A New Train Paradox:
Can Clock Time Depend on the Direction of Motion?

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Velocity is defined as distance travelled per time, hence $c = x/t$. The dilation of time in special relativity is given by the factor: $t' = t\ /\ \sqrt{(1 - v^2/c^2)}$. Moreover,due to the Lorentz-contraction $x' = x\ /\ \sqrt{(1 - v^2/c^2)}$ as well. Calculating space and time for $c' = x'/t'$, either stretching factor in the numerator and the denominator cancel out. This leads to a paradox: If the velocity of light is constant in any system, i.e. if $c = c'$, clocks will have to alter their pace depending on the direction of motion.

Mathematical verification performed in this paper shows that special relativity has created many pitfalls that violate its own fundamental hypotheses and conclusions. Let us now consider a train passing by a shining lantern at rest (as shown in the figure). Let an observer in the train wish to measure the velocity of light before and after the train had passed by the source of light and let him have a look at the clock time in either case. The clock time, obviously, could not change by the sole fact that the train was passing by.

When the train is at rest the observer in the train measures the velocity of light = distance / travel time. He/she uses as distance the length of the coach and synchronizes two clocks at both ends of the coach with light signals. The length of the coach is then $x = ct$. If the train starts to move and travels with constant velocity along the lanterns on the embankment, the length of the coach and the synchronization of his clocks do not change for the observer in the train.

Following Special Relativity the observer in the train will obtain (according to the Lorentz transformation).

Before passing the light

$$x' = (x + vt)\ /\ \sqrt{(1 - v^2/c^2)} \tag{1a}$$

$$t' = (t + vx\ /c^2)\ /\ \sqrt{(1 - v^2/c^2)} \tag{1b}$$

With $t = x/c$ and $x = ct$ one gets

$$x' = x\ (1 + v/c)\ /\ \sqrt{(1 - v^2/c^2)} \tag{2a}$$

$$t' = t\ (1 + v/c)\ /\ \sqrt{(1 - v^2/c^2)} \tag{2b}$$

Yielding of course $x'/t' = x/t = c$.

After passing the light

$$x' = (x - vt)\ /\ \sqrt{(1 - v^2/c^2)} \tag{3a}$$

$$t' = (t - vx\ /c^2)\ /\ \sqrt{(1 - v^2/c^2)} \tag{3b}$$

With $t = x/c$ and $x = c.t$ one gets

$$x' = x\ (1 - v/c)\ /\ \sqrt{(1 - v^2/c^2)} \tag{4a}$$

$$t' = t\ (1 - v/c)\ /\ \sqrt{(1 - v^2/c^2)} \tag{4b}$$

Yielding of course $x'/t' = x/t = c$.

This result is obtained under the assumption that t must be transformed by the factor $1 + v/c$ or $1 - v/c$, respectively, depending on whether the measurement was done before or after the train passed by the light. Therefore, the pace of time would be larger before and smaller after the light was passed by. This would have to mean that *the same clock* (ticking with the frequency $1/t$) was *ticking slower when the light was in front and ticking faster when the light was behind*.

The inconsistency resulting in SRT becomes even more obvious, when the train happens to travel between two lanterns.

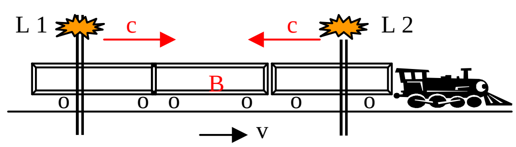

Fig. 1. Einstein's Train

If the observer B in the train measures the light from the rear source L1, she/he must transform time with - vx/c^2 , here then with *(1 - v/c),* according to the Lorentz transformation. If he/she measures at the same time the light velocity using the front lantern L2, he/she must transform time with + vx/c^2 or $(1 + v/c)$ in the present case. Even in SRT the synchronized clocks in the train on the same spot *cannot run faster and slower simultaneously, depending on the direction of measurement*.

Similar conclusions are reached with respect to the phase velocity = frequency x wavelength.

The Doppler Effect in the special relativity would read before passing the light as:

$$f' = f\,(1 + v/c)\,/\,\sqrt{(1 - v^2/c^2)} \tag{5}$$

and after passing the light as:

$$f' = f\,(1 - v/c)\,/\,\sqrt{(1 - v^2/c^2)} \tag{6}$$

Multiplying f' with the wave length (as Lorentz shrunk) one may get c again only, if the clock time is stretched and clocks are ticking, thus, slower before passing the light. In this way, the increase in frequency caused by Doppler's effect will be outweighed, giving rise to $f\lambda = c$.

Conversely, when the light is past, clock time must be shorter and, hence, the clocks of the observer will have to tick faster in order to balance the Doppler decrease of the frequencies, giving rise to $f\lambda = c$ again.

(Alternatively, in order to get c despite Doppler's effect, the wave length would have to be shorter or longer, depending on the direction of motion – this being impossible as well).