

# Relative Velocity: a Dichotomy

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 April 24, 2010

Natural Philosophy Alliance Conference  
 The State University of California at Los Angeles, Long Beach

The central concept of the (special) relativity theory is a concept of a **relative** velocity. Relativity of the velocity means that the velocity is not absolute concept. The velocity of a massive body is not an intrinsic property of this body; it depends on the free choice of the reference system, it is reference-dependent. The definition of the relative velocity does not depend on existence or absence of the privileged reference system æther. Also the extra assumption of constant relative velocity is not so important, inertial-system is also of secondary relevancy. There is no big difference with this respect among the absolute Galilean simultaneity and the relative simultaneity. We stress that the zero velocity must also be relative. Every reference system possess the own zero-velocity relative to exactly this system. Many zeroth-velocities contradict to group structure, and therefore the relativity theory in terms of relative velocities must be formulated within the groupoid structure, which is *not* a group. We discuss here the Galilean groupoid. There is a dichotomy: two different concepts of a massive body, a tetrad versus a monad.

**Keywords:** reference frame, observer, tetrad versus monad, relativity groupoid as a category

## 1 Massive body

Every reference system is a massive body. We avoid the word ‘frame’ because we consider a frame as a synonym of a basis in a vector space, and, as we explain below, see §1.2, a reference **system** need not to be obligatorily the same as a reference frame. The (special) relativity theory is about the concept of a relative velocity. The velocity of a massive body is relative to another massive body. Therefore a velocity of a body is always relative to the choice of the reference system. The (special) relativity theory in fact it is the theory of the reference systems, and the name ‘special relativity’ must read ‘velocity is relative’ or ‘velocity depends on reference systems’. Light is massless and can not be considered to be a reference systems. Therefore the light velocity must *not* be considered as the primary concept of the relativity theory. If we do not define or do not precise what means a massive body, then

the concept of velocity is meaningless. Every velocity must be a function of a two-body system, therefore the very concept of a massive body is crucial for understanding the concept of velocity. Most definitions use coordinate system to define velocity. These coordinate systems contain implicit information about massive bodies.

### 1.1 Zero velocity is also relative: groupoid versus group

Galileo observed explicitly in 1632 that every velocity is relative and our everyday experience tells the same. One can not discover own movement without looking outside for another reference system. Contrary to this experience, the group theory, the Galilean group, and as well as the Lorentz relativity group, introduced the concept of an abstract velocity  $v$ , as a group-parameter, Group  $\xrightarrow{v} \mathbb{R}$ , that one can deal with,

without the adjective ‘relative to’. All groups elements can be composed and this imply that all these abstract velocities-parameters can be added. But can we add every **relative** velocity with every other *relative* velocity? To be **relative** means that one must keep the information of what concrete target-body velocity it is relative to what source-body. A target-body is an observed body possessing the velocity, and a source-body is an observer laboratory, measuring this relative velocity. If a velocity of Sun relative to Earth,  $v(\text{Earth}, \text{Sun})$ , and the velocity of Mars relative to Jupiter, is declared to be not composable, with meaningless addition, then a set of all relative velocities is a groupoid, a relativity groupoid. The composition of **relative** velocities is not always possible.

Each zero velocity we wish to be also a relative velocity, and not an abstract unique zero group-parameter. Therefore there must be as many relative zero-velocities as there are many reference systems. Zero-velocity of the Sun relative to the Sun, we do not wish to identify with the zero-velocity of the bus relative to the bus. We propose do not accept the ‘coincidence’ of the zero-velocities of different observers.

## 1.2 What it is a mathematical model of a massive body?

It seems not yet widely recognized among relativity-group-researches that there is no unique concept of what a massive body vel a reference system means. Here is the crucial dichotomy. Albert Einstein introduced one concept of a reference system in 1905, identifying a reference system with the coordinate system of scalar fields on space-time, ‘attached’ to a massive body. The Einstein concept give rise a posterior to the name **tetrad** as the name of the basis in four-dimensional vector space, see (3)-(4) below. Inadequacy of the Einsteinian concept of a massive body was criticized by Léon Brillouin in monography ‘Relativity Reexamined’ published by Brillouin’s wife in 1970, after Brillouin’s illness. Brillouin asked ‘where is there a mass in the coordi-

nate system?’.

