and (3), induces the change of the metric on the whole], but in what follows we will overlook this detail.[7]

Note further that, at a first strike, Eq.(9) seems to allow $\kappa(r_0) = 0$, also $\kappa(r_0) < 0$. It is that, as the electron is quasistatically brought closer and closer to the nucleus, its rest mass decreases more and more, until it comes to vanish at r_0, which, for Z=1, turns out to be the classical electron radius, i.e. $e^2/(m_0 c_0^2) = 2.8\text{x}10^{-13}$ cm.

Why the electron, or any other charge, cannot fall down any further?

Recall that, if this seems to be a problem, it is not really a specific problem, arising from the approach, we propose herein; the question can come up for any dipole, and one can bring different answers to it, with respect to different cases. For instance, the dipole representing water molecule cannot go narrower than the distance it delineates, because, the electronic structure of the atoms does not allow it; at shorter distances the atoms in consideration, would repel each other more and more. If we consider an electron falling onto a nucleus, we should remember that, the electric charge of the nucleus taking place in the expression of the force exerted by the nucleus onto the electron, decreases gradually, as the electron goes beyond the nucleus wall, assuming that it can do so, without getting absorbed, etc. In this latter case, obviously, Eq.(9) should be reformulated.

Anyway let us try to answer the above question, as to why the electron, cannot fall down into, say a proton, beyond a range making the RHS of Eq.(9) vanish. It is that based on our approach, there would be no mass left to fuel the endeavor.*

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* Based on our approach just briefly discussed, and we will elaborate on below (see Reference 18), the Bohr Atom radius r_{0n}, at the nth principal level, becomes

Furthermore, one should recall that Eq.(8), along with Eq.(9), is a must imposed by the law of energy conservation, in the broader sense of the concept of *energy,* embodying the mass & energy equivalence brought by the STR. If one brings quasistatically, the electron up to an obstacle, situated at a given distance R from the heavy nucleus in consideration, he must work against the attraction force. If he wants to return with the electron, back to infinity, he must furnish to the electron, the amount of energy equal to the work he had to spend in order to get it to R. Thus, when he carries the electron back to infinity, he must deliver to the electron, the energy $\int_R^\infty (Ze^2/r^2)\,d\,r = Ze^2/R$ (see the footnotes on the bottom of pages 10 and 11). This makes that the electron, when separated from the nucleus, relativistically speaking, weighs more, and this as much as Ze^2/R. In other words *(because, as discussed, the nucleus is supposed to be much too heavy as compared to the electron, and accordingly, owing to the law of linear momentum conservation, it will stay practically in place, through, back or forth, either maneuver),* when statically bound, the electron will experience a mass deficit, and this as much as Ze^2/R. Otherwise, we believe it is clear enough that, the law of energy conservation would be violated.

Thence Eq.(8), along with Eq.(9), is a must imposed by the law of energy conservation, in the broader sense of the concept of *energy,* embodying the mass & energy equivalence brought by the STR. And if we insipidly insist on this matter, it is because we could not, for a long time, and despite enormous efforts, most important very friendly welcomes we were delighted with, but still, overcome conservative reactions.

Now suppose that the electron is engaged in a given motion around the nucleus; the motion in question can be conceived as, made of two steps:[21]

i) Bring the electron quasistatically, from infinity to a given location r, on its orbit, but keep it still at rest.

ii) Deliver to the electron at the given location, its motion on the given orbit.

$$r_{0n} = \frac{Ze^2}{m_{0\infty}c_0^2}\left(1+\frac{n^2}{Z^2\alpha^2}\right);$$

here α is the fine structure constant, i.e. $2\pi e^2/(hc)$, or about 1/137.

Note that for small Z's, the above relationship, yields well the Classical Bohr's Results. As Z increases, the orbit radius r_{0n} decreases to draw a minimum at $Z\alpha/n$ = 1. For n=1, this yields Z=137; the value of the radius r_{0nmin} at this minimum, is $2Ze^2/(m_{0\infty}c_0^2)$, where then, half of the proper mass of the electron would disappear; r_{0nmin} for n=1, becomes 0.77 x 10^{-10} cm, i.e. ~ 1/100th of the Classical Bohr Radius. We made this explanation with regards to Eq.(9) of the text. Thus, in the case of a nucleus of charge +Ze and an electron engaged into a motion around it, the electron cannot come closer to the nucleus than (137x2) x the classical electron radius.

The first step yields a decrease in the mass of m_0 as delineated by Eq.(8).* The second step yields the Lorentz dilation of the rest mass $m(r_0)$, at the location r_0, so that the overall relativistic energy $m_\gamma(r_0)c_0^2$, or the same, the total relativistic energy of the electron on the given orbit, becomes

$$m_\gamma(r_0)c_0^2 = m(r_0)c_0^2 \frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = m_0c_0^2 \frac{1-\frac{Ze^2}{r_0m_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} \quad ; \tag{10}$$

(overall relativistic energy of the bound electron on the given orbit)

v_0 is the magnitude of the local tangential velocity of the electron at r_0.†

The total energy of the electron on orbit [i.e. $m_\gamma(r_0)c_0^2$] must remain constant, so that for the motion of the object in a given orbit, one finally has

$$m_\gamma(r_0)c_0^2 = m_0c_0^2 \frac{1-\frac{Ze^2}{r_0m_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}} = \mathit{Constant} \quad . \tag{11-a}$$

(total energy written by the author, for the electron in motion around the nucleus)

This relationship is in fact the integral form of our general equation of motion, given below.

One can notice that Eq.(11-a) is different from what one would write classically, i.e.

$$m_\gamma(r_0)c_0^2 = \frac{m_0c_0^2}{\sqrt{1-\frac{v_0^2}{c_0^2}}} - \frac{Ze^2}{r_0} = \mathit{Constant}\text{: Wrong !} \tag{11-b}$$

(total energy one would write classically, for the electron in motion around the nucleus)

What is *wrong* with this latter equation?

---

* The fact that the electron is brought to the location in consideration, quasistatically, provides us with the facility of not having to deal with the *radiation problem,* that would arise otherwise.

† The velocity v_0 is not to be confused with an eventual velocity V, the atom would delineate, when brought to a uniform translational motion. If so then the overall relativistic mass $m_\gamma(r_0)$, would evidently become $m_\gamma(r_0)/\sqrt{1-V^2/c_0^2}$.

What is essentially wrong with it, is that, as silly as it may look, on the whole it delineates a violation of *conservation of energy.* Although [when compared to Eq.(11-a)] this seems trivial, it is unfortunately overlooked through several decades. The restitution of the mistake in question evidently, is to alter so very many related derivations. This may be unfortunate, but that is the way it is. We elaborate on this below.

Eq.(11-b) assumes that the *total relativistic energy* is composed of the rest relativistic energy (i.e. the rest mass x c_0^2) + the *kinetic energy* + the *potential energy.* And what is really wrong with this? To compose the total energy, that way, is what we all learned at already the high school level, and that is what most of us kept on teaching.

For one thing, in our presentation we do not make use of, or refer to the *concept of potential energy.* We only consider the mere *concept of energy,* more precisely the *concept of relativistic energy.* Thus, the energy of the statically bound electron, is decreased as much as the *static binding energy* [cf. the first step we have considered, in writing Eq.(10)], and it is the *remaining energy of the electron,* which is dilated by the Lorentz factor, while we deliver to it, its motion on the orbit [cf. the second step we have considered, in writing Eq.(10)].

