

A Comparison between Ampère's Law, Coulomb's Law and the Lorentz Force Law

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This paper describes how the Ampere Force Law has been used in order to explain the experimental results of experiments on Ampere's Bridge. A detailed derivation based upon a paper by Wesley is being referred. All the computations have also been done and they show that the original computations by Wesley had been correctly performed. This has been done, since it is often very difficult to judge the claims by authors of physics papers, due to lack of computational information. However, the results are questioned by the author, who promotes a usage of Coulomb's law to be done instead. That was already done in a paper of him in 1997. The model used by the author is successful in explaining the behaviour of the force. A reason by the author for choosing Coulomb's law is that is more original than Ampère and resembles the gravitational law. One may say that it is 'Occam's razor' that decides the choice between the laws. Going to the original texts by Ampère it becomes evident that he has more or less 'guessed' the formula for the force between currents, thereby choosing an expression consisting of two disparate terms. In the process he has also changed his opinion about the values of the coupling constants in the formula. Confessedly, this may be a good procedure at an early stage of searching for an understanding of a phenomenon, as in the early 19th century, a time when it remained two. The Lorentz force, basing itself on the research and speculations by Grassmann, is unsuccessful in explaining the behaviour of Ampère's bridge. Further on, it can also be shown that the very derivation by Grassmann is inherited with a fundamental computational error.

1. Introductionⁱ

It has widely been believed that basic electricity and magnetism was dealt with and finished until the end of the 19th century, though completed using the Special Relativity Theory in the beginning of the 20th. It is also widely believed that all the 'great masters' of electricity, as Coulomb, Ampère, Maxwell, Lorentz, Einstein et al all agree about basic matters. The new generations only add some features though defending principally the old theories.

However, it can easily be remarked that Ampère very early is presenting experimental results which are deeply inconsistent with the theories of Maxwell and Lorentz. Hence, it seems to be an important actual task to begin analysing electric circuits and the laws explaining their behaviour. Back to the 19th century again!

2. Methodology

In order to attain an expression for the force between the two parts of Ampère's Bridge it is necessary to divide the work into several steps. Since the shape of the circuit is rectangular it is convenient to integrate the contribution to the total force between each linear part of the first part and the second part. This work has already been done in preceding papers, [1-3] but the individual step have not been accounted for. Due to the tedious work needed in order to replicate all the integrals, the detailed computations will hereby be presented, including all the steps. It is more fair play between scientists if making it easy for the reader to judge the claims with the aid of easily verifiable expressions.

ⁱ This paper was excerpted from a larger, as yet unpublished work, "A Detailed 'Wesley Evaluation' of the Pappas-Moyssides Experiments on Ampere's Bridge Compared to Jonson's Evaluation Using Coulomb's Law and the Special Relativity Theory."

Below both the theory given by Wesley [1], based upon Ampère's Law [4] and the theory given by the author [2], based upon Coulomb's Law, will be used in order to attain an expression for the force between the two parts of Ampère's Bridge.

2.1. Wesley's Method

Wesley [1] basically uses Ampère's law in his analysis of a set of Ampère's bridge. It may also be mentioned that the author has been consulting Wesley himself in order to check that the integrals were performed in accordance with the method used by Wesley. Regrettably, Wesley is not more among us. However, the paper by Wesley appeared to be very usable in order to define the integrals to be done.

The numbering of the branches of the bridge obeys the paper by Wesley [1] [7]. If going counterclockwise, the order by Wesley is 1,2,5,6,7,10 whereas the author uses the order 1,2,3,4,5,6. [2]

2.2. Jonson's Method

The author [2] basically performs the integrals using the same mathematical definitions as Wesley. This is possible, since the integrations to be performed are straightforward using Cartesian coordinates. I have been able to predict measurement results with a reasonable accuracy.

All the variables being used are identical with those defined by Wesley, [1] [7] otherwise stated within the text.

3. The Check of the Wesley Result for the Force within Ampère's Bridge

Since the approach by Wesley [1] has appeared to be very usable in order to compute the contributions to the total Ampère force from each element of the circuit, it seems convenient to use the definitions of variables he gives in his article. But in the further analysis it has appeared necessary to add other completing, mainly geometric (Cartesian) variables.

