The Black Hole, the Big Bang – A Cosmology in Crisis

Stephen J. Crothers
Queensland, Australia
thenarmis@gmail.com

Introduction

It is often claimed that cosmology became a true scientific inquiry with the advent of the General Theory of Relativity. A few subsequent putative observations have been misconstrued in such a way as to support the prevailing Big Bang model by which the Universe is alleged to have burst into existence from an infinitely dense point-mass singularity. Yet it can be shown that the General Theory of Relativity and the Big Bang model are in conflict with well-established experimental facts.

Black holes are not without cosmological significance in view of the many claims routinely made for them, and so they are treated here in some detail. But the theory of black holes is riddled with contradictions and has no valid basis in observation. Nobody has ever found a black hole, even though claims for their discovery are now made on an almost daily basis. Nobody has ever found an infinitely dense point-mass singularity and nobody has ever found an event horizon, the tell-tale signatures of the black hole, and so nobody has ever found a black hole. In actuality, astrophysical scientists merely claim that there are phenomena observed about a region that they cannot see and so they illogically conclude that the unseen region must be a black hole, simply because they believe in black holes. In this way they can and do claim the presence of a black hole as they please. But that is not how science is properly done. Moreover, all black hole solutions pertain to one alleged mass in the Universe, whereas there are no known solutions to Einstein's field equations for two or more masses, such as two black holes. In other words, the astrophysics community has no solution to Einstein's field equations that can account for the presence of two or more bodies, yet they claim the existence of black holes in multitudes, interacting with one another and other matter.

Owing to the very serious problems with the Big Bang hypothesis and the theory of black holes, it is fair to say that neither meets the requirements of a valid physical theory. They are products of a peer review system that has gone awry, having all the characteristics of a closed academic club of mutual admiration and benefit into which new members are strictly by invitation only. The upshot of this is that the majority of the current astrophysics community is imbued with the dogmas of the academic club and the voice of dissent conveniently ignored or ridiculed, contrary to the true spirit of scientific inquiry. This method has protected funding interests but has done much harm to science.

Infinite Density Forbidden

Like the Big Bang progenitor, the black hole is alleged to possess an infinitely dense point-mass singularity. The black hole singularity is said to be produced by irresistible gravitational collapse (see for example [1-6]). According to Hawking [5]:

"*The work that Roger Penrose and I did between 1965 and 1970 showed that, according to general relativity, there must be a singularity of infinite density, within the black hole*."

Dodson and Poston [1] assert:

"*Once a body of matter, of any mass m, lies inside its Schwarzschild radius 2m it undergoes gravitational collapse . . . and the singularity becomes physical, not a limiting fiction*."

According to Carroll and Ostlie [3],

"*A nonrotating black hole has a particularly simple structure. At the center is the singularity, a point of zero volume and infinite density where all of the black hole's mass is located. Spacetime is infinitely curved at the singularity. . . . The black hole's singularity is a real physical entity. It is not a mathematical artifact . . .* "

Recall that an inertial frame is just somewhere Newton's First Law holds:

A body will remain at rest or move in a straight line with a constant velocity unless acted upon by an outside force.

Einstein's postulates for Special Relativity are:

1. The speed of light in vacuum is the same for all inertial frames;
2. The laws of physics are the same for all inertial frames.

It follows from these two postulates that infinite density is forbidden because infinite energy is forbidden, or equivalently, because no material body can acquire the speed of light in vacuo. General Relativity cannot violate Special Relativity since the former is a said to be a generalisation of the latter; so it too forbids infinite density. That infinite density is forbidden by the Theory of Relativity is easily proven with nothing more than simple high school algebra.

According to Einstein absolute motion does not exist; only the relative motion between bodies is meaningful. Consider two masses M_o and m_o at rest, i.e. their relative velocity is zero. These masses are therefore called 'rest masses'. Let both masses be cuboid in shape, sides L_o and X_o respectively. The rest volumes are just $L^3{}_o$ and $X^3{}_o$ respectively. Now if the relative velocity has magnitude $v > 0$, from the perspective of mass M_o the other mass increases by

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

where c is the speed of light in vacuo. In addition, from the perspective of mass M_o the length of the side of the other mass, in the direction of motion, is decreased by

$$X = X_o\sqrt{1-\frac{v^2}{c^2}} \qquad (2)$$

The other sides of the mass m_o do not change. So mass M_o sees a volume

$$V = X_o^3\sqrt{1-\frac{v^2}{c^2}} \,.$$

Now recall that density D is the mass divided by the volume. Hence, the density mass M_o sees is,

$$D = \frac{m}{V} = \frac{m_o}{X_o^3\left(1-\frac{v^2}{c^2}\right)} \,. \qquad (3)$$

These three relations are reciprocal, i.e. the perspective of m_o is described by the same equations except that M_o and L_o replace m_o and X_o in them, so it doesn't matter who watches whom; the results are the same.

Now note that according to eq. (3), as $v \rightarrow c$, $D \rightarrow \infty$. Since, according to Special Relativity, no material object can acquire the speed c (this would require an infinite energy), infinite densities are forbidden by Special Relativity, and so point-mass singularities are forbidden. Since Special Relativity must manifest in sufficiently small regions of Einstein's gravitational field, and these regions can be located anywhere in the gravitational field, General Relativity too must thereby forbid infinite densities and hence forbid point-mass singularities. It does not matter how it is alleged that a point-mass singularity is generated by General Relativity because the infinitely dense point-mass cannot be reconciled with Special Relativity. Point-charges too are therefore forbidden by the Theory of Relativity since there can be no charge without mass.

The Signatures of a Black Hole

The signatures of the black hole are an infinitely dense point-mass singularity and an event horizon. But we have already seen that infinite density is forbidden by the Theory of Relativity. So the claim for an infinitely dense point-mass singularity is false. This result is sufficient to prove that black holes are not predicted by General Relativity at all. In an attempt to escape this dilemma, astrophysical scientists are quick to resort to the argument that at the singularity General Relativity "breaks down", and so it cannot describe what happens there, so that some kind of quantum theory of gravity is needed. Nonetheless, the black hole singularity is still said to be infinitely dense. If General Relativity breaks down at the alleged singularity, as they claim, then General Relativity cannot say anything about the singularity, let alone that it is infinitely dense. And there is no quantum theory of gravity to describe it or anything else gravitational. So the singularity is either infinitely dense, as they claim, or it cannot be described by General Relativity, which "breaks down" there, as they also claim. It can't be both, either at the same time or at different times, according to fancy. But in either case it is inconsistent with the Theory of Relativity since infinite density is strictly forbidden by the Theory.

It is noteworthy at this point that Newton's theory of gravitation does not predict black holes either, although it is often claimed that it does, in some form or another: we will come back to this point later.

What about the event horizon of the black hole? According to the theory of black holes it takes an infinite amount of time for an observer to watch an object (via the light from that object, of course) to fall down to the event horizon. It therefore takes an infinite amount of time for the observer to verify the existence of an event horizon and thereby confirm the presence of a black hole. However, nobody has been and nobody will be around for an infinite amount of time in order to verify the presence of an event horizon and hence the presence of a black hole. Nevertheless, scientists claim that black holes have been found all over the place. The fact is, nobody has assuredly found a black hole anywhere - no infinitely dense point-mass singularity and no event horizon.

Some black hole proponents are more circumspect in how they claim the discovery of their black holes. They instead say that their evidence for the presence of a black hole is indirect. But such indirect "evidence" cannot be used to justify the claim of a black hole, in view of the fatal contradictions and physically meaningless properties associated with infinitely dense point-mass singularities and event horizons. One could just as well assert the existence and presence of deep space unicorns on the basis of such indirect "evidence".

Some claim that the energy of a black hole of mass m is $E = mc^2$. But then they have an infinite density associated with a finite energy, which violates Special Relativity once again.

The concept of 'point-mass' is rather meaningless. A point is a mathematical object, not a physical object. Thus, a point has no mass and a mass is not a point. One cannot go to a shop and buy a bag full of points, but one can buy a bag full of marbles. Nonetheless, the astrophysics community would have us believe that points can be material and of infinite density. The 'point-mass' is a confounding of a mathematical object with a physical object and is therefore invalid.

It is also of great importance to be mindful of the fact that no observations gave rise to the notion of a black hole in the first place, for which a theory had to be developed. The black hole was wholly spawned in the reverse, i.e. it was created by theory and observations subsequently misconstrued to legitimize the theory. Reports of black holes all over the place is just wishful thinking in support of a belief; not factual in any way.

Black Hole Escape Velocity

It is widely held by astrophysicists and astronomers that a black hole has an escape velocity c (or $\geq c$, the speed of light in vacuo) [4, 5, 7-18]. Chandrasekhar [4] remarked, "*Let me be more precise as to what one means by a black hole. One says that a black hole is formed when the gravitational forces on the surface become so strong that light cannot escape from it. ... A trapped surface is one from which light cannot escape to infinity*."

According to Hawking [5],

"*Eventually when a star has shrunk to a certain critical radius, the gravitational field at the surface becomes so strong that the light cones are bent inward so much that the light can no longer escape. According to the theory of relativity, nothing can travel faster than light. Thus, if light cannot escape, neither can anything else. Everything is dragged back by the gravitational field. So one has a set of events, a region of space-time from which it is not possible to escape to reach a distant observer. Its boundary is called the event horizon. It coincides with the paths of the light rays that just fail to escape from the black hole.*

"*A neutron star has a radius of about ten miles, only a few times the critical radius at which a star becomes a black hole.*

"*I had already discussed with Roger Penrose the idea of defining a black hole as a set of events from which it is not possible to escape to a large distance. It means that the boundary of the black hole, the event horizon, is formed by rays of light that just fail to get away from the black hole. Instead, they stay forever hovering on the edge of the black hole*."

However, according to the alleged properties of a black hole, nothing at all can even leave the black hole. In the very same paper Chandrasekhar made the following quite typical contradictory assertion:

"*The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge*."

Hughes [15] reiterates,

"*Things can go into the horizon (from $r > 2M$ to $r < 2M$), but they cannot get out; once inside, all causal trajectories (timelike or null) take us inexorably into the classical singularity at $r = 0$.*

"*The defining property of black holes is their event horizon. Rather than a true surface, black holes have a 'one-way membrane' through which stuff can go in but cannot come out*."

Taylor and Wheeler [19] assert,

"*... Einstein predicts that nothing, not even light, can be successfully launched outward from the horizon ... and that light launched outward EXACTLY at the horizon will never increase its radial position by so much as a millimeter*."

In the Dictionary of Geophysics, Astrophysics and Astronomy [9], one finds the following assertions:

"***black hole*** *A region of spacetime from which the escape velocity exceeds the velocity of light. In Newtonian gravity the escape velocity from the gravitational pull of a spherical star of mass M and radius R is*

$$v_{esc} = \sqrt{\frac{2GM}{R}} ,$$

where G is Newton's constant. Adding mass to the star (increasing M), or compressing the star (reducing R) increases v_{esc}*. When the escape velocity exceeds the speed of light c, even light cannot escape, and the star becomes a black hole.*
The required radius R_{BH} *follows from setting* v_{esc} *equal to c*:

$$R_{BH} = \frac{2GM}{c^2}$$

... "*In General Relativity for spherical black holes (Schwarzschild black holes), exactly the same expression* R_{BH} *holds for the surface of a black hole. The surface of a black hole at* R_{BH} *is a null surface, consisting of those photon trajectories (null rays) which just do not escape to infinity. This surface is also called the black hole horizon*."

