Proof of Incorrectness of General Relativity Theory

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In this paper it is shown that the General Relativity Theory (GRT), which belongs to a class of Metric Theories of Gravity (MTG), is based on a wrong assumption, contradicts the well established laws of physics and also its own postulate. It is shown that in GRT the velocity of a massive body can exceed the speed of light and that the motion of a test body in an orbit around the centrally gravitating mass does not satisfy the conservation of angular momentum. Finally, it is shown that GR theory also violates the Gauss law. The proof rests on a comparison of the Schwarzschild metric, derived from Einstein's field equations, with a new metric from which the Schwarzschild metric can be also derived as a first order approximation but which is not derived from Einstein's field equations.

1. Introduction

In order to make the proof simple and easily understandable the space-time, which the metrics describe, will be the space-time of a non-rotating centrally gravitating body. The general metric line element for such a space-time is thus as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{rr}dr^2 - g_{\phi\phi}\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) \tag{1}$$

where the metric coefficients depend only on the radial coordinate. This form of metric assumes that according to the Riemann hypothesis the motion can be represented by a curved space-time in which the test bodies move in a free fall along geodesic lines not experiencing any forces in contrast to a flat space-time with fields and forces that guide the motion. This concept forms the basis for all MTG theories and has been also adapted by Einstein in his derivation of general relativity. The Einstein's GRT, however, includes additional assumptions related to Ricci tensor that lead to the derivation of Einstein field equations with the Schwarzschild metric as a solution. The Riemann principle is thus more general than GRT and allows derivation of other metrics describing the space-time not only the Schwarzschild metric. For the purposes of this article it is not important how the metrics were obtained only whether their description of reality is reasonable and does not contradict the known laws of physics and observational facts. The reader that only studied GRT and firmly believes that all the assumptions used in its derivations are true and cannot be challenged is probably wasting his time reading this paper. It may be difficult to prove GRT incorrect using only GRT arguments. From the same assumptions one can only come to the same conclusions. So, in order to resolve the problems pointed out in this paper it is necessary to thoroughly scrutinize the assumptions used in GRT and modify them accordingly and preferably in agreement with the available data that a theory of gravity consistent with the fundamental laws of physics is developed. The departure from the classical concepts will be clearly mentioned in the text.

2. Definition of Metrics

The metrics and their metric line elements that will be studied and compared against each other and against the known and well established laws of physics are:

$$\begin{aligned} ds^2 &= (1 - R_s / r)(cdt)^2 - (1 - R_s / r)^{-1}dr^2 \\ &- r^2\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) \end{aligned} \tag{2}$$

which is the well known celebrated Schwarzschild metric with the metric coefficients defined as: $g_{tt} = 1 - R_s / r$, $g_{rr}g_{tt} = 1$, and $g_{\phi\phi} = r^2$, where $R_s = 2\kappa M / c^2$, with M being the mass of the main body and κ the gravitational constant, and the new metric, derived elsewhere [1],

$$\begin{aligned} ds^2 &= e^{-R_s/\rho}(cdt)^2 - e^{R_s/\rho}dr^2 \\ &- \rho^2 e^{-R_s/\rho}\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) \end{aligned} \tag{3}$$

where the variable ρ is found from the differential equation:

$$d\rho = e^{R_s/2\rho}dr \tag{4}$$

and where the metric coefficients are:

$$g_{tt} = \exp(-R_s / \rho),\ g_{rr}g_{tt} = 1, \quad g_{\phi\phi} = \rho^2 g_{tt}$$

3. Meaning of Coordinates and the Topology of a "Black Hole"

This section may be superfluous for the proof, but it may be helpful to some readers since it clearly defines the terms used in this paper and explains their meaning. The literature may sometimes define the same terms differently, causing confusion.