The alternative definition of a massive deformable body comes from the fluid mechanics and of fluid dynamics, and is known as the Euler derivative alias the substantial or material, or barycentric, or hydrodynamic derivative, in terms of the partial derivatives, popularized within NPA community by Greg Volk video-lecture on December 12, 2009. In fluid context the material body is not the Einsteinian set of scalar coordinate fields, but it is a **vector** field. Within the relativity of simultaneity it was Minkowski who defined in 1908 a reference system as the time-like vector field on spacetime, the reference fluid known as a **monad**. These two concepts of a massive body, an Einsteinian tetrad, and a fluid monad, are definitely different. This imply that the concept of the relative velocity among two bodies is also different, depending what concept of a reference system (= a massive body) is chosen as the first axiom of the relativity theory. The different concepts of a massive body leads to different mathematical concepts of the relative velocity, and give rise to distinct relativity theories. We are going to explain these differences and discuss some experimental consequences. Let’s illustrate the above dichotomy of massive body on example of the Galilean absolute simultaneity, with the *relative* spaces, to be in the same place is relative. A place is observer-dependent.

## 2 Massive body as a coordinate system

In this section our aim is to define the relative velocity in terms of a massive body identified with the coordinate system on two-dimensional space-time of events. The Galilean relativity postulate the absolute simultaneity given by the absolute function  $t$  that one can keep forever without the change, there is no need for another  $t' = t$ . The Galilean coordinate reference system, a massive body  $A$ , is identified with the pair of coordinate scalar functions on spacetime of events,  $A \simeq \{t, x_A\}$ , and another massive body

is  $B \simeq \{t, x_B\}$ , etc., where

$$\{\text{SpaceTime of events}\} \xrightarrow{\tau} \text{Time} \xrightarrow{\text{reloj}} \mathbb{R}, \quad (1)$$

$$\{\text{SpaceTime of events}\} \xrightarrow{\pi_A} \text{Space}_{e_A} \xrightarrow{f} \mathbb{R}, \quad (2)$$

$$t \equiv \text{reloj} \circ \tau, \quad x_A \equiv f \circ \pi_A. \quad (3)$$

Above coordinate massive body, an (Einsteinian) reference system, is called a **tetrad** given by

$$\{t, x\} \implies \{dt, dx\} \iff \left\{ \frac{\partial}{\partial t}, \frac{\partial}{\partial x} \right\} \quad (4)$$

We must see how two reference systems, say a bus  $B$  and a street  $A$ , are distinguished within coordinate bodies. Each reference system is completely defined in terms of a scalar function,  $x_A$ -system is a street, and  $x_B$ -system is a bus. Here  $x_A$  and  $x_B$  are functions on space-time of events, and two events,  $e_1$  and  $e_2$  are on the same **place** for an  $x$ -observer if and only if,  $x(e_1) = x(e_2) \in \mathbb{R}$ .

Let illustrate this in terms of the following list of three events:

- $e_1 =$  bus start from the bus stop A
- $e_2 =$  bus almost arrive to the next bus stop B
- $e_3 =$  late passenger arrived to the bus stop A

From the point of view of a driver of the bus, this is  $x_B$ -system, driver is on the same  $B$ -place inside of the bus, bus is in the  $x_B$ -‘rest’:

$$x_B(e_1) = x_B(e_2), \quad \text{but} \quad x_B(e_3) \neq x_B(e_1). \quad (5)$$

From the point of view of the crowd standing on the street, street is  $x_A$ -system:

$$x_A(e_1) = x_A(e_3), \quad \text{but} \quad x_A(e_2) \neq x_A(e_1). \quad (6)$$

Before to define the velocity of the bus relative to the street, we must suppose that we know perfectly a priori what are our (coordinate) massive bodies,  $x_A$  and  $x_B$ . We need the concept of **relative** velocity among a pair of massive bodies. No concept of relative velocity without the concept of a massive body!

