An Application of the Lorentz Transformation

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Abstract

If more than two systems are moving relatively to each other, the Lorentz Transformation leads to inconsistencies which do not occur when the Galilei Transformation is adopted.

Keywords

1. Special relativity
2. Galilei transformation
3. Lorentz transformation

1 A gedanken experiment

Suppose a rocket A is starting from earth E with velocity v at $t = 0$ (Fig. 1). At the same time a second rocket B starts at distance L, heading towards earth with the same velocity v, and arrives on earth at $t = T$:

$$T = \frac{L}{v} \tag{1}$$

This example is occasionally quoted[2] in order to explain the asymmetry of the twin paradox when acceleration is absent. The time reading of a clock in A may be transferred to a clock in B upon meeting halfway at $L/2$. This way one can show that there is a time dilation in the moving clocks as compared to a stationary clock on earth. The reason is that the rockets move on longer world lines in the Minkowski diagram.

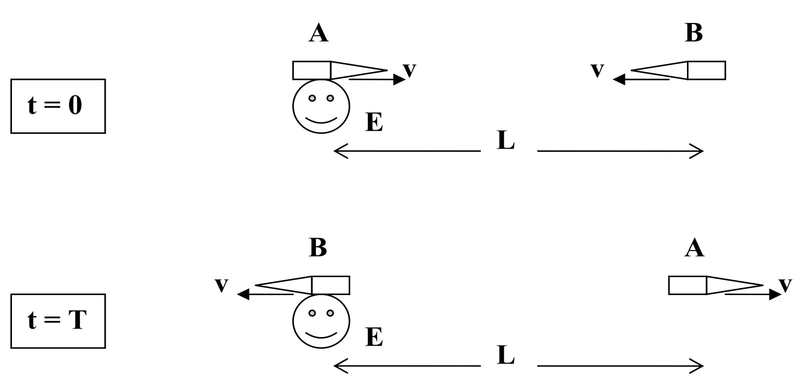

Figure 1: Two rockets, A leaving earth, B arriving on earth with constant velocity v.

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[2] M. Harder, *Einsteins Irrtümer,* Books on Demand GmbH, Norderstedt (2009), p.63 and p.77 ff

As there is only inertial motion involved, it must be possible to view this example from any of the rockets each of which may be considered at rest. Let us take A at rest, for example. The earth is then receding from rocket A, and rocket B approaches A at twice the velocity v if we apply the Galilei transformation (Fig. 2).

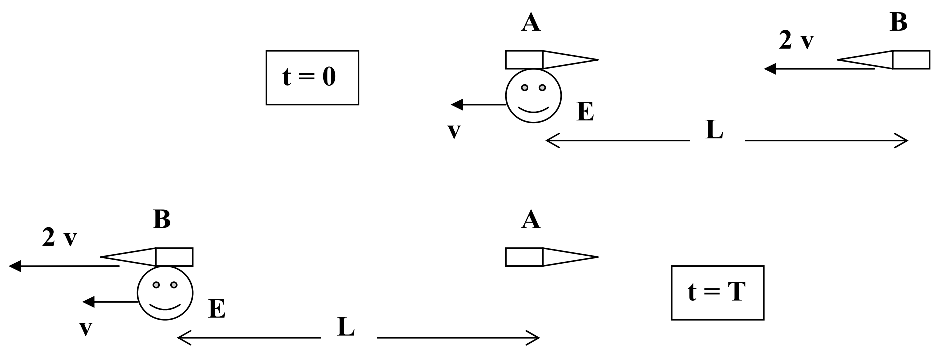

Figure 2: Same as Fig. 1, but viewed from rocket A at rest. Rocket B approaches rocket A at 2 v according to Galilei's composition law for velocities.

The time of arrival T is calculated from the relationship

$$vT + L = 2vT \qquad \rightarrow \quad T = L/v \tag{2}$$

Neither the time interval T nor the distance $L = \overline{\mathrm{EA}}$ have changed as compared to the previous case of Fig. 1 where the earth was considered at rest. This is in perfect agreement with Galilei's relativity principle, i.e. any inertial system is suited to describe the given situation.

2 Reformulation according to the Lorentz Transformation

A problem arises when the velocity of the rockets is very large, e.g. $v = 0.8\,c$. The Galilei transformation leads to twice the velocity yielding $v' = 1.6\,c$ which is not allowed by the Lorentz transformation. We must apply Einstein's composition law for

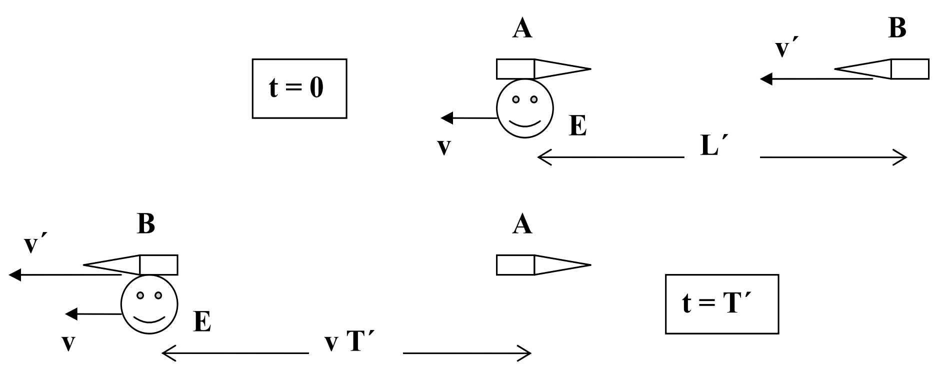

Figure 3: Same as Fig. 2, but applying the Lorentz Transformation to avoid $v' \geq c$

velocities and arrive at the picture as shown in Fig. 3. Both the earth and rocket B move in the primed system where rocket A is at rest. The composition law yields for the velocity of B:

$$v' \;=\; \frac{2\,v}{1 \,+\, v^2/c^2} \tag{3}$$

The arrival time of rocket B is now calculated from

$$v\,T' \;+\; L' \;=\; v'\,T' \qquad \rightarrow \;\; T' \;=\; \frac{L'}{v}\;\frac{1+v^2/c^2}{1-v^2/c^2} \tag{4}$$

where we have substituted (3) for v'. Both the primed intervals of time (T') and space (L') are multiplied by the dilation factor $\sqrt{1-v^2/c^2}$ which may be cancelled in (4) in order to transform this equation back into the unprimed system. Thus we obtain the final result:

$$T = \frac{L}{v}\;\frac{1+v^2/c^2}{1-v^2/c^2} \tag{5}$$

Comparing this with (1) or (2) we see that the Lorentz Transformation leads to a contradiction when we shift the reference system from the earth to rocket A. A similar contradiction would arise, if we chose rocket B as the inertial system in which both earth and rocket A move. Obviously, the Lorentz Transformation violates the mechanical relativity principle.

Einstein had claimed that he could reconcile the apparent incompatibility of the relativity principle with a constant velocity of light in vacuo as assumed by the ether theory. A careful analysis of his 1905 paper reveals, however, that he did not use the relativity principle when he derived his version of the Lorentz Transformation. Instead, he adopted Voigt's unphysical postulate $c =$ const for any arbitrarily moving observer. It is therefore not surprising that the resulting transformation is at variance with the mechanical principle of relativity.