The result we arrived at, is not any different than that we established in regards to the total relativistic energy of a H atom in the molecule of H_2Te, set to a rotational motion around Te. Thus, this energy is $(m_{0H}\, c_0^2 - E_{BH_2Te}/2)/\sqrt{1 - V_{Rot}^2/c_0^2}$, and not $m_{0H}\, c_0^2/\sqrt{1 - V_{Rot}^2/c_0^2} - E_{BH_2Te}/2$!

We can provide another way of looking at the traditional mistake we unveil, and it is the following. Eq.(11-b), assumes that Coulomb force holds, between a *static source charge (the nucleus),* and a *moving test charge (the electron).*

This is too, something we all have learned, and we all taught. What though we have assumed, to develop the present theory, is “Coulomb’s Law, reigning in between only two static charges”; this in fact, as we have shown, turns to be a requirement imposed by the STR.

“Coulomb’s Law reigning in between, only two static charges” does not, in any way, tell us how the law of force would look, if one of the charges moves, and clearly, what we used to believe “true”, is not; Coulomb’s Law does not hold in between the proton (assumed at rest, throughout) and the moving electron (the way it holds, between “the proton and the electron at rest”).

This is the crucial point. In other words, as will be specified below, Eq.(11-b) would be valid, if Coulomb’s law, were valid between the proton and the moving electron, the way it is written for the proton and the electron, both at rest. But it is not, and Eq.(11-b) is only approximate. This disclosure too, is to alter very many related derivations, but that is the way, it looks. One way or the other, it is that, the mass of the bound electron, is not the same as the mass of the free electron, and as trivial as it may look at this stage, this is what essentially had been overlooked throughout the passed century.

How Coulomb’s Law must be written for the proton and the moving electron, will soon be specified.

Thus, although Eq.(11-a) looks straightforward, to our recollection, it happens to be new. The way we write it, induces the need of elaborating on the concept of "field"; this will be undertaken below.

We show elsewhere that Eq.(11-a) furthermore constitutes, the basis of a relativistic quantum mechanical description, well equivalent to that of Dirac, if geared alike, yet established in an incomparably easier way.[5]

We have to stress that the approach in question is in full harmony with all the existing quantum electrodynamical data. The differentiation of the above equation leads to

$$-\frac{Ze^2}{m_0 r_0^2}\frac{1-\frac{v_0^2}{c_0^2}}{1-\frac{Ze^2}{r_0 m_0 c_0^2}} = v_0\frac{dv_0}{dr_0} \quad . \tag{12-a}$$

[differential form of Eq.(11-a), *equivalent to the equation of motion]*

One can transform Eq.(12-a), into a vector equation; the RHS, is accordingly transformed into the acceleration (vector) of the electron on the orbit. Thus, recalling that the LHS of Eq.(10), i.e. $m_\gamma(r_0)c_0^2$, is constant, one can write

$$-\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}\,\frac{\underline{r}_0}{r_0} = m_\gamma(r_0)\frac{d\underline{v}_0(t_0)}{dt_0}; \tag{12-b}$$

[vectorial equation written based on Eq.(12-a), *or the same, equation of motion written by the author, via the energy conservation law, extended to cover the relativistic, mass &– energy equivalence]*

here, $\underline{r}_0$ is the "radial vector" of magnitude r_0, and $\underline{v}_0$ is the "velocity vector" of the electron, at time t_0; note that $d\underline{v}_0$ and $\underline{r}_0$ lie in opposite directions.

For a small Z, thus a small v_0, the orbit would be as customary elliptical; otherwise it is open; in other words, the perihelion of it, shall precess throughout the motion.

Eq.(12-b) is anyway the same relationship as that proposed by Bohr, except that the Coulomb force intensity is now decreased by the factor $\sqrt{1-v_0^2/c_0^2}$, similarly to what empirically, but approximately proposed by Weber, by the end of nineteen century.[22,23,24,25] Note in fact that, a realistic interpretation of Eq.(12-b) should consist in considering the factor $\sqrt{1-v_0^2/c_0^2}$, at the denominator of the RHS of this equation.

Then, it is as if, the classical force, now causes a greater equivalent momentum change rate.

Recall that what we do is in no way in conflict with quantum mechanics. Quite on the contrary, through our approach soon we will land at the de Broglie relationship, which is the basis of quantum mechanics. At this stage, it seems useful to draw the following table displaying the differences between our approach and the standard approach.

Table 1 Differences Between the "Standard Approach" and "Present Approach", Based on the Electron Bound to the Proton (assumed at rest throughout)

	Standard Approach	**Present Approach**
Force Between the Proton and the Electron, Altogether at Rest	$\frac{Ze^2}{r_0^2}$	The same.
Total Energy of the Statically Bound Electron	$m(r_0)c_0^2 = m_0c_0^2 - \frac{Ze^2}{r_0}$	The same; but classically the mass of the bound electron is not considered to be altered; it is the overall *field energy* which is believed to decrease, as much as the *potential energy,* coming into play.
Total Dynamic Energy of the Electron	Rest Energy + Potential Energy + Kinetic Energy	The concept of *potential energy,* as considered classically, is misleading.
Mathematical Expression of the Total Dynamic Energy of the Electron	$m_\gamma(r_0)c_0^2 = \frac{m_0c_0^2}{\sqrt{1-\frac{v_0^2}{c_0^2}}} - \frac{Ze^2}{r_0}$	$m_\gamma(r_0)c_0^2 = m_0c_0^2 \frac{1-\frac{Ze^2}{r_0m_0c_0^2}}{\sqrt{1-\frac{v_0^2}{c_0^2}}}$
Force Between the Proton and the Moving Electron	$\frac{Ze^2}{r_0^2}$	$\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}$

Discussion With Regards to the Total Dynamic Energy We Proposed For the Electron

Eq.(12-b), that we have just derived, and primarily Eq.(11-a) leading to it, are being subject to severe criticisms. We should discuss them.

One objection concerns the kinetic energy acquired by the electron, freely falling onto to a proton at rest, along our way.

Thus, consider the electron of mass m_0 measured in empty space. Let v_1 the electron's velocity at r_1, and v_2 the electron's velocity at r_2 (the proton being taken at the origin of our coordinate system).

Based on Eq.(11-a) we can write

$$m_0 c_0^2 \frac{1-\frac{Ze^2}{r_1 m_0 c_0^2}}{\sqrt{1-\frac{v_1^2}{c_0^2}}} = m_0 c_0^2 \frac{1-\frac{Ze^2}{r_2 m_0 c_0^2}}{\sqrt{1-\frac{v_2^2}{c_0^2}}} = m_0 c_0^2 \ , \tag{12-c}$$

(free fall of the electron described within the frame of our model)

which anyway makes that for a free fall

$$\frac{1-\frac{Ze^2}{r_2 m_0 c_0^2}}{\sqrt{1-\frac{v_2^2}{c_0^2}}} = 1 \ . \tag{12-d}$$

(basic relationship about the free fall of the electron which started with zero velocity, at a practically infinite distance from the proton, within the frame of our model)

For small velocities this yields

$$m_0\left(\frac{v_2^2}{2}-\frac{v_1^2}{2}\right)=\frac{e^2}{r_2}-\frac{e^2}{r_1}+\frac{e^2}{r_2}\frac{v_2^2}{c_0^2}-\frac{e^2}{r_1}\frac{v_1^2}{c_0^2} \ , \tag{12-e}$$

(difference between kinetic energies of the electron at two different altitudes, based on the present approach)

whereas from the classical Eq.(11-b), one as usual, writes

$$m_0\left(\frac{v_2^2}{2}-\frac{v_1^2}{2}\right)=\frac{e^2}{r_2}-\frac{e^2}{r_1}. \tag{12-f}$$

(classical difference between kinetic energies of the electron at two different altitudes, based on the present approach)

Thus, the criticism is that, the former result is larger than the latter, whereas there is no such extra source of energy in nature.

