The Fundamental Assumptions of Relativity

Jaroslav Hynecek
Isetex, Inc., Pampa Drive, Allen, TX 75013
e-mail: jhynecek@netscape.net

The paper discusses in detail the fundamental assumptions that are necessary for the derivation of special relativity theory and in particular for the derivation of Lorentz coordinate transformation. It is shown that the usual postulate of the constancy of speed of light is not needed. This is a generalization that is useful for studying the space-times with gravitational fields present in them, including the space-time of the Universe, since it is well known that the gravitational potential affects not only the clock rates but also the speed of light.

1. Introduction

There have been many books published since 1905 when Einstein introduced his Special Relativity Theory (SRT) [1] with various explanations and derivations that are attempting to clarify the concept and teach it to interested scientist. In this paper SRT will be derived from the point of view of fundamental laws of physic that are believed to be firmly established and undisputable rather than using the standard methods that are typically taught in courses on the relativity theory. The approach chosen in this paper will thus use more practical and direct path that can justify the theory more clearly. It will be accepted that the theory is correct, since it has been verified by many experiments, so it is only the interpretation that is sometimes not clear and creates confusion. To make this point the article will rely on the thought experiments that are not often found in the standard text books. In any case the motivation for this work originates in the author's desire for more clarification in SRT derivation and in strengthening the conviction that the theory is correct including some subtle points that the author wants to share with the interested readers, in particular the elimination of the need for the postulate of the constancy of speed of light.

2. The Lorentz Length Contraction

The length contraction of moving bodies is one of the key conclusions of SRT. In this section this effect will be derived first based on the Maxwell's theory of electromagnetic (EM) fields. In order to accomplish this let's consider a setup described below in Fig. 1. In this drawing two insulating plates of length L and width W (W is perpendicular to the drawing plane) are positioned parallel to each other and charged by charge $+mq$ and $-mq$ respectively. Charge is embedded into the plates and cannot move relative to them. The bottom plate has a small slit opening in it with a fast electronic shutter that can close after n photons have been injected into the space between the plates. The plate's inner surfaces, including the shutter surface, are highly reflective, so that the injected photons can bounce between the upper plate and the closed shutter indefinitely. Since the plates are charged they will be attracted to each other by the electrostatic force calculated from the Maxwell's field equations according to the following formula:

$$F_q = \frac{m^2 q^2}{2\varepsilon_0 LW} \tag{1}$$

where ε_0 is the vacuum dielectric constant. The field peripheral effects at the edges of the plates can be for the purposes of this derivation neglected. The injected photons exert a repulsive force on the plates that can be calculated from the Newton inertial force equation as follows:

$$F_p = \frac{\Delta p}{\Delta t} \tag{2}$$

The change in momentum after the photon collisions with the plate or the shutter is equal to: $\Delta p = 2hf_0 / c$ and the time between collisions at one plate is equal to: $\Delta t = 2d / c$. The final formula for the photon force is thus:

$$F_p = \frac{nhf_0}{d} \tag{3}$$

It will be further considered that the number of injected photons between the plates is just enough to exactly compensate for the attractive electrostatic force.

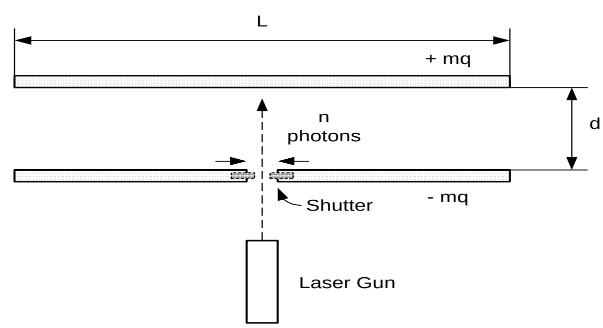

Fig. 1. Testing setup consists of two parallel plates fabricated from an insulating material and charged by charge $+mq$ and $-mq$ respectively. Bottom plate has an opening in it equipped with a fast electronic shutter that allows a certain number of photons to be injected from the Laser Gun into the space between the plates. The inner surfaces of the plates and the shutter are highly reflective. The plate area is equal to $A = LW$ and the plate spacing is d.

The forces will be equal and the setup balanced with the plates kept at the constant distance d when the following condition is satisfied:

$$L = \frac{m^2 \lambda_0 d}{nW} \frac{q^2}{2\varepsilon_0 hc} = \frac{m^2 \lambda_0 d}{nW} \alpha_S \tag{4}$$

where α_S is the famous Sommerfeld fine structure constant, c the vacuum speed of light, and λ_0 the wavelength of the injected photons. The photon wavelength is determined from the well known relation: $f_0 \lambda_0 = c$.