A. Guth [20] tells us,

"*. . . classically the gravitational field of a black hole is so strong that not even light can escape from its interior . . .* "

And according to the Collins Encyclopaedia of the Universe [17],

"***black hole*** *A massive object so dense that no light or any other radiation can escape from it; its escape velocity exceeds the speed of light*."

Now, if its escape velocity is really that of light in vacuo, then, by definition of escape velocity, light would escape from a black hole, and therefore be seen by all observers. If the escape velocity of the black hole is greater than that of light in vacuo, then light could emerge but not escape; there would therefore always be a class of observers that could see it. Not only that, if the black hole had an escape velocity, then material objects with an initial velocity less than the alleged escape velocity could leave the black hole, and therefore be seen by a class of observers, but not escape (just go out, come to a stop and then fall back), even if the escape velocity is $\geq c$. Escape velocity does not mean that objects cannot leave; it only means they cannot escape if they have an initial velocity less than the escape velocity. Hence, on the one hand it is alleged that black holes have an escape velocity $\geq c$, but on the other hand that nothing, including light, can even leave the black hole. The claims are contradictory - nothing but a meaningless play on the words "escape velocity" [24, 25]. Furthermore, escape velocity is a two-body concept (one body escapes from another), whereas the black hole is derived not from a two-body gravitational interaction, but from an alleged one-body concept (but which is in fact a no-body situation). The black hole has no escape velocity.

The Michell-Laplace Dark Body

It is also routinely asserted that the theoretical Michell-Laplace (M-L) dark body of Newton's theory, which has an escape velocity ≥ *c*, is a kind of black hole [3-5, 7, 9, 10, 22] or that Newton's theory somehow predicts "*the radius of a black hole*" [19]. Hawking remarks [5],

"*On this assumption a Cambridge don, John Michell, wrote a paper in 1783 in the Philosophical Transactions of the Royal Society of London. In it, he pointed out that a star that was sufficiently massive and compact would have such a strong gravitational field that light could not escape. Any light emitted from the surface of the star would be dragged back by the star's gravitational attraction before it could get very far. Michell suggested that there might be a large number of stars like this. Although we would not be able to see them because light from them would not reach us, we could still feel their gravitational attraction. Such objects are what we now call black holes, because that is what they are – black voids in space*."

In the Cambridge Illustrated History of Astronomy [23] it is asserted that,

"*Eighteenth-century speculators had discussed the characteristics of stars so dense that light would be prevented from leaving them by the strength of their gravitational attraction; and according to Einstein's General Relativity, such bizarre objects (today's 'black holes') were theoretically possible as end-products of stellar evolution, provided the stars were massive enough for their inward gravitational attraction to overwhelm the repulsive forces at work.*"

But the M-L dark body is not a black hole. The M-L dark body possesses an escape velocity, whereas the black hole has no escape velocity. Objects can leave the M-L dark body, but nothing can leave the black hole. There is no upper limit of the speed of a body in Newton's theory, so masses can always escape from the M-L dark body, provided they leave at or greater than the escape velocity. The M-L dark body does not require irresistible gravitational collapse, whereas the black hole does. It has no infinitely dense point-mass singularity, whereas the black hole does. It has no event horizon, whereas the black hole does. There is always a class of observers that can see the M-L dark body [24, 25], but there is no class of observers that can see the black hole. The M-L dark body can persist in a space which contains other matter and interact with that matter, but the spacetime of the black hole is devoid of other masses by construction and consequently cannot interact with anything. Thus, the M-L dark body does not possess the characteristics of the hypothesized black hole and so it is not a black hole.

Gravitational Collapse

Much of the justification for the notion of irresistible gravitational collapse into an infinitely dense point-mass singularity, and hence the formation of a black hole, is given to the analysis due to Oppenheimer and Snyder [30]. Hughes [15] relates it as follows;

"*In an idealized but illustrative calculation, Oppenheimer and Snyder ... showed in 1939 that a black hole in fact does form in the collapse of ordinary matter. They considered a 'star' constructed out of a pressureless 'dustball'. By Birkhof's Theorem, the entire exterior of this dustball is given by the Schwarzschild metric ... Due to the self-gravity of this 'star', it immediately begins to collapse. Each mass element of the pressureless star follows a geodesic trajectory toward the star's center; as the collapse proceeds, the star's density increases and more of the spacetime is described by the Schwarzschild metric. Eventually, the surface passes through* $r = 2M$. *At this point, the Schwarzschild exterior includes an event horizon: A black hole has formed. Meanwhile, the matter which formerly constituted the star continues collapsing to ever smaller radii. In short order, all of the original matter reaches* $r = 0$ *and is compressed (classically!) into a singularity* [4].

[4] *Since all of the matter is squashed into a point of zero size, this classical singularity must be modified in a complete, quantum description. However, since all the singular nastiness is hidden behind an event horizon where it is causally disconnected from us, we need not worry about it (at least for astrophysical black holes).*"

Note that the Principle of Superposition has been arbitrarily applied by Oppenheimer and Snyder, from the outset. They first *assume* a relativistic universe in which there are multiple mass elements present *a priori*, where the Principle of Superposition however, does not apply, and despite there being no solution or existence theorem for such configurations of matter in General Relativity. Then, all these mass elements "collapse" into a central point (zero volume; infinite density). However, such a collapse has not been given any specific general relativistic mechanism in this argument; it is simply asserted that the "collapse" is due to self-gravity. But the "collapse" cannot be due to Newtonian gravitation, given the resulting black hole, which does not occur in Newton's theory of gravitation. A Newtonian universe cannot "collapse" into a non-Newtonian universe. Moreover, the black hole so formed is in an empty universe, since the "Schwarzschild black hole" relates to *Ric* = 0, a spacetime that by construction contains no matter. Nonetheless, Oppenheimer and Snyder permit, within the context of General Relativity, the presence of and the gravitational interaction of many mass elements, which coalesce and collapse into a point of zero volume to form an infinitely dense point-mass singularity, when there is no demonstrated general relativistic mechanism by which any of this can occur.

Furthermore, nobody has ever observed a celestial body undergo irresistible gravitational collapse and there is no laboratory evidence whatsoever for such a phenomenon.

Einstein's Field Equations

In this section we will have need of the so-called "Schwarzschild solution":

$$ds^2 = \left(c^2 - \frac{2Gm}{r}\right)dt^2 - c^2\left(c^2 - \frac{2Gm}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{4}$$

The quantity c in expression (4) is the speed of light in vacuum and G is Newton's constant of universal gravitation.

The components of the metric tensor are easily read off expression (4), as follows:

$$g_{00} = \left(c^2 - \frac{2Gm}{r}\right), \quad g_{11} = -c^2\left(c^2 - \frac{2Gm}{r}\right)^{-1}, \quad g_{22} = -r^2, \quad g_{33} - r^2 \sin^2\theta .$$

The signature is the ordered set of the signs of the components of the metric tensor. In the above case the signature is $(+,-,-,-)$. Some writers use the equivalent signature $(-,+,+,+)$ and write the metric as

$$ds^2 = -\left(c^2 - \frac{2Gm}{r}\right)dt^2 + c^2\left(c^2 - \frac{2Gm}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

In either case the signature is correspondingly fixed and cannot change.

The astrophysics community usually sets $c = 1$ and $G = 1$ in equation (4) to get

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (5)$$

with corresponding changes in the expressions for the components of the metric tensor. The quantity *m* in these expressions is alleged to be mass – the mass causing the associated gravitational field. The quantity *r* appearing in these expressions has never been correctly identified by the theoreticians, as we shall soon see. It is from expressions (4) and (5) that the black hole was first conjured. Note that only one mass appears in these expressions and that the said mass *m* is alone in the Universe and so the associated black hole is alone in the Universe. This expression is *not* a solution for two or more masses. Nonetheless, the astrophysics community talks of the existence of black holes in multitudes.

It should be noted that neither expression (4) nor expression (5) is in fact Schwarzschild's solution. They are a corruption of Schwarzschild's solution, due to the mathematician David Hilbert. For comparison here is Schwarzschild's actual solution:

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$
$$R = \left(r^3 + \alpha^3\right)^{1/3}, \qquad 0 < r < \infty, \qquad \alpha = const.$$

There is no black hole in Schwarzschild's solution. Indeed, his solution precludes the black hole, and for this reason he never spoke of the black hole.

We will also have need of the following expression for Minkowski spacetime, in which the "laws" of Special Relativity operate:

$$ds^2 = c^2 dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

Once again it is usual to set $c = 1$ to get,

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \qquad (6)$$

Note there is no appearance of any quantity representing matter in this expression (called a metric). It contains no matter. The quantity *c* is a speed, not a photon, the maximum speed that a moving point is allowed to acquire in this geometry. Note that the signature of Minkowski spacetime is $(+,-,-,-)$ and cannot change: for instance, to $(-,+,-,-)$ (otherwise it would not be Minkowski spacetime).

The foregoing expressions are called line-elements, or metrics, which are nothing but fancy names for a distance equation, like that learnt in high school. In the foregoing expressions *ds* denotes an element of distance in space-time. In each metric *ds* is made up of a time-like quantity, *t*, and three space-like quantities, *r*, *θ*, *φ*. In this way it is claimed in astrophysical circles that time *t* is the fourth dimension of a 4-dimensional space-time continuum. The terms containing *r*, *θ* and *φ* collectively constitute the spatial section of the space-time. Thus, in expression (6) the spatial section is described by $d\rho^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$. In the case of expression (5) the spatial section is given by

$$d\rho^2 = \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

In the usual interpretation of Hilbert's [34-37] corrupted version of Schwarzschild's solution, the quantity *r* has never been properly identified by astrophysics. It has been variously and vaguely called a *"distance"* [7, 38] "*the radius*" [1, 2, 7, 8, 9-14, 21, 28, 39, 40-42], the "*radius of a 2-sphere*" [42, 43] the "*coordinate radius*" [44], the "*radial coordinate*" [3, 4, 14, 19, 28, 45], the "*Schwarzschild r-coordinate*" [45] , the "*radial space coordinate*" [46], the "*areal radius*" [3, 4, 15, 22, 44], the "*reduced circumference*" [19], and even "*a gauge choice: it defines the coordinate r*". In the particular case of $r = 2Gm/c^2$ it is almost invariably referred to as the "*Schwarzschild radius*" or the "*gravitational radius*" [45]. However, none of these various and vague concepts of *r* are correct because the *irrefutable* geometrical fact is that *r*, in the spatial section of Hilbert's version of the Schwarzschild/Droste line-elements, is the inverse square root of the Gaussian curvature (see Appendix) of the spherically symmetric geodesic surface in the spatial section [32-34], and as such, it *does not* itself determine the geodesic radial distance from the centre of spherical symmetry located at an arbitrary point in the related pseudo-Riemannian metric manifold. It does not denote any distance at all in the spherically symmetric metric manifold for "Schwarzschild" spacetime. It must also be emphasized that a geometry is completely determined by the *form* of its line-element [50, 51], of which signature is a characteristic.

The correct geometric identification of the quantity *r* in Hilbert's solution *completely subverts* all claims for black holes.