Wesley gives two fundamental expressions for the computation of the force, one using line integrals, the other using volume integrals. The choice of form depends on whether the distances between the two parts of the circuit are close to each other or distant. In the first case the line approximation is inappropriate; in the other it is usable.

The expressions, referred to as 'Ampere's original differential force law', [1] (a more complete discussion about the origin of Ampère's law and the choice of coupling constants is given by Hofmann [22]) are as follows:

$$\frac{d^6 \vec{F}}{d^3 r_2 d^3 r_1} = \vec{r} \left(-2 \frac{\vec{J}_2 \cdot \vec{J}_1}{r^3} + 3 \frac{(\vec{J}_2 \cdot \vec{r})(\vec{J}_1 \cdot \vec{r})}{r^5} \right) \quad (1)$$

$$d^2 \vec{F} = I_2 I_1 \vec{r} \left(-2 \frac{(d\vec{s}_2 \cdot d\vec{s}_1)}{r^3} + 3 \frac{(d\vec{s}_2 \cdot \vec{r})(d\vec{s}_1 \cdot \vec{r})}{r^5} \right) \quad (2)$$

It has to be remarked that Wesley prefers to use the coupling constant one instead of $\mu_0/4\pi$ which is customary. That causes some work at the very end when comparing the theoretical results due to the formula and the measurement results, but that works, too, of course.

The first term will hereafter be referred to as the 'a term' and the second term the 'b term'. To be noted is also that since the same current goes through the whole circuit (excluding the current source), $I_1 = I_2$ and $|\vec{J}_1| = |\vec{J}_2|$, more simply written

$$I_1 = I_2 \quad (3)$$

Further on, since the whole circuit is supposed to lie in one and the same geometrical plane, coordinates may be chosen so that

$$z = 0 \quad (4)$$

Actual coordinates of parts of bridge being analyzed will directly be picked from the figure in respective case without further reasoning, as is for example being done below in section 3.1.1 when inserting M as y variable.

Since in all cases the y component of the force is being analyzed, for simplicity all the results due to Eq. (1) and (2) treated below will mean the y component. Thus, instead of always writing $d^2 \vec{F} \cdot \vec{u}_y = \dots\dots\dots$, it will more simply be written $d^2 \vec{F} = \dots\dots\dots$

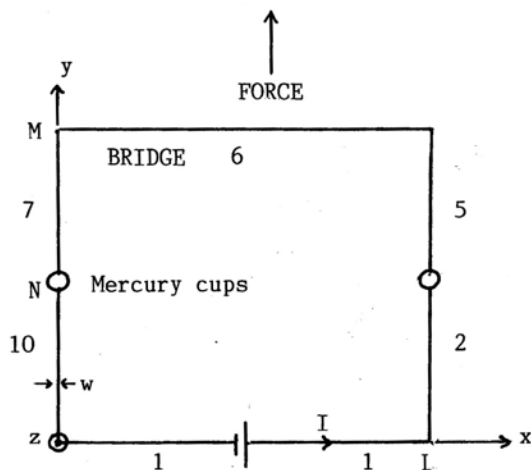


Fig. 1. Ampère's Bridge [1], [7] (Wesley), [2] (Jonson)

As can be inferred from the figure, the integrals involving the branches 2-5 and 10-7 respectively demand usage of Eq. (1) while all the other combinations demand Eq. (2). Please observe that the current source in the middle of branch 1 is denoted by the

abbreviation 'CS'.