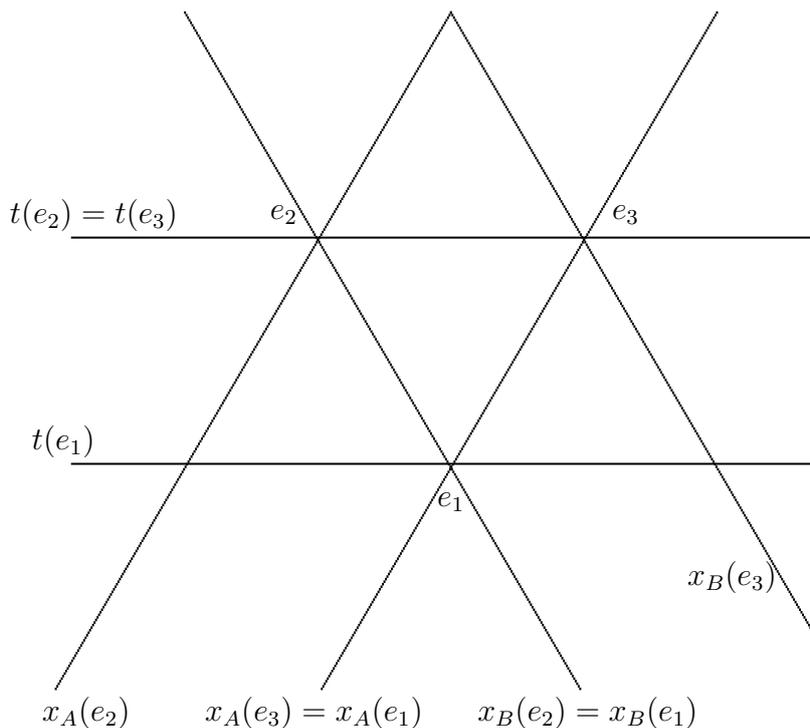


Figure 1: Two-body coordinate systems within absolute Galilean simultaneity,  $t(e_2) > t(e_1) \in \mathbb{R}$ . To be in the same place is relative,  $B$ -places are different from  $A$ -places. Entire spacetime line  $(x_B)^{-1}(x_B e_3)$  denotes a **one** single  $B$ -place.

Instead, the common dogmatic practice is to define the velocity in terms of the boost transformation, considering that the ‘well known’ concept of an abstract velocity is nothing but a ‘group-parameter  $v$ ’ in a adored group,  $x \mapsto x + vt$ . This virus-dogma insists that the concept of a velocity can be defined somewhere independently from the very concept of the reference system. All group elements can be composed and all such abstract group parameters can be added. The Galilean boost,  $\{\text{Group;Body}\} = \{v; t, x_A\} \mapsto x_A + vt$ , presuppose that the abstract concept of the velocity-parameter  $v$  is defined a priori. The name boost introduced Eugene Wigner in 1939, and this name suggest the dynamical process of acceleration, but there is no dynamical Newton second law within the concept of the boost.

The velocity of the bus relative to bus is zero,

and street velocity relative to street is also zero,

$$\begin{aligned} v_{BB}(e_2, e_1) &= \frac{\text{distance}}{\text{time}} \\ &= \frac{x_B(e_2) - x_B(e_1)}{t(e_2) - t(e_1)} = 0, \\ v_{AA}(e_3, e_1) &= \frac{x_A(e_3) - x_A(e_1)}{t(e_3) - t(e_1)} = 0, \end{aligned} \quad (7)$$

$$\begin{aligned} v_{BA}(e_3, e_2, e_1) &= \frac{x_A(e_2) - x_A(e_1)}{t(e_3) - t(e_1)}, \\ v_{AB}(e_2, e_3, e_1) &= \frac{x_B(e_3) - x_B(e_1)}{t(e_2) - t(e_1)}, \\ v_{BA}(e_3, e_2, e_1) + v_{AB}(e_2, e_3, e_1) &= 0 \in \mathbb{R}. \end{aligned} \quad (8)$$

As we see in (7)-(8), each relative velocity use only one coordinate, one observer only,  $x_A$  or  $x_B$ , but all three absolute events. However if we suppose additionally the following initial conditions,  $t(e_1) = 0$ , and *doubtful*  $x_A(e_1) = x_B(e_1)$ , then one can re-express relative velocity in terms of only one event but two coordinate-observers. This leads to velocity as a **scalar function on space-time** (and not on relative-space of individual places),

$$\begin{aligned} v_{BA}(e_3, e_2, e_1) &= \frac{x_A(e_2) - x_B(e_2)}{t(e_2)} \\ &= \frac{x_A - x_B}{t} e_2 = v_{BA}(e_2), \end{aligned} \quad (9)$$

$$\begin{aligned} v_{AB}(e_2, e_3, e_1) &= \frac{x_B(e_3) - x_B(e_3)}{t(e_3)} \\ &= \frac{x_B - x_A}{t} e_3 = v_{AB}(e_3). \end{aligned} \quad (10)$$