According to Einstein, matter is the cause of the gravitational field and the causative matter is described in his theory by a mathematical object called the energy-momentum tensor, which is coupled to geometry (i.e. spacetime) by his field equations, so that matter causes spacetime curvature (his gravitational field) and spacetime constrains motion of matter when gravity alone acts. According to the astrophysics community, Einstein's field equations,

"*... couple the gravitational field (contained in the curvature of spacetime) with its sources*." [31]

"*Since gravitation is determined by the matter present, the same must then be postulated for geometry, too. The geometry of space is not given a priori, but is only determined by matter."* [52]

"*Again, just as the electric field, for its part, depends upon the charges and is instrumental in producing mechanical interaction between the charges, so we must assume here that the metrical field (or, in mathematical language, the tensor with components g_{ik}) is related to the material filling the world*." [38]

"... we have, in following the ideas set out just above, to discover the invariant law of gravitation, according to which matter determines the components $\Gamma^{\alpha}_{\beta\iota}$ of the gravitational field, and which replaces the Newtonian law of attraction in Einstein's Theory." [38]

"*Thus the equations of the gravitational field also contain the equations for the matter (material particles and electromagnetic fields) which produces this field.*" [29]

"*Clearly, the mass density, or equivalently, energy density $\rho(\vec{x},t)$ must play the role as a source. However, it is the 00 component of a tensor $T_{\mu\nu}(x)$, the mass-energy-momentum distribution of matter. So, this tensor must act as the source of the gravitational field.* [41]

"*In general relativity, the stress-energy or energy-momentum tensor T^{ab} acts as the source of the gravitational field. It is related to the Einstein tensor and hence to the curvature of spacetime via the Einstein equation*." [28]

"*Mass acts on spacetime, telling it how to curve. Spacetime in turn acts on mass, telling it how to move*." [3]

Qualitatively Einstein's field equations are:

Spacetime geometry = -ô â *causative matter*

where *causative matter* is described by the energy-momentum tensor and κ is a constant. The *spacetime geometry* is described by a mathematical object called Einstein's tensor, $G_{\mu\nu}$, (μ, ν = 0, 1, 2, 3) and the energy-momentum tensor is $T_{\mu\nu}$. Einstein's field equations are therefore[1]:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} \,. \tag{7}$$

[1] The so-called "cosmological constant" is not included.

$R_{\mu\nu}$ is called the Ricci tensor and R the Ricci curvature. If $T_{\mu\nu} = 0$ then one finds that $R = 0$ and this expression allegedly reduces to

$$R_{\mu\nu} = 0 \tag{8}$$

and describes a universe that contains no matter.

In the transition from Minkowski spacetime of Special Relativity to Schwarzschild spacetime for the black hole, matter is not involved. The speed of light c that appears in the Minkowski spacetime line-element is a speed, not a photon. For this speed to be assigned to a photon, the photon must be present *a priori*. Similarly, for the relations of Special Relativity to hold, multiple arbitrarily large finite masses must also be present *a priori*. Minkowski spacetime is not Special Relativity because the latter requires the presence of matter, whereas the former does not. Similarly, the presence of the constant c in the line-element for Schwarzschild spacetime does not mean that a photon is present. The transition from Minkowski spacetime to Schwarzschild spacetime is thus not a generalisation of Special Relativity at all, merely a generalisation of the geometry of Minkowski spacetime. In the usual derivation of Schwarzschild spacetime, mass is included by means of a sophistic argument, viz. $R_{\mu\nu} = 0$ describes the gravitational field "outside a body". When one inquires of the astrophysics community as to what is the source of this alleged gravitational field "outside a body", one is told that it is *the body*, in which case the body must be described by a non-zero energy-momentum tensor since Einstein's field equations "… *couple the gravitational field ... with its sources*" [31]. Dirac [32] tells us that

"*…the constant of integration m that has appeared ... is just the mass of the central body that is producing the gravitational field.*"

We are told by Einstein [53] that,

"*… M denotes the sun's mass centrally symmetrically placed about the origin of coordinates*."

According to Weyl [38],

"*… the quantity m_o introduced by the equation $m=km_o$ occurs as the field-producing mass in it; we call m the gravitational radius of the matter causing the disturbances of the field*."

Foster and Nightingale [31] assert that

"*…the corresponding Newtonian potential is $V = -GM/r$, where M is the mass of the body producing the field, and G is the gravitational constant ... we conclude that $k = -2GM/c^2$ and Schwarzschild's solution for the empty space outside a spherical body of mass M is …*"

After the "Schwarzschild" solution is obtained there is no matter present. This is because the energy-momentum tensor is set to zero and Minkowski spacetime is not

Special Relativity. The astrophysics community merely inserts (Weyl says "*introduced*") mass and photons by erroneously appealing to Newton's theory through which they also get any number of masses and any amount of radiation by applying the Principle of Superposition. This is done despite the fact that the Principle of Superposition does not apply in General Relativity. However, Newton's relations, as explained above, involve two bodies and the Principle of Superposition. Conversely, $R_{\mu\nu} = 0$ contains *no bodies* and cannot accommodate the Principle of Superposition. The astrophysics community removes all matter on the one hand by setting $R_{\mu\nu} = 0$ and then puts it back in at the end with the other hand by means of Newton's theory. The whole procedure constitutes a violation of elementary logic.

Einstein asserted that his Principle of Equivalence and his laws of Special Relativity must hold in sufficiently small regions of his gravitational field, and that these regions can be located anywhere in his gravitational field. Here is what Einstein [53] said in 1954, the year before his death:

"*Let now K be an inertial system. Masses which are sufficiently far from each other and from other bodies are then, with respect to K, free from acceleration. We shall also refer these masses to a system of co-ordinates K', uniformly accelerated with respect to K. Relatively to K' all the masses have equal and parallel accelerations; with respect to K' they behave just as if a gravitational field were present and K' were unaccelerated. Overlooking for the present the question as to the 'cause' of such a gravitational field, which will occupy us later, there is nothing to prevent our conceiving this gravitational field as real, that is, the conception that K' is 'at rest' and a gravitational field is present we may consider as equivalent to the conception that only K is an 'allowable' system of co-ordinates and no gravitational field is present. The assumption of the complete physical equivalence of the systems of coordinates, K and K', we call the 'principle of equivalence'; this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation. For, according to our way of looking at it, the same masses may appear to be either under the action of inertia alone (with respect to K) or under the combined action of inertia and gravitation (with respect to K').*

"*Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of special relativity, which have been developed above, hold with remarkable accuracy*."

In their textbook, Foster and Nightingale [31] succinctly state the Principle of Equivalence thus:

"*We may incorporate these ideas into the principle of equivalence, which is this: In a freely falling (nonrotating) laboratory occupying a small region of spacetime, the laws of physics are the laws of special relativity*."

According to Pauli [52],

"*We can think of the physical realization of the local coordinate system* K_o *in terms of a freely floating, sufficiently small, box which is not subjected to any external forces apart from gravity, and which is falling under the influence of the latter. ... It is evidently natural to assume that the special theory of relativity should remain valid in* K_o.

Taylor and Wheeler state in their book [19],

"*General Relativity requires more than one free-float frame*."

Carroll and Ostlie write [3],

"***The Principle of Equivalence****: All local, freely falling, nonrotating laboratories are fully equivalent for the performance of all physical experiments. ... Note that special relativity is incorporated into the principle of equivalence. ... Thus general relativity is in fact an extension of the theory of special relativity*."

In the Dictionary of Geophysics, Astrophysics and Astronomy [9],

"*Near every event in spacetime, in a sufficiently small neighborhood, in every freely falling reference frame all phenomena (including gravitational ones) are exactly as they are in the absence of external gravitational sources*."

Note that the Principle of Equivalence involves the *a priori* presence of multiple arbitrarily large finite masses. Similarly, the laws of Special Relativity involve the *a priori* presence of at least two arbitrarily large finite masses (and at least one photon); for otherwise relative motion between two bodies cannot manifest. The postulates of Special Relativity are themselves couched in terms of inertial systems, which are in turn defined in terms of mass via Newton's First Law of motion. "Schwarzschild's solution" (and indeed all black hole "solutions"), pertains to one mass in a universe that contains no other masses. According to the astrophysics community, "Schwarzschild" spacetime consists of one mass in an otherwise *totally empty universe*, and so its alleged black hole is the only matter present - it has nothing to interact with, including "observers" (on the assumption that any observer is material).
In the space of Newton's theory of gravitation, one can insert into space as many masses as desired. Although solving for the gravitational interaction of these masses rapidly becomes intractable, there is nothing to prevent us inserting masses conceptually. This is essentially the Principle of Superposition. However, one cannot do this in General Relativity, because Einstein's field equations are non-linear. In General Relativity, each and every configuration of matter must be described by a corresponding energy-momentum tensor and the field equations solved separately for each and every configuration, because matter and geometry are coupled, as eq. (7) describes. This is not the case in Newton's theory, where geometry is independent of matter. The Principle of Superposition does not apply in General Relativity:

"*In a gravitational field, the distribution and motion of the matter producing it cannot at all be assigned arbitrarily --- on the contrary it must be determined (by solving the field equations for given initial conditions) simultaneously with the field produced by the same matter*." [29]

"*An important characteristic of gravity within the framework of general relativity is that the theory is nonlinear. Mathematically, this means that if g_{ab} and γ_{ab} are two solutions of the field equations, then $ag_{ab} + b\gamma_{ab}$ (where a, b are scalars) may not be a solution. This fact manifests itself physically in two ways. First, since a linear combination may not be a solution, we cannot take the overall gravitational field of the two bodies to be the summation of the individual gravitational fields of each body*." [28]

The astrophysics community claims that the gravitational field "outside" a mass contains no matter, and thereby asserts that the energy-momentum tensor $T_{\mu\nu} = 0$. Despite this, it is routinely alleged that there is only one mass in the whole Universe with this particular problem statement. But setting the energy-momentum tensor to zero means that there is no matter present by which the gravitational field can be caused! As we have seen, when the energy-momentum tensor is set to zero, it is also claimed that the field equations then reduce to the much simpler form,

$$Ric = R_{\mu\nu} = 0.$$

"*Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time*." [9]

However, since this is a spacetime that *by construction* contains no matter, Einstein's Principle of Equivalence and his laws of Special Relativity cannot manifest, thus violating the physical requirements of the gravitational field. It has never been proven that Einstein's Principle of Equivalence and his laws of Special Relativity, both of which are defined in terms of the *a priori* presence of multiple arbitrary large finite masses and photons, can manifest in a spacetime that *by construction* contains no matter. Now eq. (4) relates to eq. (8). However, there is allegedly mass present, denoted by *m* in eq. (4). This mass is not described by an energy-momentum tensor. The reality that the *post hoc* mass *m* is responsible for the alleged gravitational field due to a black hole associated with eq. (4) is confirmed by the fact that if $m = 0$, eq. (4) reduces to Minkowski spacetime, and hence no gravitational field according to the astrophysics community. If not for the presence of the alleged mass *m* in eq. (4) there would be no cause of their gravitational field. But this contradicts Einstein's relation between geometry and matter, since *m* is introduced into eq. (4) *post hoc*, not via an energy-momentum tensor describing the matter causing the associated gravitational field.