In previous papers [6] [11] it has just been referred to the integrals that they have been performed. So also did Wesley. [1] The total force between the two halves of Ampère's Bridge according to the interpretation of Wesley is: [3]

$$\frac{F}{2I^2} = \frac{13}{12} - \frac{\pi}{3} - \frac{2}{3} \ln 2 + \sqrt{1 + \frac{L^2}{M^2}} - \ln \left(1 + \sqrt{1 + \frac{L^2}{M^2}} \right) + \ln \left(\frac{L}{\omega} \right) \quad (5)$$

Wesley also prefers to use the circular cross section d of the wire that Ampère's Bridge consists of instead of the Cartesian variable w due to a quadratic cross section. That makes the following transformation formula necessary:

$$w = \frac{\sqrt{\pi} d}{2} \quad (6)$$

4. Analysis of Wesley's Derivation

Measurements on Ampere's Bridge performed by Pappas and Moysides [24] has been presented in his papers [3] and [20], and he has succeeded in applying his formula upon that set, thus achieving some resemblance with the measurement results by Pappas and Moysides [24]. However, the author has pointed to deficits in that result [6] and proposes another model, making use of only Coulomb's Law.

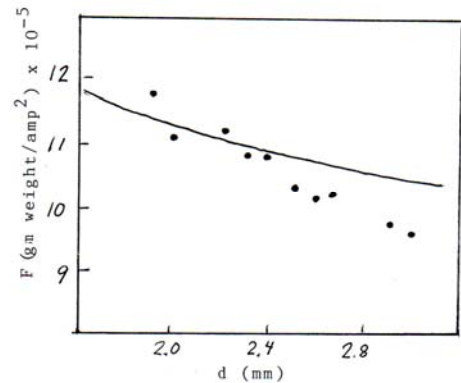


Fig. 2. Showing the force between the two halves of Ampère's Bridge as a function of the circular cross section d . [5] (Figure by Wesley redrawn by the author)

5. The Method by the Author Uses Coulomb's Law to Predict the Force within Ampère's Bridge

It has usually been assumed that Coulomb's Law, which is normally being used in order to explore electrostatic forces, is unable to account for electromagnetic forces, i.e. especially forces between electric currents.

The author has succeeded in doing that, [6] thereby taking into account the effects of the time delay that inevitably occurs with respect to all action-at-a distance.

5.1. The Expressions for the Force upon Ampère's Bridge

Please see chapter 3.1. That is, the same variables are defined by the author as by Wesley.

The same procedure with respect the choice of integral type is used as in the analysis by Wesley. For current elements far away from each other, line integrals will be used and

$$d^2\bar{F} = \frac{\mu_0}{4\pi} \frac{I^2 \bar{R}(d\bar{s}_1 \cdot \bar{R})(d\bar{s}_2 \cdot \bar{R})}{R^5} \quad (7)$$

and for parts of the bridge in close contact with each other, volume integrals have to be used:

$$d^6\bar{F} = \frac{\mu_0}{4\pi} \frac{(\bar{J}_1 \cdot \bar{R})(\bar{J}_2 \cdot \bar{R})}{R^5} \quad (8)$$

These formulas resemble the earlier defined Eqs. (1) and (2). Both formulas can straightforwardly be derived from the result in an earlier paper on the subject. [2] Also a third thing must be done, which has thus far been unrecognized by everybody: to take into account the impact of the current source (battery) of the circuit. This was originally done in the 1997 paper of this author. [2]

A difference between the expressions by Wesley and the author is also the different way they define the coupling constant before the spatial equations: The experiment reports were reported [1] with the unit abampere instead of ampere, which causes a need to divide the whole force by 100 (10 for each current) and they used gram-weight for the force, which in turn makes a division by 980 mm/s² needed. However, the author follows the scaling procedure by Wesley, when dealing with the Coulomb model of his. But when the magnetic force law is treated, the commonplace

$$\mu_0/4\pi = 10^{-7} \text{ H-m} \quad (9)$$

is being used as a coupling constant.