Here is our answer: We should obviously not expect to obtain the same results, given that the equations with which we started are different. The kinetic energy difference we end up with, is indeed a little greater than that classically predicted. Yes, but so what? This occurs because the rest mass of the electron, decreases on the way, while the total relativistic energy stays constant [cf. Eq.(12-d)], since the kinetic energy acquired by the electron, is *(as we shall elaborate below),* by the mass deficiency the electron undergoes, on the way. Otherwise, it is not question of an *unnatural source* of an extra energy, that we invent. We simply follow the law of conservation of energy *(extended to embody the mass & energy equivalence drawn by the STR).* And, it should be by now become clear that, the law of conservation of energy is broken if one states [cf. Eq. (12-f)],

[difference of kinetic energies] = [difference of potential energies] . (12-g)

(classical statement breaking the law of conservation of energy)

Such a statement badly yields, as small as it may be, but still, the wiping out from nature, of an amount of energy as much as the difference between the right hand sides of Eqs. (12-e) and (12-f).

The correct statement instead, thus, is [cf. Eq.(12-d)],

[difference of kinetic energies] = [difference of rest masses]

(correct statement obtained within the frame of our approach)

.

*

An other criticism is the following: Suppose the electron is falling between the plates of a parallel plate capacitor delineating a *difference of electric potential* of 1 Volt. Then, the philosophy behind Eq.(12-c), as the criticism claims, should yield that the electron would acquire, the *extra relativistic energy* of $1\text{eV}/\sqrt{1-v^2/c_0^2}$, instead of 1 eV, as it reaches the positively charges plate, and this is evidently erroneous. Such is, the criticism.

But, we have no right at all to use Eq.(12-c), in such a case. Let us explain.

The latter equation is written for the closed system made of an electron and a proton, exclusively; it does not allow the instantaneous creation of a source of energy in the vicinity of the electron.

Indeed, how the electron could fall from the negatively charged plate of a capacitor to the positively charged plate of it, without being carried to the negatively plate, and set free there? Or as a different case, assuming that the electron is sitting originally on the initially neutral, plate, how the electron *(not taking into account the gravitational effect on it),* could fall from there without having someone, charging this latter plate, all of a sudden negatively, and the plate across, at the same time, positively?

In both cases, the electron originally at rest, in empty space, acquires *(still at rest)* an extra energy of 1 eV *(if the "electric potential difference" between the plates of the capacitor in question is, that much).*

Thus, its rest mass must have been increased as much; this is the rest extra mass coming into play, which will fuel the electron's fall, through its fly from the negatively charged plate to the positively plate, when set free nearby the former.

Let us recall the following. Not only that, the positively charged plate attracts the electron, but the negatively charged plate repels it, as well. Hence one should not expect that the electron's relativistic energy, thus its rest mass, remains the same, right after the neutral plate it is sitting on, originally, is all of a sudden charged negatively.

Note that if the electron is sitting on the initially neutral, positively marked plate, and this latter is abruptly charged, then theoretically speaking, still a certain amount of energy, should concomitantly be retrieved from the electron's rest mass, since due to the creation of positive charges in the vicinity, the status of the electron has changed from "free" to "bound". And what would be the binding energy coming into play? Not considerable really, since here we come to talk about a conduction electron, that can move in the conduction band of the material making the capacitor's plate. Thus the electron when caught on the positively marked plate, as the plate is charged positively, can be assumed not to practically get perturbed.

Thus, if the electron is initially sitting on the negatively marked plate of a capacitor of normally 1 eV of electric potential difference between the plates, and the capacitor is charged all of a sudden, then the *rest mass* of the electron would experience an increase of 1eV. If on the other hand, the electron is initially sitting on the positively marked but neutral plate of the same capacitor, and the capacitor is charged all of a sudden, then the rest mass of the electron *practically* remains the same.

With regards to a creation of charges in the vicinity of the electron, the situation is not any different when a freely flying electron enters in between the plates of a capacitor. As soon as it crosses the border, it witnesses the creation of a source of energy and at that moment, there must occur a change in its relativistic energy; thus, its rest mass must increase accordingly. If it enters the plates of the given capacitor of 1 Volt, flying right in between the plates of it, thereafter, its relativistic energy must increase as much as 1eV /2.

The same must hold with regards to situations displayed by accelerators, and so forth. This subject is of course to be studied apart.

Our claim is anyway well supported by experimental and theoretical results, though involving totally different setups than the one we presented above.[26]

In any case, with regards to circumstances we just reviewed, one is not allowed to use, Eq.(12-c), straight. There, we have considered, the *closed system* of the electron and the proton started at a practically infinite distance from each other.

This situation does not include, say the creation of a proton, right next to the electron. Such a problem should be considered apart, and we do not propose to handle it, herein.

Discussion, About the Concept of Field

The set up of Eqs. (10) and (11-a) is clearly not achieved by just customarily used concepts, and mainly, the regular concept of "field", since the "classical field" is the same whether the test charge is at rest, or in motion. The concept of field, to us, is nothing else, but a "mathematical convenience". Indeed, the intensity of it, cannot be measured, unless, one makes use of a "test charge". In other words, the field associated by a source charge at a given location, is a definition drawn, based on the force exerted by this charge on a unit charge, situated at the given location; thus the customary field intensity is defined as the force strength divided by intensity of the test charge, coming into play. Consequently, it is not the "field intensity" that one would measure, but the strength of the "force" developed by the source charge, on a test charge.

For us, what is essential is "Coulomb Force reigning in between two static charges". This is essential in two ways: i) the electric charges are Lorentz invariant, ii) hence the 1/distance^2 dependency of the Coulomb Force between two static charges, is imposed by the STR. Thus, Coulomb Force, as it is, but reigning between only two static electric charges, is substantially imposed by the STR.

What is believed so far, is that Coulomb's force holds, if the *source charge*[*] is static, regardless whether the *test charge*[†] is at rest or in motion. This requires the validity of Eq.(11-b).

However, we have shown that Eq.(11-b) is not correct; if the test charge is in motion, then Coulomb's force is decreased, by the factor $\sqrt{1 - v_0^2 / c_0^2}$ (cf. Table 1). Those of us who have the tendency of showing strong conservative reactions, should not get panicked, since the classical field concept, can still be used, by taking into account this latter correction to it. This correction becomes important only if the test charge moves at high speeds. Nonetheless, it comes straight from the fact that the rest mass of a bound charge, such as an electron, is not the same mass this delineates in empty space.

This occurrence drives us to consider, basically the electron (contrary to what has been done, so far) not in an extreme simplistic way. That is, we sympathize by the fact that, the electron is generally, considered as a "point-like particle". It must be obvious though, as tiny as it may be, the electron cannot be reduced to a "point", given that a point cannot be a "material being". Thus, it is pointless to consider the electron as a point-like particle. The electron must embody an "internal dynamics", just like any other particle, in fact in conformity with Eq.(1). Perhaps, its "mass" is simply the "internal energy" of the "electric property", which we call "electric charge". This internal energy, is thus to be associated with (how ever it may be), the internal dynamics delineated by the electric charge.

