

Energy-time uncertainty

Abstract

Spontaneous emission is viewed as the continuous absorption of energy by an atomic oscillator followed by quantization during decay. Energy-time uncertainty can then be defined in a manifestly covariant way by establishing space-time boundaries on the action integral of the decay process; where the minimum of action is not zero, but \hbar . First order equations are derived describing the emission of a photon. Second order emission is shown to yield the Feigenbaum equation. The similarities between them are noted. It is concluded that discrete forms of time, or oscillation periods, function as operators in Lagrangian quantum mechanics because they take as their inputs a delocalized superposition state and return as their outputs a localized quantum state. It is hypothesized that period doubling must be accompanied by asymmetric geometries.

1.0 Introduction

Special relativity requires that symmetry exist between the space and time parameters. Thus it is of fundamental concern that quantum mechanics lacks an energy-time uncertainty relation. Hamiltonian quantum mechanics has long been favored in attempts at a solution because it is the most familiar and is closely related to classical theory. To demonstrate its method consider an atom whose outer electron is in an excited state $|\phi\rangle$. Then energy-time uncertainty takes the following form with respect to an observable B [1].

$$\Delta_\phi E \frac{\Delta_\phi B}{\left| \langle d\hat{B} \rangle / dt \right|} \geq \frac{\hbar}{2} \quad 1)$$

The uncertainty in time is expressed as the average time taken, starting in state $|\phi\rangle$, for the expectation of some arbitrary operator B to change by its standard deviation. This is the shortest time scale for which changes are able to be measured, which is the lifetime of the state.

There are objections to the above derivation of energy-time uncertainty. Due to its statistical nature there is a redundancy of variables built into it. The time parameter and its uncertainty should be exclusive of secondary forms of time. Also time uncertainty should have a classical interpretation so that the correspondence principle applies. In the case of position the progression from quantum to classical systems is clear. The same relationship should exist for the time parameter; however, lifetime has no classical equivalent. For transparent space-time symmetry a smooth transition should exist for both parameters.

Finally, it is often pointed out that the use of Hamiltonian quantum mechanics limits theoretical interpretation since two states correspond to a single time. Thus space and time are treated differently and emission is described by non-covariant means even though we know that it is Lorentz invariant. In the following we shall seek a method that describes time continuously in the same way as space. By treating space and time equivalently uncertainty may be defined in a manifestly covariant way.

2.0 Lagrangian quantum mechanics

2.1 Excitation

The Lagrangian method of quantum mechanics achieves Lorentz covariance by using path integrals and integrating continuously over space and time. However, because path integrals are carried out over all space and all time they cannot be used to define uncertainty relations. Energy-time uncertainty must be defined in terms of field boundaries, where field boundaries may be thought of as the continuum limits of discrete systems. The field boundaries of a particle are established by applying appropriate limits of integration to a Lagrange density.

Consider a uniform distribution of atomic oscillators immersed in a radiation field with an outer electron that can occupy either of two allowable energy states $|1\rangle$ or $|2\rangle$. The radiation field is conceived of as classical, consisting of many oscillating wave train fields superimposed on the oscillators. The amplitude of the field fluctuates randomly, either reinforcing or canceling. If the fields are of the necessary intensity to cause excitation electrons will be raised to a higher energy state. This will cause radiation to be emitted with double the frequency of the incident wave. Because absorption is classical the uncertainty relations do not apply.

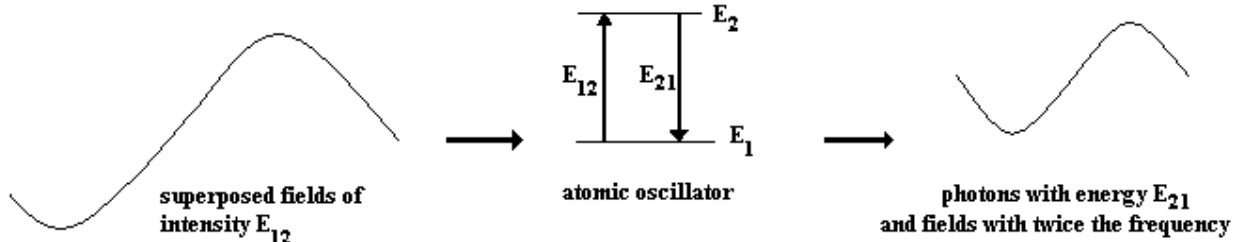


Fig.1 Frequency doubling

Radiation due to this form of excitation is referred to as “spontaneous” because the field is thought to be quantized during absorption. However, quantization will be conceived of here as a process that occurs during *decay*. The oscillatory motion of the electron is reversible prior to excitation but once the electron occupies a higher state energy is irreversibly absorbed. Clear evidence of this model of emission can be found in light scattering experiments where frequency doubling is observed in secondary radiation [2]. Continuous frequency doubling occurs in radio communications [3] and when laser light is passed through a crystal [4]. On the other hand, if a quantum of energy is absorbed the electron will be expelled due to the photoelectric effect and quantization does not occur.

