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Deriving an Atom's Stability from Classical Mechanics and from the Special Theory of Relativity

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Abstract
It is not possible to establish the ground state energy of a hydrogen atom without quantum mechanics. However, for the atom's stability only, this can be explained even without using quantum mechanics.
Even according to classical considerations, our discussion reveals that there exists an off-limit boundary r_e within the electron inside a hydrogen atom.

Key words: Atom's Stability, Special Theory of Relativity, Bohr's classical quantum theory.
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1. Introduction
When describing the total energy of a hydrogen atom according to classical mechanics, we must know the atom's potential energy and kinetic energy.
However, when discussing the energy of a hydrogen atom according to quantum mechanics, we are only concerned with the increase or decrease in total energy.
By assuming quantum conditions, Bohr derived the orbit radius of an electron. He further explained that there is a minimum value of total energy of an electron, and that an electron is not absorbed into the atomic nucleus.
However, in our discussion, we attempt to use a classical mechanics approach to consider whether an atom's stability can be explained by methods other than quantum mechanics.

2. Electron energy as described according to classical mechanics
Let us review the energy of an electron inside a hydrogen atom.
Let us suppose that the atomic nucleus is at rest because it is heavy, and consider the situation where an electron (electric charge $-e$, mass m) is orbiting at speed v along an orbit (radius r) with the atomic nucleus as its center.

A equation describing the motion is as follows:

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}. \qquad (1)$$

From this, we obtain:

$$\frac{mv^2}{2} = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r}. \qquad (2)$$

Meanwhile, the potential energy of the electron is:

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r}. \qquad (3)$$

Since the right side of Eq.(2.2) is –1/2 times the potential energy, Eq.(2.2) indicates:

$$2\left(\frac{mv^2}{2}\right) = -V(r) \qquad (4)$$

Therefore, the total electron energy:

$$E = \frac{mv^2}{2} + V(r) \qquad (5a)$$

$$= -\frac{mv^2}{2} \qquad (5b)$$

$$= -K, \qquad (5c)$$

is equal to the value when expressed as kinetic energy (K).

The reason for the difference in potential energy and kinetic energy in Eq.(2.5c) is thought to be the photonic energy $\hbar\omega$ released by the electron. Accordingly, we can establish the following law of energy conservation.

$$V(r) + K + \hbar\omega = 0.$$
$$(\text{However, } -K + \hbar\omega = 0) \qquad (6)$$

3. Electron Energy according to the Theory of Special Relativity

Let us consider a situation in which a single electron is at rest within a macroscopic space and thus holds only rest mass energy. When external energy is added to this electron, it begins moving within the macroscopic space and the kinetic energy (K) and total energy (E) increase.

However, the situation differs when the electron at rest is located within an atom. When the former emits light outside the atom, it transits to a lower energy state. Here, although the electron gains kinetic energy, its total energy decreases.

According to existing theory, the total energy of an electron is considered to be zero when the electron is separated from the atomic nucleus by a distance of infinity and remains at rest in that location. The total energy of Eq.(2.5c) is the value obtained from this perspective.

However, even if we place an electron at rest an infinite distance from its nucleus, the absolute energy of the electron is fundamentally not zero. According to Einstein, an electron in this state should have rest mass energy E_0 [1].

Let us assume that this electron at rest is attracted to the proton; in other words, it is attracted to the atomic nucleus of the hydrogen atom.

The electron tries to enter the region of the hydrogen atom. During this time, when the electron transit to a lower energy state and kinetic energy increases, an amount of energy equaling the increased kinetic energy is released outside the atom.

Considering these conditions and Eq.(2.6), we can establish the following relationship.

$$\left(E_0 + V(r)\right) + K + \hbar\omega = E_0. \tag{1}$$

In order to maintain the law of energy conservation, an energy source is needed to supply the increased kinetic energy and released photon energy.

A potential energy value is normally described in relative terms, but for an electron at rest in free space, the potential energy, in absolute terms, is zero.

The source of energy in Eq.(3.1) at first appears to be potential energy, but it seems unlikely that the physical quantity, which did not exist when the electron was originally at rest, has decreased.

Thus, in our discussion, in dealing with physical quantity, which in classical mechanics is called the potential energy of a hydrogen atom, we offer the hypothesis that this physical quantity corresponds to the reduction of the electron's rest state mass energy.

When considered in this way, it is possible for the potential energy, which did not exist when the electron was at rest, to decrease.