In this paper only two types of coordinates are used. The natural coordinates that are for example appearing in the metric line element given in Eq. (2) and the physical coordinates, sometimes called the proper coordinates, that are defined similarly as is given for example in Eq. (4). It is important to realize that the physical coordinates are not affected or changed in any way by the gravity, by adding a massive body into the space-time, while the natural coordinates are. The amount of distortion of natural coordinates is given by the metric coefficients and the gravity induced distortion affects everything in the natural space-time including our bodies since we are living in this space-time and making measurements using clocks and measuring sticks. From this description it is obvious that the physical coordinate differentials and the natural coordinate differentials become identical far away from the gravitating bodies where the gravity has no effect. It is therefore also clear that the space-time distortion can be easily evaluated against the physical coordinate system, which is forming an unchanging reference, when the metric coefficients are known. An example for the Schwarzschild metric is given below, where the physical coordinate radius is calculated, since for this metric it can be calculated analytically:

$$\rho_s = \int_{R_s}^{r} \frac{dr}{\sqrt{1 - R_s / r}} = R_s \ln\left(\sqrt{\frac{r}{R_s} - 1} + \sqrt{\frac{r}{R_s}}\right) + R_s\sqrt{\frac{r}{R_s} - 1}\sqrt{\frac{r}{R_s}} \tag{5}$$

For the purpose of this paper only the physical radius is needed. The physical radius is not distorted by gravity, as already mentioned, it is identical for either metric discussed in this paper and it actually expresses the distortion of the natural coordinate radius as an inverse of the function in Eq. (5) that cannot be analytically calculated. The graphs of dependence of the natural coordinate radius plotted as function of the physical coordinate radius are shown in Fig.1 for both metrics normalized to R_s.

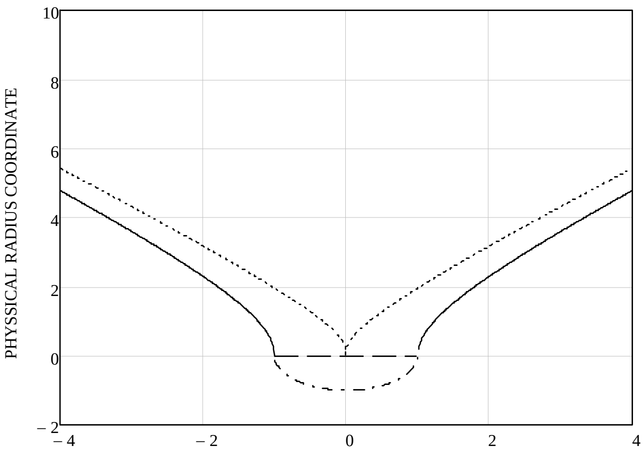

Fig.1. The solid line graph using normalized coordinates is showing the Black Hole region of the Schwarzschild metric and two possible space-time extensions into it. The left hand side of the graph results from rotation and does not mean that the natural coordinate radius rn can have negative values. The dotted line corresponds to the new metric that has no BH region.

This can be also visualized as a cut through a 2D surface of rotation around the vertical axis as it is commonly presented. From this graph it can be observed that in the Schwarzschild metric space-time the addition of a massive body compresses and stretches the natural space-time and adds into it an important inaccessible region $|r_n| \le 1$ called the Black Hole (BH). As the mass of the body changes the size of the BH also changes, but the physical coordinates and the natural coordinates must match at infinity and this helps to visualize why the physical coordinates remain unchanged. There are many papers published in the literature where through the introduction of various coordinates and coordinate transformations, for example the Kruskal-Szekeres coordinates, the space-time is analytically extended into the BH. The extension is indicated in the graph by a bowl shaped dot-dash line. It is also claimed that at the bottom of this region resides the famous Schwarzschild singularity. However, in this paper it is clear from the definition of the physical radius, that this does not make sense, since the physical radius already covers the entire space $0 \le \rho \le \infty$ and cannot be negative. The only extension of the space-time into the BH that might be meaningful can thus be made only in the positive direction of the physical coordinate radius, forming a dome shaped surface or be flat as is indicated in the graph by the dashed line. It is also clear that this extension forms a corner, a shell shaped singularity in the 3D space at the Schwarzschild radius where all the mass of the BH should then reside. In order to better understand the fundamental differences between the studied metrics it is also interesting to calculate the time of fall of a test body from a certain distance to the surface of the centrally gravitating body as observed by a distant observer and by an observer riding on the test body. For the Schwarzschild metric the time of fall is infinite when observed by a distant observer even if the test body is infinitesimally close to the BH. This does not seem reasonable and realistic. For the co-moving observer the fall time is always finite. For the new metric both times are finite as expected. The results of falling time calculations normalized to R_s / c are shown in Fig. 2.