In Schwarzschild spacetime, the components of the metric tensor are only functions of one another, and reduce to functions of just one component of the metric tensor. None of the components of the metric tensor contain matter, because the energy-momentum tensor is zero. There is no transformation of matter in Minkowski spacetime into Schwarzschild spacetime, and so the laws of Special Relativity are not transformed into a gravitational field by *Ric* = 0. The transformation is merely from a pseudo-Euclidean geometry containing no matter into a pseudo-Riemannian (non-Euclidean) geometry containing no matter. Matter is introduced into the spacetime of *Ric* = 0 by means of a vicious circle, as follows. It is stated at the outset that *Ric* = 0 describes spacetime “outside a body”. The words “outside a body” introduce matter, contrary to the energy-momentum tensor, $T_{\mu\nu} = 0$, that describes the causative matter as being absent. There is no matter involved in the transformation of Minkowski spacetime into Schwarzschild spacetime via *Ric* = 0, since the energy-momentum tensor is zero, making the components of the resulting metric tensor functions solely of one another, and reducible to functions of just one component of the metric tensor. To satisfy the initial claim that *Ric* = 0 describes spacetime “outside a body”, so that the resulting spacetime curvature is caused by the alleged mass present, the alleged causative mass is *inserted* into the resulting metric *ad hoc*. This is achieved by means of a contrived analogy with Newton’s theory and his expression for escape velocity (a *two-body* relation in what is alleged to be a one-body problem – see Appendix), thus closing the vicious circle. Here is how Chandrasekhar [4] presents the vicious circle:

“*That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass M and radius R, it must be projected with a velocity v such that* $\frac{1}{2}v^2 > GM/R$*; and it cannot escape if* $v^2 < 2GM/R$ *. On the basis of this last inequality, Laplace concluded that if* $R < 2GM/c^2 = R_s$ *(say) where c denotes the velocity of light, then light will not be able to escape from such a body and we will not be able to see it!*

“By a curious coincidence, the limit R_s discovered by Laplace is exactly the same that general relativity gives for the occurrence of the trapped surface around a spherical mass.”

But it is not surprising that general relativity gives the same R_s “*discovered by Laplace*” because the Newtonian expression for escape velocity is deliberately inserted *post hoc* by the astrophysicists and astronomers, into the “Schwarzschild solution” (equation (4) above). Newton’s escape velocity does not drop out of any of the calculations to Schwarzschild spacetime. Furthermore, although $R_{\mu\nu} = 0$ is said to describe spacetime “outside a body”, the resulting metric (4) is nonetheless used to describe the *interior* of a black hole, since the black hole begins at the alleged “event horizon”, not at its infinitely dense point-mass singularity (allegedly at *r* = 0 in equation (4)).

In the case of a totally empty Universe, what would be the relevant energy-momentum tensor? It must be $T_{\mu\nu} = 0$. Indeed, it is also maintained by the astrophysics community that spacetimes can be intrinsically curved, i.e. that there are gravitational fields that have no material cause. An example is de Sitter’s empty spherical Universe, based upon the following “field” equations [50, 54]:

$$R_{\mu\nu} = \lambda g_{\mu\nu} \tag{9}$$

where λ is the so-called “cosmological constant”. In the case of metric (4) the field equations are given by expression (8). On the one hand, de Sitter’s empty world is devoid of matter ($T_{\mu\nu} = 0$) and therefore has no material cause for the alleged associated gravitational field. On the other hand, it is stated that the spacetime described by eq. (8) has a material cause, *post hoc* as m in metric (4), even though $T_{\mu\nu} = 0$ there as well: a contradiction. This is amplified by the so-called "Schwarzschild-de Sitter" line-element,

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{2m}{r} - \frac{\lambda}{3}r^2\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (10)$$

which is the standard solution for eq. (9). Once again, m is inserted *post hoc* as mass at the centre of spherical symmetry of the manifold, said to be at $r = 0$. The completely empty universe of de Sitter [50, 54] can be obtained by setting $m = 0$ in eq. (10) to yield,

$$ds^2 = \left(1 - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{\lambda}{3}r^2\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \qquad (11)$$

Also, if $\lambda = 0$, eq. (9) reduces to eq. (8) and eq. (10) reduces to eq. (5). If both $\lambda = 0$ and $m = 0$, eqs. (10) and (11) reduce to Minkowski spacetime. Now in eq. (10), the term $\lambda g_{\mu\nu}$ is not an energy-momentum tensor, since according to the astrophysics community, expression (11) describes a world devoid of matter [35, 41]. The universe described by eq. (11), which also satisfies eq. (9), is completely empty and so its curvature has no material cause; in eq. (11), just as in eq. (9), $T_{\mu\nu} = 0$. Thus, eq. (11) is alleged to describe a gravitational field that has no material cause. Furthermore, although in eq. (9), $T_{\mu\nu} = 0$, its usual solution, eq. (5), is said to contain a (*post hoc*) material cause, denoted by m therein. Thus, for eq. (5), it is postulated that $T_{\mu\nu} = 0$ supports a material cause of a gravitational field. At the same time, for eq. (11), $T_{\mu\nu} = 0$ precludes material cause of a gravitational field. $T_{\mu\nu} = 0$ therefore includes and excludes material cause. This is not possible. The contradiction is due to the *post hoc* introduction of mass, as m, in equations (4) and (5), by means of the Newtonian expression for escape velocity (which is an implicit two-body relation). Furthermore, there is no experimental evidence to support the claim that a gravitational field can be generated without a material cause. Material cause is codified theoretically in eq. (7).

Black Hole Interactions

The literature abounds with claims that black holes can interact in such situations as binary systems, mergers, collisions, and with surrounding matter generally. Bearing in mind that all black holes "solutions" pertain to a universe that contains only one mass (the black hole itself), concepts involving multiple black holes tacitly assume application of the Principle of Superposition, which however, as we have seen, does not apply in General Relativity. According to Chandrasekhar [4],

"*From what I have said, collapse of the kind I have described must be of frequent occurrence in the Galaxy; and black-holes must be present in numbers comparable to, if not exceeding, those of the pulsars. While the black-holes will not be visible to external observers, they can nevertheless interact with one another and with the outside world through their external fields.*

"*In considering the energy that could be released by interactions with black holes, a theorem of Hawking is useful. Hawking's theorem states that in the interactions involving black holes, the total surface area of the boundaries of the black holes can never decrease; it can at best remain unchanged (if the conditions are stationary*).

"*Another example illustrating Hawking's theorem (and considered by him) is the following. Imagine two spherical (Schwarzschild) black holes, each of mass ½M, coalescing to form a single black hole; and let the black hole that is eventually left be, again, spherical and have a mass M. Then Hawking's theorem requires that*

$$16\pi\overline{M}^2 \geq 16\pi\left[2\left(\frac{1}{2}M\right)^2\right] = 8\pi M^2$$

or

$$\overline{M} \geq \frac{M}{\sqrt{2}}\,.$$

Hence the maximum amount of energy that can be released in such a coalescence is $M\left(1-1/\sqrt{2}\right)c^2 = 0.293M^2$.

Hawking [5] says,

"*Also, suppose two black holes collided and merged together to form a single black hole. Then the area of the event horizon of the final black hole would be greater than the sum of the areas of the event horizons of the original black holes*."

According to Schutz [26],

"*... Hawking's area theorem: in any physical process involving a horizon, the area of the horizon cannot decrease in time. ... This fundamental theorem has the result that, while two black holes can collide and coalesce, a single black hole can never bifurcate spontaneously into two smaller ones*.

"*Black holes produced by supernovae would be much harder to observe unless they were part of a binary system which survived the explosion and in which the other star was not so highly evolved*."

Townsend [27] also arbitrarily and incorrectly applies the Principle of Superposition to obtain multiple charged black hole (Reissner-Nordström) interactions as follows:

"*The extreme RN in isotropic coordinates is*

$$ds^2 = V^{-2}dt^2 + V^2\left(d\rho^2 + \rho^2 d\Omega^2\right)$$

where

$$V = 1 + \frac{M}{\rho}\,.$$

This is a special case of the multi black hole solution

$$ds^2 = V^{-2}dt^2 + V^2 d\vec{x} \cdot d\vec{x}$$

where $ds^2 = V^2 d\vec{x} \cdot d\vec{x}$ *is the Euclidean 3-metric and V is any solution of* $\nabla^2 V = 0$. *In particular*

$$V = 1 + \Sigma_{i=1}^{N} \frac{M_i}{\left|\vec{x} - \vec{\vec{x}}_i\right|}$$

yields the metric for N extreme black holes of masses M_i *at positions* $\vec{\vec{x}}_i$."

Carroll and Ostlie remark [3],

"*The best hope of astronomers has been to find a black hole in a close binary system. ... If a black hole coalesces with any other object, the result is an even larger black hole. ... If one of the stars in a close binary system explodes as a supernova, the result may be either a neutron star or a black hole orbiting the companion star. ... the procedure for detecting a black hole in a binary x-ray system is similar to that used to measure the masses of neutron stars in these systems. ... What is the fate of a binary x-ray system? As it reaches the endpoint of its evolution, the secondary star will end up as a white dwarf, neutron star, or black hole.*"

But Einstein's field equations are non-linear. Thus, despite the claims of the astrophysics community, the Principle of Superposition *does not apply* [24, 28, 29]. Therefore, before one can talk of black hole binary systems and the like it must first be proven that the two-body system is theoretically well-defined by General Relativity. This can be accomplished in only two ways:

1. Derivation of an exact solution to Einstein's field equations for the two-body configuration of matter; or
2. Proof of an existence theorem.

However, there are no known solutions to Einstein's field equations for the interaction of two (or more) masses (charged or not). Furthermore, no existence theorem has ever been proven, by which Einstein's field equations can even be said to admit of latent solutions for such configurations of matter. The "Schwarzschild" black hole is allegedly obtained from a line-element satisfying *Ric* = 0. For the sake of argument, assume that black holes are predicted by General Relativity as alleged in relation to metric (5). Since *Ric* = 0 is a statement that there is no matter in the Universe, one cannot simply insert a second black hole into the spacetime of *Ric* = 0 of a given black hole, so that the resulting two black holes (each obtained separately from *Ric* = 0) simultaneously persist in, and mutually interact in, a mutual spacetime that *by construction contains no matter*! One cannot simply assert by an analogy with Newton's theory that two black holes can be components of binary systems, collide, or merge [24, 25, 29], again because the Principle of Superposition does not apply in Einstein's theory. Moreover, General Relativity has to date been unable to account for the simple experimental fact that two fixed bodies will approach one another upon release. Thus, black hole binaries, collisions, mergers, black holes from supernovae, and other black hole interactions are all invalid concepts.

Consequences of $R_{\mu\nu} = 0$

Since $R_{\mu\nu} = 0$ cannot describe Einstein's gravitational field, Einstein's field equations cannot reduce to $R_{\mu\nu} = 0$ when $T_{\mu\nu} = 0$. In other words, if $T_{\mu\nu} = 0$ (i.e. there is no matter present) then there is no gravitational field. Consequently Einstein's field equations must take the form [55],

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0. \tag{12}$$

The $G_{\mu\nu} / \kappa$ are the components of a gravitational energy tensor. Thus the total energy of Einstein's gravitational field is *always zero*; the $G_{\mu\nu} / \kappa$ and the $T_{\mu\nu}$ *must vanish identically*; there is *no possibility* for the localization of gravitational energy (i.e. there are no Einstein gravitational waves). This also means that Einstein's gravitational field violates the experimentally well-established usual conservation of energy and momentum [53]. Since there is no experimental evidence that the usual conservation of energy and momentum is invalid, Einstein's General Theory of Relativity violates the experimental evidence, and so it is invalid.

In an attempt to circumvent the foregoing conservation problem, Einstein invented his gravitational pseudo-tensor, the components of which he says are '*the* "*energy components*" *of the gravitational field*' [60]. His invention had a two-fold purpose (a) to bring his theory into line with the usual conservation of energy and momentum, (b) to enable him to get gravitational waves that propagate with speed *c*. First, Einstein's gravitational pseudo-tensor is not a tensor, and is therefore not in keeping with his theory that all equations be tensorial. Second, he constructed his pseudo-tensor in such a way that it behaves like a tensor in one particular situation, that in which he could get gravitational waves with speed *c*. Now Einstein's pseudo-tensor is claimed to represent the energy and momentum of the gravitational field and it is routinely applied in relation to the localization of gravitational energy, the conservation of energy and the flow of energy and momentum.