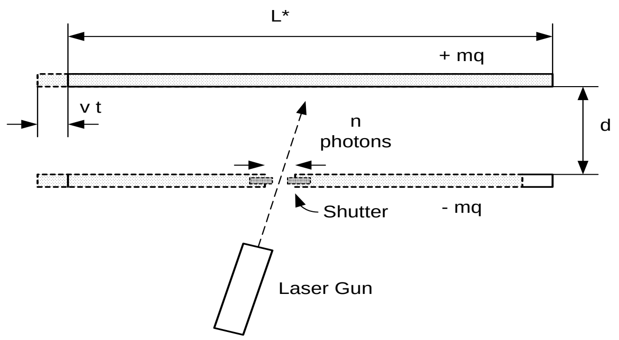

Fig. 2. The same testing setup as in Fig. 1, but with the plates moving relative to the laboratory coordinate system. The Laser Gun, however, is stationary. The shutter is timed such that when passing the beam from the Laser Gun it opens and lets n photons be injected between the plates. The photons are injected under angle α so that their drift speed matches the plate's velocity v .

Let's consider further that the same setup is now moving relative to the laboratory observer. This is shown in a more detail in Fig. 2. Since the plates are moving it is necessary to aim the laser gun to the opening of the shutter at a certain angle α and the shutter needs to also be opened with a slightly wider aperture so that enough photons can pass through. The drift velocity of photons between the plates is matched with the velocity of the plates so that the photons are again confined between the plate and the shutter and do not escape. Since the charged plates are moving the electrostatic force must be modified and the Lorentz force component added to it. The resulting formula for the EM force, as explained in more detail in the Appendix, is thus as follows:

$$F_q = \frac{m^2 q^2}{2\varepsilon_0 L(v) W}\left(1 - \frac{v^2}{c^2}\right) \tag{5}$$

Here it was assumed that the length of the plates may depend on velocity. There are other choices such as charge or the plate's width being dependent on velocity, but the length seems the most natural since it is in the direction of the velocity vector. Similarly as in the previous case the photon force will be derived from the Newton inertial force law. However, the photons must now travel a longer distance between the plates so their travel time $\Delta t = 2d / \sqrt{c^2 - v^2}$ is longer and the force derived from their momentum change reduced by the cosine of the impact angle α .

$$F_p = \frac{nhf \cos\alpha}{d}\sqrt{1 - \frac{v^2}{c^2}} \tag{6}$$

where for the $\cos\alpha$ it holds: $\cos\alpha = \sqrt{1 - v^2 / c^2}$. In order to compensate for the reduction in the transversal momentum due to the impact angle and keeping the photon number the same the frequency needs to be increased as follows: $f = f_0 / \cos\alpha$, since this would be the frequency required if the plates were stationary and the laser gun aimed at them at an angle. The required change in the photon frequency can be related to the different time rates in the moving and stationary coordinate systems, as will be discussed later. The final formula for the photon force is:

$$F_p = \frac{nhf_0}{d}\sqrt{1 - \frac{v^2}{c^2}} \tag{7}$$

Again, to keep the forces in equilibrium and the plates at a constant distance the following condition needs to be satisfied:

$$L(v) = \frac{m^2 \lambda_0 d}{nW} \frac{q^2}{2\varepsilon_0 hc}\sqrt{1 - \frac{v^2}{c^2}} = \frac{m^2 \lambda_0 d}{nW}\alpha_S\sqrt{1 - \frac{v^2}{c^2}} = L\sqrt{1 - \frac{v^2}{c^2}} \tag{8}$$

From this result it is clear that the length of the plates in the direction of motion must be shortened. This is the famous Lorentz length contraction effect. Up to this point all the considerations were made relative to the laboratory coordinate system. If we now associate a primed coordinate system with the moving plates and want to assign to a certain point along the x direction both the primed and unprimed coordinates it is necessary to consider that the length of the measuring stick in the primed coordinate system is now also contracted. The distance to the fixed point measured in the moving, primed, coordinate system and referenced to that system will therefore have to be divided by the contraction factor of the measuring stick and thus the following relation must hold:

$$x' = \frac{x - vt}{\sqrt{1 - v^2 / c^2}} \tag{9}$$

This is the Lorentz coordinate transformation for the x coordinate. However, it is also necessary to understand that the observer placed in the moving, primed, coordinate system does not actually notice any change in the object's length or distances within his coordinate system when moving. This is because all the objects that were previously stationary and are measured by him are now contracted as well as his measuring stick. This also includes his body. The length contraction is observed only from the laboratory coordinate system. No change in length of moving objects can be internally detected. So, sometimes claims appear in the literature that SRT length contraction is only an illusion. This is not so, the Lorentz length contraction is real as derived above. If there were events that do not obey the causality principle, where the cause and effect are delayed by the propagation time derived from the speed of light, then the Lorentz contraction could be internally detectable. Experimental setups that might possibly be investigated for this purpose are the setups used in quantum entanglement experiments. However, as long as all the macroscopic objects at our disposal are held together by the EM forces, then we are stuck with the impossibility to internally detect the Lorentz contraction by simple length measurements using the measuring sticks and thus internally detect the inertial motion. This is one of the important axioms of SRT.