5.2. The Sum of All Correction Terms

Adding the results above, Eq. (3,4,5), double Eq. (3,4,6) makes:

$$I^2 \frac{L+M}{6} \left[\frac{-LM}{\left(\left(\frac{L}{2}\right)^2 + M^2\right)^{3/2}} + \left(\frac{2L}{M} - \frac{2M}{L}\right) \frac{1}{\sqrt{\left(\frac{L}{2}\right)^2 + M^2}} \right. \\ \left. - \frac{NL}{\left(\left(\frac{L}{2}\right)^2 + N^2\right)^{3/2}} + \frac{2N}{L} \frac{1}{\sqrt{\left(\frac{L}{2}\right)^2 + M^2}} \right] \quad (10)$$

Evaluation gives $F_{corr} \cong -0.51I^2$ (11)

$$F_J = \frac{1}{3} 2I^2 \left[-\sqrt{1 + \left(\frac{L}{M}\right)^2} + 2 - \ln \frac{M-N}{M} - \ln \frac{N}{L} - \ln \left(1 + \sqrt{1 + \left(\frac{L}{M}\right)^2}\right) \right. \\ \left. - 2 \ln \frac{M-N + \sqrt{(M-N)^2 + L^2}}{L} - 2 \ln \frac{N + \sqrt{N^2 + L^2}}{M + \sqrt{M^2 + L^2}} \right] \\ + 2 \left[I^2 \left(-\frac{1}{3} - \ln \frac{M}{M-N} + \ln 2N \right) \right. \\ \left. - 2I^2 \int_{z_2=0}^t dz_2 \int_{x_2=0}^w dx_2 \int_{z_1=0}^t dz_1 \int_{x_1=0}^w dx_1 \ln \left((x_2 - x_1)^2 + (z_2 - z_1)^2 \right) \right] \quad (12)$$

F_J indicating a force according to Jonson's interpretation. Using Eq. (12) and taking into account Eq. (11) gives a result that fairly well fits with measured values. 10.7×10^{-5} at the left side of the

diagram, 9.3×10^{-5} at the right side, thereby using Wesley's scaling.

6. Comparison with the Magnetic Force Law (Lorentz Force)

Jonson has also performed a calculation of the force based upon the traditional 'Magnetic Force Law', the so-called Lorentz force [2]. In the DC cases the traditional expressions for the so-called magnetic field and the magnetic 'Lorentz' force will be used. The undergraduate course book this author has used during his MSc studies [8] expressed the laws as follows (indices adjusted to fit with this paper):

$$\bar{F}_m = \int I_2 d\bar{s}_2 \times \bar{B} \quad (13)$$

$$\bar{B} = \frac{\mu_0}{4\pi} \int \frac{I_1 d\bar{s}_1 \times \bar{r}}{r^3} \quad (14)$$

Now the contribution to the magnetic field at each branch of the upper bridge will be derived, provided it gives rise to a force component along the y axis. It can easily be stated which parts of the bridge will not produce y components, namely those aligned with the y axis, i.e. branch 5 and 7. Hence, the task before us is to derive expressions for the magnetic field at branch 6 only. All the three branches of the lower part of the bridge will contribute.

The total magnetic force is attained as the sum of the three contributions above. Using the values given by Wesley, [1] [10] $L = 0.48$ m, $M = 1.20$ m and $N = 0.43$ m gives a resulting magnetic force

$$\bar{F}_m \cong 0.29 \times 10^{-5} \bar{u}_y \quad (15)$$

using Wesley's scaling.

Apparently, the force is attractive but constant, with no dependence of the thickness of the circuit. Hence, the so-called magnetic (Lorentz) force is completely unable to account for the behavior of the force within Ampère's bridge.

7. Forces between Conductors

7.1. The Case of the Force between Only Two Parallel Conductors: A Nonrelativistic Analysis

This case is a classical one being taught within undergraduate courses. The most common way of explaining the attractive force between two current carrying conductors is using the so-called $F=BiL$ rule, based upon a usage of the Lorentz force. Now it can be easily be tested, whether Ampère's law is an alternative, using the calculations in this very paper. There is a suitable formula for the part of Ampère's bridge, containing two parallel conductors,

$$d^2\bar{F}_{1 \rightarrow 6} = I^2 \left(2 \sqrt{1 + \left(\frac{L}{M}\right)^2} - \frac{2M}{\sqrt{L^2 + M^2}} \right) \quad (16)$$

The most convenient circuit demonstrating the force was described by Neumann. [13]