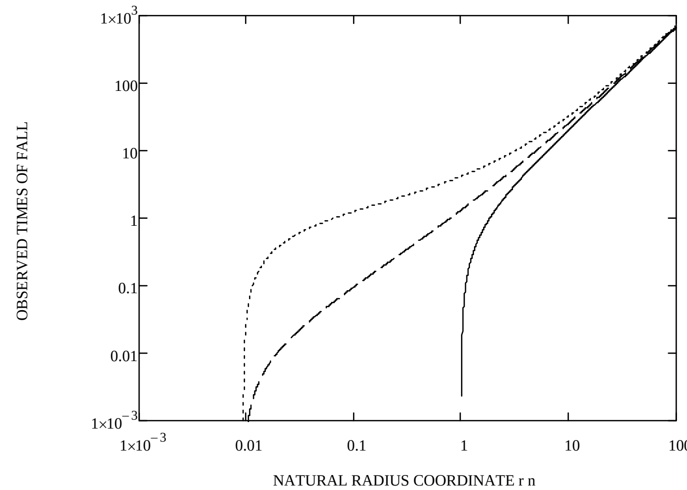

Fig.2. Normalized times of fall from a given distance to the surface of the gravitating body for a distant observer and for an observer co-moving with the test body. The dotted and dashed lines represent the new metric falling times for a distant and co-moving observer respectively. The solid line represents the falling time of the co-moving observer for the Schwarzschild metric. The distant observer falling time for the Schwarzschild metric is infinite, therefore not shown.

Therefore, in this paper it is considered that at the Schwarzschild radius the singularity is real, not only a coordinate singularity as is commonly believed, and that there is an empty void inside of the BH that cannot be reached from the exterior region. The disadvantage of various coordinate transformations that are used for mapping the interior of BHs, particularly when the time and space coordinates are mixed together, is that their meaning and the relation to the natural and physical coordinates used in this paper is lost. When new coordinates or coordinate transformations are introduced it is necessary to always clearly identify whether they can be observed and how they are measured. This is often not done and this leads to only mathematical manipulations without any physical meaning behind them or a connection to reality. This paper makes an effort to avoid this problem.

There is no impact of the modified BH model on the proof presented in this paper, since it is clear that the interior of the BH does not have to be mapped by any coordinate system and taken into account, in particular when it is shown that other metrics exist describing the space-time of the centrally gravitating body without the BH singularity.

4. General MTG Equations

The introduced metrics will be tested and evaluated by studying the motion of small test bodies in the space-times that these metrics describe. In order to simplify this task only two kinds of motion will be investigated: the radial motion and an orbital motion in a circular orbit. It is therefore advantageous to first derive general equations for these motions based on Eq. (1) and then apply them to the respective metrics.

