

## **New Transformation Equations which Replaces Lorentz-Einstein Transformations to be Applied inside an Atom**

**Koshun Suto**

[Koshun\\_suto129@mbr.nifty.com](mailto:Koshun_suto129@mbr.nifty.com)

### **Abstract**

We have derived that, between the total energy ( $E_{ab}$ ) and momentum of an electron inside a hydrogen atom, the following relationship is obtained:  $E_{ab}^2 + c^2 p_n^2 = E_0^2$ , (Here,  $E_{ab} = E_0 + E_n$ ,  $n = 1, 2, \dots$ ).

From this relationship it is easily derived that the mass of an electron inside a hydrogen atom decreases as the velocity increases.

Consequently, we reach the recognition that, in the space inside an atom, we need new transformation equations which replaces Lorentz-Einstein transformations.

We can also predict that a particle moving inside an atom or passing through an atom will expand in the moving direction and that the time which passes in the coordinate system of the particle will pass earlier.

Furthermore, we will find that, inside an atom, light velocity doesn't function as the upper velocity in the nature. From this, we can predict the existence of a tachyon moving at superluminal velocity.

**Key words:** Special Relativity, Lorentz-Einstein transformations, Tachyon, Superluminal velocity.

**PACS codes:** 03.30.+p.

### **1. Introduction**

We have derived that, inside a hydrogen atom, Einstein's energy-momentum relationship can't be applied [1].

Instead the following relationship can be applied:

$$E_{ab}^2 + c^2 p_n^2 = E_0^2, \quad (\text{Here, } E_{ab} = E_0 + E_n, n = 1, 2, \dots). \quad (1)$$

Here,  $E_{ab}$  means the total energy of an electron expressed by absolute standard,  $p$  is the momentum of an electron,  $E_0$  means rest mass energy, and  $n$  is a principal quantum number.

By the way, in Special Relativity, we can obtain:

$$m = \frac{E}{c^2}. \quad (2)$$

In classical mechanics, we can obtain:

$$m = \frac{p}{v}. \quad (3)$$

From the two equations above, we can obtain:

$$cp = \frac{Ev}{c}. \quad (4)$$

Suppose Eq. (1.4), which holds true in the macroscopic space, also holds true when we replace  $E$  with  $E_{ab}$ .

When we substitute  $cp$  in Eq. (1.4) into Eq. (1.1), we obtain:

$$E_{ab} = \frac{E_0}{\sqrt{1 + (v/c)^2}}. \quad (5)$$

Considering Eq. (1.2), we obtain:

$$m = \frac{m_0}{\sqrt{1 + (v/c)^2}}. \quad (6)$$

It is clear that the mass of an electron moving inside a hydrogen atom decreases when its velocity increases.

Therefore, we can't apply Lorentz-Einstein transformations to the two coordinate systems moving at uniform speed to each other in the space inside an atom. We need to introduce new transformation equations.

## **2. Introduction of new transformation equations that replaces Lorentz –Einstein transformations**

In quantum mechanics we can't draw a picture as to the movement of an electron inside an atom. So we are going to derive the new transformation equations by using the picture of classical quantum theory.

Let us refer to the famous thought experiment of “twin paradox”. It is known that

the accelerated movement of the rocket, which one of the twins traveling in the space experiences, can be described by using Special Relativity, if we consider the movement of limiting time for  $t \rightarrow 0$ .

For the same reason, let us suppose the new transformation equations we are going to obtain can be applied to an electron inside a hydrogen atom even if an electron changes its velocity moment by moment.

Let us now consider the coordinate system ( $S'$ ) of an electron moving along the elliptic orbit inside a hydrogen atom and the coordinate system ( $S$ ) of an atomic nucleus.

In quantum mechanics a complex number plays an essential role.

Therefore, let us suppose a complex number also plays an important role in deriving the new transformation equations.

In addition, we are going to derive the unknown transformation equations referring to the process of obtaining Lorentz-Einstein transformations [2].

First of all, the new transformation equations we are going to obtain have to be linear.

Furthermore, the symmetry implied by the relativity principle means that the form of the relationships must be as follows:

$$x = ax' + ibt' \quad (1)$$

$$x' = ax - ibt \quad (2)$$

The motion of the origin of  $S$  as measured in  $S'$  is defined by putting  $x=0$  in Eq. (2.1).