Dirac [32] pointed out that,

"It is not possible to obtain an expression for the energy of the gravitational field satisfying both the conditions: (i) when added to other forms of energy the total energy is conserved, and (ii) the energy within a definite (three dimensional) region at a certain time is independent of the coordinate system. Thus, in general, gravitational energy cannot be localized. The best we can do is to use the pseudotensor, which satisfies condition (i) but not condition (ii). It gives us approximate information about gravitational energy, which in some special cases can be accurate."

On gravitational waves Dirac [32] remarked,

"Let us consider the energy of these waves. Owing to the pseudo-tensor not being a real tensor, we do not get, in general, a clear result independent of the coordinate

system. But there is one special case in which we do get a clear result; namely, when the waves are all moving in the same direction."

About the propagation of gravitational waves Eddington [54] remarked $\left(g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}\right)$,

$$\frac{\partial^2 h_{\mu\nu}}{\partial t^2} - \frac{\partial^2 h_{\mu\nu}}{\partial x^2} - \frac{\partial^2 h_{\mu\nu}}{\partial y^2} - \frac{\partial^2 h_{\mu\nu}}{\partial z^2} = 0\,,$$

"... showing that the deviations of the gravitational potentials are propagated as waves with unit velocity, i.e. the velocity of light. But it must be remembered that this representation of the propagation, though always permissible, is not unique. ... All the coordinate-systems differ from Galilean coordinates by small quantities of the first order. The potentials $g_{\mu\nu}$ pertain not only to the gravitational influence which is objective reality, but also to the coordinate-system which we select arbitrarily. We can 'propagate' coordinate-changes with the speed of thought, and these may be mixed up at will with the more dilatory propagation discussed above. There does not seem to be any way of distinguishing a physical and a conventional part in the changes of the $g_{\mu\nu}$.

"The statement that in the relativity theory gravitational waves are propagated with the speed of light has, I believe, been based entirely upon the foregoing investigation; but it will be seen that it is only true in a very conventional sense. If coordinates are chosen so as to satisfy a certain condition which has no very clear geometrical importance, the speed is that of light; if the coordinates are slightly different the speed is altogether different from that of light. The result stands or falls by the choice of coordinates and, so far as can be judged, the coordinates here used were purposely introduced in order to obtain the simplification which results from representing the propagation as occurring with the speed of light. The argument thus follows a vicious circle."

Now Einstein's pseudo-tensor, $\sqrt{-g}\, t_\nu^\mu$, is defined by [7, 32, 32, 38, 42, 44, 50, 52, 54-56],

$$\sqrt{-g}\, t_\nu^\mu = \frac{1}{2}\left[\delta_\nu^\mu L - \left(\frac{\partial L}{\partial g^{\sigma\beta}_{,\mu}}\right) g^{\sigma\beta}_{,\nu}\right] \tag{13}$$

wherein L is given by

$$L = -g^{\alpha\beta}\left(\Gamma^\gamma_{\alpha\kappa}\Gamma^\kappa_{\beta\gamma} - \Gamma^\gamma_{\alpha\beta}\Gamma^\kappa_{\gamma\kappa}\right). \tag{14}$$

According to Einstein [56] his pseudo-tensor "*expresses the law of conservation of momentum and of energy for the gravitational field*."

In a remarkable paper published in 1917, T. Levi-Civita [55] provided a clear and rigorous proof that Einstein's pseudo-tensor is meaningless, and therefore any argument relying upon it is fallacious. I repeat Levi-Civita's proof. Contracting eq. (13) produces a linear invariant, thus

$$\sqrt{-g}\, t_{\mu}^{\mu} = \frac{1}{2}\left[4L - \left(\frac{\partial L}{\partial g_{,\mu}^{\sigma\beta}}\right) g_{,\mu}^{\sigma\beta}\right]. \tag{15}$$

Since L is, according to eq. (14), quadratic and homogeneous with respect to the Riemann-Christoffel symbols, and therefore also with respect to $g_{,\mu}^{\sigma\beta}$, one can apply Euler's theorem to obtain (also see [34]),

$$\left(\frac{\partial L}{\partial g_{,\mu}^{\sigma\beta}}\right) g_{,\mu}^{\sigma\beta} = 2L. \tag{16}$$

Substituting expression (16) into expression (15) yields the linear invariant at L. This is a first-order, intrinsic differential invariant, i.e. it depends only on the components of the metric tensor and their first derivatives (see expression (14) for L). However, the mathematicians G. Ricci-Curbastro and T. Levi-Civita [55] proved, in 1900, that such invariants *do not exist*. This is sufficient to render Einstein's pseudo-tensor entirely meaningless, and hence all arguments relying on it false. Consequently, Einstein's conception of the conservation of energy in the gravitational field is erroneous.

Linearization of Einstein's field equations and associated perturbations has been popular. "*The existence of exact solutions corresponding to a solution to the linearised equations must be investigated before perturbation analysis can be applied with any reliability*" [44]. Unfortunately, the astrophysical scientists have not properly investigated. Indeed, linearisation of the field equations is inadmissible, even though the astrophysical scientists write down linearised equations and proceed as though they are valid, because linearisation of the field equations implies the existence of a tensor which, except for the trivial case of being precisely zero, *does not otherwise exist*; proven by the German mathematician Hermann Weyl [57] in 1944.

Over a period of some 40 years and at great monetary expense, the international search for Einstein's gravitational waves has detected nothing. This is not surprising – the search for these waves is destined to detect none.

It follows from $R_{\mu\nu} = 0$ that not only is the black hole invalid but so too is the Big Bang and the associated expansion of the Universe. The violation of the usual conservation of energy and momentum cannot be circumvented in order to save General Relativity from the dustbin of scientific history.

Observational Evidence for the Black Hole

It is nowadays routinely reported that many black holes have been found. Yet the signatures of the black hole are (a) an infinitely dense 'point-mass' singularity and (b) an 'event horizon'. Nobody has ever found an infinitely dense 'point-mass' singularity and nobody has ever found an 'event horizon', so nobody has ever assuredly found a black hole. And we have seen that it takes an infinite amount of observer time to verify a black hole event horizon [4, 15, 26, 27, 31-33]. Nobody has been around and nobody will be around for an infinite amount of time and so no observer can ever verify the presence of an event horizon, and hence a black hole, in principle; the notion is irrelevant to physics. Moreover, an 'observer' cannot be present in a spacetime that by construction contains no matter (i.e. $R_{\mu\nu} = 0$), or in a universe that contains only one mass, by construction. All reports of black holes being found are patently false. The search for black holes is destined to detect none.

The Cosmic Microwave Background

It is now well known that the ubiquitous radiation at ~2.7 K discovered in 1965 by Penzias and Wilson [58] is said to be the afterglow of the birth of the Universe – the Big Bang. This notion has found its way into the popular press and even high school textbooks. It is routinely claimed that this afterglow is associated with an expansion of the Universe from a primordial singularity, predicted by Einstein's General Theory of Relativity. This alleged afterglow is usually referred to as the Cosmic Microwave Background (CMB).

The so-called expansion of the Universe is further alleged to be validated by the Hubble-Humason relation. However, Hubble and Humason, building upon the observational work of Slipher, proposed red-shift in spectra with distance, not a red-shift with recessional velocity. The former has been reinterpreted as the latter in order to forge a correspondence with theory. The Big Bang was spawned by theory, not observation, and has found no definite physical support.

The history of the temperature of the Universe is rather chequered. Mitchell [59] relates that Dicke predicted a temperature of 20 K, in 1946, a figure he revised to 40 K in the 1960's, and later to 45 K. Peebles, a colleague of Dicke, changed the temperature to approximately 10 K, which he later modified to ~3 K to agree with the findings of Penzias and Wilson. Gamow obtained by calculation a temperature of 50K which was, in 1948, revised to 5K by Alpher and Herman, both students of Gamow. About a year or so later Alpher and Herman changed it to 28 K.

Mitchell relates further that a number of scientists predicted a theoretical thermal background due to starlight without the influence of the General Theory of Relativity and the Big Bang dogma. The Noble prize winning chemist Walther Nernst gave 0.75 K in 1938. In 1926 Arthur Eddington proposed 3.2 K. Ernst Regener suggested 2.8 K in the 1930's, and astronomer Andrew McKellar 2.3 K in 1941. This thermal background, it is to be noted, was not attributed by these scientists to left-over radiation from cosmic genesis. All but for Penzias and Wilson are theoretical values according to a number of different theories, but it is the Big Bang interpretation that now prevails. It is claimed that the isotropic nature of the CMB detected by Penzias and Wilson attests to its origin in a Big Bang. Interestingly, the theoretical values suggested without the Big Bang are closer to that detected by Penzias and Wilson.

That the theoretical temperature of the alleged afterglow has been revised so arbitrarily and so often gives cause at least for suspicion that it has no valid basis in science. Although the Big Bang and the expansion of the Universe are physically unproven they are generally regarded by a large number of astrophysical scientists as unquestionable, contrary to the very spirit of scientific inquiry. This unscientific method now pervades much of science; astrophysics and astronomy in particular. Despite evidence, both physical and theoretical, that the Big Bang and expansion are without scientific justification, astrophysics clings steadfastly to cherished theory on the unscientific basis that consensus can't be wrong. That the Big Bang and expansion of the Universe have taken hold of astrophysics is remarkable and reflects a serious decline in scientific thought.

Professor Pierre-Marie Robitaille of Ohio State University, a leading expert in imaging science, has shown conclusively, in a series of quite brilliant papers [60-64], that the CMB has not been measured by the WMAP and COBE satellites. So riddled are they with design and data analysis flaws that neither satellite has contributed anything of value to science. He has also shown that the European Space Agency's Planck satellite is no better and predicts that Planck will find no signal at the second Lagrange point of the Sun-Earth system, where Planck is located. Moreover, Robitaille reassigns the CMB to the oceans of the Earth, pointing out that water is a powerful absorber and emitter in the microwave and far infrared, well known at sea in submarines and at home in microwave ovens, owing to the hydrogen bonds associated with water molecules. Emissions from the oceans are scattered by the atmosphere thereby producing an isotropic signal from an anisotropic source with a resulting temperature profile that does not reflect that of the source of the radiation (the oceans). It is this Earth emitted signal that Penzias and Wilson detected and which has tantalized astrophysics ever since. That the Earth has not been reported to interfere with the instruments aboard WMAP and COBE is, according to Robitaille, precisely because the Earth is the source of the signal.

Mathematical Appendix

Gaussian Curvature

Consider Hilbert's "Schwarzschild solution",

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \qquad \text{(A1)}$$

The spatial section of this metric is

$$ds^2 = \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) . \qquad \text{(A2)}$$

If r is constant, the spherically symmetric geodesic surface in the spatial section is given by

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \qquad \text{(A3)}$$

We will now identify r. The Gaussian curvature of a two dimensional surface is given by

$$K = \frac{R_{1212}}{g} \qquad \text{(A4)}$$

where R_{1212} is a component of the Riemann tensor of the 1st kind and $g = g_{11}g_{22} = g_{\theta\theta}g_{\varphi\varphi}$ (because the metric tensor of eq. (A3) is diagonal). Gaussian curvature is an intrinsic geometric property of a surface (Theorema Egregium[2]); independent of any embedding space.