What is the physical meaning of the Galilean group parameter? If  $\{t, x_A\}$  describe a massive body, therefore  $x_A + vt$  must describe another massive body  $x_B$  that moves at the given speed  $v$  relative to an observer  $x_A$ . The link-problem allows to clarify the physical meaning of this innocent group-parameter  $v$  as the relative velocity. The group-boost gives rise to the **definition** of relative velocity among two reference systems, as

the solution of the link-problem,

$$\begin{aligned} \text{given } x_A \text{ and } x_B, \quad x_B &= x_A + vt \\ \implies v(x_A, x_B) &= \frac{x_B - x_A}{t}. \end{aligned} \quad (11)$$

**2.1 Bad intuition** (Galilean ‘relative velocity’). The Galilean relative velocity of a coordinate body  $x_B$  as observed (measured, as seen) by a coordinate body  $x_A$ , can be defined as follows

$$v_{BA} \equiv \frac{x_A - x_B}{t}, \quad v_{AB} + v_{BC} = v_{AC}. \quad (12)$$

In (12) a **scalar** field  $v_{BA}$  represent a ‘relative velocity’ of  $B$  as seen by  $A$ . This is á la Einstein, that the relative velocity is defined as a scalar parameter relating the coordinate bodies.

**2.1 Comment.** We wish to have a concept of a massive body as  $A \simeq v_{AA}$ , that will automatically distinguish the zero-velocities,  $v_{AA} \neq v_{BB}$  for  $v_{AB} \neq 0$ . However the bad-intuition(12) imply  $v_{AA} = v_{BB} = \dots = 0$ . What is wrong within (9)-(12) that the relative velocity is considered to be a scalar function on **spacetime**, whereas we must try to see these relative velocities on individual relative factor-spaces. Every observer  $A$  give rise to the surjective projection  $\pi_A$  from the four-dimensional space-time onto three-dimensional relative  $A$ -space,  $A \circ \pi_A^* \equiv 0$ . Then the following substitutions, must be used in all above expressions,

$$\begin{aligned} x_A &= f \circ \pi_A, & v_{AB} &\rightsquigarrow v_{AB} \circ \pi_A, \\ x_B &= g \circ \pi_B, & v_{BA} &\rightsquigarrow v_{BA} \circ \pi_B. \end{aligned} \quad (13)$$

In this case the interpretation of the Galilean group parameter  $v$  within the relative factor-space needs to be re-thought, *i.e.* how to get rid off the different projections,  $\pi_A \neq \pi_B$ ?

The symbol of relative-velocity addition binary operation ‘+’ in (12) is misleading because ‘+’ suggest incorrectly that this is a commutative addition. This is not the case. The composition of relative velocities needs *ordered* pair of relative velocities! If  $v_{BA}$  is a velocity of a bus  $B$  relative to a street  $A$ , and  $v_{CB}$  is a velocity

of a car  $C$  relative to this bus, one can compose,  $v_{CB} \circ v_{BA} = v_{CA}$ , in analogy to the non-commutative composition of maps,  $g \circ f \neq f \circ g$ . In this order-convention  $v_{BA}$  is composed with  $v_{CB}$ , we read composition from the right to the left as in Arabic language. Therefore a symbol of a composition ‘ $\circ$ ’ must be adequate because order of summands is most crucial.

Can we consider a set of all Galilean *relative* velocities to be a group? To be **relative** means that every velocity must keep the information of what concrete targed-body velocity it is relative to what source-body, therefore there must be as many zero-velocities as many reference systems  $A \leftrightarrow 0_A$ ,  $v_{AA} \equiv 0_A \neq 0_B \equiv v_{BB}$ . Each individual  $A$ -factor-space is expected to possess own zero velocity, exactly  $0_A$ .

