

Foundations of a quantum gravity at large scales of length and its consequences for the dynamics of cosmological expansion

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We attempt to find new symmetries in the space-time structure, leading to a modified gravitation at large length scales, which provides the foundations of a quantum gravity at very low energies. This search begins by considering a unified model for electrodynamics and gravitation, so that the influence of the gravitational field on the electrodynamics at very large distances leads to a reformulation of our understanding about space-time through the elimination of the classical idea of rest at quantum level. This leads us to a modification of the relativistic theory by introducing the idea of a universal minimum speed related to Planck minimum length. Such a speed, unattainable by the particles, represents a privileged inertial reference frame associated with a universal background field (a vacuum energy), enabling a fundamental understanding of the quantum uncertainties. The structure of space-time becomes extended due to such a vacuum energy density, which leads to a negative pressure at the cosmological length scales as being an anti-gravity, playing the role of the cosmological constant. The tiny values of the vacuum energy density and the cosmological constant are obtained, being in agreement with current observational results. We estimate the very high value of inflationary energy density of vacuum at Planck length scale. After we find the critical radius of the universe, beyond which the accelerated expansion (cosmological anti-gravity) takes place. We show that such a critical radius is $R_{uc} = r_g/2$, where $r_g = 2GM/c^2$, being r_g the Schwarzschild radius of a sphere with a mass M representing the total attractive mass contained in our universe. And finally we obtain the radius $R_{u0} = 3r_g/4 (> R_{uc})$ where we find the maximum rate of accelerated expansion. For $R_u > R_{u0}$, the rate of acceleration decreases to zero at the infinite, avoiding Big Rip.

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I. INTRODUCTION

The idea that some symmetries of a fundamental theory of quantum gravity may have non trivial consequences for cosmology and particle physics at very low energies is interesting and indeed quite reasonable. Thus driven by Einstein's ideas for searching for new fundamental symmetries in Nature, our main focus is to go back to that point of the old incompatibility between mechanics and electrodynamics, by extending his reasoning in order to look for new symmetries that implement gravitation into electrodynamics at very large distances. We introduce new symmetries into the space-time geometry, where gravitation and electromagnetism become coupled with each other, in such a way to enable us to build a new modified dynamics at very low energies, being compatible with the quantum uncertainties.

Besides quantum gravity at Planck minimum length (very high energy scales), our new symmetry idea appears due to the indispensable presence of gravity at quantum level for particles with very large wavelengths (very low energy scales). This leads us to postulate a universal minimum speed related to a fundamental (privileged) reference frame of background field that breaks Lorentz symmetry[1].

Similarly to Einstein's reasoning, which has solved that old incompatibility between nature of light and motion of matter (massive objects), let us now expand it by making the following heuristic assumption based on new symmetry arguments:

If, in order to preserve the symmetry (covariance) of Maxwell's equations, c is required to be constant based on Einstein's reasoning, according to which it is impossible to find the rest reference frame for the speed of light ($c - c \neq 0 (= c)$) due to the coexistence of \vec{E} and \vec{B} in equal-footing, then now let us think that fields \vec{E} and \vec{B} may also coexist for moving charged massive particles (as electrons), which are at subluminal level ($v < c$). So, by making such an assumption, it would be also impossible to find a rest reference frame for a charged massive particle, by canceling its magnetic field, i.e., $\vec{B} = 0$ with $\vec{E} \neq 0$. This would break the coexistence of these two

fields, which would not be possible because it is impossible to find a reference frame where $v = 0$, in such a space-time. Thus we always must have $\vec{E} \neq 0$ and also $\vec{B} \neq 0$ for charged massive particles, due always to the presence of a non-null momentum for the electron, in a similar way to the photon electromagnetic wave.

The reasoning above leads to the following conclusion:

-The plane wave for free electron is an idealization impossible to conceive under physical reality. In the event of an idealized plane wave, it would be possible to find the reference frame that cancels its momentum ($p = 0$), just the same way as we can find the reference frame of rest for classical (macroscopic) objects with uniform rectilinear motion (a state of equilibrium). In such an idealized case, we could find a reference frame where $\vec{B} = 0$ for charged particle. However, the presence of gravity in quantum world emerges in order to always preserve the coexistence of \vec{E} and $\vec{B} (\neq 0)$ in electrodynamics of moving massive particles (section 3). That is the reason why we think about a lowest and unattainable speed limit V in such a space-time, in order to avoid to think about $\vec{B} = 0$ ($v = 0$). This means that there is no state of perfect equilibrium (plane wave and Galilean inertial reference frame) for moving particles in such a space-time, except the privileged inertial reference frame of a universal background field associated with an unattainable minimum limit of speed V . Such a reasoning allows us to think that the electromagnetic radiation (photon: " $c - c' = c$ ") as well as the matter (electron: " $v - v' > V (\neq 0)$ ") are in equal-footing, since now it is not possible to find a reference frame in equilibrium ($v_{relative} = 0$) for both through any velocity transformations (section 6).

The interval of velocity with two limits $V < v \leq c$ represents the fundamental symmetry that is inherent to such a space-time, where gravitation and electrodynamics become coupled. However, for classical (macroscopic) objects, the breaking of that symmetry, i.e., $V \rightarrow 0$, occurs so as to reinstate Special Relativity (SR) as a particular (classical) case, namely no uncertainties and no vacuum energy, where the idea of rest, based on the Galilean concept of reference frame is thus recovered.

In another paper, we will study the dynamics of particles in the presence of such a universal (privileged) background reference frame associated with V , within a context of the ideas of Mach[2], Schrodinger[3] and Sciama[4], where we will think about an absolute background reference frame in relation to which we have the inertia of all moving particles. However, we must emphasize that the approach we will intend to use is not classical as the machian ideas (the inertial reference frame of fixed stars), since the lowest limit of speed V , related to the privileged reference frame connected to a vacuum energy, has origin essentially from the presence of gravity at quantum level for particles with very large wavelengths.

We hope that a direct relationship should exist between the minimum speed V and Planck's minimum length $l_p = (G\hbar/c^3)^{1/2} (\sim 10^{-35}m)$ treated by Double Special Relativity theory (DSR)[20-25] (4th section).

In the next section, a heuristic model will be developed to describe the electromagnetic nature of the matter. It is based on the Maxwell theory used for investigating the electromagnetic nature of a photon when the amplitudes of electromagnetic wave fields are normalized for one single photon with energy $\hbar\omega$. Thus, due to the reciprocity and symmetry reasoning, we shall extend such a concept for the matter (electron) through the idea of pair materialization after γ -photon decay, so that we will attempt to develop a simple heuristic model of the electromagnetic nature of the electron that will experiment a background field in the presence of gravity.

The structure of space-time becomes extended due to the presence of a vacuum energy density associated with such a universal background field (a privileged reference frame connected to a zero-point energy of background field, which is associated with the minimum limit of speed V for particles moving with respect to such a background reference frame). This leads to a negative pressure at the cosmological length scales, behaving like a cosmological anti-gravity for the cosmological constant whose tiny value will be determined (section 8-B). Besides this, we are able to find the critical radius of the universe, beyond which the accelerated expansion (cosmological anti-gravity) takes place (section 8-C).

II. ELECTROMAGNETIC NATURE OF THE PHOTON AND OF THE MATTER

A. Electromagnetic nature of the photon

In accordance with some laws of Quantum Electrodynamics[5], we shall take into account the electric field of a plane electromagnetic wave whose amplitude is normalized for just one single photon[5]. To do this, consider

that the vector potential of a plane electromagnetic wave is

$$\vec{A} = a \cos(\omega t - \vec{k} \cdot \vec{r}) \vec{e}, \quad (1)$$

where $\vec{k} \cdot \vec{r} = kz$, admitting that the wave propagates in the direction of z , being \vec{e} the unitary vector of polarization. Since we are in vacuum, we must consider

$$\vec{E} = -\frac{1}{c} \frac{\partial \vec{A}}{\partial t} = \left(\frac{\omega a}{c}\right) \text{sen}(\omega t - kz) \vec{e} \quad (2)$$

In the Gaussian system of units, we have $|\vec{E}| = |\vec{B}|$. So the average energy density of the wave shall be

$$\langle \rho_{electromag} \rangle = \frac{1}{8\pi} \langle |\vec{E}|^2 + |\vec{B}|^2 \rangle = \frac{1}{4\pi} \langle |\vec{E}|^2 \rangle \quad (3)$$

Substituting (2) into (3), we obtain

$$\langle \rho_{electromag} \rangle = \frac{1}{8\pi} \frac{\omega^2 a^2}{c^2}, \quad (4)$$

where a is an amplitude that depends upon the number of photons in such a wave. Since we wish to obtain the plane wave of one single photon ($\hbar\omega$), then by making this condition for (4) and by considering an unitary volume for the photon ($v_{ph} = 1$), we have

$$a = \sqrt{\frac{8\pi\hbar c^2}{\omega}} \quad (5)$$

Substituting (5) into (2), we obtain

$$\vec{E}(z, t) = \frac{\omega}{c} \sqrt{\frac{8\pi\hbar c^2}{\omega}} \text{sen}(\omega t - kz) \vec{e}, \quad (6)$$

from where, we deduce that

$$e_0 = \frac{\omega}{c} \sqrt{\frac{8\pi\hbar c^2}{\omega}} = \sqrt{8\pi\hbar\omega}, \quad (7)$$

where e_0 could be thought of as an electric field amplitude normalized for 1 single photon, with $b_0 = e_0$ (Gaussian system), being the magnetic field amplitude normalized for 1 photon. So we may write

$$\vec{E}(z, t) = e_0 \text{sen}(\omega t - kz) \vec{e} \quad (8)$$

Substituting (8) into (3) and considering the unitary volume ($v_{ph} = 1$), we obtain

$$\langle E_{electromag} \rangle = \frac{1}{8\pi} e_0^2 \equiv \hbar\omega \quad (9)$$

Now, starting from the classical theory of Maxwell for the electromagnetic wave, let us consider an average quadratic electric field normalized for one single photon, which is $e_m = e_0/\sqrt{2} = \sqrt{\langle |\vec{E}|^2 \rangle}$. So doing such a consideration, we may write (9) in the following alternative way:

$$\langle E_{electromag} \rangle = \frac{1}{4\pi} e_m^2 \equiv \hbar w, \quad (10)$$

where we have

$$e_m = \frac{e_0}{\sqrt{2}} = \frac{w}{c} \sqrt{\frac{4\pi\hbar c^2}{w}} = \sqrt{4\pi\hbar w} \quad (11)$$

It is important to emphasize that, although the field in (8) is normalized for only one photon, it is still a classical field of Maxwell because its value oscillates like a classical wave (solution (8)). The only difference is that we have thought about a small amplitude field for one photon. Actually the amplitude of the field (e_0) cannot be measured directly. Only in the classical approximation (macroscopic case), when we have a very large number of photons ($N \rightarrow \infty$), we can somehow measure the macroscopic field E of the wave. Therefore, although we could idealize the case of just one photon as if it were an electromagnetic wave of small amplitude, the solution (8) is even a classical one, since the field \vec{E} presents oscillation.

Actually we already know that the photon wave is a quantum wave, i.e., a de-Broglie wave, where its wavelength ($\lambda = h/p$) is not interpreted classically as the oscillation frequency (wavelength due to oscillation) of a classical field. However, in a classical case, using the solution (8), we would have

$$E_{electromag} = \frac{1}{4\pi} |\vec{E}(z, t)|^2 = \frac{1}{4\pi} e_0^2 \sin^2(\omega t - kz) \quad (12)$$

In accordance with (12), if the wave of a photon were really classical, then its energy would not be fixed, as we can see in (12). Consequently, its energy $\hbar w$ would just be an average value [see (10)]. Hence, in order to achieve consistency between the result (10) and the quantum wave (de-Broglie wave), we must interpret (10) to be related to the de-Broglie wave of the photon with a fixed discrete energy value $\hbar w$ instead of an average energy value, since now we consider that the wave of one single photon is a non-classical wave, i.e., it is a de-Broglie wave. Thus we rewrite (10) as follows:

$$E_{electromag} = E = pc = \frac{hc}{\lambda} = \hbar w \equiv \frac{1}{4\pi} e_{ph}^2, \quad (13)$$

where we conclude that

$$\lambda \equiv \frac{4\pi\hbar c}{e_{ph}^2}, \quad (14)$$

where λ is the de-Broglie wavelength. Now, in this case (14), the single photon field e_{ph} should not be assumed as a mean value for oscillating classical field, and we shall preserve it in order to interpret it as a *scalar quantum electric field* (a microscopic field) of a photon. So basing on such a heuristic reasoning, let us also call it “*scalar support of electric field*”, representing a quantum (corpuscular)-mechanical aspect of electric field for the photon. As e_{ph} is responsible for the energy of the photon ($E \propto e_{ph}^2$), where $w \propto e_{ph}^2$ and $\lambda \propto 1/e_{ph}^2$, indeed we see that e_{ph} presents a quantum behavior, as it provides the dual aspects (wave-particle) of the photon, where its mechanical momentum may be written as $p = \hbar k = 2\pi\hbar/\lambda = \hbar e_{ph}^2/2hc$ [refer to (14)], or simply $p = e_{ph}^2/4\pi c$.

B. Electromagnetic nature of the matter

Our objective is to extend the idea of the photon electromagnetic energy [equation (13)] for the matter. By doing this, we shall provide heuristic arguments that rely directly on de-Broglie reciprocity postulate, which has extended the idea of wave (photon wave) for the matter (electron), behaving also like wave. Thus the relation (14)

for the photon, which is based on de-Broglie relation ($\lambda = h/p$) may also be extended for the matter (electron), in accordance with the idea of de-Broglie reciprocity. In order to strengthen such an argument, we are going to assume the phenomenon of pair formation, where the photon γ decays into two charged massive particles, namely the electron (e^-) and its anti-particle, the positron (e^+). Such an example will enable us to better understand the need of extending the idea of the photon electromagnetic mass ($m_{electromag} = E_{electromag}/c^2$) (equation 13) for the matter (e^- and e^+), by using that concept of *field scalar support*.

Now consider the phenomenon of pair formation, i.e., $\gamma \rightarrow e^- + e^+$. Then, by using the conservation of energy for γ -decay, we write the following equation:

$$E_\gamma = \hbar\omega = m_\gamma c^2 = m_0^- c^2 + m_0^+ c^2 + K^- + K^+ = 2m_0 c^2 + K^- + K^+, \quad (15)$$

where K^- and K^+ represent the kinetic energy for electron and positron respectively. We have $m_0^- c^2 = m_0^+ c^2 \cong 0, 51 \text{ Mev}$ for electron or positron.

Since γ -photon electromagnetic energy is $E_\gamma = h\nu = m_\gamma c^2 = \frac{1}{4\pi} e_\gamma^2$, or else, $E_\gamma = \epsilon_0 e_\gamma^2$ given in the International System of Units (IS), and also knowing that $e_\gamma = cb_\gamma$ (IS), where b_γ represents the *scalar support of magnetic field* for γ -photon, so we also may write

$$E_\gamma = c\epsilon_0(e_\gamma)(b_\gamma) \quad (16)$$

Photon has no charge, however, when it is materialized into the pair electron-positron, its electromagnetic content given in (16) ceases to be free or purely kinetic (purely relativistic mass) to become massive through the materialization of the pair. Since such massive particles ($v_{(+,-)} < c$) also behave like waves in accordance with de-Broglie idea, now it would also be natural to extend the relation (14) (of the photon) for representing wavelengths of the matter (electron or positron) after γ -photon decay, namely:

$$\lambda_{(+,-)} \propto \frac{hc}{\epsilon_0 [e_s^{(+,-)}]^2} = \frac{h}{\epsilon_0 [e_s^{(+,-)}][b_s^{(+,-)}]}, \quad (17)$$

where $e_s^{(+,-)}$ and $b_s^{(+,-)}$ play the role of the electromagnetic content for energy condensed into matter (*scalar support of electromagnetic field* for the matter). Such fields are associated with the total energy of the moving massive particle, whose mass has essentially an electromagnetic origin, given in the form

$$m \equiv m_{electromag} \propto e_s b_s, \quad (18)$$

where $E = mc^2 \equiv m_{electromag} c^2$.

Basing on (16) and (17), we may write (15) in the following way:

$$\begin{aligned} E_\gamma &= c\epsilon_0 e_\gamma b_\gamma = c\epsilon_0 e_s^- b_s^- v_e^- + c\epsilon_0 e_s^+ b_s^+ v_e^+ = [c\epsilon_0 e_s^- b_s^- v_e^- + K^-] + [c\epsilon_0 e_s^+ b_s^+ v_e^+ + K^+] = \\ &2c\epsilon_0 e_s^{(+,-)} b_s^{(+,-)} v_e + K^- + K^+ = 2m_0 c^2 + K^- + K^+, \end{aligned} \quad (19)$$

where $m_0 c^2 = m_0^{(+,-)} c^2 = c\epsilon_0 e_s^{(+,-)} b_s^{(+,-)} v_e \cong 0, 51 \text{ Mev}$. $e_s^{(+,-)}$ and $b_s^{(+,-)}$ represent the proper electromagnetic contents of the electron or positron. Later we will show that the mass m_0 does not represent a classic rest mass due to the inexistence of rest in such a space-time. This question shall be clarified in 5th section. The volume v_e in (19) is a free variable to be considered.

In accordance with equation (19), the present model provides a fundamental point that indicates electron is not necessarily an exact punctual particle. Quantum Electrodynamics, based on Special Relativity deals with the electron as a punctual particle. The well-known classical theory of the electron foresees for the electron radius the same order of magnitude of the radius of a proton, i.e., $R_e \sim 10^{-15} m$.

The most recent experimental evidence about scattering of electrons by electrons at very high kinetic energies indicates that electron can be considered approximately a point particle. Actually electrons have an extent less

than collision distance, which is about $R_e \sim 10^{-16}m$ [6]. Actually such an extent is negligible in comparison to the dimensions of an atom ($10^{-10}m$), or even the dimensions of a nucleus ($10^{-14}m$), but it is not exactly a point. For this reason, the present model can provide a very small non-null volume v_e of the electron. But, if we just consider $v_e = 0$ according to (19), we would have an absurd result, i.e., divergent internal fields $e_{s0} = b_{s0} \rightarrow \infty$. However, for instance, if we consider $R_e \sim 10^{-16}m$ ($v_e \propto R_e^3 \sim 10^{-48}m^3$) for our model, and knowing that $m_0c^2 \cong 0,51MeV (\sim 10^{-13}J)$, thus, in such a case (see (19)), we would obtain $e_{s0} \sim 10^{23}V/m$. Such a value is extremely high and therefore we may conclude that the electron is extraordinarily compact, with a very high energy density. So, for such an example, if we imagine over the “surface” of the electron, we would detect a field $e_{s0} \sim 10^{23}V/m$ instead of an infinite value for it. According to the present model, the field e_{s0} inside the almost punctual non-classical electron with such a radius ($\sim 10^{-16}m$) would be finite and constant ($\sim 10^{23}V/m$) instead of a function like $1/r^2$ with divergent classical behavior. Indeed, for $r > R_e$, the field E decreases like $1/r^2$, i.e., $E = e/r^2$. For $r = R_e$, $E = e/R_e^2 \equiv e_{s0}$. Actually, for $r \leq R_e$, we have $E \equiv e_{s0} = constant (\sim 10^{23}V/m)$.