Now recall from elementary differential geometry and tensor analysis that

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma} R^{\lambda}_{.\,\nu\rho\sigma}$$

$$R^{1}_{.212} = \frac{\partial \Gamma^{1}_{22}}{\partial x^{1}} - \frac{\partial \Gamma^{1}_{21}}{\partial x^{2}} + \Gamma^{k}_{22}\Gamma^{1}_{k1} - \Gamma^{k}_{21}\Gamma^{1}_{k2}$$

$$\Gamma^{i}_{ij} = \Gamma^{i}_{ji} = \frac{\partial\left(\frac{1}{2}\ln|g_{ii}|\right)}{\partial x^{j}}$$

$$\Gamma^{i}_{jj} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^{i}} \quad (i \neq j) \qquad \text{(A5)}$$

and all other Γ^{i}_{jk} vanish. In the above, $i, j, k = 1, 2; x^1 = \theta, x^2 = \varphi$. Applying expressions (A4) and (A5) to expression (A3) gives,

$$K = \frac{1}{r^2} \qquad \text{(A6)}$$

so that r is the inverse square root of the Gaussian curvature in eq. (A3) and hence also in eq. (A1). Gaussian curvature is intrinsic to a surface and a geometry is completely determined by the form of its line-element. Thus, r is not the radial geodesic distance from the centre of spherical symmetry of the spatial section of expression (A1), or any other distance therein.

The erroneous nature of the concepts '*reduced circumference*' and '*areal radius*' is now plainly evident - neither concept correctly identifies the geometric nature of the quantity r in metric (A1). The arc-length s in the spherically symmetric geodesic surface in the spatial section of eq. (A1) is a function of the curvilinear coordinate θ or φ and the surface area A_p is a function of the curvilinear coordinates θ and φ where, in both cases, r is a constant.

However, r therein has a clear and definite geometrical meaning: it is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section. The Gaussian curvature K is a positive constant bending invariant of the surface, independent of the values of θ and φ. Thus, neither s nor A_p, or the

[2] i.e. Gauss' Most Excellent Theorem.

infinite variations of them by means of the integrated values of θ and φ, rightly identify what r is in line-element (A1). To illustrate further, when θ = constant, the arc-length in the spherically symmetric geodesic surface is given by:

$$s = s(\varphi) = r\sin\theta\int_0^{\varphi} d\varphi = r\varphi\sin\theta, \qquad 0 \le \varphi \le 2\pi,$$

where r = constant. This is the equation of a straight line, of gradient $ds/d\varphi = r\sin\theta$. If θ = const. = ½ π, then $s = s(\varphi) = r\varphi$, which is the equation of a straight line of gradient $ds/d\varphi = r$. The maximum arc-length of the geodesic θ = const. = ½ π is therefore $s(2\pi) = 2\pi r = C_p$. Similarly the surface area is:

$$A = A(\theta,\varphi) = r^2\int_0^{\theta}\sin\theta\, d\theta\int_0^{\varphi} d\varphi = r^2\varphi(1-\cos\theta),$$

$$0 \le \varphi \le 2\pi, \quad 0 \le \theta \le \pi, \quad r = const.$$

The maximum area (*i.e.* the area of the entire surface) is $A_p(\pi, 2\pi) = 4\pi r^2$. Clearly, neither s nor A are functions of r, because r is a constant, not a variable. And since r appears in each expression (and so having the same value in each expression), neither s nor A rightly identify the geometrical significance of r in the first fundamental form for the spherically symmetric geodesic surface, $ds^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$, because r is *not* a distance in the surface and is *not* the 'radius' of the surface. The geometrical significance of r is intrinsic to the surface and is determined from the components of the metric tensor and their derivatives (Gauss' Theorema Egregium): it is the inverse square root of the Gaussian curvature K of the spherically symmetric surface so described (the constant is $K = 1/r^2$). Thus, s and A_p are merely platitudinous expressions containing the constant r, and so the 'reduced circumference' $r = C_p/2\pi$ and the '*areal radius*' $r = \sqrt{A_p/4\pi}$ do not identify the geometric nature of r in either metric (A3) or metric (A1); the former appearing in the latter. The claims by the astrophysical scientists that the '*areal radius*' and the '*reduced circumference*' each define (in two different ways) the constant r in eq. (A1) are entirely false. The "*reduced circumference*" and the "*areal radius*" are in fact one and the same, namely the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section of eq. (A1), as proven above. No proponent of black holes is aware of this simple geometrical fact, which completely subverts all claims made for black holes being predicted by General Relativity.

It can be shown [49] that the "Schwarzschild solution" is a particular case of an infinite set of equivalent solutions taking the form

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$R_c = \left(\left|r - r_o\right|^n + \alpha^n\right)^{\frac{1}{n}}$$

$$r \in \Re \ , \ \ n \in \Re^+$$

where α is a constant of integration and r and n are entirely arbitrary real constants. Choosing $n = 3$, $r_o = 0$, $r > r_o$ gives

$$R_c = \left(r^3 + \alpha^3\right)^{\frac{1}{3}}, \quad 0 < r < \infty$$

which is Schwarzschild's actual solution [65]. Choosing $n = 1$, $r_o = 0$, $r > r_o$ gives

$$R_c = r + \alpha\,, \quad 0 < r < \infty$$

which is Brillouin's solution [66]. Choosing $n = 1$, $r_o = \alpha$, $r > r_o$ gives

$$R_c = (r - \alpha) + \alpha = r\,, \quad \alpha < r < \infty$$

which is Droste's solution [67]. Droste's solution is the correct form of Hilbert's "Schwarzschild solution".

Choosing $n = 1$, $r_o = \alpha$, $r < r_o$ gives

$$R_c = 2\alpha - r\,, \quad -\infty < r < \alpha.$$

More exotic equivalent solutions can be easily formed. For instance, take $n = \pi$, $r_o = -e$, $r < r_o$ to get

$$R_c = \left(\left|r + e\right|^\pi + \alpha^\pi\right)^{\frac{1}{\pi}} = \left[(-e - r)^\pi + \alpha^\pi\right]^{\frac{1}{\pi}}, \quad -\infty < r < -e,$$

where e is Euler's number. Note that in every single case, $R_c(r_o) = \alpha$, irrespective of the admissible values selected for n and r_o: this is a scalar invariant. In no case can a black hole result.

The radial distance R_p from the point at the centre of spherical symmetry of Schwarzschild space is given by [49]

$$R_p = \int_o^{R_p} dR_p = \int \frac{dR_c}{\sqrt{1 - \frac{\alpha}{R_c}}} = \sqrt{R_c(R_c - \alpha)} + \alpha \ln\left(\sqrt{R_c} + \sqrt{R_c - \alpha}\right) + C$$

where the constant C takes the value $C = -\alpha\ln\sqrt{\alpha}$, so that when $R_c(r_o) = \alpha$, the radius $R_p = 0$, which occurs for all admissible values of n and r_o.

The proponents of the Standard Model admit that if $0 < r < 2m$ in eq. (A1), the rôles of t and r are interchanged. In addition, this violates their construction, which has the fixed signature $(+,-,-,-)$, and is therefore inadmissible. To further illustrate this violation, when $2m < r < \infty$ the signature of eq. (A1) is $(+,-,-,-)$; but if $0 < r < 2m$ in eq. (A1), then

$$g_{00} = \left(1 - \frac{2m}{r}\right) \text{ is negative}$$

and

$$g_{11} = \left(1 - \frac{2m}{r}\right)^{-1} \text{ is positive.}$$

So the signature of metric (A1) changes to $(-,+,-,-)$. Thus the rôles of t and r are interchanged. According to Misner, Thorne and Wheeler [45], who use the spacetime signature $(-,+,+,+)$,

"The most obvious pathology at $r = 2M$ is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region $r > 2M$, the t direction, $\partial / \partial t$, is timelike ($g_{tt} < 0$) and the r direction, $\partial / \partial r$, is spacelike ($g_{rr} > 0$); but in the region $r < 2M$, $\partial / \partial t$, is spacelike ($g_{tt} > 0$) and $\partial / \partial r$, is timelike ($g_{rr} < 0$).

"What does it mean for r to 'change in character from a spacelike coordinate to a timelike one'? The explorer in his jet-powered spaceship prior to arrival at $r = 2M$ always has the option to turn on his jets and change his motion from decreasing r (infall) to increasing r (escape). Quite the contrary in the situation when he has once allowed himself to fall inside $r = 2M$. Then the further decrease of r represents the passage of time. No command that the traveler can give to his jet engine will turn back time. That unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate $r = 2M$ to the later time coordinate $r = 0$. No human act of will, no engine, no rocket, no force (see exercise 31.3) can make time stand still. As surely as cells die, as surely as the traveler's watch ticks away 'the unforgiving minutes', with equal certainty, and with never one halt along the way, r drops from 2M to 0.

"At $r = 2M$, where r and t exchange roles as space and time coordinates, g_{tt} vanishes while g_{rr} is infinite."

Chandrasekhar [4] has expounded the same claim as follows,

'There is no alternative to the matter collapsing to an infinite density at a singularity once a point of no-return is passed. The reason is that once the event horizon is passed, all time-like trajectories must necessarily get to the singularity: "all the King's horses and all the King's men" cannot prevent it.'

Carroll [42] also says,

"This is worth stressing; not only can you not escape back to region I, you cannot even stop yourself from moving in the direction of decreasing r, since this is simply the timelike direction. (This could have been seen in our original coordinate system; for r < 2GM, t becomes spacelike and r becomes timelike.) Thus you can no more stop moving toward the singularity than you can stop getting older."

Vladmimirov, Mitskiévich and Horský [33] assert,

"For $r < 2GM/c^2$, however, the component g_{oo} becomes negative, and g_{rr}, positive, so that in this domain, the role of time-like coordinate is played by r, whereas that of space-like coordinate by t. Thus in this domain, the gravitational field depends significantly on time (r) and does not depend on the coordinate t."

To amplify this, set $t = r^*$ and $r = t^*$, and so for $0 < r < 2m$, eq. (A1) becomes,

$$ds^2 = \left(1 - \frac{2m}{t^*}\right)dr^{*2} - \left(1 - \frac{2m}{t^*}\right)^{-1} dt^{*2} - t^{*2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$0 < t^* < 2m.$$

It now becomes quite clear that this is a time-dependent metric since all the components of the metric tensor are functions of the timelike t^*, and so this metric bears no relationship to the original time-independent problem to be solved [35, 46]. In other words, this metric is a non-static solution to a static problem: contra hype! Thus, in eq. (A1), $0 < r < 2m$ is meaningless.