The mentioned equation may be applied using instead of M the radial distance between two parallel conductors r_{12} . When the distance between the conductors is very small, infinitesimal analysis allows for neglecting terms that becomes very small in comparison to the 'main term'. The $(L/M)^2$ term remains with Eq. (16) within the left square root. This gives

$$d^2\bar{F}_{1\rightarrow6} \rightarrow I^2(2\frac{L}{r_{12}}) \quad (17)$$

Realizing that there is needed a minus sign if putting the directions of the two currents the same, the force between the currents becomes negative

$$d^2\bar{F}_{1\rightarrow6} \rightarrow -I^2(2\frac{L}{r_{12}}) \quad (18)$$

which indicates a negative, hence attractive force between the currents, in good accordance with experience. This seems to imply that the 'cross product Lorentz's law' is unnecessary.

Ampère's law also has the benefit of being able to explain the repulsive force in the mercury basin between the electric poles and a copper boat described by Ampère, [3] [4] which the Lorentz' force law is unable to. It may also be mentioned that Coulomb's law could not account for the attractive force, using the equation that follows:

$$d^2\bar{F}_{1\rightarrow6,b} = I^2(-2\sqrt{(1+(\frac{L}{M})^2 + 4 - \frac{2M}{\sqrt{L^2 + M^2}})}) \quad (19)$$

The force namely becomes repulsive, as appears when looking at the equation. Making the same infinitesimal analysis as with respect to Eq. (16) above, namely gives a minus sign in front of the $(L/M)^2$ term within the left square root and after having changed sign, in analogy with above, the final sign with respect to two parallel current flowing in the same direction becomes positive, hence a repulsive force.

However, there is a problem in understanding from where does the very law by Ampère come? It is an ad hoc like formula, and Ampère himself is unable to give good reasons to the two terms. [4] But Keele try by applying the Lorentz transformation of the Special Relativity Theory (SRT). [14] [15] He arrives at just the terms Ampère's law contains. Regrettably, that effort fails, due to an incorrect calculation. [3] Hence, there remains only hopelessness with respect to the desire of understanding why Ampère's law looks as it does.

7.2. A Relativistic Analysis of the Four Contributions to the Force between the Two Conductors

Since hopelessness is completely unacceptable to scientists, the only way to come forward is to go even deeper into the problem. What has been left to be done is a relativistic analysis. If a non-relativistic approach is insufficient in order to explain a physical phenomenon involving velocities, it is reasonable to see to what extent an application of the Special Relativity Theory (SRT) will change the result. The need is in this case felt acute, since there is no explanation to Ampère's Law.

Since each conductor has both immobile positive ions and moving electrons, there will appear four separate force terms due to the four ways the charges of the first conductor may interact with charges of the second conductor. In order to make the properties of the forces easily conceivable, the analysis is restricted to the simplistic case with two parallel conductors. This means that the angle with which a distance vector crosses both the conductors, ϕ and ψ , are equal, even though the simplification is not immediately conducted. In this case the so-called Standard Configuration of the SRT may be applied, with one system K, which is stationary with respect to the positive ions of the two conductors, hence the whole circuitry, and K' following the movement of the electrons of the first conductor, along the x_1 axis, and with

velocity v_1 . The velocity of the electrons of the second conductor is assumed to be v_2 .

These definitions make it possible to compare the effects of the electrons of respective conductor on each other. The focus in the analysis is on the force perpendicular to the currents, which is also the force that acts repelling or attracting between them. The fundamental assumption that is made here is that Coulomb's Law is the only cause behind the force between electric charges, which implies that the very idea of magnetic fields is abandoned. What brings about a change of the shape of the expression of the force compared with the original Coulomb's Law is

1. The effects of the SRT (i.e. Lorentz transformation and derived expressions)
2. Delay effects concerning the retarded observation of fields being generated by charges

7.3. The Force between the Positive Charges and the Electrons Respectively

$$d^2F_{y,+to+} \cong \frac{\rho_1\rho_2\Delta x_1\Delta x_2}{r^3}y \quad (20)$$

using thus charge densities in Coulomb's Law.