The next section will be dedicated to the investigation about the electron coupled to a gravitational field.

III. ELECTRON COUPLED TO A GRAVITATIONAL FIELD

When a photon with energy $h\nu$ is subjected to a certain gravitational potential ϕ , its energy (or frequency) increases to be $E' = h\nu'$, where

$$E' = h\nu' = h\nu\left(1 + \frac{\phi}{c^2}\right) \quad (20)$$

By convention, as we have stipulated $\phi > 0$ to be attractive potential, we have $\nu' > \nu$. By considering (16) for any photon and by substituting (16) into (20), we alternatively write

$$E' = c\epsilon_0 e'_{ph} b'_{ph} = c\epsilon_0 e_{ph} b_{ph} \sqrt{g_{00}}, \quad (21)$$

where g_{00} is the first component of the metric tensor, where $\sqrt{g_{00}} = \left(1 + \frac{\phi}{c^2}\right)$ and $e_{ph} = cb_{ph}$.

From (21), we can extract the following relationships, namely:

$$e'_{ph} = e_{ph} \sqrt{\sqrt{g_{00}}}, \quad b'_{ph} = b_{ph} \sqrt{\sqrt{g_{00}}} \quad (22)$$

In the presence of gravity, such fields e_{ph} and b_{ph} of the photon increase according to (22), leading to the increasing of the photon frequency or energy, according to (20). Thus we may think about the following increments, namely:

$$\Delta e_{ph} = e'_{ph} - e_{ph} = e_{ph}(\sqrt{\sqrt{g_{00}}} - 1), \quad \Delta b_{ph} = b'_{ph} - b_{ph} = b_{ph}(\sqrt{\sqrt{g_{00}}} - 1) \quad (23)$$

In accordance with General Relativity (GR), when a massive particle of mass m_0 moves in the presence of a gravitational potential ϕ , its total energy E is given in the following way:

$$E = mc^2 = m_0c^2\sqrt{g_{00}} + K, \quad (24)$$

where we can think that $m_0 (= m_0^{(+,-)})$ is the mass of the electron or positron, emerging from γ -decay in the presence of a gravitational potential ϕ .

In order to facilitate the understanding of what we are proposing, let us consider $K \ll m_0c^2$, since we are interested only in obtaining the influence of the potential ϕ . Therefore we write

$$E = m_0c^2\sqrt{g_{00}} \quad (25)$$

As we already know that $E_0 = m_0 c^2 = c \epsilon_0 e_{s_0}^{(+,-)} b_{s_0}^{(+,-)} v_e$, we can also write the total energy E , as follows:

$$E = c \epsilon_0 e_s^{(+,-)} b_s^{(+,-)} v_e = c \epsilon_0 e_{s_0}^{(+,-)} b_{s_0}^{(+,-)} v_e \sqrt{g_{00}}, \quad (26)$$

from where we can extract

$$e_s^{(+,-)} = e_{s_0}^{(+,-)} \sqrt{\sqrt{g_{00}}}, \quad b_s^{(+,-)} = b_{s_0}^{(+,-)} \sqrt{\sqrt{g_{00}}}. \quad (27)$$

So we obtain

$$\Delta e_s = e_{s_0}^{(+,-)} (\sqrt{\sqrt{g_{00}}} - 1), \quad \Delta b_s = b_{s_0}^{(+,-)} (\sqrt{\sqrt{g_{00}}} - 1), \quad (28)$$

where we have $\Delta e_s = c \Delta b_s$.

As the energy of the particle can be represented as a condensation of electromagnetic fields in scalar forms e_s and b_s , this model is capable of assisting us to think that the well-known external fields \vec{E} and \vec{B} for the moving charged particle, by storing an energy density ($\propto |\vec{E}|^2 + |\vec{B}|^2$) should also suffer some influence (shifts) in the presence of gravitational potential. In accordance with GR, every kind of energy is also a source of gravitational field. This non-linearity that is inherent to the gravitational field leads us to think that, at least in a certain approximation in the presence of gravity, the external fields E and B should experiment small positive shifts δE and δB , which are proportional to the intrinsic increments (shifts) Δe_s and Δb_s of the particle, namely:

$$\delta E = (E' - E) \propto \Delta e_s = (e_s - e_{s_0}), \quad \delta B = (B' - B) \propto \Delta b_s = (b_s - b_{s_0}) \quad (29)$$

Here we have omitted the signs $(+, -)$ in order to simplify the notation. Since $\Delta e_s = c \Delta b_s$, then $\delta E = c \delta B$.

In accordance with (29), we may conclude that there is a constant of proportionality that couples the external electromagnetic fields E and B of the moving charge with gravity by means of the small shifts δE and δB . Such a constant works like a fine-tuning, namely:

$$\delta E = \xi \Delta e_s, \quad \delta B = \xi \Delta b_s, \quad (30)$$

where ξ is a dimensionless constant to be obtained. We expect that $\xi \ll 1$ due to the fact that the gravitational interaction is much weaker than the electromagnetic one. δE and δB depend only on ϕ over the electron.

Substituting (28) into (30), we obtain

$$\delta E = \xi e_{s_0} (\sqrt{\sqrt{g_{00}}} - 1), \quad \delta B = \xi b_{s_0} (\sqrt{\sqrt{g_{00}}} - 1). \quad (31)$$

Due to the very small positive shifts δE and δB in the presence of a weak gravitational potential ϕ , the total electromagnetic energy density in the space around the charged particle is slightly increased, as follows:

$$\rho_{electromag}^{total} = \frac{1}{2} \epsilon_0 (E + \delta E)^2 + \frac{1}{2 \mu_0} (B + \delta B)^2 \quad (32)$$

Substituting (31) into (32) and performing the calculations, we will finally obtain

$$\rho_{electromag}^{total} = \frac{1}{2} [\epsilon_0 E^2 + \frac{1}{\mu_0} B^2] + \xi [\epsilon_0 E e_{s_0} + \frac{1}{\mu_0} B b_{s_0}] (\sqrt{\sqrt{g_{00}}} - 1) + \frac{1}{2} \xi^2 [\epsilon_0 (e_{s_0})^2 + \frac{1}{\mu_0} (b_{s_0})^2] (\sqrt{\sqrt{g_{00}}} - 1)^2 \quad (33)$$

We may assume that $\rho_{electromag}^{total} = \rho_{electromag}^{(0)} + \rho_{electromag}^{(1)} + \rho_{electromag}^{(2)}$ for representing (33), where the first term $\rho_{electromag}^{(0)}$ is the free electromagnetic energy density (zero order) for the ideal case of a charged particle

uncoupled from gravity ($\xi = 0$), i.e., the ideal case of a free particle (a perfect plane wave, which does not exist in reality due always to the presence of gravity). We have $\rho^{(0)} \propto 1/r^4$ (*coulombian term*).

The coupling term $\rho^{(1)}$ (second term) represents an electromagnetic energy density of first order, that is, it contains an influence of 1st order for δE and δB , as it is proportional to δE and δB due to a certain influence of gravity. Therefore it is a mixture term that behaves essentially like a *radiation term*. Thus we have $\rho^{(1)} \propto 1/r^2$, since e_{s0} (or b_{s0}) \sim *constant* and E (or B) $\propto 1/r^2$. It is very interesting to notice that such a radiation term of a charge in a true gravitational field corresponds effectively to a certain radiation field due to a slightly accelerated charge in free space, however such an equivalence is weak due to the very small value of ξ .

The last coupling term ($\rho^{(2)}$) is purely interactive due to the presence of gravity only. This means that it is a 2nd order interactive electromagnetic energy density term, since it is proportional to $(\delta E)^2$ and to $(\delta B)^2$. Hence we have $\rho^{(2)} \propto 1/r^0 \sim$ *constant*, being $\rho^{(2)} = \frac{1}{2}\epsilon_0(\delta E)^2 + \frac{1}{2\mu_0}(\delta B)^2 = \epsilon_0(\delta E)^2 = \frac{1}{\mu_0}(\delta B)^2$, which varies only with the gravitational potential (ϕ). Since we have $\rho^{(2)} \propto 1/r^0$, it has a non-locality behavior. This means that $\rho^{(2)}$ behaves like a kind of non-local field that is inherent to the space (*a constant term for representing a background field*). It does not depend on the distance r from the charged particle. So it is a constant energy density for a fixed potential ϕ , and fills the whole space. $\rho^{(2)}$ always exists due to the inevitable presence of gravity and therefore it cannot be cancelled by any transformation. Due to this fact, the increment δB that contributes for the density of interactive energy $\rho^{(2)}$ cannot vanish since the electron is not free ($\rho^{(2)} \neq 0$). This always assures a non-zero value of magnetic field ($\delta B \neq 0$) for any transformation, and so this is the fundamental reason why the fields E and B should coexist in the presence of gravity, where the charge experiments a background field ($\rho^{(2)} \propto (\delta B)^2$) connected to a privileged reference frame of an unattainable minimum speed that justifies in a kinematic point of view the impossibility of finding $\delta B = 0$. This minimum speed (V) is a universal constant that should be related directly to gravity (G), since V is also responsible for the coexistence of E and B . We will see such a relationship in the next section.

Usually we have $\rho^{(0)} \gg \rho^{(1)} \gg \rho^{(2)}$. For a very weak gravitational field, we can consider a good practical approximation as $\rho_{electromag}^{total} \approx \rho^{(0)}$. However, from a fundamental point of view, we cannot neglect the coupling terms, specially the last one for large distances, as it has a vital importance in this work, allowing us to understand a non-local vacuum energy that is inherent to the space, i.e., $\rho^{(2)} \propto 1/r^0$. Such a background field with energy density $\rho^{(2)}$ has deep implications for our understanding of the space-time structure at very large scales of length (cosmological scales), since $\rho^{(2)}$ does not have r -dependence, i.e., it remains for $r \rightarrow \infty$.

In the next section, we will estimate the constant ξ and consequently the idea of a universal minimum speed in the space-time. Its cosmological implications will be treated in section 8.

IV. THE FINE ADJUSTMENT CONSTANT ξ AND ITS IMPLICATIONS

Let us begin this section by considering the well-known problem that deals with the electron at the bound state of a coulombian potential of a proton (Hydrogen atom). We start from this subject because it presents certain similarity with the present model for the electron coupled to a gravitational field. We know that the fine structure constant ($\alpha_F = 1/137$) plays an important role for obtaining the energy levels that bond the electron to the nucleus (proton) in the Hydrogen atom. Therefore, in a similar way to such an idea, we plan to extend it in order to see that the fine coupling constant ξ plays an even more fundamental role than the well-known fine structure α_F , by considering that ξ couples gravity to the electromagnetic field of the electron charge.

Let's initially consider the energy that bonds the electron to the proton at the fundamental state of the Hydrogen atom, as follows:

$$\Delta E = \frac{1}{2}\alpha_F^2 m_0 c^2, \quad (34)$$

where ΔE is assumed as module. We have $\Delta E \ll m_0 c^2$, where m_0 is the electron mass, which is practically the reduced mass of the system ($\mu \approx m_0$).

We have $\alpha_F = e^2/\hbar c = q_e^2/4\pi\epsilon_0\hbar c \approx 1/137$ (fine structure constant). Since $m_0 c^2 \cong 0.51$ Mev, we have $\Delta E \approx 13.6$ eV.

Since we already know that $E_0 = m_0c^2 = c\epsilon_0e_{s0}b_{s0}v_e$,so we may write (34) in the following alternative way:

$$\Delta E = \frac{1}{2}\alpha_F^2c\epsilon_0e_{s0}b_{s0}v_e = \frac{1}{2}c\epsilon_0(\alpha_Fe_{s0})(\alpha_Fb_{s0})v_e \equiv \frac{1}{2}c\epsilon_0(\Delta e_s)(\Delta b_s)v_e, \quad (35)$$

from where we extract

$$\Delta e_s \equiv \alpha_Fe_{s0}, \quad \Delta b_s \equiv \alpha_Fb_{s0}. \quad (36)$$

It is interesting to observe that (36) maintains a certain similarity with (30),however,first of all,we must emphasize that the variations Δe_s and Δb_s for the electron energy have a purely coulombian origin,since the fine structure constant α_F depends solely on the electron charge. Thus we can write the electric force between two electronic charges in the following way:

$$F_e = \frac{e^2}{r^2} = \frac{q_e^2}{4\pi\epsilon_0r^2} = \frac{\alpha_F\hbar c}{r^2}, \quad (37)$$

where $e = q_e/\sqrt{4\pi\epsilon_0}$.

If we just consider a gravitational interaction between two electrons,we would have

$$F_g = \frac{Gm_e^2}{r^2} = \frac{\beta_F\hbar c}{r^2}, \quad (38)$$

from where we obtain

$$\beta_F = \frac{Gm_e^2}{\hbar c}. \quad (39)$$

We have $\beta_F \ll \alpha_F$ due to the fact that the gravitational interaction is much weaker than the electric one,so that $F_e/F_g = \alpha_F/\beta_F \sim 10^{42}$,where $\beta_F \cong 1.75 \times 10^{-45}$. Therefore we shall call β_F^1 the *superfine structure constant* ,since gravitational interaction creates a bonding energy extremely smaller than the coulombian bonding energy considered for the fundamental state (ΔE) in the Hydrogen atom.

To sum up,whereas $\alpha_F(e^2)$ provides the adjustment for the coulombian bonding energies between two electronic charges, $\beta_F(m_e^2)$ gives the adjustment for the gravitational bonding energies between two electronic masses. Such bonding energies of electrical or gravitational origin increment the particle energy through Δe_s and Δb_s .

Now,following the above reasoning,we notice that the present model enables us to introduce the very fine-tuning (coupling) ξ between gravity (a gravitational potential generated by the mass m_e) and electrical field (electrical energy density generated by the charge q_e (refer to (30))). Thus for such more fundamental case,we have a kind of bond of the type m_eq_e (mass-charge) through the adjustment (coupling) ξ . So the subtleness here is that the bonding energy density due to ξ ,by means of the increments δE and δB (see (30), (31),(32) or (33)) occurs on the electric and magnetic fields generated by the charge q_e .

Although we could show a laborious and step by step problem for obtaining the constant ξ ,the way we follow here is shorter because it starts from important analogies by using the ideas of fine structure $\alpha_F = \alpha_F(e^2)$,i.e.,an electric interaction (*charge-charge*) and also superfine structure $\beta_F = \beta_F(m_e^2)$,i.e.,a gravitational interaction (*mass-mass*). Hence,now it is easy to conclude that the kind of mixing coupling we are proposing,of the type " m_eq_e " (*mass-charge*) represents a gravi-electrical coupling constant,which leads us naturally to think that such a constant ξ is of the form $\xi = \xi(m_eq_e)$,and therefore meaning that

¹ we must not mistake superfine structure β_F with hyperfine structure (spin interaction), as they are completely different.

$$\xi = \sqrt{\alpha_F \beta_F}, \quad (40)$$

which represents a geometrical average between electrical and gravitational couplings. Thus, from (40) we finally obtain

$$\xi = \sqrt{\frac{G}{4\pi\epsilon_0} \frac{m_e q_e}{\hbar c}}, \quad (41)$$

where indeed we have $\xi = \xi(m_e q_e) \propto m_e q_e$. From (41) we obtain $\xi \cong 3.57 \times 10^{-24}$. Let us call ξ *fine adjustment constant*. The quantity $\sqrt{G} m_e$ in (41) can be thought of as a *gravitational charge* e_g , so that $\xi = e_g e / \hbar c$.²

In the Hydrogen atom, we have the fine structure constant $\alpha_F = e^2 / \hbar c = v_B / c$, where $v_B = e^2 / \hbar = c / 137$. This is the velocity of the electron at the atom fundamental level (Bohr velocity). At this level, the electron does not radiate because it is in a kind of balance state, in spite of its electrostatic interaction with the nucleus (centripete force), namely it works effectively like an inertial system. Hence, following an analogous reasoning for the more fundamental case of the constant ξ , we may also write (41) as the ratio of two velocities, as follows:

$$\xi = \frac{V}{c}, \quad (42)$$

from where we have

$$V = \xi c = \frac{e_g e}{\hbar} = \sqrt{\frac{G}{4\pi\epsilon_0} \frac{m_e q_e}{\hbar}}, \quad (43)$$

where $V \cong 1.07 \times 10^{-15} m/s$. In the newtonian (classical) universe, where $c \rightarrow \infty$ and $V \rightarrow 0$, we have $\xi \rightarrow 0$. So the coupling of fields does not exist. Under Einstein's theory (relativistic theory), $V \rightarrow 0$ and we also have $\xi \rightarrow 0$, where, although electrodynamics is compatible with relativistic mechanics, gravitation is still not properly coupled to electrodynamics at quantum level. In the present model that breaks Lorentz symmetry, where $\xi \sim 10^{-24}$, gravitation is coupled to electrodynamics of moving particles. The quantum uncertainties should naturally arise from such a symmetric space-time structure ($V < v \leq c$), which will be denominated *Symmetrical Special Relativity* (SSR) due to the existence of two limits of speed.

Similarly to the Bohr velocity (v_B) for fundamental bound state, the speed V is also a universal fundamental constant, however the crucial difference between them is that V is associated with a more fundamental bound state in the Universe as a whole, since gravity (G), which is the weakest interaction plays now an important role for the dynamics of the electron (electrodynamics) in such a space-time. This may be observed in (43) because, if we make $G \rightarrow 0$, we would have $V \rightarrow 0$ and so we will recover the case of the classical vacuum (empty space or no background field).

Our aim is to postulate V as an unattainable universal (constant) minimum speed associated with a privileged frame of background field, but before this, we must provide a better justification of why we consider the electron mass and charge to calculate V ($V \propto m_e q_e$), instead of masses and charges of other particles. Although there are fractionary electric charges as the case of quarks, such charges are not free in Nature for bonding only with gravity. They are strongly connected by the strong force (gluons). Actually the charge of the electron is the smallest free charge in Nature. Besides this, the electron is the elementary charged particle with the smallest mass. Therefore the product $m_e q_e$ assumes a minimum value. And in addition to that, the electron is completely stable. Other charged particles such as for instance π^+ and π^- have masses that are greater than the electron mass, and they are unstable, decaying very quickly. Such a subject may be dealt with more extensively elsewhere.

² we write (33) as $\rho_{electromag}^{total} = \frac{1}{2}[\epsilon_0 E^2 + \frac{1}{\mu_0} B^2] + \frac{e_g e}{\hbar c} [\epsilon_0 E e_{s0} + \frac{1}{\mu_0} B b_{s0}] (\sqrt{\sqrt{g_{00}} - 1}) + \frac{1}{2} (\frac{e_g e}{\hbar c})^2 [\epsilon_0 (e_{s0})^2 + \frac{1}{\mu_0} (b_{s0})^2] (\sqrt{\sqrt{g_{00}} - 1})^2$.