When the electron is bound, say, to a proton, its internal dynamics is then (as a requirement of the law of energy conservation), slown down, as much as the binding energy coming into play, assuming for convenience that the proton (being much more massive than the electron), is not affected by the process of binding.

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[*] The "source charge", is by definition, the center of the frame of reference in consideration.

[†] The "test charge" status, is defined with respect to the "source charge".

Our claim regarding the weakening of the internal dynamics of the bound electron, can be, as elaborated above, checked right away through the reverse process. Briefly speaking, suppose we propose to bring, the bound electron, back to infinity. Accordingly, we have to furnish to it, an amount of energy equal to its binding energy (still supposing that, moving away the electron, would not disturb, the proton). The two particles, forming a *closed system;* furnishing energy to the electron, owing to the law of energy conservation, will increase the rest relativistic energy, thus the rest mass of the latter.

This is why we are inclined to talk about the "internal dynamics" of an electron, in fact just like anything else. If the rest relativistic energy of it increases, to us, it is that its internal dynamics, somehow gains as much extra energy, and speeds up accordingly.

We will mention it once again, when entirely detached from the interaction domain, with the proton; the electron's rest mass would get increased as much as the energy we would have furnished to it, i.e. by an amount equal to its original binding energy.

Hence, the free electron is not anymore the previous bound electron, or vice versa, the bound electron is not anymore the same as the free electron. It is indeed hard to accept that it would be, given that one cannot make an omelet, and keep the eggs as they are, prior to cooking!

The bound muon decay rate retardation may be considered as an experimental proof of our assertion.[11-18]

*

Being aware of conservative reactions, we are somewhat sorry to affirm that (not only the expression of Coulomb Force, exerted by a source charge at rest, on a moving charged is altered, but), the expression of Lorentz Force too, exerted by a moving source charge on a moving test charge, is as well altered by our approach. We are going to leave this interesting (but, based on our approach, rather straightforward) problem, for a subsequent paper.

Nonetheless, we would like to mention that, fortunately for us, we are not the first one, who landed at such an *awkward result.* Weber, more than a century and half ago, arrived to a similar result, though through barely empirical means, while trying to derive from a single formula, Coulomb Force and Ampere Force, reigning between current elements, in perfect compatibility with the law of energy conservation. Thus he introduced, the Weber Potential,[27,28] which is the usual Coulomb Electric Potential, more specifically, Qq/r, multiplied by $\left[1-v^2/(2c_0^2)\right]$, where Q is the source charge intensity, q the test charge intensity, r the instantaneous distance between Q and q, and v the velocity of the test charge. This leads to Weber's Force, in other words, the usual Coulomb Force (Qq/r^2), multiplied by $\left[1-v^2/(2c_0^2)+rdv/(dtc_0^2)\right]$, or simply $\left[1-v^2/(2c_0^2)\right]$ for a circular motion.

Historically Weber's Electric Potential had been criticized, mainly because it was leading to a negative mass behavior of the charges,[29,30] and finally discarded. This electric potential was empirically elaborated a decade and half ago, in order to surmount problems related with it.[31,32] The new potential is called the Generalized Weber's Potential; this turns out to be the usual Coulomb Potential, divided by the familiar Lorentz dilation factor, or the same, multiplied by $\sqrt{1-v^2/c_0^2}$, which strikingly happens, what we have derived above, through our approach.

Recall further that, a change in the inertial mass of a charged test particle when placed inside a charged spherical shell, was later confirmed; the order of magnitude of the measured change was in accordance with the mentioned theory, developed on Weber's Electric Potential.[21,33]

As we will soon sketch, our approach for the bound electron yields a similar result, except that, not the Coulomb Potential Energy, but the Coulomb Force comes to be multiplied by the same factor. Though, it is exciting that we arrive at this result, based on Coulomb's Force straight (reigning between static charges, exclusively), and no other ingredients. (Recall that the extra term coming to multiply both Coulomb Potential Energy, and Coulomb Force is anyway, the same if the motion of the test charge, is circular.)

*

Let us now go back to our original approach, and question: *"How the interaction between the proton and the electron occurs, if along our approach, their respective energy is not spread in the surrounding space?"* We will work out the answer, below.

3. MASS *SUBLIMES* INTO KINETIC ENERGY, AND KINETIC ENERGY *CONDENSES* INTO MASS, THROUGHOUT THE MOTION: A JET MODEL

For a closed system *(thus excluding any creation of any field, on the way),* according to our approach, the total relativistic energy $c_0^2\ m_{0\gamma}(r_0)$ of the electron, as described by Eq.(11-a), ought to remain constant, all along the electron's journey around the proton.

On an elliptic orbit this implies an alternating decrease and increase of the static binding energy of the electron and its kinetic energy. The kinetic energy decreases, as the static binding energy increases, and vice versa. But, as elaborated above, the change in the static binding energy, implies a change of the electron's rest mass. Thus, on the elliptic orbit, as the kinetic energy of the electron increases, its rest mass decreases, and vice versa.

Thereby, as the proton speeds up nearby the proton, it is that, an *infinitesimal part of its rest mass,* somehow *sublimes* into *extra kinetic energy (the electron acquires, as it accelerates).* In other words, the *extra kinetic energy* in question, is *fueled* by an *equivalent rest mass.* Conversely, as the electron slows down away from the proton, through its orbital motion, it is that, the *corresponding portion of its kinetic energy,* somehow *condenses,* into *rest mass",* on the orbit. Note that recently a *fluid model* of the bound electron is proposed, incorporating a change of the mass of the electron through an exchange of mass between the electron and the nucleus (though, in a different manner than the one proposed herein).[34]

We would like to stress that, what we do is in no way, in conflict, with the established quantum mechanical framework.

One way of conceiving the *rest mass variation* we disclosed, together with, say, the acceleration of the electron, is to think in terms of a *jet effect.*

This effect, to the first strike, seems to be the only way we can think of, to account for the rest variation of the electron, causing its acceleration or deceleration (on an elliptic orbit).

Thus, within the frame of such a modeling, in order to accelerate *(while keeping its overall relativistic energy, constant),* the electron would *throw* out an infinitesimal *net mass* from the *back,* just like an *accelerating rocket.* Conversely, in order to decelerate, it would *absorb* an infinitesimal *net mass,* from the *front.*

Whether in reality, the whole thing works out this way or not, *we do not really know.* For the present purpose, we do not need to know it, either. It is that, we came to be able to refer to a mechanism, which can provide us, with what we need, in order to take care of the variation of the kinetic energy, in relation to the variation of the electrostatic binding energy, thus in relation to the variation of the rest mass of the bound electron (imposed by the law of energy conservation). Hence, we can well base ourselves on it, to make useful predictions.

Before we continue, it is worth to analyze the situation, based on the simplest motion, i.e. the rectilinear motion.

Study of the Rectilinear Motion in conjunction with the Law of Conservation of Momentum

Thus, suppose that the electron falls onto a proton. This occurrence, according to our approach, is insured by an overall *jet mass* thrown by the *rare* of the electron.

Recall that the way we have set up Eq.(10), where the electron, as a first step, is brought to the given location, *quasistatically,* excludes at once (an otherwise expected) *radiation emission.*

Note yet that the speed to be delivered to the electron, through the "second step of the set up" [cf. Eq(10)], should be smaller, if otherwise, through a usual fall, the related "loss of energy via radiation" were to be taken into account.