Furthermore, if the signature of "Schwarzschild" spacetime is permitted to change from $(+,-,-,-)$ to $(-,+,-,-)$ in the fashion claimed for black holes then there must be for the latter signature a corresponding generalisation of the Minkowski metric, taking the fundamental form

$$ds^2 = -e^{\lambda}dt^2 + e^{\beta}dr^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

where λ, β, and R are all unknown real-valued functions of only the now time-like real variable r, and where, by construction, $e^{\lambda} > 0$ and $e^{\beta} > 0$. But this is impossible because the Minkowski spacetime metric has the fixed signature $(+,-,-,-)$, since the spatial section of Minkowski spacetime is a positive definite quadratic form; and so the foregoing generalised metric is not a generalization of Minkowski spacetime at all. The metric for Minkowski spacetime cannot be written as

$$ds^2 = -dt^2 + dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

Newtonian Relations

We will now amplify the inadmissibility of the introduction of the Newtonian potential into Schwarzschild spacetime. The Newtonian potential is a *two-body* concept; it is defined as the work done per unit mass against the gravitational field of some other mass. There is no meaning to a Newtonian potential for a single mass in an otherwise empty Universe. Newton's theory of gravitation is defined in terms of the interaction of two masses in a space for which the Principle of Superposition applies. It is clearly impossible for Schwarzschild spacetime, which is alleged by the astrophysical scientists to contain one mass in an otherwise totally empty Universe, to reduce to or otherwise contain an expression that is defined in terms of the *a priori* interaction of two masses. This is illustrated even further by examining eq. (4). The term $2Gm/r$ is recognized as the square of the Newtonian escape velocity from a mass m: and so the astrophysical scientists assert that when the 'escape velocity' is that of light in vacuum, there is an event horizon ('Schwarzschild radius') and hence a black hole. But escape velocity is a concept that implicitly involves two bodies - one body escapes from another body. Even though one mass appears in the expression for escape velocity, it cannot be determined without recourse to a fundamental two-body gravitational interaction. Recall that Newton's Universal Law of Gravitation is,

$$F_g = -\frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the centre of mass of m and the centre of mass of M. A center of mass is an infinitely dense point-mass, but it is not a physical object; merely a mathematical artifice. Newton's gravitation is clearly defined in terms of the interaction of two bodies. Newton's gravitational potential Φ is defined as,

$$\Phi = -\int_{\infty}^{r} \frac{F_g}{m}\, dr = -\frac{GM}{r},$$

which is the work done per unit mass in the gravitational field due to masses M and m, against the gravitational field of the mass M. This is a two-body concept. The potential energy P of a mass m in the gravitational field of a mass M is therefore given by,

$$P = m\Phi = -\frac{GmM}{r}$$

which is clearly a two-body concept.

Similarly, the velocity required by a mass m to escape from a mass M is determined by,

$$F_g = -\frac{GmM}{r^2} = ma = m\frac{dv}{dt} = mv\frac{dv}{dr}.$$

Separating variables and integrating gives,

$$\int_{v}^{0} mv\, dv = \lim_{\rho\to\infty} \int_{R}^{\rho} -\frac{GmM}{r^2}\, dr$$

whence

$$v = \sqrt{\frac{2GM}{R}},$$

where R is the radius of the mass M. Thus, escape velocity necessarily involves two bodies: m escapes from M. In terms of the conservation of kinetic and potential energies,

$$K_i + P_i = K_f + P_f$$

whence,

$$\frac{1}{2}mv^2 - \frac{GmM}{R} = \frac{1}{2}mv_f^2 - \frac{GmM}{r_f}.$$

Then as $r_f \to \infty$, $v_f \to 0$, and the escape velocity of m from M is,

$$v = \sqrt{\frac{2GM}{R}}.$$

Once again, the relation is derived from an *a priori* two-body gravitational interaction. That two bodies are present is implicit in these expressions.

Clearly, the introduction of Newtonian concepts into Hilbert's solution is inadmissible.

Three-Dimensional Spherically Symmetric Metric Manifolds - First Principles

To complete the purely mathematical foundations of this exposition, the differential geometry expounded in the foregoing is now developed from first principles.

Following the method suggested by Palatini, and developed by Levi-Civita [47] denote ordinary Euclidean 3-space by $\mathbf{E}^3$. Let $\mathbf{M}^3$ be a 3-dimensional metric manifold. Let there be a one-to-one correspondence between all points of $\mathbf{E}^3$ and $\mathbf{M}^3$. Let the point $O \in \mathbf{E}^3$ and the corresponding point in $\mathbf{M}^3$ be O'. Then a point transformation $\boldsymbol{T}$ of $\mathbf{E}^3$ into itself gives rise to a corresponding point transformation of $\mathbf{M}^3$ into itself.

A rigid motion in a metric manifold is a motion that leaves the metric dl'^2 unchanged. Thus, a rigid motion changes geodesics into geodesics. The metric manifold $\mathbf{M}^3$ possesses spherical symmetry around any one of its points O' if each of the ∞^3 rigid rotations in $\mathbf{E}^3$ around the corresponding arbitrary point O determines a rigid motion in $\mathbf{M}^3$.

The coefficients of dl'^2 of $\mathbf{M}^3$ constitute a metric tensor and are naturally assumed to be regular in the region around every point in $\mathbf{M}^3$, except possibly at an arbitrary point, the centre of spherical symmetry $O' \in \mathbf{M}^3$.

Let a ray i emanate from an arbitrary point $O \in \mathbf{E}^3$. There is then a corresponding geodesic $i' \in \mathbf{M}^3$ issuing from the corresponding point $O' \in \mathbf{M}^3$. Let P be any point on i other than O. There corresponds a point P' on $i' \in \mathbf{M}^3$ different to O'. Let g' be a geodesic in $\mathbf{M}^3$ that is tangential to i' at P'.

Taking i as the axis of ∞^1 rotations in $\mathbf{E}^3$, there corresponds ∞^1 rigid motions in $\mathbf{M}^3$ that leaves only all the points on i' unchanged. If g' is distinct from i', then the ∞^1 rigid rotations in $\mathbf{E}^3$ about i would cause g' to occupy an infinity of positions in $\mathbf{M}^3$ wherein g' has for each position the property of being tangential to i' at P' in the same direction, which is impossible. Hence, g' coincides with i'.

Thus, given a spherically symmetric surface Σ in $\mathbf{E}^3$ with centre of symmetry at some arbitrary point $O \in \mathbf{E}^3$, there corresponds a spherically symmetric geodesic surface Σ' $\in \mathbf{M}^3$ with centre of symmetry at the corresponding point $O' \in \mathbf{M}^3$.

Let Q be a point in $\Sigma \in \mathbf{E}^3$ and Q' the corresponding point in $\Sigma' \in \mathbf{M}^3$. Let $d\sigma^2$ be a generic line element in Σ issuing from Q. The corresponding generic line element $d\sigma'^2 \in \Sigma'$ issues from the point Q'. Let Σ be described in the usual spherical-polar coordinates r, θ, φ. Then

$$d\sigma^2 = r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (1.1)$$
$$r = \left|\overline{O}\overline{Q}\right|.$$

Clearly, if r, θ, φ are known, Q in Σ is determined and hence also Q' in Σ'. Therefore, θ and φ can be considered to be curvilinear coordinates for Q' in Σ', and the line element $d\sigma' \in \Sigma'$ will also be represented by a quadratic form similar to (1.1). To determine $d\sigma'$, consider two elementary arcs of equal length, $d\sigma_1$ and $d\sigma_2$ in Σ, drawn from the point Q in different directions. Then the homologous arcs in Σ' will be $d\sigma'_1$ and $d\sigma'_2$, drawn in different directions from the corresponding point Q'. Now $d\sigma_1$ and $d\sigma_2$ can be obtained from one another by a rotation about the axis $\overline{O}\overline{Q}$ in $\mathbf{E}^3$, and so $d\sigma'_1$ and $d\sigma'_2$ can be obtained from one another by a rigid motion in $\mathbf{M}^3$, and are therefore also of equal length, since the metric is unchanged by such a motion. It therefore follows that the ratio $d\sigma'/d\sigma$ is the same for the two different directions irrespective of $d\theta$ and $d\varphi$, and so the foregoing ratio is a function of position, i.e. of r, θ, φ. But Q is an arbitrary point in Σ, and so $d\sigma'/d\sigma$ must have the same ratio for any corresponding points Q and Q'. Therefore, $d\sigma'/d\sigma$ is a function of r alone, and thus

$$\frac{d\sigma'}{d\sigma} = H(r)\,,$$

and so

$$d\sigma'^2 = H^2(r)d\sigma^2 = H^2(r)r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (1.2)$$

where $H(r)$ is *a priori* unknown. For convenience set $R_c = R_c(r) = H(r)r$, so that (1.2) becomes

$$d\sigma'^2 = R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (1.3)$$

where R_c is a quantity associated with $\mathbf{M}^3$. Comparing (1.3) with (1.1) it is apparent that R_c is to be rightly interpreted in terms of the Gaussian curvature K at the point *Q'*, i.e. in terms of the relation $K = 1/R_c^2$ since the Gaussian curvature of (1.1) is $K = 1/r^2$. This is an intrinsic property of all line elements of the form (1.3) [47]. Accordingly, R_c, the inverse square root of the Gaussian curvature, can be regarded as the radius of Gaussian curvature. Therefore, in (1.1) the radius of Gaussian curvature is $R_c = r$. Moreover, owing to spherical symmetry, all points in the corresponding surfaces Σ and Σ' have constant Gaussian curvature relevant to their respective manifolds and centres of symmetry, so that all points in the respective surfaces are umbilics.

Let the element of radial distance from $O \in \mathbf{E}^3$ be *dr*. Clearly, the radial lines issuing from *O* cut the surface Σ orthogonally. Combining this with (1.1) by the theorem of Pythagoras gives the line element in $\mathbf{E}^3$

$$dl^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{1.4}$$

Let the corresponding radial geodesic from the point $O' \in \mathbf{M}^3$ be dR_p. Clearly the radial geodesics issuing from *O'* cut the geodesic surface Σ' orthogonally.

Combining this with (1.3) by the theorem of Pythagoras gives the line element in $\mathbf{M}^3$ as,

$$dl'^2 = dR_p^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{1.5}$$

where dR_p is, by spherical symmetry, also a function only of R_c. Set $dR_p = \sqrt{B(R_c)}dR_c$ so that (1.5) becomes

$$dl'^2 = B(R_c)dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad 1.6)$$

where $B(R_c)$ is an *a priori* unknown function.

Expression (1.6) is the most general for a metric manifold $\mathbf{M}^3$ having spherical symmetry about some arbitrary point $O' \in \mathbf{M}^3$.

Considering (1.4), the distance $R_p = \left|\overline{OQ}\right|$ from the point at the centre of spherical symmetry *O* to a point $Q \in \Sigma$, is given by

$$R_p = \int_0^r dr = r = R_c .$$

Call R_p the proper radius. Consequently, in the case of $\mathbf{E}^3$, R_p and R_c are identical, and so the Gaussian curvature of the spherically symmetric geodesic surface containing any point in $\mathbf{E}^3$ can be associated with R_p, the radial distance between the centre of spherical symmetry at the point $O \in \mathbf{E}^3$ and the point $Q \in \Sigma$. Thus, in this case, $K = 1/R_c^2 = 1/R_p^2 = 1/r^2$. However, this is not a general relation, since according to

(1.5) and (1.6), in the case of $\mathbf{M}^3$, the radial geodesic distance from the centre of spherical symmetry at the point $O' \in \mathbf{M}^3$ is not the same as the radius of Gaussian curvature of the associated spherically symmetric geodesic surface, but is given by

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{B(R_c(r))}\, dR_c = \int_0^r \sqrt{B(R_c(r))} \frac{dR_c(r)}{dr} dr$$

where $R_c(0)$ is *a priori* unknown owing to the fact that $R_c(r)$ is *a priori* unknown. One cannot simply assume that because $0 \le r < \infty$ in (1.4) that it must follow that in (1.5) and (1.6) $0 \le R_c < \infty$. In other words, one cannot simply assume that $R_c(0) = 0$. Furthermore, it is evident from (1.5) and (1.6) that R_p determines the radial geodesic distance from the centre of spherical symmetry at the arbitrary point O' in $\mathbf{M}^3$ (and correspondingly so from O in $\mathbf{E}^3$) to another point in $\mathbf{M}^3$. Clearly, R_c does not in general render the radial geodesic length from the point at the centre of spherical symmetry to some other point in a metric manifold. Only in the particular case of $\mathbf{E}^3$ does R_c render both the radius of Gaussian curvature of an associated spherically symmetric surface and the radial distance from the point at the centre of spherical symmetry, owing to the fact that R_p and R_c are identical in that special case.