We could think about a velocity Gm_e^2/\hbar ($\ll V$) that has origin from a purely gravitational interaction, however such a much lower bound state does not exist because the presence of electromagnetic interactions is essential at subatomic level. And since neutrino does not interact with electromagnetic field, it cannot be considered to estimate V .

Now we can verify that the minimum speed (V) given in (43) is directly related to the minimum length of quantum gravity (Planck length), as follows:

$$V = \frac{\sqrt{G}m_e e}{\hbar} = (m_e e \sqrt{\frac{c^3}{\hbar^3}})l_p, \quad (44)$$

where $l_p = \sqrt{G\hbar/c^3}$.

In (44), as l_p is directly related to V , if we make $l_p \rightarrow 0$ by considering $G \rightarrow 0$, this implies $V \rightarrow 0$ and thus we restore the case of the classical space-time in Relativity.

Now we can notice that the universal constant of minimum speed V in (44), associated with very low energies (very large wavelengths) is directly related to the universal constant of minimum length l_p (very high energies), whose invariance has been studied in DSR by Magueijo, Smolin, Camelia et al [20-25]. So there should be a connection between quantum gravity at very high energy scales (E_P) and a quantum gravity at large length scales (very low energy scales for $v \rightarrow V$) to be deeply investigated in the future.

The natural consequence of the presence of a more fundamental level associated with V in the space-time is the existence of a privileged reference frame of background field in the Universe. Such a frame should be connected to a kind of vacuum energy that is inherent to the space-time (refer to $\rho^{(2)}$ in equation (33)). This idea reminds us of the conceptions of Mach[2], Schroedinger[3] and Assis[7], although such conceptions are still within the classical context.

Since we are assuming an absolute and privileged reference frame (V), which is underlying and also inherent to the whole space-time geometry, we shall call it ultra-referential- S_V . By drawing inspiration from some of the non-conventional ideas of Einstein in relation to the “ether” [8], let us assume that such an ultra-referential of background field S_V , which in a way redeems his ideas, introduces a kind of relativistic “ether” of the space-time. Such a new concept has nothing to do with the so-called luminiferous ether (classical ether) established before Relativity theory.

The present idea about a relativistic “ether” for the ultra-referential S_V aims at the implementation of the quantum principles (uncertainties) in the space-time. This line of investigation resumes those non-conventional Einstein’s ideas [8][9], who attempted to bring back the idea of a new “ether” that cannot be conceived as composed of punctual particles and having a world line followed in the time.

Actually such an idea of “ether” as conceived by Einstein should be understood as a non-classical concept of ether due essentially to its non-locality feature. In this sense, such a new “ether” has a certain correspondence with the ultra-referential S_V due to its totality as a physical space, not showing any movement. In fact, as S_V would be absolutely unattainable for all particles (at local level), V would prohibit to think about a perfect plane wave ($\Delta x = \infty$), since it is an idealized case associated with the perfect equilibrium of a free particle ($\Delta p = 0$). So the ultra-referential S_V would really be non-local ($\Delta x = \infty$), which is in agreement with that Einstein’s conception about an “ether” that could not be split into isolated parts and which, due to its totality in the space, would give us the impression that it is actually stationary. *In order to understand better its non-locality feature by using a symmetry reasoning, we must perceive that such a minimum limit V works in a reciprocal way when compared with the maximum limit c , so that particles supposed in such a limit V , in contrast of what would happen in the limit c , would become completely “defrosted” in the space ($\Delta x \rightarrow \infty$) and time ($\Delta \tau \rightarrow \infty$), being in anywhere in the space-time and therefore having a non-local behavior. This super ideal condition corresponds to the ultra-referential S_V , at which the particle would have an infinite de-Broglie wavelength, being completely spread out in the whole space. This state coincides with the background field for S_V , however S_V is unattainable for all the particles.*

In vain, Einstein attempted to satisfactorily redeem the idea of a new “ether” under Relativity in various manners [9, 10, 11, 12, 13, 14] because, in effect, his theory wasn’t still able to adequately implement the quantum uncertainties as he also tried to do [15, 16, 17], and in this respect, Relativity is still a classical theory, although the

new conception of “ether” presented a few non-classical characteristics. Actually it was Einstein who coined the term *ultra-referential* as the fundamental aspect of Reality. To him, the existence of an ultra-referential cannot be identified with none of the reference frames in view of the fact that it is a privileged one in respect of the others. This seems to contradict the principle of Relativity, but, in vain, Einstein attempted to find a relativistic “ether” (physical-space), that is inherent to the geometry of the space-time, which does not contradict such a principle. That was the problem because such a new “ether” does not behave like a Galilean reference frame and therefore it has nothing to do with that absolute space filled by the luminiferous ether, although it behaves like a privileged background field in the Universe.

The present work seeks to naturally implement the quantum principles into the space-time. Thanks to the current investigation, we shall notice that Einstein’s non-conventional ideas about the relativistic “ether” and also his vision [18] of making quantum principles to emerge naturally from a unified field theory become closely related to each other.

V. A NEW CONCEPTION OF REFERENCE FRAMES AND SPACE-TIME INTERVAL: A FUNDAMENTAL EXPLANATION FOR THE UNCERTAINTY PRINCIPLE

A. Reference frames and space-time interval

The conception of background privileged reference frame (ultra-referential S_V) has deep new implications for our understanding of reference systems. That classical notion we have about the inertial (Galilean) reference frames, where the idea of rest exists, is eliminated at quantum level, where gravity plays a fundamental role for such a space-time with a vacuum energy associated with S_V ($V \propto G^{1/2}/\hbar$).

Before we deal with the implications due to the implementation of such a ultra-referential S_V in the space-time at quantum level, let us make a brief presentation of the meaning of the Galilean reference frame (reference space), well-known in Special Relativity. In accordance with that theory, when an observer assumes an infinite number of points at rest in relation to himself, he introduces his own reference space S . Thus, for another observer S' who is moving with a speed v in relation to S , there should also exist an infinite number of points at rest at his own reference frame. Therefore, for the observer S' , the reference space S is not standing still and it has its points moving at a speed $-v$. For this reason, in accordance with the principle of relativity, there is no privileged Galilean reference frame at absolute rest, since the reference space of a given observer becomes movement for another one.

The absolute space of pre-einsteinian physics, connected to the ether in the old sense, also constitutes by itself a reference space. Such a space was assumed as the privileged reference space of the absolute rest. However, as it was also essentially a Galilean reference space like any other, comprised of a set of points at rest, actually it was also subjected to the notion of movement. The idea of movement could be applied to the “absolute space” when, for instance, we assume an observer on Earth, which is moving with a speed v in relation to such a space. In this case, for an observer at rest on Earth, the points that would constitute the absolute space of reference would be moving at a speed of $-v$. Since such an absolute space was connected to the old ether, the Earth-bound observer should detect a flow of ether $-v$, however the Michelson-Morley experiment has not detected such an ether.

Einstein has denied the existence of the ether associated with a privileged reference frame because it has contradicted the principle of relativity. Therefore this idea of a Galilean ether is superfluous, as it would also merely be a reference space constituted by points at rest, as well as any other. In this respect, there is nothing special in such a classical (luminiferous) ether.

However, motivated by the provocation from H. Lorentz and Ph. Lenard Lorentz [8], Einstein attempted to introduce several new conceptions of a new “ether”, which did not contradict the principle of relativity. After 1925, he started using the word “ether” less and less frequently, although he still wrote in 1938: “*This word ‘ether’ has changed its meaning many times, in the development of Science... Its history, by no means finished, is continued by Relativity theory [10]...* ”.

In 1916, after the final formulation of GR, Einstein proposed a completely new concept of ether. Such a new “ether” was a relativistic “ether”, which described space-time as a *sui generis* material medium, which in no

way could constitute a reference space subjected to the relative notion of movement. Basically, the essential characteristics of the new “ether” as interpreted by Einstein can be summarized as follow:

-It constitutes a fundamental ultra-referential of Reality, which is identified with the physical space, being a relativistic ether, i.e., it is covariant because the notion of movement cannot be applied to it, which represents a kind of absolute background field that is inherent to the metric $g_{\mu\nu}$ of the space-time.

-It is not composed of points or particles, therefore it cannot be understood as a Galilean reference space for the hypothetical absolute space. For this reason, it does not contradict the well-known principle of Relativity.

-It is not composed of parts, thus its indivisibility reminds the idea of non-locality.

-It constitutes a medium which is really incomparable with any ponderable medium constituted of particles, atoms or molecules. Not even the background cosmic radiation of the Universe can represent exactly such a medium as an absolute reference system (ultra-referential)[19].

-It plays an active role on the physical phenomena[11] [12]. In accordance with Einstein, it is impossible to formulate a complete physical theory without the assumption of an “ether” (a kind of non-local vacuum field), because a complete physical theory must take into consideration real properties of the space-time.

The present work attempts to follow this line of reasoning that Einstein did not finish, providing a new model with respect to the fundamental idea of unification, namely the electrodynamics of a charged particle (electron) moving in a gravitational field.

As we have interpreted the lowest limit V (formulas (43) and (44)) as unattainable and constant (invariant), such a limit should be associated with a privileged non-Galilean reference system, since V must remain invariant for any frame with $v > V$. As a consequence of such a covariance of the relativistic “ether” S_V , new speed transformations will show that it is impossible to cancel the speed of a particle over its own reference frame, in such a way to always preserve the existence of a magnetic field \vec{B} for such a charged particle. Thus we should have a speed transformation that will show us that “ $v - v' > V$ for $v > V$ (see section 6), where the well-known constancy of c remains, i.e., “ $c - c' = c$ for $v = c$.”

Since it is impossible to find with certainty the rest for a given non-Galilean reference system S' with a speed v with respect to the ultra-referential S_V , i.e., “ $v - v' \neq 0 (> V)$ (section 6), consequently it is also impossible to find by symmetry a speed $-v$ for the relativistic “ether” when an “observer” finds himself at the reference system S' assumed with v . Hence, due to such an asymmetry, the flow $-v$ of the “ether” S_V does not exist and therefore, in this sense, it maintains covariant (V). This asymmetry breaks that equivalence by exchange of reference frame S for S' through an inverse transformation. Such a breakdown of symmetry by an inverse transformation breaks Lorentz symmetry due to the presence of the background field for S_V (section 6).

There is no Galilean reference system in such a space-time, where the ultra-referential S_V is a non-Galilean reference system and in addition a privileged one (covariant), exactly as is the speed of light c . Thus the new transformations of speed shall also show that “ $v \pm V' = v$ (section 6) and “ $V \pm V' = V$ (section 6).

Actually, if we make $V \rightarrow 0$, we therefore recover the validity of the Galilean reference frame of Special Relativity (SR), where only the invariance of c remains. In this classical case (SR), we have reference systems constituted by a set of points at rest or essentially by macroscopic objects. Now, it is interesting to notice that SR contains two postulates which conceptually exclude each other in a certain sense, namely:

1) -the equivalence of the inertial reference frames (with $v < c$) is essentially due to the fact that we have Galilean reference frames, where $v_{rel} = v - v = 0$, since it is always possible to introduce a set of points at relative rest and, consequently, for this reason, we can exchange v for $-v$ by symmetry through inverse transformations.

2) -the constancy of c , which is unattainable by massive particles and therefore it could never be related to a set of infinite points at relative rest. In this sense, such “referential”(c), contrary to the 1st. one, is not Galilean because we have “ $c - c' \neq 0 (= c)$ and, for this reason, we can never exchange c for $-c$.”

However, the covariance of a relativistic “ether” S_V places the photon (c) in a certain condition of equality with the motion of other particles ($v < c$), just in the sense that we have completely eliminated the classical idea of rest for reference space (Galilean reference frame) in such a space-time. Since we cannot think about a reference system constituted by a set of infinite points at rest in such a space-time, we should define a non-Galilean reference system essentially as a set of all those particles which have the same state of motion (v) in relation to the ultra-referential- S_V of the relativistic “ether”. Thus SSR should contain 3 postulates as follow:

1) -the constancy of the speed of light (c).

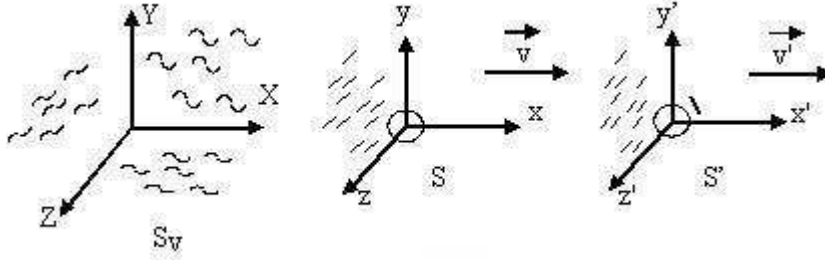


FIG. 1: S_V is the covariant ultra-referential of background field (relativistic “ether”). S represents the non-Galilean reference frame for a massive particle with speed v in relation to S_V , where $V < v < c$. S' represents the non-Galilean reference frame for a massive particle with speed v' in relation to S_V . In this case, we consider $V < v \leq v' \leq c$.

2) -the non-equivalence (asymmetry) of the non-Galilean reference frames, i.e., we cannot exchange v for $-v$ by the inverse transformations, since “ $v - v'' > V (\propto \sqrt{G/\hbar})$ ”, which breaks Lorentz symmetry due to the universal background field associated with S_V .

3) -the covariance of a relativistic “ether” (ultra-referential S_V) associated with the unattainable minimum limit of speed V .

The three postulates described above are compatible among themselves, in the sense that we completely eliminate any kind of Galilean reference system for the space-time of SSR.

Figure 1 illustrates a new conception of reference systems in SSR.

Under SR, there is no ultra-referential S_V , i.e., $V \rightarrow 0$. Hence, the starting point for observing S' is the reference frame S , at which the classic observer thinks he is at rest (Galilean reference frame S).

Under SSR, the starting point for obtaining the actual motion of all particles of S' is the ultra-referential S_V (see Fig.1). However, due to the non-locality of S_V , that is unattainable by the particles, the existence of an observer (local level) at it (S_V) becomes inconceivable. Hence, let us think about a non-Galilean frame S_0 for a certain intermediate speed mode ($V \ll v_0 \ll c$) in order to represent the starting point at local level for “observing” the motion of S' across the ultra-referential S_V . Such a frame S_0 (for v_0 with respect to S_V (Fig.2)) plays the similar role of a “rest”, in the sense that we restore all the newtonian parameters of the particles, such as the proper time interval $\Delta\tau$, i.e., $\Delta t(v = v_0) = \Delta\tau$, the mass m_0 , i.e., $m(v = v_0) = m_0$, among others. Therefore S_0 plays a role that is similar to the frame S under SR, where $\Delta t(v = 0) = \Delta\tau$, $m(v = 0) = m_0$, etc. However, here in SSR, the classical relative rest ($v = 0$) of S should be replaced by a universal “quantum rest” $v_0 (\neq 0)$ of the non-Galilean frame S_0 . We will show that v_0 is also a universal constant. *In short*, S_0 is a universal non-Galilean reference frame with speed v_0 given with respect to S_V . At S_0 , the well-known proper mass (m_0) or proper energy $E_0 = m_0c^2$ of a particle is restored. This means that, at such a frame S_0 , we have the proper energy $E = E_0 = m_0c^2 = m_0c^2\Psi(v_0)$, such that $\Psi(v_0) = 1$, as well as $\gamma(v = 0) = 1$ for the particular case of Lorentz transformations, where $V \rightarrow 0$. So we will look for the general function $\Psi(v)$ of SSR, where we have $E = m_0c^2\Psi(v)$. In the limit $V \rightarrow 0$, indeed we expect that the function $\Psi(v) \rightarrow \gamma(v) = (1 - v^2/c^2)^{-1/2}$ (see Fig.7).

By making the non-Galilean reference frame S (Fig.1) coincide with S_0 , we get Figure 2.

In general, we should have the total interval $V < v < c$ for S' (Fig.2). In short we say that both of the frames S_V and S_0 are already fixed or universal, whereas S' is a rolling frame to describe the variations of the moving state of the particle within such a total interval. Since the rolling frame S' is not a Galilean one due to the impossibility to find a set of points at rest on it, we cannot place the particle exactly on the origin O' , since there would be no exact location on $x' = 0$ (O') (an uncertainty $\Delta x' = \overline{O'C}$: see Figure 3). Actually we want to show that $\Delta x'$ (Fig.3) is a function which should depend on speed v of S' with respect to S_V , namely, for example, if $S' \rightarrow S_V$ ($v \rightarrow V$), then we should have $\Delta x' \rightarrow \infty$ (infinite uncertainty), which is due to the non-local aspect of the ultra-referential S_V . On the other hand, if $S' \rightarrow S_c$ ($v \rightarrow c$), then we should have $\Delta x' \rightarrow 0$ (much better located on O'). Thus let us search for a function $\Delta x' = \Delta x'(v) = \Delta x'_v$, starting from Figure 3.

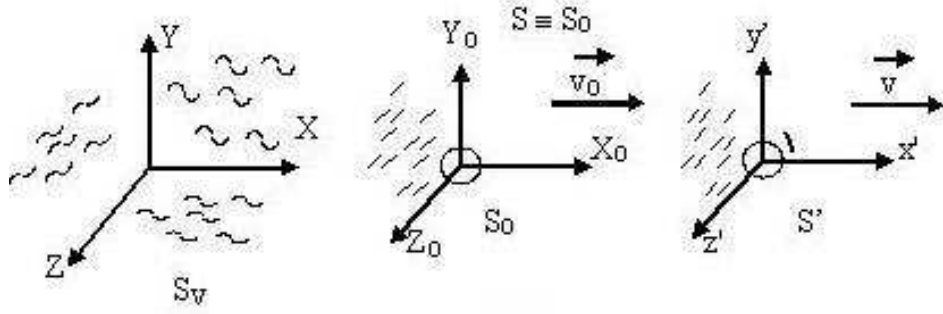


FIG. 2: As S_0 is fixed (universal), being $v_0 \gg V$ given with respect to S_V , we should also consider the new interval V ($S_V < v(S') \leq v_0(S_0)$). This non-classical regime for v introduces a new symmetry in the space-time, leading to SSR. Thus we expect that new and interesting results take place. In such an interval ($V < v \leq v_0$), we will see that $0 < \Psi(v) \leq 1$ (see equations (60), (72) and Fig.7).