2.2 Electron decay

In order to maintain the generality of a field theory we introduce a Lagrange density $\mathcal{L}(\varphi, \dot{\varphi})$ describing the atomic oscillator. The field boundaries will determine how to define uncertainty relations. The oscillator boundaries are the diagonalized coordinates of steady states [5]. Decay initiates at the invariant boundary of $|2\rangle$ given by (x_2, y_2, z_2) at exact initial time t_2 and finalizes at $|1\rangle$ with coordinates (x_1, y_1, z_1) at exact final time t_1 . Thus fields are continuous in space vanishing at infinity, but are discontinuous at times t_2 and t_1 . If Hamilton's principle is applied to electron decay for initial time t_2 and final time t_1 the action minimum of the electron is not zero, but h . The change in action due to emission is obtained by integrating the Lagrange

density four-dimensionally. We use the field boundaries to define limits of integration.

$$S(\varphi) = \int_{r_2}^{t_1} \int \mathcal{L}(\varphi, r) dr, dt = h \quad 2)$$

The action is determined by integrating φ_r between $|2\rangle$ and $|1\rangle$. The spatial components of the field yield the energy difference E_{21} while the temporal contribution is simply the period of oscillation τ . Together they form the action which assumes the minimum value, h .

$$E \cdot \tau \geq h \quad 3)$$

The above equation describes the minimum energy required to excite an electron. It is a type of uncertainty relation because it gives a lower limit for the detection of bound electrons. The field energy is spatially bounded to infinity and temporally bounded by the period τ so it defines a volume of space-time. It is equal to the universal constant h so it is Lorentz invariant.

2.3 Photon detection

The photon's fields may be described by introducing a Lagrange density of the field's first derivatives $\mathcal{L}(\partial_\mu \varphi)$. Let the space-time boundaries of the photon initially coincide with those of the atomic oscillator. Then the action of the oscillator is unchanged to the first order if the photon's fields are wave-like. The creation of the photon is defined by the exact times t_2 and t_1 which introduce space-like discontinuities.

Suppose that the photon is aligned with the x -axis when it is created. Due to special relativity theory its fields cannot extend outside the initial and final points of creation (x_2, t_2) and (x_1, t_1) . Therefore the field boundaries and limits of integration defining the Lagrange density are determined by the parallel surfaces (y_2, z_2, t_2) and (y_1, z_1, t_1) . They are spaced apart a distance equal to the wavelength $\lambda = (x_2 - x_1)$ and period $\tau = (t_2 - t_1)$. The period gives a lower bound for the measurement of time because partial wave cycles cannot be detected.

We define energy-time uncertainty to be the minimum action of a photon, thereby acting as the equivalent of Heisenberg's microscope experiment for position measurements. To determine the action we evaluate the photon's Lagrange density in space-time between its infinitely extended parallel surfaces, where the minimum action is again h .

$$S(\varphi) = \int_{y, z, t_2}^{t_1} \int \mathcal{L}(\partial_\mu \varphi) dy, dz, dt = h \quad 4)$$

Whereas 2) was evaluated between space-time volumes the integration in 4) is taken between space-time surfaces. The action minimum is simply the energy of the sinusoidal fields evaluated over all y and z for the time interval $(t_2 - t_1)$, where it is assumed that a single wave cycle is the minimum that can be detected.

$$2\pi E \cdot \tau \geq h \quad 5)$$

This equation tells us that even if we have exact knowledge of a photon's space-time surfaces and therefore λ and τ , a photon's field energy cannot be localized in a volume smaller than h . We

interpret this to mean that the uncertainty of an energy quantum is due to its invariant space-time volume. The fact that \hbar is a universal constant means that the space-time volume of all photons is identical.