Similarly, the motion of the origin of  $S'$  as measured in  $S$  is defined by putting  $x'=0$  in Eq. (2.2).

The velocities are equal and opposite and both of magnitude  $v$ .

This gives us the condition:

$$\frac{ib}{a} = v \quad (3)$$

Next we consider the descriptions according to  $S$  and  $S'$  of a light signal traveling in the positive  $x$  direction.

When the origins of the  $x$ -axis of the two coordinate systems agree, a light signal is emitted from the origin  $O$ . It is then described by the following very simple equations in  $S$  coordinates and  $S'$ , respectively:

$$x = ict \quad (4)$$

$$x' = ict' \quad (5)$$

Substitute these particular expression for  $x$  and  $x'$  in Eqs. (2.1) and (2.2), and we get the following:

$$ct = (ac + b)t'. \quad (6)$$

$$ct' = (ac - b)t. \quad (7)$$

Eliminating  $t$  and  $t'$  of these two equations and using the condition  $b = -iav$  from Eq. (2.3), we find:

$$c^2 = a^2(c^2 + v^2). \quad (8)$$

Therefore,

$$a = \frac{1}{\sqrt{1 + (v/c)^2}}. \quad (9)$$

Now rewriting the coefficient  $a$  as  $\gamma_a$ , Eqs. (2.1) and (2.2) will be:

$$\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1 + (v/c)^2}} \\ &= \gamma_a (x' + vt'). \end{aligned} \quad (10)$$

and

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 + (v/c)^2}} \\ &= \gamma_a (x - vt). \end{aligned} \quad (11)$$

where

$$\gamma_a(v) = \frac{1}{\sqrt{1 + (v/c)^2}}. \quad (12)$$

### 3. Discussion

The coefficient  $\gamma(v)$  in Lorentz-Einstein transformations was given in the following relation:

$$\gamma(v) = \frac{1}{\sqrt{1 - (v/c)^2}}. \quad (1)$$

The coefficient of Eq. (2.12) obtained in this paper is  $\gamma_a(v) < 1$ , whereas the coefficient in Lorentz-Einstein transformations is  $\gamma(v) > 1$ .

If we can obtain Eqs. (2.10) and (2.11), it is a matter of elementary algebra to obtain the following expressions for  $t$  and  $t'$ :

$$t = \gamma_a \left( t' + \frac{vx'}{c^2} \right). \quad (2)$$

and

$$t' = \gamma_a \left( t - \frac{vx}{c^2} \right). \quad (3)$$

Here, we give the complete set of transformations below, expressed both ways, i.e.  $S'$  coordinates in terms of  $S$  and vice versa:

<p>THE SUTO TRANSFORMATIONS</p> $\begin{aligned} x' &= \gamma_a (x - vt) & , & & x &= \gamma_a (x' + vt') \\ y' &= y & , & & y &= y' \\ z' &= z & , & & z &= z' \\ t' &= \gamma_a \left( t - \frac{vx}{c^2} \right) & , & & t &= \gamma_a \left( t' + \frac{vx'}{c^2} \right) \end{aligned} \quad (4)$ <p style="text-align: center;">with <math>\gamma_a = \frac{1}{\sqrt{1+(v/c)^2}}</math>, where <math>v</math> is the velocity of <math>S'</math> as measured in <math>S</math>.</p>	
---	--

The only difference between the new transformation equations (3.4) and Lorentz-Einstein transformations is coefficients, i.e.  $\gamma_a$  and  $\gamma$ .

Therefore, the new transformation equations are, as Lorenz-Einstein transformations, covariant concerning transformation of coordinate systems.

Then, what about the length of an object moving inside an atom?

Let us consider the situation where a particle with the length  $l_0$  at rest moves at velocity  $v$  inside an atom. Suppose that an observer in  $S$  measures the length of the particle in the moving direction and gets the value  $l$ .

According to the new transformation equations, we can get the relationship between  $l_0$  and  $l$  as follows:

$$l = \frac{l_0}{\gamma_a} = l_0 \sqrt{1+(v/c)^2}. \quad (5)$$

From this equation, we can find that a particle moving inside an atom at velocity  $v$  expands in the moving direction.