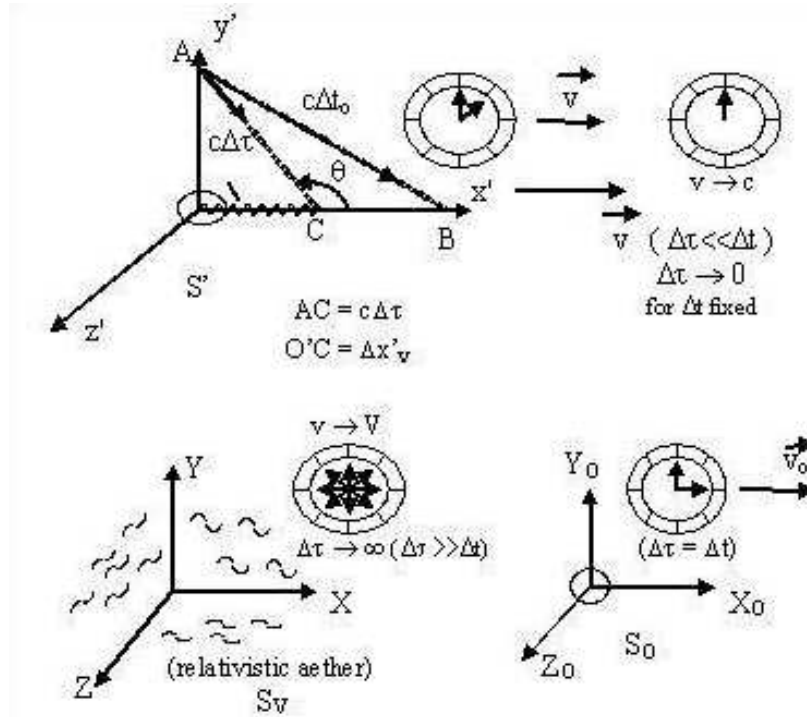


FIG. 3: We have four imaginary clocks associated with non-Galilean reference frames S_0 , S' , the ultra-referential S_V (for V) and also S_c (for c). We observe a new result, namely the proper time (interval $\Delta\tau$) elapses much faster closer to infinite ($\Delta\tau \rightarrow \infty$) when one approaches to S_V . On the other hand, it tends to stop ($\Delta\tau \rightarrow 0$) when $v \rightarrow c$, providing the strong symmetry for SSR. Here we are fixing Δt ($\Delta(t_0)$) and letting $\Delta\tau$ vary.

At the frame S' in Fig.3, let us consider that a photon is emitted from a point A at y' , in the direction $\overline{AO'}$. This occurs only if S' were Galilean (at rest over itself). However, since the electron cannot be thought of as a point at rest on its proper non-Galilean frame S' , and cannot be located exactly on O' , its non-location $\overline{O'C}$ ($= \Delta x'_v$) (see Fig.3) causes the photon to deviate from the direction $\overline{AO'}$ to \overline{AC} . Hence, instead of just the segment $\overline{AO'}$, a rectangular triangle $AO'C$ is formed at the proper non-Galilean reference frame S' , where it is not possible to find a set of points at rest.

From the non-Galilean frame S_0 ("quantum rest"), which plays the role of S , from where one "observes" the motion of S' across S_V , one can see the trajectory \overline{AB} for the photon. Thus the rectangular triangle $AO'B$ is

formed. Since the vertical leg $\overline{AO'}$ is common to the triangles $AO'C$ (for S') and $AO'B$ (for $S_0 \equiv S$), we have

$$(\overline{AO'})^2 = (\overline{AC})^2 - (\overline{O'C})^2 = (\overline{AB})^2 - (\overline{O'B})^2, \quad (45)$$

or else

$$(c\Delta\tau)^2 - (\Delta x'_v)^2 = (c\Delta t_0)^2 - (v\Delta t_0)^2. \quad (46)$$

If $\Delta x'(v) = \Delta x'_v = 0$ ($V \rightarrow 0 \Rightarrow S_V \equiv S_0 \equiv S$), we go back to the classical case (SR), where we consider for instance a train wagon (S'), which is moving in relation to a fixed rail (S). At a point A on the ceiling of the wagon, there is a laser that releases photons toward y' , reaching the point O' assumed in the origin of S' (on the floor of the train wagon). For Galilean- S' , the trajectory of the photon is $\overline{AO'}$. For Galilean- S , its trajectory is \overline{AB} .

Since $\Delta x'_v$ is a function of v , assumed as a kind of “displacement” (uncertainty) given on the proper non-Galilean reference frame S' , we may write it in the following way:

$$(\Delta x'_v) = f(v)\Delta\tau, \quad (47)$$

where $f(v)$ is a function of v , which also presents dimension of velocity, i.e., it is a certain velocity in SSR, which could be thought of as a kind of *internal motion* v_{int} of the particle, being responsible for the increasing or dilation (stretch) of an internal dimension of the particle on its own non-Galilean frame S' . Such an internal dilation is given by the non-classical “displacement” $\Delta x'_v = \overline{O'C}$ (see Fig.3). This leads us to think that there is an uncertainty of position for the particle, as we will see later. Hence, substituting (47) into (46), we obtain

$$\Delta\tau \left[1 - \frac{(f(v))^2}{c^2} \right]^{\frac{1}{2}} = \Delta t \left(1 - \frac{v^2}{c^2} \right)^{\frac{1}{2}}, \quad (48)$$

where we use the notation Δt_0 or Δt ($S_0 \equiv S$), and where we have $f(v) = v_{int}$ to be duly interpreted.

Thus, since we have $v \leq c$, we should have $f(v) \leq c$ in order to avoid an imaginary number in the 1st. member of (48).

The domain of $f(v)$ is such that $V \leq v \leq c$. Thus, let us also think that its image is $V \leq f(v) \leq c$, since $f(v)$ has dimension of velocity and also represents a speed v_{int} (internal motion), which also must be limited for the extremities V and c .

Let us make $[f(v)]^2/c^2 = f^2/c^2 = v_{int}^2/c^2 = \alpha^2$, whereas we already know that $v^2/c^2 = \beta^2$. v is the well-known external motion (spatial velocity). Thus we have the following cases originated from (48), namely:

- (i) When $v \rightarrow c$ ($\beta \rightarrow \beta_{max} = 1$), the relativistic correction in its 2nd. member (right-hand side) prevails, whereas the correction on the left-hand side becomes practically neglected, i.e., we should have $v_{int} = f(v) \ll c$, where $\lim_{v \rightarrow c} f(v) = f_{min} = (v_{int})_{min} = V$ ($\alpha \rightarrow \alpha_{min} = V/c = \xi$). $\xi \cong 3.57 \times 10^{-24}$ (refer to (41)).

- (ii) On the other hand, due to idea of symmetry, if $v \rightarrow V$ ($\beta \rightarrow \beta_{min} = V/c = \xi$), there is no substantial relativistic correction on the right-hand side of (48), whereas the correction on the left-hand side becomes now considerable, namely we should have $\lim_{v \rightarrow V} f(v) = f_{max} = (v_{int})_{max} = c$ ($\alpha \rightarrow \alpha_{max} = 1$).

In short, from (i) and (ii), we observe that, if $v \rightarrow v_{max} = c$, then $f \rightarrow f_{min} = (v_{int})_{min} = V$, and if $v \rightarrow v_{min} = V$, then $f \rightarrow f_{max} = (v_{int})_{max} = c$. So now we perceive that the internal motion v_{int} ($= f(v)$) works like a reciprocal speed (v_{Rec}) in such a symmetrical structure of space-time in SSR. In other words, we notice that the (external or spatial) velocity v increases to c whereas the internal (reciprocal) one ($v_{int} = v_{Rec}$) decreases to V . On the other hand, when v tends to V (S_V), v_{int} tends to c , leading to a large internal stretch (uncertainty $\Delta x'_v$) due to a non-locality behavior much closer to the ultra-referential S_V . Due to this fact, we reason that

$$f(v) = v_{int} = v_{Rec} = \frac{a}{v}, \quad (49)$$

where a is a constant that has dimension of square speed. Such a reciprocal velocity v_{Rec} will be better understood later. It is interesting to know that a similar idea of considering an internal motion for microparticles was also thought by Natarajan[26].

In addition to (48) and (49),we already know that,at the referential S_0 (see Fig.2 and Fig.3),we should have the condition of equality of the time intervals,namely $\Delta t = \Delta\tau$ for $v = v_0$,which,in accordance with (48),occurs only if

$$\frac{[f(v_0)]^2}{c^2} = \frac{v_0^2}{c^2} \Leftrightarrow f(v_0) = v_0 \quad (50)$$

By comparing (50) with (49) for the case $v = v_0$,we obtain

$$a = v_0^2 \quad (51)$$

Substituting (51) into (49),we obtain

$$f(v) = v_{int} = v_{Rec} = \frac{v_0^2}{v} \quad (52)$$

According to (52) and also considering (i) and (ii),indeed we observe respectively that $f(c) = V = v_0^2/c$ (V is the reciprocal velocity of c) and $f(V) = c = v_0^2/V$ (c is the reciprocal velocity of V),from where we immediately obtain

$$v_0 = \sqrt{cV} \quad (53)$$

As we already know the value of V (refer to (43)) and c ,we obtain the velocity of ‘‘quantum rest’’ $v_0 \cong 5.65 \times 10^{-4}m/s$,which is also universal just because it depends on the universal constants c and V . However,we must stress that only c and V remain invariant under speed transformations in such a space-time of SSR.

Finally,by substituting (53) into (52) and after into (48),we finally obtain

$$\Delta\tau \sqrt{1 - \frac{V^2}{v^2}} = \Delta t \sqrt{1 - \frac{v^2}{c^2}}, \quad (54)$$

where $\alpha = f(v)/c = v_{int}/c = V/v$ and $\beta = v/c$ inside (54). In fact,if $v = v_0 = \sqrt{cV}$ in (54),so we have $\Delta\tau = \Delta t$. Therefore we conclude that S_0 (v_0) is the intermediate (non-Galilean) reference frame such that,if:

a) $v \gg v_0$ ($v \rightarrow c$) $\Rightarrow \Delta t \gg \Delta\tau$: It is the well-known *time dilation* .

b) $v \ll v_0$ ($v \rightarrow V$) $\Rightarrow \Delta t \ll \Delta\tau$: Let us call this new result *contraction of time* . This shows us the novelty that the proper time interval ($\Delta\tau$) is variable,so that it may expand in relation to the improper one (Δt in S_0). $\Delta\tau$ is an intrinsic variable for the particle on its proper non-Galilean frame S' . Such an effect of dilation of $\Delta\tau$ with respect to Δt would become more evident only for $v \rightarrow V$ (S_V),since we would have $\Delta\tau \rightarrow \infty$ in such a limit S_V . In other words,this means that the proper time (S') would elapse much faster than the improper one at S_0 .

In SSR,it is interesting to notice that we restore the newtonian regime when $V \ll v \ll c$,which represents an intermediate regime of speeds,where we can make the approximation $\Delta\tau \approx \Delta t$.

Substituting (52) into (47) and also considering (53),we obtain

$$\overline{O'C} = \Delta x'_v = v_{int}\Delta\tau = v_{Rec}\Delta\tau = \frac{v_0^2}{v}\Delta\tau = \frac{Vc}{v}\Delta\tau = \alpha c\Delta\tau \quad (55)$$

Actually we can verify that,if $V \rightarrow 0$ or $v_0 \rightarrow 0$,this implies $\overline{O'C} = \Delta x'_v = 0$, restoring the classical case (SR),where there is no such an internal motion. And also,if $v \gg v_0$,this implies $\Delta x'_v \approx 0$,i.e.,we have an approximation where the internal motion is neglected.

From (55),it is important to observe that,if $v \rightarrow c$,we have $\Delta x'(c) = V\Delta\tau$ and,if $v \rightarrow V$ (S_V), we have $\Delta x'(V) = c\Delta\tau$. This means that,when the particle momentum with respect to S_V increases ($v \rightarrow c$),it becomes much more localized upon itself over O' ($V\Delta\tau \rightarrow 0$) and,when its momentum decreases ($v \rightarrow V$),it becomes much less localized over O' ,because it gets much closer to the non-local ultra-referential S_V ,where $\Delta x'_v = \Delta x'_{max} = \overline{O'C}_{max} = c\Delta\tau \rightarrow \infty$. Thus,now we begin to perceive that the velocity v (momentum) and the position (non-localization $\Delta x'_v = v_{Rec}\Delta\tau$) operate like mutually reciprocal quantities in such a space-time of SSR,since the non-localization is $\Delta x'_v \propto v_{Rec} \propto v^{-1}$ (see (49) or (52)). This really provides a basis for the fundamental comprehension of the quantum uncertainties in a context of objective reality of the space-time,according to Einstein's vision[18].

It is very interesting to observe that we may write $\Delta x'_v$ in the following way:

$$\Delta x'_v = \frac{(V\Delta\tau)(c\Delta\tau)}{v\Delta\tau} \equiv \frac{\Delta x'_5 \Delta x'_4}{\Delta x'_1}, \quad (56)$$

where $V\Delta\tau = \Delta x'_5$, $c\Delta\tau = \Delta x'_4$ and $v\Delta\tau = \Delta x'_1$. We also know that $c\Delta t_0 \equiv c\Delta t = \Delta x_4$ and $v\Delta t_0 \equiv v\Delta t = \Delta x_1$ for the frame $S(\equiv S_0)$. So we write (46) in the following way:

$$\Delta x_4'^2 - \frac{\Delta x_5'^2 \Delta x_4'^2}{\Delta x_1'^2} = \Delta x_4^2 - \Delta x_1^2, \quad (57)$$

where $\Delta x'_5$ corresponds to a fifth dimension of temporal nature. Therefore we may already conclude that the new geometry of space-time has three spatial dimensions (x_1, x_2, x_3) plus two temporal dimensions ($c\Delta t, V\Delta\tau$),being $V\Delta\tau$ normally hidden. However,we will perceive elsewhere that we can also describe such a space-time in a compact form as effectively a 4-dimensional structure,because $V\Delta\tau$ and $c\Delta t$ represents two complementary aspects of the same temporal nature, and also mainly because $V\Delta\tau$ appears as an implicit variable for the space-time interval $c\Delta\tau$ (see (61), (62) or (63)).

If $\Delta x'_5 \rightarrow 0$ ($V \rightarrow 0$),we restore the invariance of the 4-dimensional interval in Minkowski space as a particular case,that is, $\Delta S^2 = \Delta x_4^2 - \Delta x_1^2 = \Delta S'^2 = \Delta x_4'^2$.

As we have $\Delta x'_v > 0$,we observe that $\Delta S'^2 = \Delta x_4'^2 > \Delta S^2 = \Delta x_4^2 - \Delta x_1^2$. Hence,we may write (57),as follows:

$$\Delta S'^2 = \Delta S^2 + \Delta x_v'^2, \quad (58)$$

where $\Delta S' = \overline{AC}$, $\Delta x'_v = \overline{O'C}$ and $\Delta S = \overline{AO'}$ (refer to Fig.3).

For $v \gg V$ or also $v \rightarrow c$,we have $\Delta S' \approx \Delta S$, hence $\theta \approx \frac{\pi}{2}$ (see Fig.3). In macroscopic world (or very large masses),we have $\Delta x'_v = \Delta x'_5 = 0$ (hidden dimension),hence $\theta = \frac{\pi}{2} \Rightarrow \Delta S' = \Delta S$. The quantum uncertainties can be neglected in such a particular regime (Galilean reference frames of SR).

For $v \rightarrow V$,we would have $\Delta S' \gg \Delta S$, where $\Delta S' \approx c\Delta\tau$,with $\Delta\tau \rightarrow \infty$ and $\theta \rightarrow \pi$. In this new relativistic limit (relativistic "ether" S_V),due to the maximum non-localization $\Delta x'_v \rightarrow \infty$,the 4-dimensional interval $\Delta S'$ loses completely its equivalence in respect to ΔS ,because 5th dimension ($V\Delta\tau$) increases drastically much closer to such a limit,i.e., $\Delta x'_5 \rightarrow \infty$. So it ceases to be hidden for such very special case.

Equation (58) or (57) shows us a break of the 4-interval invariance ($\Delta S' \neq \Delta S$),which becomes noticeable only at the limit $v \rightarrow V$ (S_V). However,a new invariance is restored when we implement a 5th.dimension (x'_5) to be intrinsic to the particle (frame S') through the definition of a new (effective) general interval,where the interval $V\Delta\tau$ appears as an implicit variable,namely:

$$\Delta S_5 = \sqrt{\Delta S'^2 - \Delta x_v'^2} = \Delta x_4' \sqrt{1 - \frac{\Delta x_5'^2}{\Delta x_1'^2}} = c\Delta\tau \sqrt{1 - \frac{V^2}{v^2}}, \quad (59)$$

such that $\Delta S_5 \equiv \Delta S$ (see (58)).

We have omitted the index l for Δx_5 ,as such an interval is given only at the non-Galilean proper reference frame (S'),that is intrinsic to the particle. Actually such a 5-interval or simply an effective 4-interval $c\Delta\tau^* =$

$c\Delta\tau\sqrt{1-\alpha^2}$ guarantees the existence of a certain effective internal dimension for the electron. However, from a practical viewpoint, for experiments of higher energies, the electron approximates more and more to a punctual particle, since Δx_5 becomes hidden. So in order to detect its internal dimension, it should be at very low energies, namely very close to S_V .

Comparing (59) with the left side of equation (54), we may alternatively write

$$\Delta t = \Psi \Delta\tau = \frac{\Delta S_5}{c\sqrt{1-\frac{v^2}{c^2}}} = \Delta\tau \frac{\sqrt{1-\frac{V^2}{v^2}}}{\sqrt{1-\frac{v^2}{c^2}}}, \quad (60)$$

where ΔS_5 is the invariant effective interval given at the frame S' . We have $\Psi = \frac{\sqrt{1-\alpha^2}}{\sqrt{1-\beta^2}} = \frac{\sqrt{1-\frac{V^2}{v^2}}}{\sqrt{1-\frac{v^2}{c^2}}}$ and,

alternatively, we can also write $\Psi = \frac{\sqrt{1-\beta_{int}^2}}{\sqrt{1-\alpha_{int}^2}} = \frac{\sqrt{1-\frac{v_{int}^2}{c^2}}}{\sqrt{1-\frac{V^2}{v_{int}^2}}}$, since $\alpha = V/v = \beta_{int} = v_{int}/c$ and $\beta = v/c = \alpha_{int} = V/v_{int}$, from where we get $v_{int} = v_{Rec} = cV/v = v_0^2/v$ (see (52)). Only for $v = v_0$, we obtain $v_{int} = v = v_0$. Although we cannot obtain directly v_{int} by any experiment (just the uncertainty Δx is obtained), we could also use Ψ in its alternative form $\Psi(v_{int})$. However, let us use $\Psi(v)$.

For $v \gg V$, we get $\Delta t \approx \gamma \Delta\tau$, where $\Psi \approx \gamma = (1-\beta^2)^{-1/2}$.

Substituting (55) into (46) and using the notation $\Delta t_0 \equiv \Delta t$, we obtain

$$c^2 \Delta\tau^2 = \frac{1}{(1-\frac{V^2}{v^2})} [c^2 \Delta t^2 - v^2 \Delta t^2], \quad (61)$$

from where, we also obtain the equation (54).

By placing (61) in a differential form and manipulating it, we will obtain

$$c^2 \left(1 - \frac{V^2}{v^2}\right) \frac{d\tau^2}{dt^2} + v^2 = c^2 \quad (62)$$

We may write (62) in the following alternative way:

$$\frac{dS_5^2}{dt^2} + v^2 = c^2, \quad (63)$$

where $dS_5 = c\sqrt{1-\frac{V^2}{v^2}}d\tau$.

