Correspondence:

Theoretical Derivation of Ampère's Law

Andre Ampère is often credited with establishing the field of electrodynamics. In about the year 1822, he experimentally determined a law describing the force of interaction between two current elements. Employing RMKS units and the system of notation used by Moon and Spencer in [1], Ampère's Law reads

$$d^2\mathbf{f} = -\hat{\mathbf{r}}\frac{kI_tI_sds_tds_s}{c^2r^2}\left(2\sin\theta_t\sin\theta_s\cos\eta - \cos\theta_t\cos\theta_s\right) \tag{1}$$

where $\hat{\boldsymbol{r}}$ = unit vector in the direction of, and r = magnitude of, the vector $\boldsymbol{r}$ joining the two current elements. The constants are $k = 1/4\pi\varepsilon_0$ (ε_0 = permittivity of free space) and c = speed of light. The I_t and I_s are current magnitudes and ds_t and ds_s are current-element lengths. The angles are: θ_t = angle between ds_t and $\mathbf{r}$; θ_s = angle between ds_s and $\boldsymbol{r}$; η = angle between the plane of ds_t with $\boldsymbol{r}$ and the plane of ds_s with $\boldsymbol{r}$.

Ampère's Law has never been shown to fail in any situation, no matter how exotic. Most recently, it has been shown to describe the observed dramatic repulsion between current elements in a single current-carrying wire[2].

Ampère's Law is a mathematical codification of experimental data, independent of whatever interpretation might be given it. The interpretation I wish to offer here is that Ampère's Law is the consequence of a standard formula that originates in Special Relativity Theory (SRT). The derivation begins with formula for the electric field at a location specified by a three-vector $\boldsymbol{r}$ that emanates from a charge q_s that moves at velocity $\boldsymbol{v}$ relative to a stationary point [3]:

$$\mathbf{e_s} = \frac{kq_s\mathbf{r}/\gamma^2r^3}{\left[1-\left(v^2/c^2\right)\sin^2\theta\right]^{\frac{3}{2}}} \tag{2}$$

where v = magnitude of $\boldsymbol{v}$, $\gamma = 1/\sqrt{1-v^2/c^2}$, θ = angle between $\boldsymbol{r}$ and $\boldsymbol{v}$. First, the steps in the derivation of the magnetic force formula between isolated charges moving slowly relative to each other are presented. These steps are similar to the steps in the derivation of Ampère's Law, which then follows.

The Coulomb electric field is defined as (2) evaluated with $v = 0$ and $\gamma = 1$. The Coulomb electric field is subtracted from (2) leaving a residual electric field. In comparison to the Coulomb electric field intensity, $\boldsymbol{e_s}$ (2) is reduced in the direction of the velocity vector $\boldsymbol{v}$ and augmented in the direction perpendicular to the velocity vector $\boldsymbol{v}$. So the residual electric field has a pattern of sign reversal that is recognizable as consistent with the magnetic force (1).

The next step in this derivation is to place a stationary test charge q_t at $\mathbf{r}$ from q_s to make a force equation from the electric field equation. Then v is taken to be much less than the speed of light. This produces the formula for the magnetic force between two isolated charges moving slowly relative to each other:

$$\mathbf{f} = \frac{kq_tq_s}{r^3}\frac{v^2}{c^2}\mathbf{r}\left(0.5 - 1.5\cos^2\theta\right) \tag{3}$$

The variables in ***f*** (3) are the same as defined for (2). Eq. (3) may be useful, for example, in describing magnetic forces between hydrogen atoms or between rotating spherical charges. However, this equation does not lead directly to Ampère's Law.

To arrive at Ampère's Law, an additional magnetic effect associated with wire current elements must now be included. This effect is length contraction of the space between the electrons comprising the current, thus increasing the charge line density as seen by the other wire element. The q_s in (2) is replaced by $-\gamma\sigma_s ds_s$, where σ_s is charge line density and $\sigma_s ds_s$ is the current element charge. By subtracting the corresponding Coulomb electric field from the modified (2), taking v to be much less than c, and including the test charge q_t, we find:

$$d\mathbf{f} = -\frac{kq_t\sigma_s ds_s}{r^3}\frac{v^2}{c^2}\mathbf{r}\left(1\text{-}1.5\cos^2\theta\right) \tag{4}$$

Here q_t is either an isolated charge, or the charge $\sigma_t ds_t$ of another wire current element with charge line density σ_t.

Eq. (4) is good for current-carrying wire elements. It is applied four times to the cross combinations of charges in two wire elements in order to arrive at Ampère's Law (1). The Ids/v's are substituted for σds's. Note that the direction of the magnetic force lies essentially in the direction of a vector connecting the two charges or current-carrying wire elements. This applies only to slow moving charges. For fast moving charges and contact force action, the direction of the force vector would not lie on the vector connecting the two charges. Note also that Eq. (4) describes a force between a stationary charge and a stationary wire element carrying a current, as has been noted in [4].

A mathematical process similar to that employed to derive Ampère's Law was used to derive (3). Therefore I claim that (3) is the correct law for describing the magnetic force between two unaccelerated charges moving slowly relative to each other.

Acknowledgment

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References

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