When this decrease in energy is expressed as $-\Delta E_0$, we can establish the following two equations.

$$\begin{aligned} V(r) &= -\Delta E_0 \\ &= -\left(K + \hbar\omega\right). \end{aligned} \tag{2}$$

$$-\Delta E_0 + K + \hbar\omega = 0. \tag{3}$$

The above points may be summarized according to the following table:

i	ii	iii	iv	v	vi	vii
a	Absorbed	$E = E_0 + K$	$K = \hbar\omega$	Established	None	—
b	Emitted	$E = E_0 - K$	$-K = -\hbar\omega$	Not established	$K + \hbar\omega$	Rest mass energy

Table. Comparison of the electron energy inside and outside an atom

i. Electron at rest in an isolated system (rest mass energy E_0)

a. When the photon is absorbed at rest and motion begins in macroscopic space

b. When the photon is emitted from a resting state and absorbed into a hydrogen atom

ii. Photon energy $\hbar\omega$ transfer

iii. Electron total energy E

iv. Electron's gained kinetic energy K

v. Law of energy conservation

vi. Difference from the law of energy conservation

vii. Source of energy to compensate for the difference in vi

4. Discussion

From the definition of Eq.(3.2), the value $V(r)$ must satisfy the following inequality.

$$0 \geq V(r) \geq -m_0 c^2. \tag{1}$$

Therefore, there exists a minimum value of potential energy, whereupon the following relationship is established.

$$-\frac{e^2}{4\pi\varepsilon_0 r_{\mathrm{e}}} = -m_0 c^2. \tag{2}$$

The location r that satisfies this relationship is the distance of closest approach r_{e}, which indicates how close the electron comes to the center of the atom.

From Eq.(4.2), r_{e} is the following value.

$$r_{\mathrm{e}} = \frac{e^2}{4\pi\varepsilon_0 m_0 c^2} \tag{3a}$$

$$= r_{\mathrm{c}}. \tag{3b}$$

Here, r_{c} is the classical electron radius.

Thus, in the prediction based on classical mechanics, it is clear that an electron is not absorbed in an atomic nucleus.

Furthermore, the distance r_{e} agrees with the distance of closest approach of α -particle in Rutherford scattering[2].

Incidentally, according to classical mechanics, the orbit radius of a hydrogen atom is expressed by the following equation.

$$r_n = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{m_0 e^2}, \qquad (n=1, 2, \cdots). \tag{4}$$

Here, the value n=1 is the Bohr radius, which corresponds to the ground state of the hydrogen atom. By plugging r_{c} into Eq.(4.4), we obtain the following equation.

$$r_n = \frac{e^2}{4\pi\varepsilon_0 m_0 c^2}\left(\frac{4\pi\varepsilon_0 \hbar c}{e^2}\right)^2 n^2 \tag{5a}$$

$$= \frac{r_c}{\alpha^2} n^2, \qquad (n=1,2,\cdots). \tag{5b}$$

However, α is a fine structure constant and defined as below.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}. \tag{6}$$

The hydrogen atom orbit radius, which is a concept of classical mechanics, can be simply explained using the classical electron radius, fine structure constant, and principal quantum number.

Next, we shall investigate the energy of a hydrogen atom.

Now, E_0 and E_n are expressed as follows:

$$E_0 = m_0 c^2. \tag{7}$$

$$E_n = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{m_0 e^4}{\hbar^2 n^2}. \tag{8}$$

The following values can be obtained from these two equations:

$$E_n = -\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)^2 \frac{m_0 c^2}{2n^2} \tag{9a}$$

$$= -\frac{\alpha^2 E_0}{2n^2}, \qquad (n=1,2,\cdots). \tag{9b}$$

5. Conclusion

In this discussion, in considering the physical quantity, which in classical mechanics is called the potential energy, we offered the hypothesis that this physical quantity corresponds to the decrease in the electron's rest mass energy.

According to Eq.(3.3), half decrease in rest mass energy was released outside the atom as photonic energy, while the other half was converted into the electron's kinetic energy.

When each of photonic energy and electron kinetic energy reach $m_0c^2/2$, the rest mass energy of the electron has been all converted into other energy. The electron cannot obtain more kinetic energy than this, and it is also unable to decrease its potential energy.

The stability of an atom was first explained according to Bohr's classical quantum theory.

However, in the classical considerations of our discussion, it becomes clear that within the electron inside a hydrogen atom, there exists an off-limits boundary r_e. Furthermore, this distance of closest approach corresponds to the classical electron radius.

This value is different from the value predicted by classical quantum theory, or the Bohr radius, but the atom stability can be explained even by a classical mechanics approach.

However, this does not necessarily mean that our discussion has casts doubt onto quantum mechanics.

Our discussion shows only that it is possible to explain the stability of an atom according to theories other than quantum mechanics.

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