Equation (62) shows us that the speed related to the marching of time (“temporal-speed”), which is $v_t = c\sqrt{1-\frac{V^2}{v^2}}\frac{d\tau}{dt}$, and the spatial speed, which is v in relation to the background field for S_V form respectively the vertical and horizontal legs of a rectangular triangle.

We have $c = (v_t^2 + v^2)^{1/2}$, which represents the spatio-temporal velocity of any particle (hypotenuse of the triangle = c). The novelty here is that such a space-time implements the ultra-referential S_V . Such an implementation arises at the vertical leg v_t of such a rectangular triangle.

We should consider 3 important cases as follow:

- If $v \approx c$, then $v_t \approx 0$, where $\Psi \gg 1$, since $\Delta t \gg \Delta\tau$ (*dilation of time*).
- If $v = v_0 = \sqrt{cV}$, then $v_t = \sqrt{c^2 - v_0^2}$, where $\Psi = \Psi_0 = \Psi(v_0) = 1$, since $\Delta t = \Delta\tau$ (“*quantum rest*” S_0).
- If $v \approx V$, then $v_t \approx \sqrt{c^2 - V^2} = c\sqrt{1-\xi^2}$, where $\Psi \ll 1$, since $\Delta t \ll \Delta\tau$ (*contraction of time*).

In SR, when $v = 0$, we have $v_t = v_{tmax} = c$. However, in accordance with SSR, due to the existence of a minimum limit V of spatial speed for the horizontal leg of the triangle, we see that the maximum temporal-speed is $v_{tmax} < c$. This means that we have $v_{tmax} = c\sqrt{1-\xi^2}$. Such a result introduces a strong symmetry in such a space-time of SSR, in the sense that both of spatial and temporal speeds c become unattainable for all massive particles.

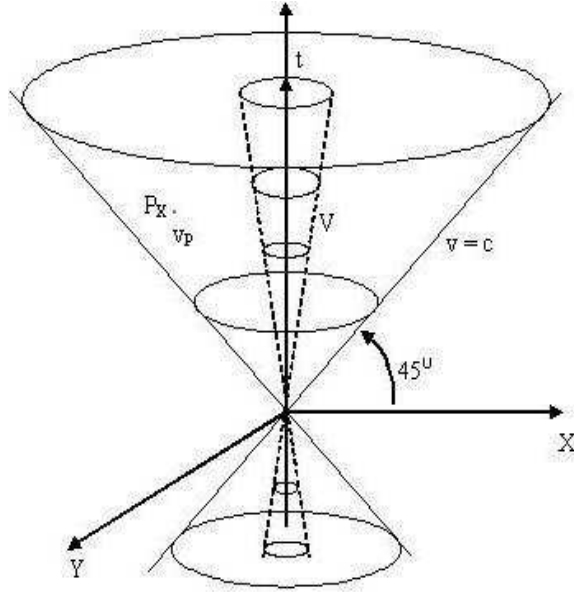


FIG. 4: The external and internal conical surfaces represent respectively c and V , where V is represented by the dashed line, that is a definitely prohibited boundary. For a point P in the interior of the two conical surfaces, there is a corresponding internal conical surface, such that $V < v_P \leq c$.

The speed $v = c$ is represented by the photon (massless particle), whereas $v = V$ is definitely inaccessible for any particle. Actually we have $V < v \leq c$, but, in this sense, we have a certain asymmetry, as there is no particle at the ultra-referential S_V where there should be a kind of *sui generis* vacuum energy density ($\rho^{(2)}$) to be studied elsewhere.

In order to produce a geometric representation for that problem ($V < v \leq c$), let us assume the world line of a particle limited by the surfaces of two cones, as shown in Figure 4.

A spatial speed $v = v_P$ in the representation of light cone shown in Figure 4 (horizontal leg of the rectangular triangle) is associated with a temporal speed $v_t = v_{tP} = \sqrt{c^2 - v_P^2}$ (vertical leg of the same triangle) given in another cone representation, which could be denominated *temporal cone* (Figure 5).

We must observe that a particle moving just at one spatial dimension always goes only to right or left, since the unattainable non-null minimum limit of speed V forbids it to stop its spatial velocity ($v = 0$) in order to return at this same spatial dimension. On the other hand, in a complementary way to V , the limit c is temporal in the sense that it forbids to stop the time (temporal velocity $v_t = 0$) and also to come back to the past. However, if we consider more than one spatial dimension, at least 2 spatial dimensions (xy), the particle can now return by moving at the additional dimension(s). So SSR provides the reason why we must have more than one (1) spatial dimension ($d > 1$) for representing movement in reality, although we could consider $1d$ just as a good approximation for some cases in classical space-time of SR (classical objects). Such a minimum limit V has deep implications for understanding the irreversible aspect of time connected to movement, since we can now distinguish the motions to right and left in the time. Such an asymmetry generated by SSR really deserves a deeper treatment elsewhere.

Based on (61) or also by substituting (55) into (46), we obtain

$$c^2 \Delta t^2 - v^2 \Delta t^2 = c^2 \Delta \tau^2 - \frac{v_0^4}{v^2} \Delta \tau^2 \quad (64)$$

In (64), when we transpose the 2nd. term from the left side to the right side and divide the equation by Δt^2 , we obtain (62) in the differential form. Now, it is important to observe that, upon transposing the 2nd. term from the

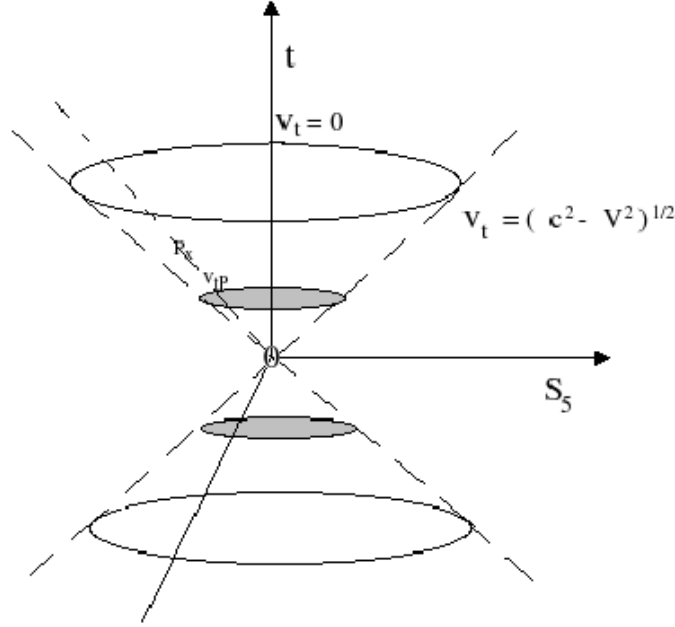


FIG. 5: Comparing this Figure 5 with Figure 4, we notice that the dashed line on the internal cone of Figure 4 ($v = V$) corresponds to the dashed line on the surface of the external cone of this Figure 5, where $v_t = \sqrt{c^2 - V^2}$, which represents a definitely forbidden boundary in this cone representation of temporal speed v_t . On the other hand, $v = c$ (photon) is represented by the solid line of Figure 4, which corresponds to the temporal speed $v_t = 0$ in this Figure 5, coinciding with the vertical axis t . In short, we always have $v^2 + v_t^2 = c^2$, being v for spatial (light) cone (Figure 4) and v_t for temporal cone represented in this Figure 5, such that an internal point P_x is related to a temporal velocity v_{tP} , where 0 (photon) $\leq v_{tP} (= \sqrt{c^2 - v_P^2}) < \sqrt{c^2 - V^2}$. The horizontal axis is $S_5 = c\sqrt{1 - V^2/v^2}\tau$, so that $v_t = dS_5/dt = c\sqrt{1 - V^2/v^2}d\tau/dt = \sqrt{c^2 - v^2}$ (see equation (54)).

right side to the left one and dividing the equation by $\Delta\tau^2$, we obtain the following equation in the differential form, namely:

$$c^2\left(1 - \frac{v^2}{c^2}\right)\frac{dt^2}{d\tau^2} + \frac{v_0^4}{v^2} = c^2 \quad (65)$$

From (59) and (54), we obtain $dS_5 = cd\tau\sqrt{1 - \alpha^2} = cdt\sqrt{1 - \beta^2}$. Hence we can write (65) in the following alternative way:

$$\frac{dS_5^2}{d\tau^2} + v_{Rec}^2 = \frac{dS_5^2}{dt^2} + \frac{v_0^4}{v^2} = c^2 \quad (66)$$

We see that equation (65) or (66) reveals a complementary way of viewing equation (62) or (63). This leads us to that idea of reciprocal space for conjugate quantities. Thus let us write (65) or (66) in the following way:

$$v_{tRec}^2 + v_{Rec}^2 = c^2, \quad (67)$$

where $v_{tRec} = (v_t)_{int} = dS_5/d\tau = c\sqrt{1 - \frac{v^2}{c^2}\frac{dt}{d\tau}}$, which represents an *internal (reciprocal) temporal velocity*. The internal (reciprocal) spatial velocity is $v_{int} = v_{Rec} = f(v) = \frac{v_0^2}{v}$. Therefore we can also represent a rectangular triangle, but now displayed in a reciprocal space. For example, if we assume $v \rightarrow c$ (equation (62)), we obtain

$v_{Rec} = \lim_{v \rightarrow c} f(v) \rightarrow \frac{v_0^2}{c} = V$ (equation (65)). In this same case, we have $v_t \rightarrow 0$ (equation (62)) and $v_{tRec} = \frac{dS_5}{d\tau} \rightarrow \sqrt{c^2 - V^2}$ (equation (65) or (66)). On the other hand, if $v \rightarrow V$ (eq.(62)), we have $v_{Rec} \rightarrow \frac{v_0^2}{V} = c$ (eq.(65)), where $v_t \rightarrow \sqrt{c^2 - V^2}$ (eq.(62)) and $(v_t)_{int} = v_{tRec} \rightarrow 0$ (eq.(65)). Thus we should observe that there are altogether four cone representations in such a symmetrical structure of space-time in SSR, namely:

$$\text{two spatial representations : } \begin{cases} a_1)v = \frac{dx}{dt}, \text{ in equation (62),} \\ \text{represented in Fig.4;} \\ b_1)v_{Rec} = \frac{dx'_v}{d\tau} = \frac{v_0^2}{v}, \text{ in equation (65).} \end{cases} \quad (68)$$

$$\text{two temporal representations : } \begin{cases} a_2)v_t = \frac{dS_5}{dt} = c\sqrt{1 - \frac{V^2}{v^2}} \frac{d\tau}{dt} = c\sqrt{1 - \frac{v^2}{c^2}}, \\ \text{in equation (62), represented in Fig.5;} \\ b_2)v_{tRec} = \frac{dS_5}{d\tau} = c\sqrt{1 - \frac{v^2}{c^2}} \frac{dt}{d\tau} = c\sqrt{1 - \frac{V^2}{v^2}}, \\ \text{in equation (65).} \end{cases} \quad (69)$$

The chart given in Figure 6 shows us those four representations.

Now, by considering (54),(60),(69) and also looking at a_2 and b_2 in Fig.6, we may observe that

$$\Psi^{-1} = \frac{\Delta\tau}{\Delta t} = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \frac{V^2}{v^2}}} = \frac{v_t}{c\sqrt{1 - \frac{V^2}{v^2}}} = \frac{v_t}{v_{tRec}} \propto (\text{time}) \quad (70)$$

and

$$\Psi = \frac{\Delta t}{\Delta\tau} = \frac{\sqrt{1 - \frac{V^2}{v^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{v_{tRec}}{c\sqrt{1 - \frac{v^2}{c^2}}} = \frac{v_{tRec}}{v_t} \propto E \text{ (Energy } \propto (\text{time})^{-1}) \quad (71)$$

From (71), since we have energy $E \propto \Psi$, we write $E = E_0\Psi$, where E_0 is a constant of proportionality. Hence, if we consider $E_0 = m_0c^2$, we obtain

$$E = m_0c^2 \frac{\sqrt{1 - \frac{V^2}{v^2}}}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (72)$$

where E is the total energy of the particle in relation to the absolute inertial frame of universal background field S_V . Such a result shall be explored in a coming article about the dynamics of the particles in SSR. In (71) and (72), we observe that, if $v \rightarrow c \Rightarrow E \rightarrow \infty$ and $\Delta\tau \rightarrow 0$ for Δt fixed. If $v \rightarrow V \Rightarrow E \rightarrow 0$ and $\Delta\tau \rightarrow \infty$, also for Δt fixed. If $v = v_0 = \sqrt{cV} \Rightarrow E = E_0 = m_0c^2$ (energy of “quantum “rest”). Figure 7 shows us the graph for the energy E in (72).

B. The Uncertainty Principle

The particle actual momentum (in relation to S_V) is $P = \Psi m_0 v$, whose conjugate value is $\Delta x'_v = \frac{v_0^2}{v} \Delta\tau = \frac{v_0^2}{v} \Delta t \Psi^{-1}$, where $\Delta\tau = \Psi^{-1} \Delta t$ (refer to (54)). From S_V it would be possible to know exactly the actual momentum P and the total energy E of the particle, however, since S_V represents an ultra-referential which is unattainable (non-local) and also inaccessible for us, so one becomes impossible to measure such quantities with accuracy. And for this reason, as a classical observer (local and macroscopic) is always at rest ($v = 0$) in his proper reference frame S , he measures and interprets E without accuracy because his frame is Galilean, *being*

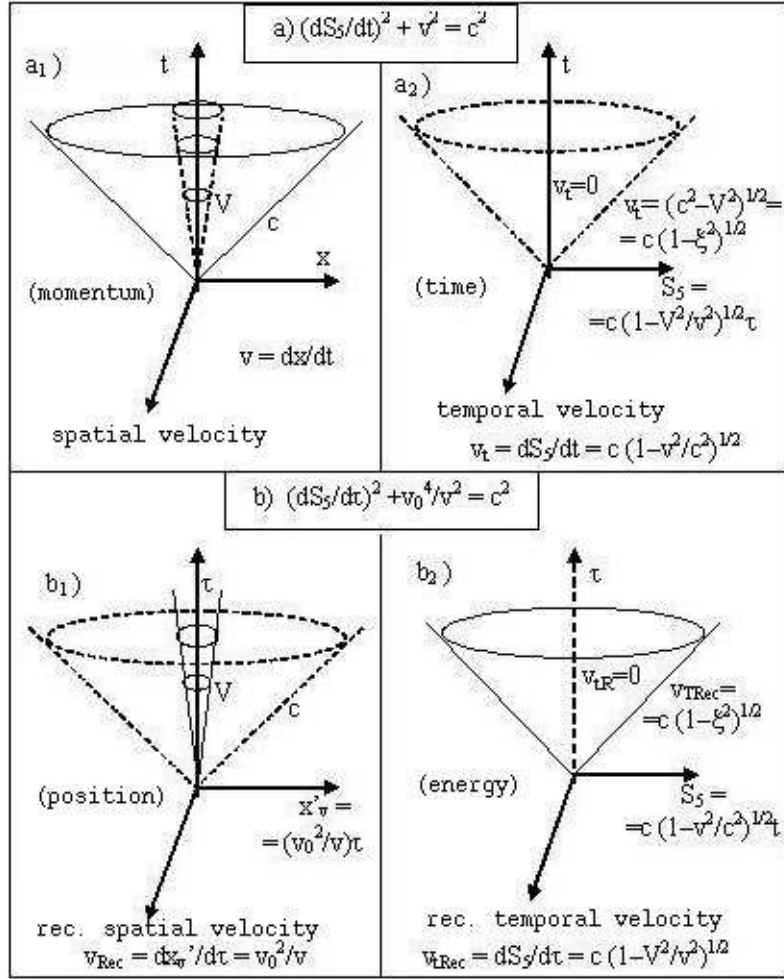


FIG. 6: The spatial representations in a_1 (also shown in Figure 4) and b_1 are related respectively to velocity v (momentum) and position (non-localization $\Delta x'_v = f(v)\Delta\tau = v_{int}\Delta\tau = v_{Rec}\Delta\tau = (v_0^2/v)\Delta\tau$), which represent conjugate (reciprocal) quantities in space. On the other hand, the temporal representations in a_2 (also shown in Figure 5) and b_2 are related respectively to time ($\propto v_t$) and energy ($\propto v_{tRec} = (v_t)_{int} \propto v_t^{-1}$), which represent conjugate (reciprocal) quantities in the time. Hence we can perceive that such four cone representations of SSR provide a basis for the fundamental understanding of the two uncertainty relations.

related essentially to macroscopic systems (a set of points at rest), whereas on the other hand, non-Galilean reference frames for representing subatomic world in SSR are really always moving for any transformation in such a space-time and therefore cannot be related to a set of points at rest. Due to this conceptual discrepancy between the nature of non-Galilean reference frames in SSR (no rest) and the nature of Galilean reference frames in SR for classical observers (with rest), the total energy E in SSR (eq.(72)) behaves as an uncertainty ΔE for such classical observers at rest, i.e., E (for S_V) \equiv ΔE (for any Galilean- S at rest). Similarly P also behaves as an uncertainty Δp (P (S_V) \equiv Δp (Galilean- S)) and, in addition, the non-localization $\Delta x'_v$ as simply an uncertainty Δx . Hence we have

$$\Delta x'_v P \equiv (\Delta x \Delta p)_{classical\ observer\ S} = \frac{v_0^2}{v} \Delta t \Psi^{-1} \Psi m_0 v = (m_0 v_0)(v_0 \Delta t) \quad (73)$$

and

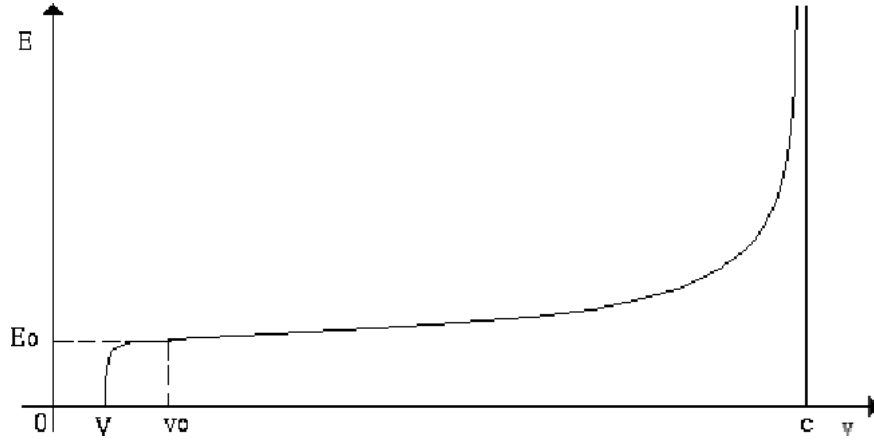


FIG. 7: v_0 represents the velocity of “quantum rest” in SSR, from where we get $E = E_0 = m_0c^2$, being $\Psi_0 = \Psi(v_0) = 1$.

$$\Delta\tau E \equiv (\Delta\tau \Delta E)_{\text{classical observer } S} = \Delta t \Psi^{-1} \Psi m_0 c^2 = (m_0 c)(c \Delta t), \quad (74)$$

where we consider again Δt fixed and let $\Delta\tau$ vary for each case. In obtaining (73) and (74), we also have considered the relations $\Delta x'_v = \frac{v_0^2}{v} \Delta\tau$, $\Delta\tau = \Delta t \Psi^{-1}$, $P = \Psi m_0 v$ and $E = \Psi m_0 c^2$.