It should also be noted that in writing expressions (1.4) and (1.5) it is implicit that $O \in \mathbf{E}^3$ is defined as being located at the origin of the coordinate system of (1.4), i.e. O is located where $r = 0$, and by correspondence O' is defined as being located at the origin of the coordinate system of (1.5) and of (1.6); $O' \in \mathbf{M}^3$ is located where $R_p = 0$. Furthermore, since it is well known that a geometry is completely determined by the form of the line element describing it [50], expressions (1.4), (1.5) and (1.6) share the very same fundamental geometry because they are line elements of the same metrical groundform.

Expression (1.6) plays an important rôle in Einstein's alleged gravitational field.

References

[1] Dodson C. T. J. and Poston T. Tensor Geometry -- The Geometric Viewpoint and its Uses, 2nd Ed., Springer--Verlag, 1991.

[2] NASA Goddard Space Flight Center, http://imagine.gsfc.nasa.gov/docs/science/know_l1/black_holes.html, http://imagine.gsfc.nasa.gov/docs/science/know_l2/black_holes.html

[3] Carroll B. W. and Ostlie D. A. An Introduction to Modern Astrophysics, Addison--Wesley Publishing Company Inc., 1996.

[4] Chandrasekhar S. The increasing role of general relativity in astronomy, *The Observatory*, 92, 168, 1972.

[5] Hawking S. W. The Theory of Everything, The Origin and Fate of the Universe; New Millennium Press, Beverly Hills, CA., 2002.

[6] National Center for Supercomputing Applications, The Board of Trustees of the University of Illinois, http://archive.ncsa.uiuc.edu/Cyberia/NumRel/BlackHoleFormation.html

[7] Percy J. University of Toronto, Astronomical Society of the Pacific, 390 Ashton Avenue, San

Francisco, CA 94112, www.astrosociety.org/education/publications/tnl/24/24.html

[8] Cambridge University, UK, www.damtp.cam.ac.uk/user/gr/public/bh_home.html

[9] DICTIONARY OF GEOPHYSICS, ASTROPHYSICS, and ASTRONOMY, Edited by Richard A. Matzner, CRC Press LLC,, Boca Raton, USA, 2001, http://www.deu.edu.tr/userweb/emre.timur/dosyalar/Dictionary\%20of \%20Geophysics,\%20Astrophysics\%20and\%20Astronomy.pdf

[10] Harrison D. M. Black holes, University of Toronto, www.upscale.utoronto.ca/GeneralInterest/Harrison/BlackHoles/BlackHoles.html

[11] Kunstatter, G. Black holes, Physics Department University of Winnipeg, http://theory.uwinnipeg.ca/users/gabor/black_holes/

[12] http://design.lbl.gov/education/blackholes/index.html

[13] National Maritime Museum, London, UK, www.nmm.ac.uk/explore/astronomy-and-time/astronomy-facts/stars/black-holes/*/viewPage/2

[14] Information Leaflet No. 9: ‘Blackholes’, Science and Engineering Research Council Royal Greenwich Observatory, http://www.nmm.ac.uk/?PHPSESSID=0093df5b6c376566dfcf1e17a6c4b117

[15] Hughes S. A. Trust but verify: The case for astrophysical black holes, Department of Physics and MIT Kavli Institute, 77 Massachusetts Avenue, Cambridge, MA 02139, SLAC Summer Institute 2005.

[16] Center for Supercomputing Applications, The Board of Trustees of the University of Illinois, http://archive.ncsa.uiuc.edu/Cyberia/NumRel/BlackHoles.html

[17] Collins Encyclopedia of the Universe, HarperCollins Publishers, London, 2001.

[18] The Penguin Dictionary of Physics, 3rd Ed., Penguin Books Ltd., London, 2000.

[19] Taylor E. F. and Wheeler J. A. Exploring Black Holes - Introduction to General Relativity, Addison Wesley Longman, 2000 (in draft).

[20] Guth A. The Inflationary Universe (The quest for a new theory of cosmic origins), Jonathan Cape, London, 1977.

[21] Celloti A., Miller J. C., Sciama D. W. Astrophysical evidence for the existence of black holes, (1999), http://arxiv.org/abs/astro-ph/9912186

[22] Ludvigsen M. General Relativity -- A Geometric Approach, Cambridge University Press, Cambridge, UK, 1999.

[23] The Cambridge Illustrated History of Astronomy, Edited by Michael Hoskin, Cambridge University Press, Cambridge, UK, 1997.

[24] McVittie G. C. Laplace's alleged “black hole”. *The Observatory*, v.98, 272, 1978.

[25] Crothers S. J. A Brief History of Black Holes. *Progress in Physics*, v.2, 54--57, 2006, www.ptep-online.com/index_files/2006/PP-05-10.PDF

[26] Schutz B. F. A first course in general relativity, Cambridge University Press, UK, 1990.

[27] Townsend P. K. Black Holes, lecture notes, DAMTP, University of Cambridge, Silver St.,

Cambridge, U.K., 4 July, 1977, http://arxiv.org/abs/gr-qc/9707012

[28] McMahon D. Relativity Demystified, A Self-teaching guide; McGraw-Hill, New York, 2006.

[29] Landau L. and Lifshitz E. The Classical Theory of Fields, Addison-Wesley Publishing Company, Inc., Reading, Massachusettes, 1951.

[30] Oppenheimer J. R. and Snyder H. *Phys. Rev*. 56, 455 (1939).

[31] Foster J. and Nightingale J. D. A short course in General Relativity, Springer-Verlag, New York, Inc., 1995.

[32] Dirac P. A. M. General Theory of Relativity. Princeton Landmarks in Physics Series, Princeton University Press, Princeton, New Jersey, 1996.

[33] Vladimirov Yu., Mitskiévich, N., Horský, J. Space Time Gravitation, Mir Publishers, Moscow, 1984.

[34] Abrams L. S. Black holes: the legacy of Hilbert's error. *Can. J. Phys*., v.\,67, 919, 1989, http://arxiv.org/pdf/gr-qc/0102055

[35] Antoci S. David Hilbert and the origin of the "Schwarzschild" solution. 2001, http://arxiv.org/pdf/physics/0310104

[36] Loinger A. On black holes and gravitational waves. La Goliardica Paves, Pavia, 2002.

[37] Dunning-Davies J. Exploding a myth, Conventional wisdom or scientific truth?, Horwood Publishing Limited, Chichester, 2007 (ISBN: 978-1-904275-30-5)

[38] Weyl H. Space Time Matter, Dover Publications Inc., New York, 1952.

[39] O'Neill B. www.math.ucla.edu/~bon/kerrhistory.html

[40] Nemiroff R. Virtual Trips to Black Holes and Neutron Stars, Michigan Technological University and NASA Goddard, http://antwrp.gsfc.nasa.gov/htmltest/rjn_bht.html

[41] 't Hooft G. Introduction to the theory of black holes, Version February 5, 2009, (Lectures presented at Utrecht University, 2009), Institute for Theoretical Physics Utrecht University, Princetonplein 5, 3584 CC Utrecht, the Netherlands, www.phys.uu.nl/~thooft/lectures/blackholes/BH_lecturenotes.pdf

[42] Carroll S. Lecture Notes on General Relativity, http://arxiv.org/abs/gr-qc/9712019

[43] Bruhn G. W. http://www.mathematik.tu-darmstadt.de/~bruhn/CrothersViews.html

[44] Wald R. M. General Relativity, The University of Chicago Press, Chicago, 1984.

[45] Misner C. W., Thorne K. S., Wheeler J. A. Gravitation, W. H. Freeman and Company, New York, 1970.

[46] Zel'dovich, Ya. B. and Novikov, I. D. Stars and Relativity, Dover Publications Inc., New York, 1996.

[47] Levi-Civita T. The Absolute Differential Calculus, Dover Publications Inc., New York, 1977.

[48] Schwarzschild K. On the gravitational field of a sphere of incompressible fluid according to Einstein's theory. *Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl*., 424, 1916, http://arxiv.org/pdf/physics/9912033

[49] Crothers S. J. On the geometry of the general solution for the vacuum field of the point-mass.

Progress in Physics, v.2, 3--14, 2005, www.ptep-online.com/index_ files/2005/PP-02-01.PDF

[50] Tolman R. C. Relativity Thermodynamics and Cosmology, Dover Publications Inc., New York, 1987.

[51] Efimov N. V. Higher Geometry, Mir Publishers, Moscow, 1980.

[52] Pauli W. The Theory of Relativity, Dover Publications, Inc., New York, 1981.

[53] Einstein A. The Meaning of Relativity, Science Paperbacks and Methuen & Co. Ltd., 56--57, 1967.

[54] Eddington A. S. The mathematical theory of relativity, Cambridge University Press, Cambridge, 2nd edition, 1960.

[55] Levi-Civita T. Mechanics. - On the analytical expression that must be given to the gravitational tensor in Einstein's theory.*Rendiconti della Reale Accadmeia dei Lincei*, 26, 381, (1917), http://arxiv.org/pdf/physics/9906004

[56] Einstein A. The Foundation of the General Theory of Relativity, *Annalen der Physik*, 49, 1916, The Principle of Relativity (A collection of original memoirs on the special and general theory of relativity), Dover Publications Inc., New York, 1952.

[57] Weyl H. How far can one get with a linear field theory of gravitation in flat space-time?, *Amer. J. Math.*, 66, 591, (1944).

[58] Penzias A.A. and Wilson R.W. A measurement of excess antenna temperature at 4080 Mc/s. *Astrophys. J.*, 1965, v.1, 419-421.

[59] Mitchell W.C. Bye Bye Big Bang Hello Reality, Cosmic Sense Books, Nevada, 2002.

[60] Robitaille, P.-M.L. WMAP: A Radiological Analysis, *Prog. in Phys.*, 2007, v.1, 3-18. http://www.ptep-online.com/index_files/2007/PP-08-01.PDF

[61] Robitaille, P.-M.L. COBE: A Radiological Analysis, *Prog. in Phys.*, 2009, v.4, 17-42. http://www.ptep-online.com/index_files/2009/PP-19-03.PDF

[62] Robitaille, P.-M.L. The Earth Microwave Background (EMB), Atmospheric Scattering and the Generation of Isotropy, *Prog. in Phys.*, 2008, v.2, 164-165. http://www.ptep-online.com/index_files/2007/PP-10-01.PDF

[63] Robitaille P.-M. Calibration of Microwave Reference Blackbodies and Targets for Use in Satellite Observations: An Analysis of Errors in Theoretical Outlooks and Testing Procedures, *Progress in Physics*, v.3, p.3-10, 2010, http://www.ptep-online.com/index_files/2010/PP-22-01.PDF

[64] Robitaille P.-M. The Planck Satellite LFI and the Microwave Background: Importance of the 4K Reference Targets, http://www.ptep-online.com/index_files/2010/PP-22-02.PDF

[65] Schwarzschild K. "On the gravitational field of a mass point according to Einstein's theory", *Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl.*, 189 (1916). www.sjcrothers.plasmaresources.com/schwarzschild.pdf

[66] M. Brillouin, "The singular points of Einstein's Universe", *Journ Phys. Radium* **23**, 43 (1923), www.sjcrothers.plasmaresources.com/brillouin.pdf

[67] Droste J. "The field of a single centre in Einstein's theory of gravitation, and the motion of a particle in that field", *Ned. Acad. Wet., S.A.*, **19**, 197 (1917), www.sjcrothers.plasmaresources.com/Droste.pdf