First, for the circular orbital motion, using the well known Lagrange formalism, considering for simplicity the motion only in the equatorial plane, the Lagrangian describing such a motion of a small test body is as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 - g_{\phi\phi}\left(\frac{d\phi}{d\tau}\right)^2 \tag{6}$$

The first integral of Euler-Lagrange (EL) equation derived from the variational principle $\delta\int_\tau L d\tau = 0$ that corresponds to the time coordinate is:

$$g_{tt}\left(\frac{dt}{d\tau}\right) = k \tag{7}$$

where k is an arbitrary constant of integration typically set to $k = 1$ to satisfy the initial condition at infinity. The EL equation of motion corresponding to the radial coordinate is as follows:

$$-\frac{d}{d\tau}\left(2g_{rr}\frac{dr}{d\tau}\right) = \dot{g}_{tt}\left(\frac{cdt}{d\tau}\right)^2 - \dot{g}_{rr}\left(\frac{dr}{d\tau}\right)^2 - \dot{g}_{\phi\phi}\left(\frac{d\phi}{d\tau}\right)^2 \tag{8}$$

where the dot represents the partial derivative with respect to the radial coordinate. Since for the circular orbit the radial coordinate is constant, Eq. (8) simplifies to read:

$$\left(\frac{d\phi}{dt}\right)^2 = c^2\frac{\dot{g}_{tt}}{\dot{g}_{\phi\phi}} \tag{9}$$

where Eq. (7) was used to eliminate the variable τ. Considering now that the natural coordinate orbital time t_o, which is the observable quantity referenced to the central mass coordinate system, is found when the angle is set to: $\phi = 2\pi$, the following equation results:

$$t_o = \frac{2\pi}{c}\sqrt{\frac{\dot{g}_{\phi\phi}}{\dot{g}_{tt}}} \tag{10}$$

This is a general formula that can be used for any metric of the form given in Eq. (1) describing the space-time of a non-rotating centrally gravitating body. From the Lagrangian in Eq. (6) also follows the general formula for the conservation of angular momentum. The first integral corresponding to the angular coordinate is:

$$g_{\phi\phi}\frac{d\phi}{d\tau} = k\alpha \tag{11}$$

Dividing Eq. (11) by Eq. (7) the general formula for the conservation of angular momentum in any MTG whose metric satisfies the form given in Eq. (1) is thus as follows:

$$\frac{g_{\phi\phi}}{g_{tt}}\frac{d\phi}{dt} = \alpha \tag{12}$$

In the second step, turning the attention to the tests where the test body moves only in the radial direction, the Lagrangian describing such a motion is as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 \tag{13}$$

Since the Lagrangian itself is also a first integral equal to $L = c^2$ it is simple to derive the following relations:

$$g_{tt}\left(\frac{dt}{d\tau}\right) = k \tag{14}$$

$$g_{rr}g_{tt}\left(\frac{dr}{d\tau}\right)^2 = c^2k^2 - c^2g_{tt} \tag{15}$$

Rearranging Eq. (15) and differentiating the result with respect to τ, assuming that the metric coefficients are functions of the gravitational potential, the following equation is obtained:

$$\frac{d^2r}{d\tau^2} = -\frac{c^2}{2}\frac{\partial}{\partial\phi_n}\left(\frac{1}{g_{rr}} - \frac{k^2}{g_{rr}g_{tt}}\right)\frac{\partial\phi_n}{\partial r} \tag{16}$$

In order to maintain the same contravariance character on both sides of Eq. (16) it is easily seen that the partial derivative of the bracket must be a component of a contravariant tensor. There are only two possibilities how to satisfy this requirement, the bracket derivative equal either to g^{rr} for the spherical coordinates or to g^{tt} for the cylindrical coordinates. For the spherical coordinates the solution is: $g_{rr}g_{tt} = 1$, or alternately written as: $g_{tt} = g^{rr}$, simplifying Eq. (16) into:

$$\frac{d^2r}{d\tau^2} = -\left(\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial\phi_n}\right)g^{rr}\frac{\partial\phi_n}{\partial r} \tag{17}$$