Since we know the actual momentum P of the particle moving across the relativistic “ether”- S_V , its de-Broglie wavelength is

$$\lambda = \frac{h}{P} = \frac{h}{\Psi m_0 v} = \frac{h}{m_0 v} \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \frac{V^2}{v^2}}} \quad (75)$$

If $v \rightarrow c \Rightarrow \lambda \rightarrow 0$ (*spatial contraction* or *temporal dilation*), and if $v \rightarrow V \Rightarrow \lambda \rightarrow \infty$ (*spatial dilation* or *temporal contraction*). In such a space-time of SSR, actually we should interpret the spatial scales as wavelengths λ given at the background frame S_V , in accordance with (75).

The relationship (75) shows us a strong symmetry that enables us to understand the space as an elastic structure, which is capable of contracting ($\lambda \rightarrow 0$ for $v \rightarrow c$) and also expanding ($\lambda \rightarrow \infty$ for $v \rightarrow V$ (S_V)).

The wavelength λ in (75) may be thought of as being related to the non-localization $\Delta x'_v$, namely $\lambda \propto \Delta x'_v$. Such a proportionality is verified by comparing (55) with (75) and also by considering $\Delta\tau = \Psi^{-1} \Delta t$. Hence we have

$$\lambda \propto \Delta x'_v = \frac{v_0^2}{v} \Delta\tau = \frac{v_0^2}{v} \Delta t \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\sqrt{1 - \frac{V^2}{v^2}}}, \quad (76)$$

where $\lambda \propto \Delta x'_v$ ($\equiv \Delta x$) = $v_{int} \Delta\tau = v_{Rec} \Delta\tau \propto (v \Psi)^{-1}$. We also make Δt fixed and let $\Delta\tau$ vary, such that $0 < \Delta\tau < \infty$. Now, we can perceive that the quantum nature of the wave is derived from the internal motion $v_{int} = v_{Rec}$ of the proper particle, since its wavelength for S_V is $\lambda \propto v_{Rec}$. This leads to a fundamental explanation for the wave-particle duality in such a space-time of SSR. Natarajan[26] also used a kind of internal motion v_{in} [26] of the microparticle to explain in alternative way such a dual aspect of the matter. In approximation for SR, we have $V \rightarrow 0$ (or also $v_0 \rightarrow 0$), so that $v_{Rec} = 0 \Rightarrow \lambda = 0$. Indeed this means that the wave nature of the matter is not included in SR.

Now let us observe that, if we make $v = v_0$ in (76) and (75), and then compare these two results, we obtain

$$v_0 \Delta t \equiv v_0 T_0 \sim \lambda_0 = \frac{h}{m_0 v_0} \sim 1m, \quad (77)$$

where we fix $\Delta t \equiv T_0 \sim \frac{h}{m_0 v_0^2}, m_0$ being the electron mass. T_0 represents the period of the wave with length λ_0 , such that $T_0 \sim 10^3 s$. λ_0 is a special standard intermediate scale for the frame S_0 . Since $\lambda_0 \sim 1m$, indeed it represents a typical scale of a classical observer (human scale).

Finally, by substituting (77) into (73), we obtain

$$\Delta x'_v P \equiv \Delta x \Delta p \sim m_0 v_0 \lambda_0 = h \quad (78)$$

Now, it is easy to conclude that

$$\Delta \tau E \equiv \Delta \tau \Delta E \sim m_0 c \lambda_c = h, \quad (79)$$

where $c \Delta t \equiv c T_c \sim \lambda_c = \frac{h}{m_0 c}$ (refer to (74)). $\lambda_c \sim 10^{-12} m$ (Compton wavelength for the photon whose energy $m c^2$ ($\propto e_s b_s$) must be equivalent to the electron energy $m_0 c^2$ ($\propto e_{s0} b_{s0}$), that is, $m \equiv m_0$. In this case, $\Delta t \equiv T_c \sim \frac{h}{m_0 c^2}$).

It is interesting to notice that $\frac{\lambda_c}{\lambda_0} = \frac{v_0}{c}$, where $\lambda_0 \sim 1m$. It is also very curious to observe that $\lambda_c = \frac{v_0}{c} \lambda_0 = \frac{V}{v_0} \lambda_0 \iff v_0 = \sqrt{cV}$, which in fact represents a special intermediate point (a kind of *aurum point*), namely it represents a geometric average between c and V , where the human scale ($\lambda_0 \sim 10^0 m$) is really found as an intermediate scale. Thus we may write $\lambda_c^2 = \beta_0^2 \lambda_0^2 = \alpha_0^2 \lambda_0^2$, such that $\lambda_c^2 = \xi \lambda_0^2 \sim 10^{-24} m^2$, where we have $\beta_0^2 = \alpha_0^2 = v_0^2/c^2 = V^2/v_0^2 = V/c = \xi \sim 10^{-24}$.

As we already know the total energy $E = m_0 c^2 \Psi$ and the momentum $\vec{P} = m_0 \vec{v} \Psi$ at S_V , we can demonstrate that $E^2 = c^2 \vec{P}^2 + m_0^2 c^4 (1 - V^2/v^2)$, where Ψ is shown in (71).

VI. TRANSFORMATIONS OF SPACE-TIME AND VELOCITY IN THE PRESENCE OF THE ULTRA-REFERENTIAL S_V

Let us assume the reference frame S' with a speed v in relation to the ultra-referential S_V . To simplify, consider the motion only at one spatial dimension, namely $(1+1)D$ -space-time with background field S_V . So we write the following transformations:

$$dx' = \Psi(dX - \beta_* c dt) = \Psi(dX - v dt + V dt), \quad (80)$$

where $\beta_* = \beta \epsilon = \beta(1 - \alpha)$, being $\beta = v/c$ and $\alpha = V/v$, so that $\beta_* \rightarrow 0$ for $v \rightarrow V$ or $\alpha \rightarrow 1$.³

³ Let us assume the following more general transformations: $x' = \theta \gamma (X - \epsilon_1 v t)$ and $t' = \theta \gamma (t - \frac{\epsilon_2 v X}{c^2})$, where θ, ϵ_1 and ϵ_2 are factors (functions) to be determined. We hope all these factors depend on α , such that, for $\alpha \rightarrow 0$ ($V \rightarrow 0$), we recover Lorentz transformations ($\theta = 1, \epsilon_1 = 1$ and $\epsilon_2 = 1$). By using those transformations to perform $[c^2 t'^2 - x'^2]$, we find the identity: $[c^2 t'^2 - x'^2] = \theta^2 \gamma^2 [c^2 t^2 - 2\epsilon_1 v t X + 2\epsilon_2 v t X - \epsilon_1^2 v^2 t^2 + \frac{\epsilon_2^2 v^2 X^2}{c^2} - X^2]$. Since the metric tensor is diagonal, the crossed terms must vanish and so we assure that $\epsilon_1 = \epsilon_2 = \epsilon$. Due to this fact, the crossed terms ($2\epsilon v t X$) are cancelled between themselves and finally we obtain $[c^2 t'^2 - x'^2] = \theta^2 \gamma^2 (1 - \frac{\epsilon^2 v^2}{c^2}) [c^2 t^2 - X^2]$. For $\alpha \rightarrow 0$ ($\epsilon = 1$ and $\theta = 1$), we reinstate $[c^2 t'^2 - x'^2] = [c^2 t^2 - x^2]$ of SR. Now we write the following transformations: $x' = \theta \gamma (X - \epsilon v t) \equiv \theta \gamma (X - v t + \delta)$ and $t' = \theta \gamma (t - \frac{\epsilon v X}{c^2}) \equiv \theta \gamma (t - \frac{v X}{c^2} + \Delta)$, where we assume $\delta = \delta(V)$ and $\Delta = \Delta(V)$, such that $\delta = \Delta = 0$ for $V \rightarrow 0$, which implies $\epsilon = 1$. So from such transformations we extract: $-v t + \delta(V) \equiv -\epsilon v t$ and $-\frac{v X}{c^2} + \Delta(V) \equiv -\frac{\epsilon v X}{c^2}$, from where we obtain $\epsilon = (1 - \frac{\delta(V)}{v t}) = (1 - \frac{c^2 \Delta(V)}{v X})$. As ϵ is a dimensionless factor, we immediately conclude that $\delta(V) = V t$ and $\Delta(V) = \frac{V X}{c^2}$, such that we find $\epsilon = (1 - \frac{V}{v}) = (1 - \alpha)$. On the other hand, we can determine θ as follows: θ is a function of α ($\theta(\alpha)$), such that $\theta = 1$ for $\alpha = 0$, which also leads to $\epsilon = 1$ in

$$dt' = \Psi(dt - \frac{\beta_* dX}{c}) = \Psi(dt - \frac{vdX}{c^2} + \frac{VdX}{c^2}), \quad (81)$$

being $\vec{v} = v_x \mathbf{x}$. We have $\Psi = \frac{\sqrt{1-\alpha^2}}{\sqrt{1-\beta^2}}$. If we make $V \rightarrow 0$ ($\alpha \rightarrow 0$), we recover Lorentz transformations, where the ultra-referential S_V is eliminated and simply replaced by the Galilean frame S at rest for the observer.

The transformations shown in (80) and (81) are the direct transformations from S_V [$X^\mu = (X, ict)$] to S' [$x'^\nu = (x', ict')$], where we have $x'^\nu = \Omega'_\mu X^\mu$ ($x' = \Omega X$), so that we obtain the following matrix of transformation:

$$\Omega = \begin{pmatrix} \Psi & i\beta(1-\alpha)\Psi \\ -i\beta(1-\alpha)\Psi & \Psi \end{pmatrix}, \quad (82)$$

such that $\Omega \rightarrow L$ (Lorentz matrix of rotation) for $\alpha \rightarrow 0$ ($\Psi \rightarrow \gamma$).

We obtain $\det\Omega = \frac{(1-\alpha^2)}{(1-\beta^2)}[1-\beta^2(1-\alpha)^2]$, where $0 < \det\Omega < 1$. Since V (S_V) is unattainable ($v > V$), this assures that $\alpha = V/v < 1$ and therefore the matrix Ω admits inverse ($\det\Omega \neq 0$ (> 0)). However Ω is a non-orthogonal matrix ($\det\Omega \neq \pm 1$) and so it does not represent a rotation matrix ($\det\Omega \neq 1$) in such a space-time due to the presence of the privileged frame of background field S_V that breaks the invariance of the norm of 4-vector. Such a break occurs strongly closer to S_V because the particle experiments an enormous “dislocation” (uncertainty) from the origin O' of the frame S' (see Fig.3). This leads to the strong inequality $\Delta S'^2 \gg \Delta S^2$, where $\Delta x'_v \rightarrow \infty$ for $v \rightarrow V$ (see (55),(56),(57) and (58)). Actually such an effect ($\det\Omega \approx 0$ for $\alpha \approx 1$) emerges from such a new relativistic physics for treating much lower energies at infrared regime (very large wavelengths), where a new implicit dimension (Δx_5) ceases to be hidden and then stretches drastically to the infinite closer to S_V (see (56)). In the limit S_V , the “particle” would loose its identity, by dissolving completely in the background field ($\Delta x'_v = \infty$). So the matrix Ω would become singular ($\det\Omega = 0$), however, as such a limit V is unattainable, this really assures the existence of an inverse matrix for Ω .

We notice that $\det\Omega$ is a function of the speed v with respect to S_V . In the approximation for $v \gg V$ ($\alpha \approx 0$), we obtain $\det\Omega \approx 1$ and so we practically reinstate the rotation behavior of Lorentz matrix as a particular regime for higher energies. In this case, we find the particle with a more determined position ($\Delta x'_v \approx 0$), which leads to $\Delta S' \approx \Delta S$. Alternatively, if we make $V \rightarrow 0$ ($\alpha \rightarrow 0$), we exactly recover $\det\Omega = 1$ ($\Delta S' = \Delta S, \Delta x'_v = 0$).

The inverse transformations (from S' to S_V) are

$$dX = \Psi'(dx' + \beta_* c dt') = \Psi'(dx' + v dt' - V dt'), \quad (83)$$

$$dt = \Psi'(dt' + \frac{\beta_* dx'}{c}) = \Psi'(dt' + \frac{v dx'}{c^2} - \frac{V dx'}{c^2}). \quad (84)$$

In matrix form, we have the inverse transformation $X^\mu = \Omega'^\mu_\nu x'^\nu$ ($X = \Omega^{-1} x'$), so that the inverse matrix is

$$\Omega^{-1} = \begin{pmatrix} \Psi' & -i\beta(1-\alpha)\Psi' \\ i\beta(1-\alpha)\Psi' & \Psi' \end{pmatrix}, \quad (85)$$

order to recover Lorentz transformations. So, as ϵ depends on α , we conclude that θ can also be expressed in terms of ϵ , namely $\theta = \theta(\epsilon) = \theta[(1-\alpha)]$, where $\epsilon = (1-\alpha)$. Therefore we can write $\theta = \theta[(1-\alpha)] = [f(\alpha)(1-\alpha)]^k$, where the exponent $k > 0$. The function $f(\alpha)$ and k will be estimated by satisfying the following conditions: i) as $\theta = 1$ for $\alpha = 0$ ($V = 0$), this implies $f(0) = 1$.

ii) the function $\theta\gamma = \frac{[f(\alpha)(1-\alpha)]^k}{(1-\beta^2)^{\frac{1}{2}}} = \frac{[f(\alpha)(1-\alpha)]^k}{[(1+\beta)(1-\beta)]^{\frac{1}{2}}}$ should have a symmetry behavior, that is to say it goes to zero closer to V ($\alpha \rightarrow 1$) in the same way it goes to infinite closer to c ($\beta \rightarrow 1$). This means that the numerator of the function $\theta\gamma$, which depends on α should have the same shape of its denominator, which depends on β . Due to both conditions, we naturally conclude that $k = 1/2$ and $f(\alpha) = (1+\alpha)$, so that $\theta\gamma = \frac{[(1+\alpha)(1-\alpha)]^{\frac{1}{2}}}{[(1+\beta)(1-\beta)]^{\frac{1}{2}}} = \frac{(1-\alpha^2)^{\frac{1}{2}}}{(1-\beta^2)^{\frac{1}{2}}} = \frac{\sqrt{1-V^2/v^2}}{\sqrt{1-v^2/c^2}} = \Psi$, where $\theta = \sqrt{1-\alpha^2} = \sqrt{1-V^2/v^2}$.

where we can show that $\Psi' = \Psi^{-1}/[1 - \beta^2(1 - \alpha)^2]$, so that $\Omega^{-1}\Omega = I$.

Indeed we have $\Psi' \neq \Psi$ and therefore $\Omega^{-1} \neq \Omega^T$. This non-orthogonal aspect of Ω has an important physical implication. In order to understand such an implication, let us consider firstly the orthogonal (e.g: rotation) aspect of Lorentz matrix in SR. Under SR, we have $\alpha = 0$, so that $\Psi' \rightarrow \gamma' = \gamma = (1 - \beta^2)^{-1/2}$. This symmetry ($\gamma' = \gamma$, $L^{-1} = L^T$) happens because the Galilean reference frames allow us to exchange the speed v (of S') for $-v$ (of S) when we are at rest at S' . However, under SSR, since there is no rest at S' (non-Galilean frame), we cannot exchange v (of S') for $-v$ (of S_V) due to that asymmetry ($\Psi' \neq \Psi$, $\Omega^{-1} \neq \Omega^T$). Due to this fact, S_V must be covariant, namely V remains invariant for any change of non-Galilean frame. Thus we can notice that the paradox of twins, which appears due to that symmetry by exchange of v for $-v$ in SR should be naturally eliminated in SSR, because only the non-Galilean reference frame S' can move with respect to S_V that remains covariant (invariable for any change of reference frame).

We have $\det\Omega = \Psi^2[1 - \beta^2(1 - \alpha)^2] \Rightarrow [(\det\Omega)\Psi^{-2}] = [1 - \beta^2(1 - \alpha)^2]$. So we can alternatively write $\Psi' = \Psi^{-1}/[1 - \beta^2(1 - \alpha)^2] = \Psi^{-1}/[(\det\Omega)\Psi^{-2}] = \Psi/\det\Omega$. By inserting this result in (85) for Ψ' , we will obtain the relationship between the inverse matrix and the transposed matrix of Ω , namely $\Omega^{-1} = \Omega^T/\det\Omega$. Indeed Ω is a non-orthogonal matrix, since we have $\det\Omega \neq \pm 1$.

By dividing (80) by (81), we obtain the following speed transformation:

$$v_{Rel} = \frac{v' - v + V}{1 - \frac{v'v}{c^2} + \frac{v'V}{c^2}}, \quad (86)$$

where we have considered $v_{Rel} = v_{Relative} \equiv dx'/dt'$ and $v' \equiv dX/dt$. v' and v are given with respect to S_V , with v_{Rel} being related between them. Let us consider $v' > v$. If $V \rightarrow 0$, the transformation (86) recovers the Lorentz velocity transformation, where v' and v are given in relation to a certain Galilean frame S at rest. Since (86) implements the ultra-referential S_V , the speeds v' and v are now given with respect to S_V , which is covariant (absolute). Such a covariance is verified if we assume that $v' = v = V$ in (86). Thus, for this case, we obtain $v_{Rel} = "V - V" = V$. Let us also consider the following cases:

a) $v' = c$ and $v \leq c \Rightarrow v_{Rel} = c$. This just verifies the well-known invariance of c .

b) if $v' > v (= V) \Rightarrow v_{Rel} = "v' - V" = v'$. For example, if $v' = 2V$ and $v = V \Rightarrow v_{Rel} = "2V - V" = 2V$. This means that V really has no influence on the speed of the particles. So V works as if it were an "absolute zero of movement", being invariant.

c) if $v' = v \Rightarrow v_{Rel} = "v - v" (\neq 0) = \frac{v - v}{1 - \frac{v^2}{c^2}(1 - \frac{v}{v})}$. From (c) let us consider two specific cases, namely:

-c₁) assuming $v = V \Rightarrow v_{Rel} = "V - V" = V$ as mentioned before.

-c₂) if $v = c \Rightarrow v_{Rel} = c$, where we have the interval $V \leq v_{Rel} \leq c$ for $V \leq v \leq c$.