Now, one can show that, just like in the case of the classical approach; within the frame of the approach we summarized herein too, the "law of linear momentum conservation" holds, and this, essentially as a result of the "law of energy conservation", along with "Coulomb's law", leading altogether, to the general equation of motion [cf. Eq.(12-b)].

Note on the other hand that, the "law of angular momentum conservation", too can be deduced, directly from our set up, just like it can be deduced from the "classical approach".*
Thus, what is essential is, as evermore, the law of energy conservation.

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* One can see this readily, by multiplying the vector Eq.(12-b), by the location vector $\underline{r}_0$. Thus the related cross product becomes

$$-\underline{r}_0 \times \frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}\,\frac{\underline{r}_0}{r_0} = 0 = m_\gamma(r_0)\,\underline{r}_0 \times \frac{d\underline{v}_0(t_0)}{dt_0} \quad .$$

[the cross product of Eq.(12-b) *by the location vector]*

The integration of this equation yields

$$\underline{r}_0 \times \underline{v}_0(t_0) = \text{Constant} \quad \text{(c.q.f.d.)}.$$

(Angular Momentum Conservation law derived from the author's set up)

One can indeed check right away that the differential of the LHS of the above equation turns out to be zero.

Suppose that the electron of rest mass $m(r_0)$, and velocity v_0, through the free fall in consideration, speeds up as much as dv_0, through an infinitely small period of time dt_0, around the time t_0. It should be recalled that, when we say, “the linear momentum is conserved, thorough the fall of the electron onto the proton”, we mean, “the total linear momentum of the system made of the electron and the proton is conserved”. It is that, after all, the center of mass of the system stays in place.

Within the frame of our model though, where we simulate the fall of the electron through the jet effect, no matter whether this is a fictitious assumption or not, we should concentrate on the electron along with the electron’s jet on the one hand, and the proton along with the proton’s jet on the other, instead of the system made of the electron and the proton.

The conservation of the linear momentum through the free fall of the closed system made of electron onto the proton, under these circumstances, can be considered as

i) the conservation of the linear momentum of the system made of the electron and its jet, on the one hand, and

ii) the conservation of the linear momentum of the system made of the proton, and its accompanying jet on the other hand

Based on this assertion, because the proton is much too heavy as compared to the electron, and accordingly it will practically stay in place through the motion in consideration, thus displaying practically no jet effect, through the fall of the electron, we can overlook, the motion of the proton, together with the jet we associate with it.

Hence, we will only have to worry about the electron’s motion along with its jet.

Note that, the linear momentum of the system made of the jet associated with the electron and the jet associated with the proton too, should be conserved throughout.

As proposed, below, we will write the law of momentum conservation, just with regards to the system made of the falling electron and its jet. Thus, the magnitude of the linear momentum $P(t_0)$ of the electron, assumed to fall right onto the proton with the velocity v_0, at time t_0, at the given location r_0, is

$$P(t_0) = m_\gamma(r_0)v_0 \ . \tag{13}$$

(magnitude of the momentum of the electron in free fall, at the given location and the given time)

Recall that, in any case, this momentum alone, is in no way conserved. *(Unfortunately, we are to spell this triviality, because it became the center of attraction of conservative reactions.)* What is conserved, is once again, the sum of the momentum of the electron and that of the proton, or along our model, instead, the sum of the momentum of the electron and that of the jet, given that we neglect the motion of the proton toward the electron, throughout.

Thus, through the period of time dt_0, in order to accelerate, the electron should throw out from its back, the net rest mass $-dm(r_0)$, with a *jet speed* U. This shall produce a kick forward, on the overall mass $m_\gamma(r_0)$, which accordingly acquires the velocity $v_0 + dv_0$. Let us precise that this mass is taken away from the rest mass of the electron, measured at the given location. It should be stressed that, although the overall relativistic energy $m_\gamma(r_0)c_0^2$ stays constant throughout, the rest mass of the electron, at a given location and its kinetic energy vary reciprocally, and in opposite directions [cf. Eq.(11-a)].

Since $m_\gamma(r_0)$ remains constant, all the way through, to be brief, we propose to call it m_γ.

On the other hand, recall that through the acceleration process in consideration, the quantity $dm(r_0)$, by definition,* is negative [so that $-dm(r_0)$ is a positive quantity].

Thus, at time $t_0 + dt_0$, the magnitude of the net linear momentum of the system made of the *jet* of mass $-dm(r_0)$, and the *electron,* becomes

$$P(t_0 + dt_0) = dm(r_0)U + m_\gamma(v_0 + dv_0)\,. \tag{14}$$

(magnitude of the momentum of the electron, an infinitely small period of time, after the given time)

Because of the law of conservation of momentum *(for the closed system made of the electron and the jet in question),* we must have:

$$P(t_0) = m_\gamma v_0 = P(t_0 + dt_0) = dm(r_0)U + m_\gamma(v_0 + dv_0)\,, \tag{15}$$

(equation of conservation of momentum for the rectilinear motion with regards to the of the fall of the electron assuming that the proton stays in place, all the way through)

which yields

$$m_\gamma dv_0 = -dm(r_0)U\,. \tag{16}$$

(kick received by the electron due to the jet effect, on a rectilinear motion)

This equation tells us that, an infinitely small mass $|dm(r_0)|$ has to be thrown out by the falling electron, in the opposite direction, with a speed U (as referred to the fixed proton), in order to provide the electron with an extra speed dv_0.

Note that above, we happened to have associated the jet speed U with the *rest mass variation* $|dm(r_0)|$, the electron displays on the way. We did it on purpose, given that as we will see, it is the rest mass $|dm(r_0)|$ that can be determined directly, from the related *"infinitesimal electrostatic binding energy change"*. Moreover, as we shall soon pin down, $|dm(r_0)|$ may well be zero, whereas U can still be defined. In short, it can be shown that U may well come into play, as a specific quantity, in one piece, as such.

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* $dm(r_0) = m(r_0+dr_0) - m(r_0) < 0$, when the electron accelerates via throwing out mass; $m(r_0)$ is the electron's mass at r_0.

At any rate, not to yield misinterpretations, the *"jet momentum"* $U\left|dm(r_0)\right|$, should better be written, as

$$\left|dm(r_0)\right|U = \frac{\left|dm(r_0)\right|}{\sqrt{1-\frac{V^2}{c_0^2}}}V = \gamma_V\left|dm(r_0)\right|V = \left(\gamma_V\left|dm(r_0)\right|\right)V = \left|dm(r_0)\right|\left(\gamma_V V\right) , \qquad (17\text{-a})$$

(momentum of the jet expressed in different terms)

where γ_V is

$$\gamma_V = \frac{1}{\sqrt{1-\frac{V^2}{c_0^2}}} \quad , \qquad (17\text{-b})$$

and V is the jet speed of the "relativistic mass" $\gamma_V\left|dm(r_0)\right|$, so that

$$\gamma_V V = U \ . \qquad (17\text{-c})$$

The RHS of Eq.(17-a), i.e. $\gamma_V\left|dm(r_0)\right|V$, can visibly be read either as $\left(\gamma_V\left|dm(r_0)\right|\right)V$, or as $\left(\gamma_V V\right)\left|dm(r_0)\right|$.