Furthermore, let us suppose that, when time  $t_0$  passes by the clock in  $S$ , the time  $t$  passes by the clock in  $S'$ . The relationship between  $t_0$  and  $t$  becomes

$$t = \frac{t_0}{\gamma_a} = t_0 \sqrt{1 + (v/c)^2}. \quad (6)$$

From this equation, we can find that, in the coordinate system of a moving particle, the time passes earlier.

#### 4. Conclusion

1. So far, we have found that, as its velocity increases, the total energy and mass of an electron moving inside a hydrogen atom decrease. This gives a different result from the one predicted in Special Relativity.

However, this paper can't express the clear standpoint as to whether an electron really expands and whether the time in an electron's coordinate system really passes earlier.

That is because the physical meaning of the expansion of the electron is unclear, since the electron is considered a particle without extent.

As to the passage of time, too, the meaning of the fact that time passes earlier is unclear, since the electron is considered a particle without life.

However, when a particle moving in from outside an atom passes through an atom, it is possible that Eqs. (3.5) and (3.6) can be applied.

Now, when we obtain the value of  $ml$  and  $mt$  from Eqs. (1.6), (3.5) and (3.6), we have:

$$ml = m_0 l_0 = \text{const.} \quad (1)$$

$$mt = m_0 t_0 = \text{const.} \quad (2)$$

From the equations above, it follows that, whether a particle is moving in or outside an atom, when its mass or total energy becomes  $n$  times larger its length in the moving direction and the time passing in its coordinate system become  $1/n$ .

If Eqs. (4.1) and (4.2) can be obtained in every case, we can't exclude the possibility that an electron may be a particle with extent and life.

Special Relativity insists that the contraction of the length of an object and the time dilation passing in the coordinate system of the object depend on the velocity of the object. However, this paper concludes it is more essential to think that the length of an object and the time passing in the coordinate system depend on the mass or total energy of the object.

2. When the velocity of an electron inside an atom increases, its mass decreases.

Therefore inside an atom, light velocity doesn't function as the upper velocity in the

nature.

Here, let us obtain the velocity of an electron when its total energy  $E_{ab}$  is the minimum value  $m_0c^2/2$  [3].

Substitute  $m_0c^2/2$  in  $E_{ab}$  in Eq. (1.1):

$$\left(\frac{m_0c^2}{2}\right)^2 + c^2p^2 = (m_0c^2)^2. \quad (3)$$

Also, from Eq. (1.3):

$$p^2 = m^2v^2. \quad (4)$$

Substitute  $p^2$  of Eq. (4.4) and  $m_0$  of Eq. (1.6) into Eq. (4.3). Adopting the plus value, we have:

$$v = \sqrt{3}c. \quad (5)$$

We can predict that, when an electron gets closer to an atomic nucleus, the velocity of an electron surpasses light velocity. Then an electron behaves as a tachyon, a particle moving at superluminal velocity. A tachyon is not an unknown particle but an existing particle behaves as one under a certain condition.

When describing the state of an electron inside an atom by means of wave function, the probability of detecting an electron is not zero even in the area where an electron behaves as a tachyon. Therefore, we can predict the existence of a tachyon.

## Acknowledgments

I am very grateful to Takahiro Yasui for putting this paper into English.

The text of reference [2] was borrowed in chapter 2. I want to express gratitude to the authors of this book.

## References

- [1] K.Suto, Limitation of Applicability of Einstein's Energy-Momentum Relationship  
[http://koshun.cool.ne.jp/physics/limit\\_2005\\_6/e\\_1.pdf](http://koshun.cool.ne.jp/physics/limit_2005_6/e_1.pdf)
- [2] A.P.French, Special Relativity:THE M.I.T INTRODUCTORY PHYSICS SERIES,  
W.W.NORTON&COMPANY, New York. London, 80 (1968).
- [3] K.Suto, Deriving an Atom's Stability from Classical Mechanics and from the  
Theory of Special Relativity,  
[http://koshun.cool.ne.jp/physics/deriving/deriving\\_e01.pdf](http://koshun.cool.ne.jp/physics/deriving/deriving_e01.pdf)