Furthermore, the bracket in Eq. (17) must be equal to unity, so it must hold that:

$$\frac{c^2}{2}\frac{1}{g_{tt}}\frac{\partial g_{tt}}{\partial\phi_n} = 1 \tag{18}$$

in order to obtain the Newton-like equation with the gravitating force being equal to acceleration according to the equivalence principle:

$$\frac{d^2r}{d\tau^2} = -g^{rr}\frac{\partial\phi_n}{\partial r} \tag{19}$$

The relation $g_{rr}g_{tt} = 1$ is well known from the Schwarzschild metric and can be also derived for the new metric from the condition that the natural space deformation by the gravity is locally isotropic. The solution of Eq. (18) is easily found, assuming zero potential boundary condition at infinity:

$$g_{tt} = e^{2\phi_n/c^2} \tag{20}$$

This equation thus allows to calculate the gravitational potential and therefore the gravitational field intensity from the metric coefficient standing by the time coordinate. It is interesting to note that Eq. (20) and the condition $g_{rr}g_{tt} = 1$ follow uniquely from the metric in Eq. (1) when it is assumed that the gravitational field has a potential and that the metric coefficients depend on it.

Finally, once the gravitational field intensity is known it should also satisfy the Gauss law as any other standard field intensity. The Gauss law is a conservation law, in this case the law of conservation of rest mass, similarly as the conservation of charge in the Maxwell's EM field theory. This can be written in the static curved natural space-time coordinates as:

$$\oint_S \vec{E}_g \cdot \vec{n}\, dS = 4\pi\; g_{\phi\phi}\sqrt{g_{rr}}\frac{\partial \phi_n}{\partial r} = 4\pi\;\kappa\; M \tag{21}$$

Since the integrating surface S could be chosen a spherical surface the integration was carried out, and this allows the Gauss law to be re-written in terms of the metric coefficients as follows:

$$\frac{g_{\phi\phi}}{g_{tt}\sqrt{g_{tt}}}\frac{\partial g_{tt}}{\partial r} = R_s \tag{22}$$

Any metric of MTG describing the non-rotating centrally gravitating body with mass M should thus also satisfy the condition in Eq. (22). The validity of the Gauss law suggests that the field energy, which is negative or removed from the space around the gravitating body, was converted into the tangible gravitating mass of the body. This also implies that the total energy W of the field plus the energy equivalent of the mass of the body could be zero and that the gravitation field around the central body does not have any tangible gravitating mass. The total energy can be calculated as follows:

$$W = Mc^2 + \int_0^M \phi_n dM \tag{23}$$

This becomes for the new metric at the mass equivalent radius $\rho_e = R_s/4$ or equivalently for the natural coordinate mass equivalent radius $r_e = 0.009384\cdot R_s$ equal to:

$$W = Mc^2 - \frac{\kappa\, M^2}{2\rho_e} = 0 \tag{24}$$

For the Schwarzschild metric, however, the result is:

$$W = \frac{1}{2}Mc^2 - \frac{1}{2}Mc^2\left(\frac{r_e}{R_s}-1\right)\ln\left(1-\frac{R_s}{r_e}\right) > 0 \tag{25}$$

There is no mass equivalent radius that would make the total energy zero for the Schwarzschild space-time, so the energy had to be supplied from somewhere else to create this space-time with masses and fields in them. To correct this problem those who believe in the existence of BHs should consider that half of the BH mass is trapped in the void in form of negative energy [2].

To summarize the results of this section, it is convenient to arrange all the conditions that were found and that any MTG must satisfy into a Table 1. These are the general formulas independent of the GRT assumption about the Ricci tensor or any other assumptions that could be construed as a field theory, in addition to the metric such as for example in Ref. [3], that the metrics must satisfy. There are five conditions in Table 1 related to metric coefficients and only three metric coefficients in the metric line elements. It is therefore obvious that there must be some interdependency between the coefficients. By inspecting the formulas it is easy to conclude that only one parameter is free to be chosen or determined from the experiments, for example the gravitational potential or the metric coefficient standing by the time coordinate, the rest are derived parameters. It is also necessary that at large distances the potential approaches the Newton gravitational potential. If more than one metric coefficient is specified, this violates the mathematical consistency of the metric or some well known law of physics as is discussed in more detail in the next section.