Consider four reference systems (in two-dimensional space-time) given by spacetime coordinates:  $x_A, x_B, x_Y, x_Z$ , appropriately related to the notation on Figure 2. The false relative velocities tangent to iso-time instantaneous three-dimensional subspace, not yet projected onto relative factor-spaces, can all be composed. The composition of the true **relative** velocities, tangent to factor-spaces, is not always possible. We are not allowed to compose the projected relative velocity of  $A$  to  $B$ , in  $B$ -space, with the projected velocity-morphism of  $Y$  to  $Z$  in  $Z$ -space (where  $A \neq Z$  as shown on Figure 2). Therefore the set of all projected relative velocities among all reference systems is not a group, it is a groupoid:

$$\text{Galilean groupoid} = \{v_{AB} | \text{for all coordinate reference systems } \{t, x_A\}\}. \quad (14)$$

We believe that the relativity theory can be formulated as a groupoid of relative velocities-morphisms, and not as a group parameterized in terms of abstract velocities.

### 3 A massive body as a reference fluid á la Euler

The mainstream of Physics prefer the Einsteinian-like definition of the relative veloc-

ity (12) in terms of the scalar spacetime measurements of events  $x_A, x_B, t$ . Leonard Euler in 1754, and Minkowski in 1908 had another idea how to represent a deformable massive body, that leads to another definition of the relative velocity among reference systems. The Euler definition is not equivalent to the Einsteinian coordinate speed. Instead, the Euler reference system is called a monad and is given solely by *one alone* vector field.

**3.1 Definition** (The Euler definition of a reference fluid). According to Euler the massive body is defined as the deformable reference fluid given in terms of a vector field  $A$  on space-time,

$$At \equiv 1, \text{ and } Ax_A \equiv 0 \iff A = \left(\frac{\partial}{\partial t}\right)_{x_A} \quad (15)$$

$$\mathbf{v}_{AB} \equiv \left(\frac{\partial}{\partial t}\right)_{x_B} - \left(\frac{\partial}{\partial t}\right)_{x_A} \implies \mathbf{v}_{AB}x_A = \left(\frac{\partial x_A}{\partial t}\right)_{x_B} \quad (16)$$

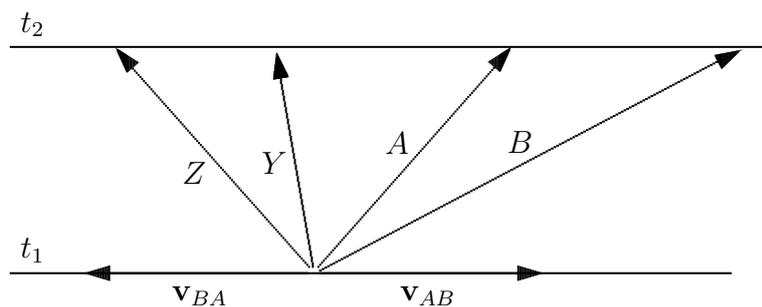


Figure 2: Four body system,  $A, B, Y, Z$ , as the future-pointed Eulerian vector fields,  $t_2 > t_1$ , within absolute Galilean simultaneity. To be in the same place is relative,  $Z$ -places are different from  $A$ -places, etc. Eulerian observer is a connection. The false relative velocities are tangent to iso-time instantaneous three-dimensional subspace, not yet projected onto relative factor-spaces.

During our absolute travel in a time, time can not be stopped, every body ‘at rest’ in fact is moving in our future. A coordinate  $x_A$  is an integral of motion of a street, whereas  $x_B$  is an integral of motion of a bus. The derivative of  $x_A$  along the street-vector-field  $A$  is zero,  $Ax_A \equiv 0$  and  $At = 1$ .

A bus  $B$ , as a vector field in a space-time, is defined by another two conditions,  $Bx_B \equiv 0$  and  $Bt \equiv 1$ . There is the conceptual difference among the coordinate speed given in terms of the scalar fields (12), and the Eulerian relative velocity (16) given in terms of the **vector** field. Sometimes some bad textbooks define a vector as a set of scalar components. However a set of scalars is a set of scalars, and a vector does not possess scalar components if the basis was not chosen. We prefer the Euler definition of the relative velocity (on space-time) in terms of the vector fields (16), and this is not equivalent to the coordinate tetrad (12).