This last case (c) shows us in fact that it is impossible to find the rest for the particle on its own non-Galilean frame S' , where $v_{Rel}(v) (\equiv \Delta v(v))$ is an increasing function. However, if we make $V \rightarrow 0$, then we have $v_{Rel} \equiv \Delta v = 0$ and therefore it would be possible to find the rest for S' , which becomes a Galilean reference frame ($v < c$) of SR.

By dividing (83) by (84), we obtain

$$v_{Rel} = \frac{v' + v - V}{1 + \frac{v'v}{c^2} - \frac{v'V}{c^2}} \quad (87)$$

In (87), if $v' = v = V \Rightarrow "V + V" = V$. Indeed V is invariant, working like an *absolute zero point* in SSR. If $v' = c$ and $v \leq c$, this implies $v_{Rel} = c$. For $v' > V$ and considering $v = V$, this leads to $v_{Rel} = v'$. As a specific example, if $v' = 2V$ and assuming $v = V$, we would have $v_{Rel} = "2V + V" = 2V$. And if $v' = v \Rightarrow v_{Rel} = "v + v" = \frac{2v - V}{1 + \frac{v^2}{c^2}(1 - \frac{v}{v})}$. In newtonian regime ($V \ll v \ll c$), we recover $v_{Rel} = "v + v" = 2v$. In relativistic (einsteinian) regime ($v \rightarrow c$), we reinstate Lorentz transformation for this case ($v' = v$), i.e., $v_{Rel} = "v + v" = 2v/(1 + v^2/c^2)$.

By joining both transformations (86) and (87) into just one, we write the following compact form:

$$v_{Rel} = \frac{v' \mp \epsilon v}{1 \mp \frac{v'\epsilon v}{c^2}} = \frac{v' \mp v(1 - \alpha)}{1 \mp \frac{v'v(1 - \alpha)}{c^2}} = \frac{v' \mp v \pm V}{1 \mp \frac{v'v}{c^2} \pm \frac{v'V}{c^2}}, \quad (88)$$

being $\alpha = V/v$ and $\epsilon = (1 - \alpha)$. For $\alpha = 0$ ($V = 0$) or $\epsilon = 1$, we recover Lorentz speed transformations.

In a more realistic case for motion of the electron in SSR, due to the non-zero minimum limit of speed V for all directions in the space, actually we should also consider the existence of non-null transverse components v_y and v_z , such that $\vec{v}_T = v_y \mathbf{j} + v_z \mathbf{k}$. So, if we also assume that such a transverse motion in 2d (yz) oscillates in the time ($\vec{v}_T(t) = v_y(t) \mathbf{j} + v_z(t) \mathbf{k}$) around x , where the particle has a constant longitudinal motion $v = v_x$, we obtain an oscillatory (jittery) motion for the electron. This so-called *zitterbewegung* (*zbw*) of the electron was introduced by Schroedinger[27] who proposed the electron spin to be a consequence of a local circulatory motion, constituting *zbw* and resulting from the interference between positive and negative energy solutions of the Dirac equation. Such an issue turned out to be of renewed interest[28] [29]. The present work provides naturally a more fundamental vision for *zbw*, whose origin is connected to the vacuum energy from the ultra-referential S_V , where now gravity also plays an essential role ($V \propto \sqrt{G}$). We intend to go deeper into such a subject about more general transformations elsewhere.

VII. COVARIANCE OF THE MAXWELL WAVE EQUATION IN PRESENCE OF THE ULTRA-REFERENTIAL S_V

Let us assume a light ray emitted from the frame S' . Its equation of electrical wave in this reference frame is

$$\frac{\partial^2 \vec{E}(x', t')}{\partial x'^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}(x', t')}{\partial t'^2} = 0 \quad (89)$$

As it is already known, when we make the exchange by conjugation on the spatial and temporal coordinates, we obtain respectively the following operators: $X \rightarrow \partial/\partial t$ and $t \rightarrow \partial/\partial X$; also $x' \rightarrow \partial/\partial t'$ and $t' \rightarrow \partial/\partial x'$. Thus the transformations (80) and (81) for such differential operators are

$$\frac{\partial}{\partial t'} = \Psi \left(\frac{\partial}{\partial t} - \beta c \frac{\partial}{\partial X} + \xi c \frac{\partial}{\partial X} \right) = \Psi \left[\frac{\partial}{\partial t} - \beta c (1 - \alpha) \frac{\partial}{\partial X} \right], \quad (90)$$

$$\frac{\partial}{\partial x'} = \Psi \left(\frac{\partial}{\partial X} - \frac{\beta}{c} \frac{\partial}{\partial t} + \frac{\xi}{c} \frac{\partial}{\partial t} \right) = \Psi \left[\frac{\partial}{\partial X} - \frac{\beta}{c} (1 - \alpha) \frac{\partial}{\partial t} \right], \quad (91)$$

where $v = \beta c$, $V = \xi c$ and $\xi = \alpha \beta$, being $\alpha = V/v$.

By squaring (90) and (91), inserting into (89) and after performing the calculations, we will finally obtain

$$\Psi^2 [1 - \beta^2 (1 - \alpha)^2] \left(\frac{\partial^2 \vec{E}}{\partial X^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \right) = \det \Omega \left(\frac{\partial^2 \vec{E}}{\partial X^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \right) = 0 \quad (92)$$

As the ultra-referential S_V is definitely inaccessible for any particle, we always have $\alpha < 1$ (or $v > V$), which always implies $\det \Omega = \Psi^2 [1 - \beta^2 (1 - \alpha)^2] > 0$. And as we already have shown in section 6, such a result is in agreement with the fact that we must have $\det \Omega > 0$. Therefore this will always assure

$$\frac{\partial^2 \vec{E}}{\partial X^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (93)$$

By comparing (93) with (89), we verify the covariance of the equation of the electromagnetic wave propagating in the relativistic “ether” (background field) S_V .

VIII. COSMOLOGICAL IMPLICATIONS

A. Energy-momentum tensor in the presence of the ultra-referential- S_V

Let us write the 4-velocity in the presence of S_V , as follows:

$$U^\mu = \left[\frac{\sqrt{1 - \frac{V^2}{v^2}}}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{v_\alpha \sqrt{1 - \frac{V^2}{v^2}}}{c \sqrt{1 - \frac{v^2}{c^2}}} \right], \quad (94)$$

where $\mu = 0, 1, 2, 3$ and $\alpha = 1, 2, 3$. If $V \rightarrow 0$, we recover the 4-velocity of SR.

The well-known energy-momentum tensor to deal with perfect fluid has the form

$$T^{\mu\nu} = (p + \epsilon)U^\mu U^\nu - pg^{\mu\nu}, \quad (95)$$

where now U^μ is given in (94). p represents a pressure and ϵ an energy density.

From (94) and (95), by calculating the new component T^{00} , we obtain

$$T^{00} = \frac{\epsilon(1 - \frac{V^2}{v^2}) + p(\frac{v^2}{c^2} - \frac{V^2}{v^2})}{(1 - \frac{v^2}{c^2})} \quad (96)$$

If $V \rightarrow 0$, we recover the old component T^{00} of the Relativity theory.

Now, in order to obtain T^{00} in (96) for vacuum limit in the ultra-referential- S_V , we perform

$$\lim_{v \rightarrow V} T^{00} = T_{vacuum}^{00} = \frac{p(\xi^2 - 1)}{(1 - \xi^2)} = -p, \quad (97)$$

where $\xi = V/c$ (see (42)).

As we always must have $T^{00} > 0$, we have $p < 0$ in (97), which implies a negative pressure for vacuum energy density of the ultra-referential S_V . So we verify that a negative pressure emerges naturally from such new tensor in the limit of S_V .

We can obtain $T_{vacuum}^{\mu\nu}$ by calculating the following limit:

$$T_{vacuum}^{\mu\nu} = \lim_{v \rightarrow V} T^{\mu\nu} = -pg^{\mu\nu}, \quad (98)$$

where we naturally conclude that $\epsilon = -p$. $T_{vac.}^{\mu\nu}$ is in fact a diagonalized tensor as we hope to be. So the vacuum- S_V , which is inherent to such a space-time works like a *sui generis* fluid at equilibrium and with negative pressure, leading to a cosmological anti-gravity connected to the cosmological constant.

B. Cosmological constant Λ

Let us begin by writing the Einstein equation in the presence of the cosmological constant Λ , namely:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^2}T_{\mu\nu} + \Lambda g_{\mu\nu}, \quad (99)$$

where we think that the anti-gravitational effect due to the vacuum energy has origin from the last term $\Lambda g_{\mu\nu}$. In the absence of matter ($T_{\mu\nu} = 0$), we have

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 0 \quad (100)$$

For very large scales of space-time, the presence of the term $\Lambda g_{\mu\nu}$ is considerable and the accelerated expansion of the universe is governed by vacuum energy density. So we can relate Λ to the vacuum energy density. To do that, we just use the energy-momentum tensor (95) (from (94)) given in vacuum limit of the ultra-referential S_V (see (98)). Thus we can rewrite equation (100) in its equivalent form for the energy-momentum tensor given in the limit of vacuum- S_V , as follows:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{8\pi G}{c^2}T_{\mu\nu}^{vac.} = 0, \quad (101)$$

where $T_{\mu\nu}^{vac.} = \lim_{v \rightarrow V} T_{\mu\nu} = -pg_{\mu\nu}$ (see (98)). And as $p = -\epsilon = -\epsilon_{vac.} = -\rho_{(\Lambda)}$ ($p = w\epsilon$ with $w = -1$), we write (101) in the following way:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{8\pi G}{c^2}\rho_{(\Lambda)}g_{\mu\nu} = 0 \quad (102)$$

Finally, by comparing (102) with (100), we obtain

$$\rho_{(\Lambda)} = \frac{\Lambda c^2}{8\pi G}, \quad (103)$$

which gives the direct relationship between cosmological constant Λ and vacuum energy density $\rho_{(\Lambda)}$.

Our aim is to estimate Λ and $\rho_{(\Lambda)}$ by using the idea of such a universal minimum speed V and its influence on gravitation at very large scales of length. In order to study such an influence, let us firstly start from the well-known simple model of a massive particle that escapes from a classical gravitational potential ϕ , where its total relativistic energy for an escape velocity v is due to the presence of such a potential ϕ , namely $E = m_0c^2(1 - v^2/c^2)^{-1/2} \equiv m_0c^2(1 + \phi/c^2)$. Here the interval of velocity $0 \leq v < c$ is associated with the interval of potential $0 \leq \phi < \infty$, where we stipulate $\phi > 0$ to be attractive potential. Now it is very important to notice that the breakdown of Lorentz symmetry due to S_V of background field has origin in a non-classical (non-local) aspect of gravitation that leads to a repulsive gravitational potential ($\phi < 0$) for very large distances (cosmological anti-gravity). In order to see such a modified aspect of gravitation [30], let us consider the total energy of the particle with respect to S_V , shown in (72), namely:

$$E = m_0c^2 \frac{\sqrt{1 - \frac{V^2}{v^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv m_0c^2(1 + \phi/c^2), \quad (104)$$

from where we obtain

$$\phi \equiv c^2 \left[\frac{\sqrt{1 - \frac{V^2}{v^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right] \quad (105)$$

From (105), we observe two regimes of gravitational potential, namely:

$$\phi = \begin{cases} \phi_R : -c^2 < \phi \leq 0 \text{ for } V (= \xi c) < v \leq v_0, \\ \phi_A : 0 \leq \phi < \infty \text{ for } v_0 (= \sqrt{\xi}c) \leq v < c. \end{cases} \quad (106)$$

ϕ_A and ϕ_R are respectively the attractive (classical) and repulsive (non-classical) potentials. We observe that the strongest repulsive potential is $\phi = -c^2$, which is associated with a vacuum energy for the ultra-referential S_V of the universe as a whole (consider $v = V$ in (105)). Therefore such most negative potential is related to the cosmological constant (see (97)), and so we write:

$$\phi_\Lambda = \phi(V) = -c^2 \quad (107)$$

The negative potential above depends directly on Λ , namely $\phi_\Lambda = \phi(\Lambda) = \phi(V) = -c^2$. To show that, let us consider a simple model of spherical universe with a radius R_u , being filled by a uniform vacuum energy density $\rho_{(\Lambda)}$, so that the total vacuum energy inside the sphere is $E_\Lambda = \rho_{(\Lambda)}V_u = -pV_u = M_\Lambda c^2$. V_u is its volume and M_Λ is the total dark mass associated with the dark energy for Λ ($w = -1$). Therefore the repulsive gravitational potential on the surface of such a sphere is

$$\phi_\Lambda = -\frac{GM_\Lambda}{R_u} = -\frac{G\rho_{(\Lambda)}V_u}{R_u c^2} = \frac{4\pi G p R_u^2}{3c^2} \quad (108)$$

By inserting (103) in (108), we find

$$\phi_\Lambda = \phi(\Lambda) = -\frac{\Lambda R_u^2}{6} \quad (109)$$

Finally, by comparing (109) with (107), we obtain

$$\Lambda = \frac{6c^2}{R_u^2}, \quad (110)$$

where $\Lambda S_u = 24\pi c^2$, being $S_u = 4\pi R_u^2$.

And also by comparing (108) with (107), we have

$$\rho_{(\Lambda)} = -p = \frac{3c^4}{4\pi G R_u^2}, \quad (111)$$

where $\rho_{(\Lambda)} S_u = 3c^4/G$. (111) and (110) satisfy (103).

Λ is a kind of *cosmological scalar field*, extending the old concept of Einstein about the cosmological constant for stationary universe. From (110), by considering the Hubble radius, with $R_u = R_{H_0} \sim 10^{26}m$, we obtain $\Lambda = \Lambda_0 \sim (10^{17}m^2s^{-2}/10^{52}m^2) \sim 10^{-35}s^{-2}$. To be more accurate, we know the age of the universe $T_0 = 13.7$ Gyr, being $R_{H_0} = cT_0 \approx 1.3 \times 10^{26}m$, which leads to $\Lambda_0 \approx 3 \times 10^{-35}s^{-2}$. This result is very close to the observational results[31][32][33][34][35]. The tiny vacuum energy density[36][37] shown in (111) for R_{H_0} is $\rho_{(\Lambda_0)} \approx 2 \times 10^{-29}g/cm^3$, which is also in agreement with observations. For scale of the Planck length, where $R_u = l_P = (G\hbar/c^3)^{1/2}$, from (110) we find $\Lambda = \Lambda_P = 6c^5/G\hbar \sim 10^{87}s^{-2}$, and from (111) $\rho_{(\Lambda)} = \rho_{(\Lambda_P)} = T_{vac.P}^{00} = \Lambda_P c^2/8\pi G = 3c^7/4\pi G^2\hbar \sim 10^{113}J/m^3 (= 3c^4/4\pi l_P^2 G \sim 10^{43}kgf/S_P \sim 10^{108}atm \sim 10^{93}g/cm^3)$. So just at that past time, Λ_P or $\rho_{(\Lambda_P)}$ played the role of an inflationary vacuum field with 122 orders of magnitude[38] beyond of those ones (Λ_0 and $\rho_{(\Lambda_0)}$) for the present time.

It must be emphasized that our assumption for obtaining Λ starts from fundamental principles (universal minimum speed V and its ultra-referential S_V for vacuum energy), and thus it does not depend on detailed adjustments with cosmological models.

C. The critical radius of exchange of gravity for anti-gravity during expansion of the universe

Let us consider a plane universe, where $\Omega_{(\Lambda_0)} + \Omega_{matter} = 1$, being $\Omega_{(\Lambda_0)} = \rho_{(\Lambda_0)}/\rho_{critical} \approx 0.7$ and $\Omega_{matter} = \rho_{matter}/\rho_{critical} \approx 0.3$. As we have obtained $\rho_{(\Lambda_0)} \approx 2 \times 10^{-29}g/cm^3$, we have $\rho_{critical} \approx 2.85 \times 10^{-29}g/cm^3$, which leads to $\rho_{matter} \approx 0.85 \times 10^{-29}g/cm^3$. Actually we see that both $\rho_{(\Lambda_0)}$ and ρ_{matter} have the same order of magnitude ($\sim 10^{-29}g/cm^3$) and therefore both of the total masses of dark energy (Λ_0) and matter inside the Hubble volume $V_{H_0} \sim 10^{78}m^3$ are $M_{(\Lambda_0)} \sim M_{matter} \sim 10^{52}Kg$. This result is valid just for the present time of the universe because, as we can see from (111), $M_{(\Lambda)} = \rho_{(\Lambda)}V_u \propto R_u$. On the other hand, if M_{matter} is conserved, so there was a long time in the past when $M_{matter} > M_\Lambda$, which means that gravity governed the expansion of the universe. But at the present time, as $M_{(\Lambda)}$ has been increasing with R_u , anti-gravity overcomes gravity, leading to an accelerated expansion of the universe. We will attempt to find a critical radius R_{uc} so that gravity is balanced by anti-gravity and thus anti-gravity begins to govern the expansion just for $R_u > R_{uc}$.

In order to simplify our notation, let us just write $M_{matter} = M$ and $\rho_{matter} = \rho$, being $V_H = V_u$. Now if we consider that the whole universe is filled by a uniform energy density of attractive matter ρ , we have $M = \rho V_u / c^2 = 4\pi\rho R_u^3 / 3c^2$, where V_u is a Hubble spherical volume. On the other hand, the attractive gravitational potential over the surface of such a sphere (without rotation) is $\Phi(R_u) = GM/R_u = 4\pi G\rho R_u^2 / 3c^2$, from where we obtain the average energy density of matter (ρ), namely:

$$\rho = \frac{3Mc^2}{4\pi R_u^3} \quad (112)$$

The Einstein equation without cosmological constant and with the form of mixing components is written as follows:

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \frac{8\pi G}{c^2} T_\mu^\nu \quad (113)$$

Making $\mu = \nu$ in (113), we find

$$R = -\frac{8\pi G}{c^2} T, \quad (114)$$

where $\delta_\mu^\mu = 4$. $T = T_\mu^\mu = T_\nu^\nu$ is the trace of the energy-momentum tensor, $R = R_\mu^\mu = R_\nu^\nu$ is the trace of the Ricci tensor, which represents a scalar curvature. If $T = 0$ (the case of the electromagnetic field, i.e., a massless field), this implies $R = 0$ (null scalar curvature). For (attractive) matter, we obtain $T = \epsilon = \rho$ and from (114) we obtain

$$\rho = -\frac{Rc^2}{8\pi G} \quad (115)$$

Since $\rho > 0$, we have $R < 0$, which means attractive energy of matter (gravity) in contrast to $\Lambda > 0$ (anti-gravity due to repulsive energy of vacuum).