Time coordinate metric coefficient	$g_{tt} = e^{2\phi_n/c^2}$
Gravitational potential from g_{tt}	$\phi_n = c^2 \ln\sqrt{g_{tt}}$
Radius and time metric coefficients relation	$g_{rr}g_{tt} = 1$
Conservation of angular momentum	$\frac{g_{\phi\phi}}{g_{tt}}\frac{d\phi}{dt} = \alpha$
Orbital time, the Kepler's third law	$t_o = \frac{2\pi}{c}\sqrt{\frac{\dot{g}_{\phi\phi}}{\dot{g}_{tt}}}$
Gauss law of rest mass conservation	$\frac{g_{\phi\phi}\dot{g}_{tt}}{g_{tt}\sqrt{g_{tt}}} = R_s$
Acceleration-force equivalence formula	$\frac{d^2r}{d\tau^2} = -g^{rr}\frac{\partial\phi_n}{\partial r}$
Total energy, field plus mass equivalent	$Mc^2 + \int_0^M \phi_n dM = 0$

Table 1.

Evaluation Parameter	**Schwarzschild Metric**	**New Metric**
Time coordinate metric coefficient	$g_{tt} = 1 - \frac{R_s}{r}$	$g_{tt} = e^{\frac{-R_s}{\rho}}$
Gravitational potential from g_{tt}	$\varphi_s = \frac{c^2}{2}\ln\left(1-\frac{R_s}{r}\right)$	$\varphi_n = -\frac{\kappa\, M}{\rho(r)}$
Radius and time coefficients relation	$g_{rr}g_{tt} = 1$	$g_{rr}g_{tt} = 1$
Conservation of angular momentum	$r^2\frac{d\varphi}{dt} = \alpha\left(1-\frac{R_s}{r}\right)$	$\rho^2\frac{d\varphi}{dt} = \alpha$
Orbital time, the Kepler's third law	$t_{os} = 2\pi\sqrt{\frac{r^3}{\kappa\, M}}$	$t_{oh} = 2\pi\sqrt{\frac{\rho^3}{\kappa\, M}+\frac{\rho^2}{c^2}}$
Gauss law of rest mass conservation	$\frac{g_{\varphi\varphi}\dot{g}_{tt}}{g_{tt}\sqrt{g_{tt}}} = R_s\left(1-\frac{R_s}{r}\right)^{\frac{-3}{2}}$	$\frac{g_{\varphi\varphi}\dot{g}_{tt}}{g_{tt}\sqrt{g_{tt}}} = R_s$
Acceleration-force equivalence formula	$\frac{d^2r}{d\tau^2} = -\frac{\kappa\, M}{r^2}$	$\frac{d^2r}{d\tau^2} = -\frac{\kappa\, M}{\rho^2}\sqrt{g^{rr}}$
Total energy, field plus mass equivalent	$Mc^2 + \int_0^M \varphi_s dM > 0$	$Mc^2 - \frac{\kappa\, M^2}{2\rho_e} = 0$

Table 2.

5. Metrics Evaluation.

The two metrics introduced in the metric definition section of the paper will be now investigated and compared according to the criteria in Table 1. Since the criteria formulas are simple it is easy to substitute the corresponding metric coefficients into them and construct another table, Table 2 for the metrics. By inspecting the results it is now easy to evaluate the metrics properties and arrive at the following conclusions:

1. The Schwarzschild metric has a problem at the Schwarzschild radius called the Event Horizon (EH). There is no problem for the new metric at the Schwarzschild radius, no Black Holes exist. The metric coefficient standing by the time coordinate must be an exponential function of the gravitational potential, the Schwarzschild metric does not satisfy this requirement.
2. The Schwarzschild gravitational potential at the EH is infinitely negative. No radiation or BH evaporation is thus possible. There is no problem for the new metric at the EH. The potential for the new metric resembles the Newton gravitational potential with a difference of natural coordinate radius being replaced by the physical coordinate radius following Eq. (4).
3. The relations between the time and the radius metric coefficients are the same. This is the only condition that the Schwarzschild metric satisfies exactly and correctly.
4. The Schwarzschild metric does not conserve the angular momentum of orbiting test bodies. There is zero angular momentum at the Schwarzschild radius. The BH cannot rotate [2]. The new metric maintains the conservation of angular momentum, which is a well know and many times verified fundamental law of physics.
5. For the Schwarzschild metric the orbital time (the Kepler's third law) approaches zero when the mass of the centrally gravitating body tends to very large values. This implies an infinite orbital speed. For example: at the radius $r = R_s / 2$ the test bodies would whiz around with the vacuum speed of light even inside of the BH, assuming that BHs have any interior regions. For a smaller radius the speed of light is obviously exceeded. It is also strange that this formula is identical to formula derived classically form the Newton inertial and gravitational laws without any effects from the curvature of space-time. For the new metric, however, there is a limit equal to the physical orbital length divided by the vacuum speed of light and an effect from the space-time curvature. This is what would be expected from any reasonable theory of gravity, but fails in GRT.
6. An important finding that should be disturbing to any physicist is that the gravitational field of the Schwarzschild metric does not satisfy the Gauss law. This is a fatal flaw of GR theory particularly when the new metric has no problem with it. The new metric result that the gravitational field has no tangible gravitating rest mass is interesting and in some sense similar to claims used in GRT that in the empty space around the gravitating body the mass energy tensor T_{jk} is zero.
7. It is surprising and strange that for the Schwarzschild metric the inertial and gravitational forces follow the classical Newton laws formulas, except for the natural time being replaced by the metric invariant τ, without any effect from the curvature of space-time. The new metric formula includes these effects. It is also necessary that the contravariant character of geometric objects from the natural space-time is equal on both sides of the equation. The Schwarzschild metric formula does not seem to satisfy this simple requirement.
8. Finally, the field energy plus the mass equivalent energy of the centrally gravitating body being zero for the new metric is an appealing property also from the philosophical point of view, since it balances the energy of the entire visible mass of the Universe to zero. The current Schwarzschild metric model obviously does not offer this possibility.

From this summary it is clear that the Schwarzschild metric is not physical and cannot correspond to reality. From this conclusion also follows that GRT is not a correct theory of gravity. The assumption about the Ricci tensor being zero used in the derivation of Einstein field equations and from that the Schwarzschild metric do not correspond to reality. The Schwarzschild metric is only an approximation for the weak gravitational fields as is shown in the next section.

6. Weak Field Approximation

The first order approximation for the new metric coefficient g_{tt} and thus for the new metric line element of the centrally gravitating non-rotating body is found by integrating Eq. (4) as follows:

$$\begin{aligned} r &= \int_0^{\rho} e^{\frac{-R_s}{2\rho}} d\rho = \frac{R_s}{2} \int_{\frac{R_s}{2\rho}}^{\infty} \frac{e^{-x}}{x^2} dx \\ &= \rho e^{\frac{-R_s}{2\rho}} - \frac{R_s}{2} \int_{\frac{R_s}{2\rho}}^{\infty} \frac{e^{-x}}{x} dx = \rho e^{\frac{-R_s}{2\rho}} + \frac{R_s}{2} Ei\left(-\frac{R_s}{2\rho}\right) \end{aligned} \tag{26}$$

where $Ei(x)$ is the Euler exponential integral function. Unfortunately there is no analytic expression for ρ as function of r, so the approximation needs to be found iteratively. For large distances (0<x<<1), the Euler exponential integral is approximated as:

$$Ei(-x) = \ln(\gamma' x) + ... \tag{27}$$

where $\gamma' = 1.781072...$ and $\ln(\gamma') = \gamma$ is the famous Euler constant $\gamma = 0.577215...$. It is therefore possible to write:

$$\rho e^{\frac{-R_s}{2\rho}} = r - \frac{R_s}{2} \ln\left(\frac{\gamma' R_s}{2\rho}\right) + ... \tag{28}$$