One way to treat this case is to turn to the K' system, in which the electrons of the first conductor are at rest. Since the force as it is being felt within K shall be computed, it must be realized the distance Δx_1 is felt shorter by K', namely

$$\Delta x_1' = \frac{\Delta x_1}{\gamma(v_1)} \quad (21)$$

The same holds also for the charge element of the second conductor with moving electrons. They will regard

$$\Delta x_2' = \frac{\Delta x_2}{\gamma(v_2)} \quad (22)$$

Further, the distance vector between the charge elements will not be $\bar{r} = (x, y, z)$. Instead it will be

$$\bar{r}' = (\frac{x}{\gamma(v_1)}, y, z) \quad (23)$$

It must also be taken into account that if having a force in K', that too will be transformed. The Coulomb force between the electrons of respective conductor will ___?___. Now it is the force that shall be transformed from K' to K and Resnick shows how: [16] this must be done according to the SRT:

$$F_x = F'_x \quad F_y = \frac{F'_y}{\gamma(v)} \quad F_z = \frac{F'_z}{\gamma(v)} \quad (24-26)$$

Finally, the effects of retardation mentioned above imply that if there is a difference between the two velocities, a delay factor

$(1 - \frac{v_2 - v_1}{c} \cos \theta)$ must be multiplied. [2] However, this term can

be neglected if they are assumed to be equal, which will be done here in the simplified case. However, there inevitably will appear two delay effects due to the fact that in K, the signal from a moving charge element is delayed dependent on direction, simultaneously the field arrives at the moving charge element of the second conductor with a delay that differs with respect to position at that charge element. [2]

The resulting force can now be written:

$$d^2F_{y,-to-} \cong \frac{\rho_1 \rho_2 \frac{\Delta x_1}{\gamma(v_1)} \frac{\Delta x_2}{\gamma(v_2)} (1 - \frac{v_2 - v_1}{c} \cos \theta)}{\left(\left(\frac{x}{\gamma(v_1)} \right)^2 + y^2 + z^2 \right)^{\frac{3}{2}}} y \quad (27)$$

$$\bullet \frac{1}{\gamma(v_1)} \left(1 - \frac{v_1}{c} \cos \theta \right) \left(1 - \frac{v_2}{c} \cos \psi \right)$$

After some manipulations, assuming $v_1/c \ll 1$ and allowing for series expansion, (27) gives:

$$d^2F_{y,-to-} \cong \frac{\rho_1 \rho_2 \Delta x_1 \Delta x_2}{r^3} y \left[1 - \frac{3}{2} \left(\frac{v_1}{c} \right)^2 \cos^2 \theta - \left(\frac{v_1}{c} \right)^2 - \frac{1}{2} \left(\frac{v_2}{c} \right)^2 + \frac{v_1}{c} \cos \theta \cdot \frac{v_2}{c} \cos \psi \right] \quad (28)$$

7.4. The Force from the Electrons of the First Conductor to the Positive Ions of the Second Conductor

The expression for the force can rather easily be constructed using the just given equation, if replacing $\rho_2 \Delta x_2'$ with $-\rho_2 \Delta x_2$, the minus sign due to the opposite sign of the positive ions with respect to the electrons. Further, there will be no delay term, since the ions of the second conductor are at rest with respect to K. There will also in this case be a division by the gamma factor at the end, since the electrons of the first conductor are at rest in K'. Hence,

$$d^2F_{y,-to+} \cong \frac{-\rho_1 \rho_2 \frac{\Delta x_2}{\gamma(v_2)} \Delta x_2}{\left(\left(\frac{x}{\gamma(v_1)} \right)^2 + y^2 + z^2 \right)^{\frac{3}{2}}} y \bullet \frac{1}{\gamma(v_1)} \quad (29)$$

After some manipulations with the expression, assuming $v_1/c \ll 1$, allowing for series expansion, gives:

$$d^2F_{y,-to+} \cong \frac{-\rho_1 \rho_2 \Delta x_1 \Delta x_2}{r^3} y \bullet \left(1 - \frac{3}{2} \left(\frac{v_1}{c} \right)^2 \cos^2 \theta \right) - \frac{1}{2} \left(\frac{v_1}{c} \right)^2 \quad (30)$$