The writing $\left(\gamma_V\left|dm(r_0)\right|\right)V$ is the customary one, given that it embodies the *relativistic mass* $\left(\gamma_V\left|dm(r_0)\right|\right)$ multiplying as usual the speed V, to yield the relativistic momentum of the jet in consideration. The second writing, i.e. $\left(\gamma_V V\right)\left|dm(r_0)\right|$ *(in which the Lorentz dilation factor, and the related mass are decoupled from each other),* is obviously unusual, but becomes very interesting, as we will see, for the case where $\left|dm(r_0)\right|$ is zero, pointing to an interaction with *no net mass variation*; in this case, the product $\gamma_V V = U$ is to be considered *en bloc;* in any case, the product $\gamma_V V$, comes into play as *a whole,* since it turns out that we will end up with U, as just *one single quantity,* and not with separately γ_V and V.

Eq.(17-a) is remarkable, because, as simple as it may look, here may be a clue for the *wave-particle duality:* The relativistic momentum $\left(\gamma_V\left|dm(r_0)\right|\right)V$, evidently points to the *particle character* of the electron, whereas $\gamma_V V = U$ as *a whole,* taking place in the product $\left|dm(r_0)\right|\left(\gamma_V V\right)$ *(as we will soon discover)* indeed works as the key of the *wave-like character* of the electron; this becomes particularly evident, when $\left|dm(r_0)\right|$ vanishes.

We will discover that U, more precisely, its component U_t along the trajectory of concern, which is still U for a rectilinear motion, becomes $U_t = (c_0^2/v_0)\sqrt{1-v_0^2/c_0^2}$, and $V = c_0\sqrt{1-v_0^2/c_0^2}$; for this reason we propose to call U the *wave-like jet speed,* or the *superluminal jet speed (associated with not a certain amount of energy, but a given information),* and V the *relativistic jet speed (associated with the mass of the jet);* to refer to U shortly, we may just write, the "jet speed U".

What we mean by the denomination *"superluminal",* is "faster than the speed of light". Though, when we say *"superluminal",* we do not mean that *"energy is carried faster than the speed of light";* what we certainly mean nonetheless, is that that *"a given information somehow is conveyed with a speed, in effect, faster than the speed of light.*

*

Now, suppose that the falling electron is stopped at a given distance to the proton, and thrown backward with a given initial kinetic energy. Let us assume that this energy is less than the *escape kinetic energy.* The electron would then get elevated, while it has to spend its kinetic energy as *work* achieved against the *attraction force.* What happens according to our approach is that, owing to the law of energy conservation, the electron transforms gradually its *kinetic energy,* into *additional internal energy,* or the same, *additional rest mass*.

In order to achieve this end, we conjecture that, somehow, it receives momentum from the outside, in conformity with Eq.(16). Thus, its speed decreases, until it exhausts all its initial kinetic energy (after which it will undergo over again, the free fall). Through the elevation process, the electron receives momentum from the front. Through the fall, it throws out an overall momentum from the rare. (In fact, the two sides of concern happen to be the same side, that is, the *dark side* as referred to the proton.) At the highest elevation, the direction of $\underline{U}$ (the jet velocity vector) is reversed. The direction of $\underline{U}$, in any case, opposes the direction of the motion.

One can as well conjecture that, if thrown, the elevating electron slows down, not because it receives momentum from the front, but because it throws out momentum from the front. The latter process can evidently explain the slowing down of the elevating electron, but not a concurrent rest mass increase. By the same token, one can conjecture that, the falling electron accelerates not because it throws out momentum by the rare, but because it receives momentum from the rare. In this case, the latter process can evidently explain the acceleration of the falling electron, but not a concurrent rest mass loss.

Thus as the electron falls, according to our approach, it should throw out momentum from the rare; this way its rest mass can decrease, or the same, a minimal part of its rest mass is transformed into kinetic energy.

Likewise, if thrown away from the proton, while the electron elevates, it should receive momentum from the front, which slows it down; this way its rest mass increases, or the same, a minimal part of its kinetic energy condenses into rest mass.

And how does the electron know when to accelerate, or when to decelerate? The answer is easy; it is merely the Coulomb Force which tells the electron how to behave. The proton and the electron pull each other (or they seem doing so, i.e. they may as well be pushed toward each other, by a certain property of their surrounding). Anyway, the answer we look for, is not any different from the classical answer.

As we will soon see, the wave-like jet speed U depends only on the speed of the electron. The smaller the speed of the electron, the greater U is. It is infinite when the electron's speed is zero. It would be zero, if in an extreme case, the electron's speed were equal to the speed of light.

At the highest elevation, where the speed of the electron is vanished, the magnitude of U is thus infinite. In fact, at this level, the *momentum reception by the electron, from the outside,* is switched into a *momentum ejaculation from the electron to the outside,* each of these processes *(at the highest elevation in consideration),* delineating, an infinite magnitude for U.

Under the given circumstances, it seems legitimate to interpret U, as the *interaction speed*, given that the manifestation of the electric interaction, is well taken care by the *momentum reception,* or the *momentum ejaculation* processes, we have introduced.

Thus, it seems legitimate to admit that the electric force is somehow created, by the processes we just described.

As one can notice right away, nothing would change, if we considered gravitationally interacting bodies, instead of electrically interacting charges.

Now, let us analyze the more complicated problem of an orbital motion, say, that of the rotation of the electron around the proton.

Orbital Motion: Equation of the Kick due to the Jet Effect

Eq.(16) can be written as a vector equation, i.e.

$$m_\gamma d\underline{v}_0 = \underline{U} dm(r_0)\,, \tag{18}$$

(vector equation delineating the conservation of momentum throughout the jet effect, for a rectilinear motion)

where $\underline{U}$ is the jet velocity, and $d\underline{v}_0$ is the vectorial increase of the velocity of the object, corresponding to the ejection or the absorption of the infinitely small mass $|dm(r_0)|$, through the period of time dt_0. Note that $\underline{U}$ is opposed to the direction of motion, whereas, $d\underline{v}_0$ is directed inward, i.e. toward the proton.

Recall again that $dm(r_0)$ is negative for the free fall, through which $\underline{U}$ is directed outward, and $dm(r_0)$ is positive in the case the electron is thrown away from the proton, through which $\underline{U}$ is directed inward.

The division of Eq.(18) by dt_0, may constitute a clue about the root of the *electric force:*

$$m_\gamma \frac{d\underline{v}_0}{dt_0} = \underline{U}\frac{dm(r_0)}{dt_0}\,, \tag{21}$$

(equation depicting the creation of the force via the jet model)

or via Eq.(12-b),

$$-\frac{Ze^2}{r_0^2}\sqrt{1-\frac{v_0^2}{c_0^2}}\,\frac{\underline{r}_0}{r_0} = \underline{U}\frac{dm(r_0)}{dt_0}\,. \tag{22}$$

(the force expression via the proposed jet model)

The foregoing three equations should be expected to hold generally. Thus, it should well hold regarding an elliptic motion, which we have considered originally. [In fact, recall that the orbit drawn by the above equation, is not exactly elliptic; instead its perihelion precesses. But we do not need to elaborate on this piece of detail. Thus below, we will call the resulting orbital motion, straight, "elliptic motion".]