3.0 Macroscopic emission

If we desire both space-time symmetry and adherence to the correspondence principle in a quantization model then there should be classical emission processes that can be compared to atomic emission. In fact the onset of turbulence in fluids exhibits behavior similar to that of an atomic oscillator. Continuous increases in energy amplitude that are delocalized due to energy equipartition suddenly become localized as a result of transitions known as period doubling bifurcations. If the period doubling is due to classical excitation as described in 2.1 they may be interpreted as the result of two successive wave amplitudes as shown in Figure 2 below, where the field amplitudes are directed in opposite directions. Although vastly oversimplified in only two dimensions it indicates that the oscillations generated by period doubling are vectorially distinct. This is supported by evidence from linear systems such as the pendulum where distinguishable patterns of motion are repeated every *alternate* period of the external drive [6]. Thus period doubling and frequency doubling are both the result of wave behavior. It validates the hypothesis in 2.1 that excitation is due to *instantaneous* wave amplitude and also provides insight into the physical changes that occur during quantization.

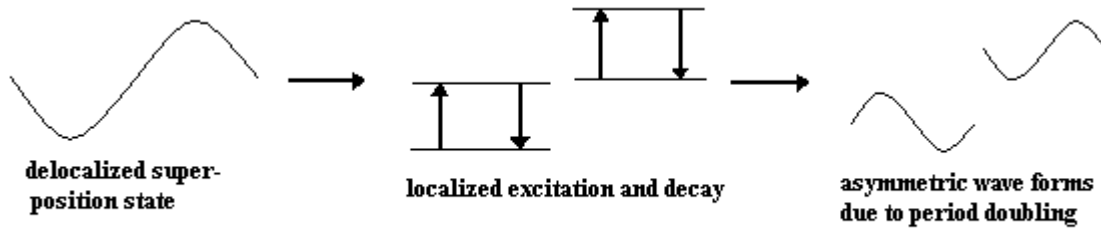


Fig. 2 Period Doubling

The uncertainty relation 5) is determined by first order field equations while thermal flows satisfy second order kinetic equations. If a macroscopic equivalent of uncertainty can be derived then it will be second order. In fact there is a second order equation that predicts the appearance of period doubling bifurcations. It is expressed in terms of the dimensionless constants Q , or quality factors, obtained by dividing wave amplitude by the energy of a single period, E_τ . When the Q from three successive bifurcations Q_n , Q_{n-1} , and Q_{n-2} are combined in the Feigenbaum equation they define a universal constant [7].

$$F_n(\dot{E}) = \frac{Q_{n-1} - Q_{n-2}}{Q_n - Q_{n-1}} \quad 6)$$

Because τ is constant E_τ is the second order equivalent of photon energy. Thus the quality factor of an atomic oscillator is the instantaneous (and also indeterminate) amplitude of the radiation field divided by the discrete energy of the photon.

A description of emission by Lagrangian quantum mechanics has shown that energy cannot be localized more precisely than a region of space-time defined by \hbar . Period doubling phenomena confirm this requirement for second order macroscopic phenomena. In other words, microscopic and macroscopic processes are governed by the same natural laws. This comparison shows how classical and quantum processes blend seamlessly thereby validating the correspondence principle.

5. Conclusion

When time is used as a continuous parameter we see that it has the characteristics of an operator. It takes as its input a pure superposition state that is delocalized in space and time. It returns as its output a localized quantum state that is defined by space-time constraints. Thus quantization at its most elemental level is conceived of as a field transformation process whereby exact space-time boundaries are imposed upon infinitely extended superposed fields. The delocalizing influence of energy equipartition occurs macroscopically so that when space-time constraints are applied it can be distinguished from the surroundings. Whereas the statistics of Hamiltonian quantum mechanics are obtained by summing many independent states linearly in space and time, in Lagrangian theory the statistics are immediately evident in the form of a superposition state. It is now clear why 1) does not successfully define energy-time uncertainty. The wave function $|\phi\rangle$ describes the superposition of all possible states at a *particular point in time*. On the other hand, Lorentz and Lagrangian transformations use continuous time in their equations causing the relationship between space and time to be manifest.

Conventional theory assumes that the absorption of energy occurs in discrete lumps or quanta. However, the evidence presented here suggests that energy absorption is a continuous process that is determined by instantaneous wave amplitude and quantization occurs during decay. The wave nature of energy is clearly evident in non-linear emissions. Frequency doubling occurs when a wave's periods are too small to detect (fig. 1), and when wave frequencies cannot be observed period doubling occurs (fig. 2). Other wave properties are also evident. Temporal symmetry breaking is a result of the alternating vector fields of waves. As a result it is predicted that period doubling will *always* be accompanied by asymmetries. Although continuous/discrete, wave-like/particle-like, and classical/quantum mechanical seem to be incompatible terms they are inextricably linked by the emission and absorption of energy.

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