7.5. The Force from Positive Ions of the First Conductor to the Electrons of the Second Conductor

In this case, there will be no Lorentz transformation from K' to K, since the positive ions of the first conductor that are giving rise to a force at the second conductor are already situated in K. The case is in that case similar to the first case above. The difference is now that the electrons of the second conductor are moving away from the observer, which makes the need for a delay term apparent. Please observe also the need for a change of sign due to the opposite sign of the positive ions with respect to the electrons. Hence

$$d^2F_{y,+to-} \cong \frac{-\rho_1 \rho_2 \Delta x_1 \Delta x_2 (1 - \frac{v_2}{c} \cos \psi)}{r^3} y \quad (31)$$

7.6. The Sum of the Four Contributions to the Force between the Two Conductors

Summing all four contributions, assuming also the simplest case with equal velocities, gives after some boring steps

$$d^2F_{y,total} \cong \frac{\rho_1 \rho_2 \Delta x_1 \Delta x_2}{r^3} y \left(-\left(\frac{v}{c} \right)^2 + \left(\frac{v}{c} \right)^2 \cos \theta \cos \psi \right) \quad (32)$$

which may also be written

$$d^2F_{y,total} \cong \frac{1}{c^2 r^3} y \left(-\frac{I_1 I_2}{c^2} + \frac{I_1 I_2}{c^2} \cos \theta \cos \psi \right) \quad (33)$$

In fact this expression has the same asymptotic properties as Eq. (2), even though the coupling constants are different. The expressions will differ, however, if allowing for different velocities and directions, but in the time of Ampère, it seems reasonable that there were no opportunities to neither vary nor measure the electron velocities. Not even the electron had been discovered. Therefore one can say that the above result (385n) is in appropriate accordance with Ampère's Law.

The appropriate conclusion to be drawn is that Ampère's Law appears as a consequence of applying the SRT to Coulomb's Law in the case of two current carrying conductors. And since this law is sufficient, the Lorentz force is no more needed. Ampère's Law can namely account for the attractive force between two parallel conductors.

7.7. An Observation Concerning the Lorentz Force

P. Graneau [16] [17] has written in some papers that H. Grassmann [18] has laid the foundations to the expression of the 'Lorentz force'. If closer studying the derivations of the Lorentz force in its premature stage, made by Grassmann, it becomes evident that the theoretical basis has been fallaciously derived due to a simplistic fatal computational error.

The error is that he performs an integral wrongly. [21] By integrating the expression

$$-(ib/l)(\cos^2 \beta \cdot \cos \alpha \cdot d\alpha - 2 \sin \alpha \cdot \sin \beta \cdot \cos \beta \cdot d\beta) \quad (34)$$

Over the whole line, $d\beta = d\alpha$, he attains from the first term $-(ib/l) \sin \alpha \cdot \cos^2 \beta$ but from the second term he attains zero, without giving any explanation to the zero result.

He uses the definitions given by Ampère. A more detailed text including a figure will follow. Since this derivation is said to lay the foundation to the Lorentz force theory, that seems to be false due to this first elementary computational error. Or - can anybody else verify the zero result?

8. Conclusion

As has already been stated, the author results basically confirm the slope of the measurement dependence of the cross section of the wire that Ampère's Bridge consists of, whereas Wesley partially fails in this respect. However, the Lorentz force is completely unable to predict any spatial dependence.

It is also necessary to be stated that the strength of the current one measures, presumes the Lorentz force participating in the torque that forces the arrow of a volt (ampere) meter to move.

$$\vec{\tau} = \vec{r} \times \vec{F}_m \quad (35)$$

Hence, any measurement thus far contains a 'circular proof moment.

However, an obstacle to the model that has been proposed by the author arises when looking at the very traditional case of two parallel conductors only. Deriving an expression for the force namely gives rise to a positive value, contrary to experience. This made it necessary to analyze what implications the Special Relativity Theory has for the case. In fact, that usage turned the sign negative and hence, the theory by the author is again applicable, provided it is completed with a relativistic analysis.

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