By comparing (115) with (112), we find

$$R = -\frac{6GM}{R_u^3}, \quad (116)$$

being $M \sim 10^{52} Kg$. R (116) can be thought of as a universal scalar curvature for the total attractive mass M , whereas Λ (110) represents a cosmological repulsive scalar field (vacuum energy). So finally the dynamics of expansion of the universe is governed by an effective scalar field (curvature) R_{eff} as a result of competition between gravity (R) and anti-gravity (Λ), namely:

$$R_{eff} = R + \Lambda = -\frac{6GM}{R_u^3} + \frac{6c^2}{R_u^2}, \quad (117)$$

where $R \propto -1/R_u^3 (< 0)$ and $\Lambda \propto 1/R_u^2 (> 0)$. When gravity is balanced by anti-gravity during the cosmological expansion, we have $R_{eff} = 0$, leading to a critical radius R_{uc} that satisfies such a condition, namely:

$$R + \Lambda = -\frac{6GM}{R_{uc}^3} + \frac{6c^2}{R_{uc}^2} = 0, \quad (118)$$

from where we obtain $R_{uc} = \infty$ (trivial solution) and $R_{uc} = r_g/2$, where $r_g = 2GM/c^2$ represents the Schwarzschild radius of a sphere with mass M . As we have $M \sim 10^{52} Kg$, we find the critical radius

$R_{uc} \sim r_g \sim 10^{25}m$, beyond which the accelerated cosmological expansion (anti-gravity) takes place. Thereby, at the present time, since our universe has a Hubble radius $R_{H_0}(\sim 10^{26}m) > R_{uc}(\sim 10^{25}m)$, indeed it is governed by an accelerated expansion according to observations.

According to (117) we can show that there is a radius $R_{u0} > R_{uc}$ so that R_{eff} reaches a maximum value before decreasing. For such a radius R_{u0} , the universe has a maximum accelerated expansion. In order to satisfy this condition, we must have

$$\frac{dR_{eff}}{dR_u} = \frac{18GM}{R_u^4} - \frac{12c^2}{R_u^3} = 0, \quad (119)$$

from where we obtain the non-trivial solution $R_u = R_{u0} = 3r_g/4 = 3GM/2c^2 = (3/2)R_{uc}$, being $R_{uc} = r_g/2$. This leads to the expansion rate $R_{eff} = R_{eff,max} = R_{eff,0} = 32c^2/9r_g^2 \sim 10^{-33}s^{-2}$, which has two orders of magnitude beyond the cosmological constant. In short we conclude that some time after the critical time (R_{uc}) of transition from gravity to anti-gravity, accelerated expansion of the universe (anti-gravity) reached its maximum value for $R_{u0} = 1.5R_{uc}$, but after this time it was decreasing until the present time, which is represented by the cosmological constant. Fig.8 shows the dynamics of cosmological expansion through temporal evolution of the competition between gravity and anti-gravity.

The function R_{eff} given in (117) should have a correction for very small radius since, close to origin, a very strong anti-gravity instead of gravity was responsible by the rapid expansion of the universe. Due to this fact, in order to implement a corrective term into R_{eff} , let us consider the following boundary condition: $R_{eff}(R_u = l_P) = \Lambda_P = 6c^2/l_P^2$ (see Fig.8). This means that anti-gravity prevails at the origin whereas gravity (first term of the function (117)) should vanish. Indeed due to strong fluctuations at Planck length scale l_P , we think that gravity should vanish since we have a null average of M ($\langle M \rangle = 0$) at such a minimum scale of length. So we have to satisfy the conditions as follow: (i) $\langle M \rangle = 0$ for $R_u = l_P$ and (ii) $\langle M \rangle = M$ for $R_u \gg l_P$ so that we recover (117) as a good approximation for large scales of length. Although there are an infinite number of corrective functions that satisfy those conditions, let us consider a more simple function of the form $+1/R_u^4$ in order to implement a repulsive gravity just for very small distances, in addition to those known terms $-1/R_u^3$ (gravity) and $+1/R_u^2$ (anti-gravity for large distances) given in (117). So we write

$$R_{eff} = -\frac{6GM}{R_u^3} \left(1 - \frac{l_P}{R_u}\right) + \frac{6c^2}{R_u^2} = \frac{6GMl_P}{R_u^4} - \frac{6GM}{R_u^3} + \frac{6c^2}{R_u^2}, \quad (120)$$

where we can think that the first (positive) term is associated with ultra-microscopic length scale, the second (negative) one is associated with intermediate (micro and mesoscopic) scales and the last (positive) one has a large reach (cosmological scales). We can conclude that $\langle M \rangle = M(1 - \frac{l_P}{R_u})$ as a more simple way for representing our assumption. If $R_u \gg l_P$, we recover (117).

According to (120), by considering $R_{eff} = 0$, we obtain two transition points (two critical radius), namely R_{uc} already obtained from (117) and R'_{uc} as follow:

$$R_{uc} = \frac{1}{2} \frac{GM}{c^2} + \frac{1}{2} \frac{GM}{c^2} \sqrt{1 - \frac{4l_P c^2}{GM}} \approx \frac{GM}{c^2} = \frac{r_g}{2}, \quad (121)$$

being $l_P \ll GM/c^2$.

$$R'_{uc} = \frac{1}{2} \frac{GM}{c^2} \left(1 - \sqrt{1 - \frac{4l_P c^2}{GM}}\right) = l_P \left(1 + \frac{l_P c^2}{GM} + \frac{2l_P^2 c^4}{G^2 M^2} + \frac{5l_P^3 c^6}{G^3 M^3} + \dots\right) \quad (122)$$

Now by considering the derivative $dR_{eff}/dR_u = 0$ for (120), we find two maximum points, namely the radius R_{u0} for the maximum acceleration $R_{eff,0} = R_{eff,max}$ (transition from 4th. step to 5th. step: see Fig.9) and the radius R'_{u0} for the maximum deceleration $R'_{eff,0} = R_{eff,min}$ (transition from 2nd. step to 3rd. step: see also Fig.9), as follow:

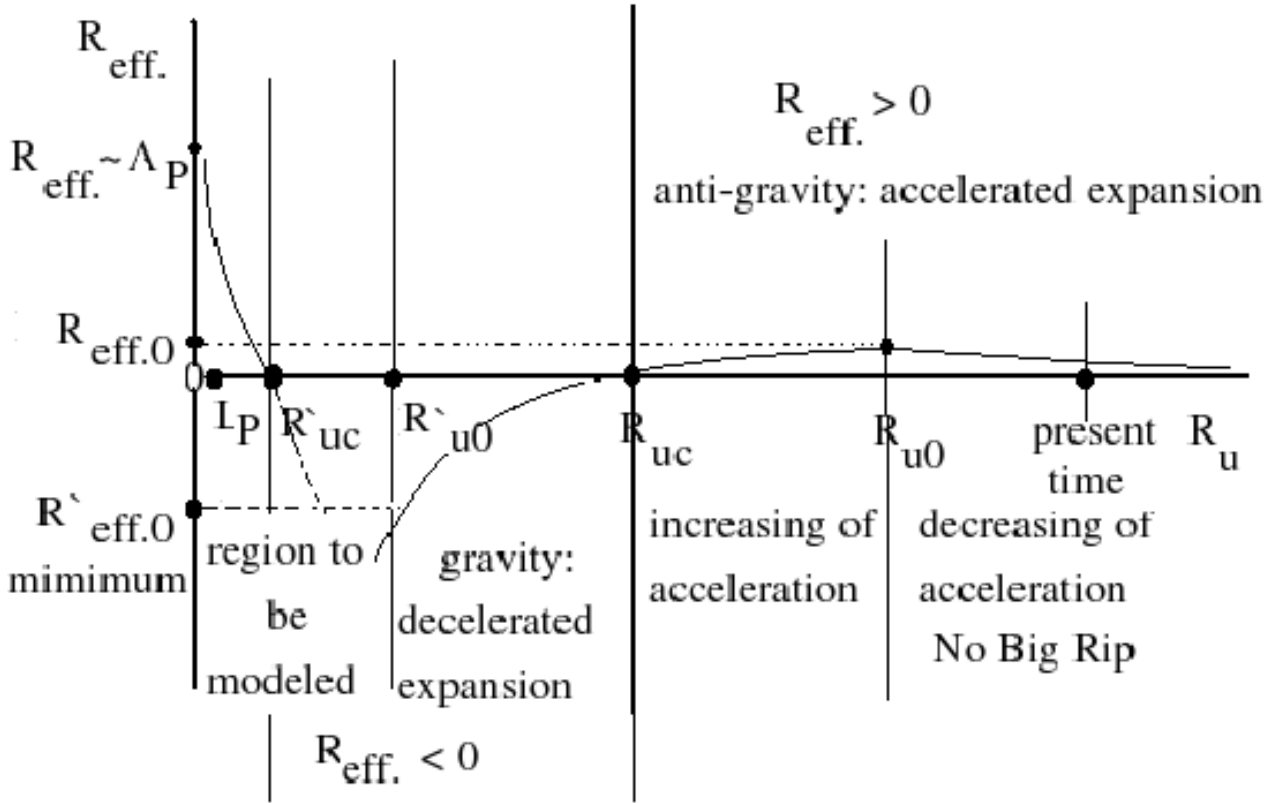


FIG. 8: The dynamics of expansion of the universe since its origin is given by 5 steps (first step from anti-gravity; second and third one from gravity; fourth and fifth one from anti-gravity again), namely in detail: (a) $l_P \leq R_u \leq R'_{uc}$, which corresponds to a short initial period of time with acceleration decreasing rapidly until a null value at R'_{uc} (first critical radius of transition: from an initial anti-gravity - “a rapid inflation” to gravity). Such an initial period is not foreseen by the function (117) and thus it will be modeled through a correction in (117) to be made at the origin l_P . (b) $R'_{uc} \leq R_u \leq R'_{u0}$: In this period of time, gravity increasing rapidly so that the rate of decelerated expansion increases rapidly in order to reduce the very high velocities of expansion. Such a deceleration due to gravity reaches its maximum value in module ($R_{eff.0}$) for $R_u = R'_{u0}$. This region will be modeled according to the correction to be made in (117). (c) $R'_{u0} \leq R_u \leq R_{uc}$: This period of time is much larger than the last one. It also corresponds to a decelerated expansion, however such a deceleration decreases slowly in the time (see Fig.9). So it needs a very long time to reach a null value ($R_{eff} = 0$) at $R_{uc} = GM/c^2 \sim 10^{25}m$ (second critical radius of transition: from gravity to anti-gravity). (d) $R_{uc} \leq R_u \leq R_{u0}$: the acceleration of expansion is increasing until reaching its maximum value $R_{eff.0}$ for $R_u = R_{u0} = 3GM/c^2$. (e) $R_{u0} \leq R_u < \infty$: the acceleration of expansion is decreasing to zero at the infinite and therefore there is no Big Rip. At the present time (in the fifth step), such an acceleration corresponds to the cosmological constant for $R_u \sim 10^{26}m$.

$$R_{u0} = \frac{3GM}{4c^2} + \frac{3GM}{4c^2} \sqrt{1 - \frac{32l_P c^2}{9GM}} \approx \frac{3GM}{2c^2} = \frac{3}{4} r_g \quad (123)$$

$$R'_{u0} = \frac{3GM}{4c^2} - \frac{3GM}{4c^2} \sqrt{1 - \frac{32l_P c^2}{9GM}} = l_P \left(\frac{4}{3} + \frac{32l_P c^2}{27GM} + \frac{512l_P^2 c^4}{243G^2 M^2} + \frac{5120l_P^3 c^6}{2187G^3 M^3} + \dots \right) \quad (124)$$

Now by substituting (123) and (124) into (120) we obtain respectively $R_{eff.max}$ and $R_{eff.min}$, namely:

$$R_{eff.max} = R_{eff.0} \approx \frac{32c^2}{9r_g^2}, \quad (125)$$

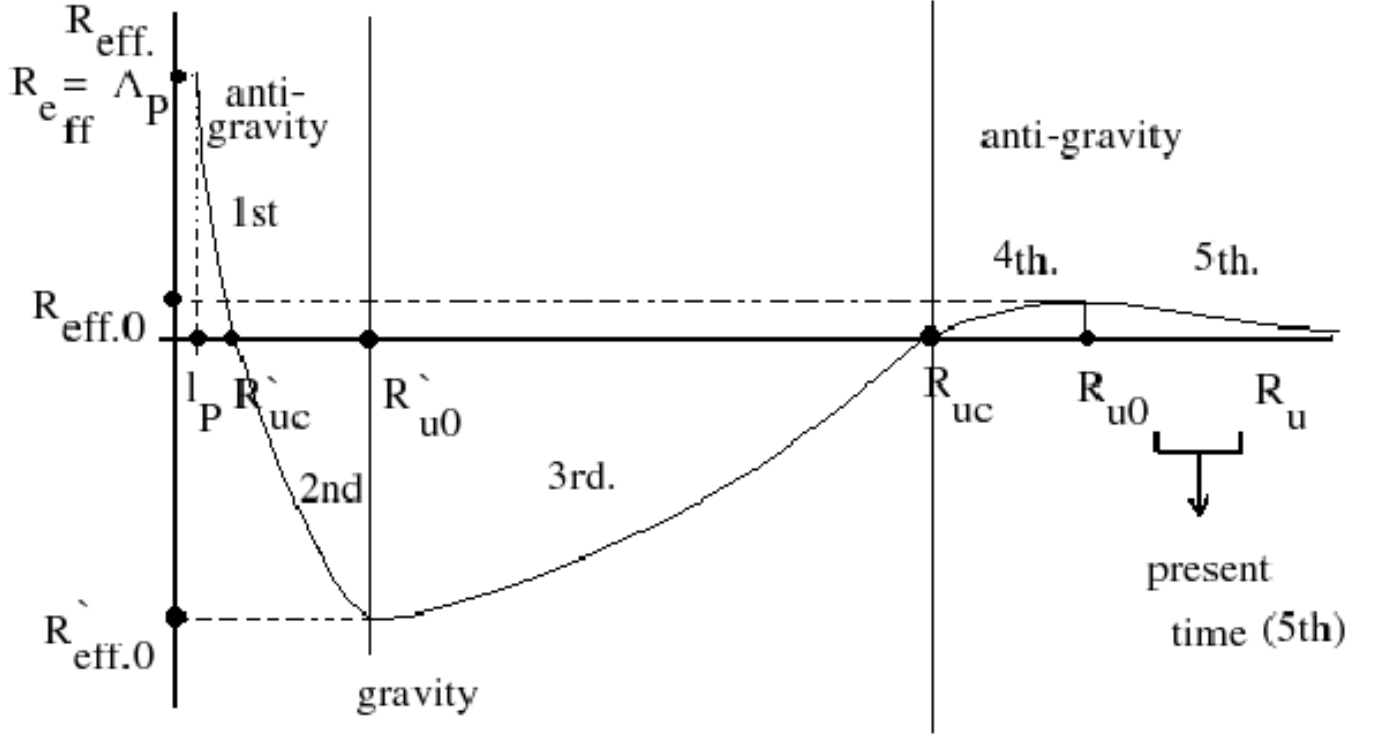


FIG. 9: As we can see in this figure, the first step represents a rapid decreasing of acceleration from the “big bang” of the repulsive kernel at Planck scale l_P and the second one is a rapid increasing of deceleration. Both of them occurred in order to avoid very high accelerations and velocities of expansion, and thus avoiding a big rip close to the origin of the universe. The third step represents a decreasing of deceleration until reaching a null deceleration at the critical radius R_{uc} , beyond which we have a soft increasing of acceleration in the fourth step. In the fifth step, the acceleration begins to decrease, having a null value at the infinite. Due to such a decreasing of acceleration, there is no big rip.

$$R_{eff.min} = R'_{eff.0} \approx -\frac{3GM}{2l_P^3 \left(\frac{4}{3} + \frac{32 l_P c^2}{27 GM} + \frac{512 l_P^2 c^4}{243 G^2 M^2} + \frac{5120 l_P^3 c^6}{2187 G^3 M^3} + \dots \right)^3} + \frac{6c^2}{l_P^2 \left(\frac{4}{3} + \frac{32 l_P c^2}{27 GM} + \frac{512 l_P^2 c^4}{243 G^2 M^2} + \frac{5120 l_P^3 c^6}{2187 G^3 M^3} + \dots \right)^2} \quad (126)$$

If we neglect the very small terms of l_P, l_P^2, \dots in the denominator of (126), we find another good approximation as follows:

$$R_{eff.min} \approx -\frac{81 GM}{128 l_P^3} + \frac{27c^2}{8l_P^2} = -\frac{81 r_g c^2}{256 l_P^3} + \frac{27c^2}{8l_P^2} \quad (127)$$

IX. CONCLUSIONS AND PROSPECTS

We have introduced a space-time with symmetry, so that $V < v \leq c$, where V is an inferior and unattainable limit of speed associated with a privileged inertial reference frame of universal background field. So we have essentially concluded that the space-time structure where gravity is coupled to electromagnetism at quantum level naturally contains the fundamental ingredients for comprehension of the quantum uncertainties through that mentioned symmetry ($V < v \leq c$), where gravity plays a crucial role due to the minimum velocity $V (\propto G^{1/2})$ related to the minimum length (Planck scale) of DSR[20][21][22][23] [24][25] by Magueijo, Smolin, Camelia, et al.

We have studied the cosmological implications of S_V , by estimating the tiny values of the vacuum energy density ($\rho_{(\Lambda)} \sim 10^{-29} g/cm^3$) and the current cosmological constant ($\Lambda \sim 10^{-35} s^{-2}$), which are still not well understood by quantum field theories for quantum vacuum [38], because such theories foresee a very high value for Λ , whereas, on the other hand, exact supersymmetric theories foresee an exact null value for it, which also does not agree with Reality. Besides this, we were able to find the critical radius of the universe, beyond which the accelerated expansion (cosmological anti-gravity) takes place. In short we have studied the dynamics of expansion of the universe since its origin until the present time and also its future time (see Fig.9).

The present theory has various implications which shall be investigated in coming articles. A new transformation group for such a space-time will be explored in details. We will propose the development of a new relativistic dynamics, where the energy of vacuum (ultra-referential S_V) plays a crucial role for understanding the origin of the inertia, including the problem of mass anisotropy.

Another relevant investigation is with respect to the problem of the absolute zero temperature in thermodynamics of a gas. We intend to make a connection between the 3rd. law of Thermodynamics and the new dynamics, through a relationship between the absolute zero temperature ($T = 0K$) and the minimum average speed ($\langle v \rangle_N = V$) for N particles. Since $T = 0K$ is thermodynamically unattainable, this is due to the impossibility of reaching $\langle v \rangle_N = V$ from the new dynamics standpoint. This leads still to other important implications, such as for example, Einstein-Bose condensate and the problem of the high refraction index of ultracold gases, where we intend to estimate that the speed of light would approach to V inside the condensate medium for $T \rightarrow 0K$. So the maximum refraction index would be $n_{max} = c/v_{min} = c/V = \xi^{-1} = \sigma \sim 10^{23}$ to be shown elsewhere. Thus we will be in a condition to propose an experimental manner of making an extrapolation in order to obtain $v_{lightMin.} = c'_{min} \rightarrow V$ for $T \rightarrow 0K$, through a mathematical function obtained by the theory applied to ultracold systems.

In sum, we begin to open up a new fundamental research field for various areas of Physics, by including condensed matter, quantum field theories, cosmology (dark energy and cosmological constant) and specially a new exploration for quantum gravity at very low energies (very large wavelengths).

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