For such a motion, $d\underline{v}_0$ is directed radially. Thus, $\underline{U}$ must be directed accordingly, were $dm(r_0) > 0$. As the electron accelerates toward the proton, $\underline{U}$ is directed outward, since $dm(r_0) < 0$; thus, the electron ejaculates a minimal part of its rest mass, or the same, that much of rest mass, is transformed into extra kinetic energy. As the electron decelerates away from the proton, $\underline{U}$ is directed inward, since $dm(r_0) > 0$; thus the electron receives momentum from the outside, killing a fraction of its the kinetic energy, or the same, that much of energy is restored as *extra rest mass.*

Let us now define θ, as the angle between $d\underline{v}_0$ (directed inward), and the electron's velocity $\underline{v}_0$ (tangent to the orbit).

Thus, one can show that

$$dv_0 = -|d\underline{v}_0|\cos(\pi - \theta) = |d\underline{v}_0|\cos\theta \,. \tag{21}$$

Based on this relationship, one can transform Eq.(18), into

$$m_\gamma dv_0 = \cos\theta|\underline{U}dm(r_0)| = -U_t dm(r_0)\,, \tag{22}$$

(scalar equation delineating the conservation of momentum throughout the jet effect, on a given orbit)

where we call U_t, the quantity $|\cos\theta\underline{U}|$, i.e. the *magnitude of the tangential component* of $\underline{U}$.

This equation is well compatible with Eq.(16), written for a rectilinear motion. Hence, it is the general equation describing the "kick", the electron receives, throughout the centripetal motion, owing to the jet effect we introduced.

Let see what happens in the extreme case of a circular motion.

Circular Motion: The Electric Interaction Can Be Achieved, Without Any "Energy", or "Mass" Exchange, Whatsoever

The circular motion appears to be more peculiar. Yet we will show that Eq.(18), as should be expected, still holds. In the case of a circular motion, dv_0 is null; $dm(r_0)$ too should be null, since *kinetic energy* is obviously not altered throughout. *(Recall that we have conceived the jet effect in order to take care of the variation of the kinetic energy, in relation to the variation of the electrostatic binding energy, thus in relation to the variation of rest mass of the bound electron, as imposed by the law of energy conservation.)*

Eq.(22) fulfills well, the *condition* of $dv_0 \Leftrightarrow dm(r_0)$. But then, there seems to arise a few problems. First of all, for the circular motion, $\cos\theta$ vanishes. Already because of this occurrence, the RHS of Eq.(22), would vanish. Thereby, what would it mean that the RHS of Eq.(22) disappears, not only due to $dm(r_0)$=0, but also due to $\cos\theta$=0? The answer is tricky, and will be clarified below. It is that, although for a circular motion $\cos\theta$=0, the RHS of Eq.(22) does not actually vanish, because of this occurrence, and the reason is that, $|\cos\theta\underline{U}|$ (i.e. the tangential component of the jet velocity, as will soon be proven), turns out to be *finite,* even if $\cos\theta$=0. Thus $\underline{U}$ in this case, must turn to be *infinite.* This will constitute a clue to clarify the next problem, which is the following.

If $dm(r_0)$=0, then, the RHS of Eq.(18), at the first strike would vanish. But, the LHS of this equation does not (since the variation of the velocity vector, constitutes the basis of the acceleration, and the acceleration is a finite quantity for the circular motion)! Then, how come that the RHS of Eq.(18) seems to vanish, and the LHS of it remains finite? The answer to this question too, is tricky. In fact, it is that, the physical requirements we have considered regarding the rectilinear accelerational motion, and especially elliptic motion, have forced us, first to write Eq.(16), then Eq.(18), and then Eq.(22), by considering either the *mass ejaculation,* or the *mass absorption,* through corresponding jets, and *not* the two processes, simultaneously. But, for a circular motion, to get an infinitely small kick (momentum change) inward, through an *infinitely short period of time*, we are allowed to consider, both the *"ejaculation of an infinitesimal mass to the outside",* and the *"absorption of an infinitesimal mass from the outside",* concurrently, given that both of these processes yield the same effect. Under the circumstances, we do not have to watch, at one fell swoop, to secure the increase or the decrease of the speed of the object through the motion (since the speed remains the same, throughout), but just the kick inward. The only other requirement is that the mass ejaculated to secure the kick inward is balanced by the mass received from the outside, still to secure the kick inward, so that the two kicks amount to the expected overall kick. Fortunately we will not have to formulate these anticipations.

At any rate, the result is that, for a circular motion, there is *no net rest mass gain, or loss.* In other words, regarding an orbital motion, one can interpret Eq.(18), in two different ways. Thus, this equation can be viewed as the description of the kick due to either the *absorption of an infinitesimal rest mass,* or the *ejaculation of an infinitesimal rest mass,* coming respectively into play, along an elliptic motion.

It can also be seen as the description of the *"resultant kick",* due to the superimposed processes of the reception of an *infinitesimal momentum by the electron from the outside,* and the *simultaneous ejaculation of the same amount of momentum by the electron to the outside,* through a circular motion.

This latter process, as expected, depicts a *zero net mass change.* This is important for, on the one hand, it points to the fact that, the interaction in question occurs, without any energy exchange *(or the same, without any net mass exchange),* with the attraction center, *whatsoever.* On the other hand, $dm(r_0)$=0, arising on the RHS of Eq.(18), along with a *finite* LHS taking place in this equation, can only be tolerable if $\underline{U}$ is infinite, and this is exactly what we have just established.

We will see in fact that it is the quantity $|\cos\theta\underline{U}|$, we can calculate, and not $\underline{U}$ alone.

*

It is worth to stress that the approach we have developed on the basis of the jet effect, points to a possible mechanism of the interaction in consideration. Thus, we conjecture that, the faster the jet speed U, the quicker the interaction takes place.

U, or more precisely $|\cos\theta \underline{U}|$, can be taken as the *speed of the transmission of the information* assuring the electric motion. The same holds, if it were question of a gravitational motion.

Now for a circular motion the overall mass change throughout is null. The information assuring the interaction must be there, though.

Thus, there seems reasons to believe that, even when the net mass gained or lost by the jet effect is null, whatever is the information we anticipate to be carried by the jet of speed U, this information is still transferred. In other words, whatever is the information carried by the jet speed U, this information can be transferred, along with *no mass,* thus *no energy* involved, at all. This is interesting since we came to say that *information can well be transferred with no need of any usage of energy.*

4. DERIVATION OF THE DE BROGLIE RELATIONSHIP, AND SUPERLUMINAL SPEEDS

Let us now multiply Eq.(16) *(for a rectilinear motion),* or Eq.(18) *(for an elliptic motion),* by c_0^2:

$$c_0^2 m_\gamma(r_0) dv_0 = -c_0^2 U_t dm(r_0) , \tag{23}$$

The law of energy conservation requires that, the quantity $-c_0^2 dm(r_0)$, appearing at the RHS of this equation, must come to be equal, to the change in the corresponding kinetic energy, which in return, must be equal to the change in the corresponding electrostatic binding energy [cf. Eq.(8)].

Thus

$$c_0^2 dm(r_0) = \frac{Ze^2}{r_0^2} dr_0 \quad , \tag{24}$$

(variation of the rest mass, in terms of the static, electrostatic binding energy)

written in CGS unit system

Note, on the other hand that, when the electron *(either through a head on free fall, or an elliptic motion)* speeds up; it gets closer to the proton; in this case dr_0 [just like, $dm(r_0)$], turns out to be a negative quantity.

Equating the LHS of Eq.(23), with the product of the RHS of Eq.(24) by U_t [via Eq.(10)], leads to

$$c_0^2 m_\gamma dv_0 = -U_t \frac{Ze^2}{r_0^2} dr_0 \quad . \tag{25}$$

Here we can replace dv_0, by the same quantity, furnished by Eq.(12-a).