Rearranging this result as follows:

$$\begin{aligned} \frac{1}{\rho} &= \frac{1}{r} e^{\frac{-R_s}{2\rho}} + \frac{R_s}{2r\rho} \ln\left(\frac{\gamma' R_s}{2\rho}\right) + ... \\ &= \frac{1}{r} - \frac{R_s}{2r\rho} + \frac{R_s}{2r\rho} \ln\left(\frac{\gamma' R_s}{2\rho}\right) + ... \end{aligned} \tag{29}$$

and substituting for $1/\rho$ from the left hand side of Eq. (29) to the right hand side, the iterative expression for $1/\rho$ valid for large distances becomes:

$$\frac{1}{\rho} = \frac{1}{r} - \frac{1}{2} \frac{R_s}{r^2} \left(1 + \ln\left(\frac{2r}{\gamma' R_s}\right)\right) + ... \tag{30}$$

From this formula then follows the approximation for g_{tt} :

$$g_{tt} = e^{\frac{-R_s}{\rho}} = 1 - \frac{R_s}{r} + \frac{R_s^2}{r^2}\left(1 + \ln\sqrt{\frac{2r}{\gamma' R_s}}\right) + \ldots \qquad (31)$$

In the next step the second order term in Eq. (31) can be neglected since the logarithmic function of $\sqrt{r}$ increases very slowly and R_s is for all practical purposes always very much smaller than r. This leads to the familiar formula for the metric coefficient g_{tt}:

$$g_{tt} = 1 - \frac{R_s}{r} + \ldots \qquad (32)$$

Substituting this approximation into Eq. (3), and considering that the logarithmic term multiplied by $R_s/2$ in Eq. (28) can also be for large distances neglected in comparison to r, the formula for the metric line element becomes the celebrated Schwarzschild metric:

$$ds^2 = \left(1 - \frac{R_s}{r}\right)(cdt)^2 - \left(1 - \frac{R_s}{r}\right)^{-1} dr^2 - r^2\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) \qquad (33)$$

The derived approximation also allows finding a condition that needs to be satisfied to obtain a reasonable description of the space-time geometry by this metric $(r > R_s)$. The condition is as follows:

$$\frac{r}{R_s} - \ln\sqrt{\frac{r}{R_s}} >> 1 \qquad (34)$$

From the above derivations there is no doubt that the Schwarzschild metric is only the first order approximation of the metric introduced in Eq. (3) and thus only the first order approximation of reality. The reality is being defined here as a space-time described by a metric whose time metric coefficient is an exponential function of gravitational potential, metric that does not permit larger than the speed of light velocities, and where the test body trajectories conserve the angular momentum. Of course it is also necessary that the four tests of GRT are satisfied: Mercury perihelion advance, the light bending by the Sun, the Shapiro delay, and the gravitational red shift. Since the Schwarzschild metric satisfies these tests and it is the weak field approximation of the new metric it is clear that the new metric also satisfies them. The Schwarzschild metric is the correct and unique solution of Einstein field equations for the spherical case, according to the well known Birkhoff theorem, [4] therefore, there can be only one inescapable conclusion that Einstein field equations yield only the first order approximations of the correct metrics when the energy-momentum tensor T_{jk} and thus the Ricci tensor are set to zero. While the study of Einstein field equations and various Einstein Spaces described by these equations can be an interesting and intellectually rewarding experience with a large amount of work already devoted to this topic, [5] it is clear that very little of this work can actually be applied to reality.

7. Conclusion

In this paper it has been clearly shown that GRT is only a first order approximation of reality and thus it can be concluded that Einstein field equations and their various derivatives should not be used to search for the metrics to model the strong gravitational fields, be used in various string theories, or be used in modeling of the Universe. The assumption about the Ricci tensor being zero in an empty space around the gravitating body is not correct, the zero is not a permissible value for this tensor when the centrally gravitating mass generating the field is not zero and consequently the Schwarzschild metric does not describe the reality accurately and correctly.

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