Tamás Matolcsi published in 1994 a monograph *Spacetime Without Reference Frames*, where ‘without reference frame’ must read ‘without Einsteinian coordinate system’, but **with** the concept of a reference system as a monad. The first part of Matolcsi’s monograph, 30% of entire book, is devoted to Galilean absolute simultaneity called non-relativistic model = the absolute simultaneity. The central subject of the Matolcsi monograph is the concept of relative velocity and the above Euler & Minkowski definition (16) is exactly the same as the Matolcsi definition in Part I, §6.2.2. Tamás Matolcsi in Part I, §7.1.3 is stressing

It is an important fact that the spaces of different ... observers are *different ... spaces ...*

The relative velocity must be seen not on space-time as in Matolcsi §6.2.2 (3.1), but as projected to the relative factor-spaces.

**3.2 Definition (Main).** Let  $\pi_A$  denotes a projection from four-dimensional spacetime onto three-dimensional  $A$ -quotient-space, and  $s_A$  be some section for  $\pi_A$ , *i.e.*  $\pi_A \circ s_A \equiv \text{id}_A$ . Then  $\pi_A^*$  denotes the algebra submersion (pull-back) of functions on  $A$ -factor-space, into functions on spacetime,  $A \circ \pi_A^* \equiv 0$ . The relative velocity vector tangent to  $A$ -factor-space,  $A$  is observing  $B$ , is defined as follows,

$$\gamma \mathbf{v}_{AB} \equiv S_A^* \circ B \circ \pi_A^* \in \text{der } \mathcal{F}_A. \quad (17)$$

In particular, if  $x$  is a coordinate function on  $A$ -factor-space, then

$$\gamma \mathbf{v}_{AB} x = \{B(x \circ \pi_A)\} \circ s_A, \quad \gamma \equiv B t_A. \quad (18)$$

These expressions, (17)-(18), show that the relative velocity depends on the free choice of the  $\pi$ -section  $S$ , *i.e.*  $\mathbf{v}_{AB}(s_A)$  is time dependent.

**3.3 Definition (Boost).** Given an observer  $A$ ,  $A t_A \equiv 1$ , together with a velocity  $\mathbf{v}$  in  $A$ -factor-space, then the body  $B$  possessing this velocity, a boost,  $A \xrightarrow{\mathbf{v}} B(\mathbf{v})$ , is given by the following expression

$$A \xrightarrow{\mathbf{v}} B(\mathbf{v}) \equiv (B t_A)(A + \pi_A^* \circ \mathbf{v} \circ S_A^*). \quad (19)$$

**3.4 Definition (Composition of velocities).** The following expression for addition/composition of relative velocities on quotient-space is deduced from the associative addition of relative velocities on space-time derived in [Oziewicz 2005],

$$\begin{aligned} \mathbf{v}_{SC} &= \mathbf{v}_{BC} \circ \mathbf{v}_{SB} \\ &= \frac{(s_S^* \gamma_{\mathbf{u}}) \mathbf{v}_{SB} + (s_S^* \circ \pi_B^*) \circ \mathbf{v}_{BC} \circ (s_B^* \circ \pi_S^*)}{(s_S^* \gamma_{\mathbf{u}}) \{1 - s_S^*(\mathbf{v} \cdot \mathbf{u}^{-1})/c^2\}} \end{aligned} \quad (20)$$

We propose to consider the relativity theory as a **groupoid** of relative velocities, and not as a group, for the following reasons.

1. We prefer to keep a separate zero-velocity for each reference system,  $A \mapsto \mathbf{v}_{AA} = \mathbf{0}_A$ . Every reference system possesses own zero velocity,

$$\mathbf{0}_A = \mathbf{v}_{AA} \neq \mathbf{v}_{BB} = \mathbf{0}_B. \quad (21)$$

This groupoid possesses as many units=neutrals as many different reference systems,

$$\mathbf{v}_{BA} \circ \mathbf{0}_A = \mathbf{v}_{BA} = \mathbf{0}_B \circ \mathbf{v}_{BA} \quad (22)$$

2. The order of composition is important,

$$\mathbf{v}_{AB} \circ \mathbf{v}_{BA} = \mathbf{0}_A \neq \mathbf{0}_B = \mathbf{v}_{BC} \circ \mathbf{v}_{CB} \quad (23)$$

The Galilean composition of velocities does not commute.