Thence, the *tangential jet speed* U_t, as assessed by the distant observer, turns out to be*

$$U_t = \frac{c_0^2}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \quad . \tag{26}$$

(the wave-like jet speed as referred to the outside fixed observer)

Note that, as mentioned the approach allows us to determine straight, the magnitude of the tangential component of $\underline{U}$.

Eq.(26), is amazingly the same as Eq.(7-c), if the tangential jet speed U_t is taken to be same as U_B, of this latter equation. It is *rigorous.* It only depends on the speed of the object of concern.

We can, anyway, write Eq.(26) as

$$U_t = \frac{\lambda_0}{T_0}\frac{c_0}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \quad , \tag{27}$$

via the usual definition of the speed of light, i.e. Eq.(2).

Now, if we propose to write the *tangential jet speed* U_t, in question, in terms of the period of time T_0, of the electromagnetic wave, we associate with the mass m_0, along Eq.(1); we come to the expression of a wavelength λ_t, in terms of λ_0, i.e.

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* It is worth to redo the derivation in question, in *vector form,* starting fort he vectorial equation, Eq.(18). Thus, Eq.(25) of the text becomes

$$c_0^2 m_\gamma d\underline{v}_0 = \underline{U}\frac{Ze^2}{r_0^2} dr_0 \quad . \tag{i}$$

Let us take the absolute values of both sides, and use Eq.(19):

$$c_0^2 m_\gamma dv_0 = \cos\theta \left| \underline{U}\frac{Ze^2}{r_0^2} dr_0 \right| = -U_t \frac{Ze^2}{r_0^2} dr_0 \quad , \tag{ii}$$

where U_t is the *magnitude of the tangential component of the vector* $\underline{U}$.

Finally , using Eq.(12-a) to replace dv_0, one lands straight, at Eq.(26).

$$\frac{\lambda_t}{T_0} = \frac{\lambda_0}{T_0}\frac{c_0}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \quad . \tag{28}$$

Thus, λ_t is nothing else, but the de Broglie wavelength [cf. Eq.(6-a)]:

$$\lambda_t \equiv \lambda_B = \lambda_0 \frac{c_0}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}} \quad . \tag{29}$$

(de Broglie wavelength obtained from the wave-like jet speed, derived in here)

It holds generally, thus through a rectilinear motion or, as well through an elliptic motion, or as an exception, through a circular motion.

Since, it is velocity dependent, it is practical to consider it, first, for a circular orbit *(embodying a constant speed).*

Yet, under the form delineated by Eq.(29), it ought to be valid only for the ground state (then leading to the original Bohr postulate).[1,3]

If the motion is circular, for the nth level, one should instead write (cf. the closing footnote in the Introduction, above):

$$\lambda_{Bn} = n\lambda_0 \frac{c_0}{v_{0n}}\sqrt{1-\frac{v_{0n}^2}{c_0^2}} \quad . \tag{30}$$

(de Broglie wavelength on the n^{th} orbit)

We should further determine the *"relativistic jet velocity"* V, appearing in Eq.(17-c). Thus via Eqs. (17-b) and (26), one can readily write

$$V = c_0\sqrt{1-\frac{v_0^2}{c_0^2}} \quad , \tag{31}$$

(the relativistic jet velocity)

which indeed happens to be below the velocity of light.

Thereby γ_V defined along Eq.(17-b), becomes

$$\gamma_V = \frac{U_t}{V} = \frac{c_0}{v_0} \quad . \tag{32}$$

We can on the other hand, calculate the jet speed u, with respect to the electron. As a rough approximation, one can write,

$$u \approx U_t + v_0, \tag{33}$$

(the wave-like jet speed as referred to the "moving electron")

where v_0 is the speed of the electron, with respect to the proton (assumed at rest); we will call u the *superluminal jet speed,* since, as clarified right below, it is always greater than the speed of light. Note that in our approach the ceiling c_0 cannot be reached, unless the photon bears an infinite amount of energy.[4]

At this stage, we do not know the rule regarding the addition of superluminal velocities, with ordinary velocities (taking place, below the speed of light). Nonetheless, the examination of Eq.(26), makes our task easy. The two interesting cases indeed occur for $v_0 = 0$ and $v_0 = c_0$. For $v_0 = 0$, $U_t = \infty$; thus one can right away guess that, in this case, u must be infinite. For $v_0 = c_0$, $U_t = 0$; thus one can guess that, in this case u must be c_0.

Hence, we can well establish that, the superluminal interaction speed u *(with respect to the object in question),* varies between ∞ *(for the object of concern, at rest),* and c_0 *(for the object moving with the speed of light).* As tautological as it may seem, this yields the fact that, light cannot interact with anything, via a speed above the speed of light (since its "superluminal jet speed" is, at best, c_0).

It is interesting to note that, our results is somewhat in conformity with what had been established with *tachyons,* particles moving faster than light.[35,36] Though tachyons are *imaginary particles,* whereas we have no energy or mass moving with a superluminal speed; only *information* may be transferred with a speed faster than that of light.

To us, this is like *quantum mechanics,* which is, next to the de Broglie relationship, based on the law energy conservation, but allowing well an infinite uncertainty about energy, if the uncertainty on time is null.

Thus, while the STR does not allow any speed faster than that of light, amazingly, it appears to allow even an infinite speed of *information transfer,* if the mass or energy, involved, is missing.

It seems useful to summarize different velocities we introduced, along with different values of interest they would assume. This is done in Table 2.

Table 2 Different Velocities Introduced Throughout

Velocity	Explanation	Expression	Special Case 1	Special Case 2
v_0	*Electron's speed as assessed by the outside fixed observer*	v_0	$v_0 = 0$	$v_0 = c_0$
c_0	*Speed of light in "empty space"*	c_0	c_0	c_0
U_t	*The "magnitude of the tangential component of the superluminal jet speed", as assessed by the outside fixed observer*	$U_t = \frac{c_0^2}{v_0}\sqrt{1-\frac{v_0^2}{c_0^2}}$ $= \gamma_V V$	$U_t = \infty$	$U_t = 0$
V γ_V	*The "relativistic jet speed", as assessed by the outside fixed observer* *Lorentz Dilation Factor*	$V = c_0\sqrt{1-\frac{v_0^2}{c_0^2}}$ $\frac{1}{\sqrt{1-\frac{V^2}{c_0^2}}}$ $= \frac{U_t}{V} = \frac{c_0}{v_0}$	$V = c_0$ $\gamma_V = \infty$	$V = 0$ $\gamma_V = 1$
u	*The "superluminal jet speed", as referred to the electron*	$u \approx U_t + v_0$	$u = \infty$	$u = c_0$

CONCLUSION

Herein, based on essentially energy conservation, we figured out that, a motion driven by electric attraction depicts some sort of mass exchange, throughout.

One way to conceive this phenomenon is to consider a “jet effect”. Accordingly, an object on a given orbit, through its journey, must eject mass to speed up, or must pile up mass, to slow down.

The component of the speed U of the jet, tangent to the trajectory in question (as referred to the proton), strikingly delineates the de Broglie wavelength, coupled with the period of time T_0, delineated by the corresponding electromagnetic energy content of the object [as required by Eq.(1)].

This result seems to be important, in many ways. A more detailed conclusion will be drawn at the end of Part II, where we will deal with the gravitational field.

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