3. Not all velocities-morphisms are allowed to be composable.

**3.1 Comment** (Center of mass). Let  $0 < m_X$  be a mass of a body  $X$ , and  $0 < m_Y$  be a mass of a body  $Y$ . The Galilean center of mass of  $X$  and  $Y$  is the convex sum  $Z$  (analogous to the convex sum of mixed states in quantum mechanics),

$$Z = \left( \frac{\partial}{\partial t} \right)_z = \frac{m_X \left( \frac{\partial}{\partial t} \right)_x + m_Y \left( \frac{\partial}{\partial t} \right)_y}{m_X + m_Y} \quad (24)$$

It is not trivial to find a coordinate function  $z$  describing a center-of-mass body without some extra assumptions. However under the doubtful extra axiom we have

$$\left( \frac{\partial x}{\partial t} \right)_y = \left( \frac{\partial y}{\partial t} \right)_x \implies z = m_X x - m_Y y. \quad (25)$$

The commutative addition symbol ‘+’ must be used for addition of relative momenta on space-time (and *not* for composition of velocities on relative spaces!),  $\mathbf{p}_{BA} \equiv m_B \mathbf{v}_{BA}$ , when the source-body  $A$  is the same,

$$\mathbf{p}_{XA} + \mathbf{p}_{YA} = \mathbf{p}_{ZA}. \quad (26)$$

**3.2 Comment** (Groupoid versus group). Every relative velocity (not a transformation), Definition 3.2 has an inverse but not all velocities, **indexed** by source-system and target-system, can be composed.

If we consider the velocities  $\mathbf{v}$  and  $\mathbf{u}$  just in the abstract space of ‘velocities’ then looks like every two velocities are composable and therefore we have a group. However if  $\mathbf{v}$  is concretely  $\mathbf{v}_{BA}$  a velocity of a body  $B$  relative to  $A$  (then  $\mathbf{v}_{AB}$  is relative to the source-body  $B$ , the minus sign  $\mathbf{v}_{BA} = -\mathbf{v}_{AB}$  is misleading), and similarly  $\mathbf{u}$  is concretely  $\mathbf{u}_{YZ}$ , as shown on Figure 2, then the composition of these velocities,  $\mathbf{v}_{BA} \circ \mathbf{v}_{YZ}$ , can not be interpreted as the relative velocity in case that  $A \neq Y$ .

If the composition of a velocity of Sun relative to Earth  $\mathbf{v}_{SE}$ , with the velocity of the Mars relative to Jupiter  $\mathbf{v}_{MJ}$ , is not a relative velocity, we are no more within group, we are within groupoid.

**3.3 Comment** (Monad). The Euler reference systems is a time-like future-pointed vector field in space-time. The Euler fluid-observer is a vector-monad, whereas the Einstein coordinate observer is a tetrad, a system of coordinates.

We have the following relation among space-time differentials, with two unknown scalar functions,  $a$  and  $b$ , both are implicitly  $\mathbf{v}_{AB}$ -dependent, where  $\mathbf{v}_{AB}$  is on space-time, not projected,

$$dx_B = a dx_A + b dt. \quad (27)$$

$$\text{Normalization: } At = Bt \equiv 1, \quad (28)$$

$$\begin{aligned} x_A \text{ is an } A\text{-space} &\iff Ax_A = 0, \\ x_B \text{ is an } B\text{-space} &\iff Bx_B = 0. \end{aligned} \quad (29)$$

Set  $v \equiv \mathbf{v}_{AB}x_A$ . This imply

$$b = -a(v)v, \quad dx_B = a(v)(dx_A - vdt), \quad (30)$$

$$a|_{v=0} = 1. \quad (31)$$

It is not obvious that a scalar  $a$  must be obligatorily  $a \equiv 1$ .

## Acknowledgments

Many thanks to Greg Volk, for inviting me to video-conference in December 2009, and to Long Beach 17th Annual Natural Philosophy Alliance Conference in June 2010.

I am grateful to Bill Page, at Kingston, Canada, and to Cynthia Kolb Whitney, NPA-officer, for very stimulating correspondence, for critics, and useful email discussions.

## References

- Brillouin Léon, *Relativity Reexamined*, New York: Academic Press 1970, Chapters 4–5
- Matolcsi Tamás, *Spacetime Without Reference Frames*, Budapest: Akadémiai Kiadó 1994, ISBN 9630564335. Part I: Non-Relativistic Model
- Volk Greg, *Reference-frame independent dynamics, or how to get off Einstein’s train*, Video-Lecture, December 12, 2009