3. Time Dilation

The time dilation effect will be derived from the assumption that for the inverse transform of the distance the same formula as given in Eq. (9) should apply. It will be also considered that the speed of light in the primed coordinate system is not necessarily equal to the speed of light in the unprimed coordinate system and that a more general linear relationship $v / c = -v' / c'$ between the velocities is valid. It should therefore hold that:

$$x = \frac{x' - v't'}{\sqrt{1 - v'^2 / c'^2}} \tag{10}$$

By substituting for x from Eq. (10) into Eq. (9) the results for the forward and inverse time transformations are:

$$t' = \frac{c}{c'} \frac{t - x\frac{v}{c^2}}{\sqrt{1 - v^2/c^2}} \tag{11}$$

$$t = \frac{c'}{c} \frac{t' - x'\frac{v'}{c'^2}}{\sqrt{1 - v'^2/c'^2}} \tag{12}$$

There is no need to introduce any clock synchronization procedures as is typically discussed in the standard derivations. From Eq. (9) and Eq. (12) it is then possible to recover the length contraction effect by differentiating Eq. (9) while keeping t' constant.

$$dx'_{t'=\text{const}} = \frac{dx}{\sqrt{1 - v^2/c^2}} + \frac{vv'}{cc'} \frac{dx'_{t'=\text{const}}}{\left(1 - v^2/c^2\right)} \tag{13}$$

This is a view from the laboratory coordinate system. After rearrangement and including for completeness also the differentials of x with t' and t kept constant the results become:

$$dx'_{t'=\text{const}} = dx\sqrt{1 - v^2/c^2} \tag{14a}$$

$$dx_{t'=\text{const}} = \frac{dx'}{\sqrt{1 - v'^2/c'^2}} \tag{14b}$$

$$dx'_{t=\text{const}} = \frac{dx}{\sqrt{1 - v^2/c^2}} \tag{15a}$$

$$dx_{t=\text{const}} = dx'\sqrt{1 - v'^2/c'^2} \tag{15b}$$

For the time differentials from Eq. (10) and Eq. (11) it follows that

$$dt'_{x=\text{const}} = \frac{c}{c'} \frac{dt}{\sqrt{1 - v^2/c^2}} \tag{16a}$$

$$dt_{x=\text{const}} = \frac{c'}{c} dt' \sqrt{1 - v'^2/c'^2} \tag{16b}$$

This is the famous time dilation effect. The formulas in Eq. (16) can be derived directly by considering that the light bouncing between the moving plates as shown in Fig. 2 represents a clock. The relation for the time intervals between two collisions observed in the laboratory coordinate system is then as follows:

$$\Delta t' = \frac{d}{c}\frac{c}{c'} \frac{1}{\sqrt{1 - v'^2/c'^2}} = \frac{c}{c'} \frac{\Delta t}{\sqrt{1 - v^2/c^2}} \tag{17}$$

It is also interesting to derive the inverse expressions for the time differential for an observer in a moving coordinate system.

$$dt'_{x'=\text{const}} = \frac{c}{c'} dt \sqrt{1 - v^2/c^2} \tag{18a}$$

$$dt_{x'=\text{const}} = \frac{c'}{c} \frac{dt'}{\sqrt{1 - v'^2/c'^2}} \tag{18b}$$

This result follows from differentiating Eq. (12) and will be needed later. However, again, the observer moving with the clocks in the primed coordinate system will not internally notice any change in the time rate, since all the processes in his system will be slowed down in the same proportion. The time dilation effect is only observable from the other coordinate system not within the system itself.

The formulas were derived here by using the following three propositions that are presumed valid: the Maxwell EM field equations including the Lorentz force equation, the photon force equation, and the group property of existence of an identical inverse transform. The crucial and questionable assumption, typically used in the traditional derivations, is the assumption of constancy of speed of light. As is apparent from the above derivation this assumption is not necessary and its relaxation results only in the additional factors of c'/c or c/c' appearing in front of the corresponding time transformation equations. The constancy of speed of light is not satisfied in reality, since in the space around Earth, the Sun, any other star, or the entire Universe, the gravitational potential is not constant as is well known, and this affects the speed of light. This fact would limit the usefulness and range of applications of standard LCT. It is therefore important to know how LCT changes if the assumption of constancy of speed of light is relaxed. However, before the next steps in the derivation of the generalized LCT are addressed it is interesting to establish the metric line element of the laboratory coordinate system and investigate its behavior during the transformation to the moving coordinate system.

For the following choice of metric, called the Minkowski space-time metric, it can be easily shown using Eq. (10) and Eq. (12) that:

$$ds^2 = c^2 dt^2 - dx^2 = c'^2 dt'^2 - dx'^2 = ds'^2 \tag{19}$$

As can be seen this metric line element is an invariant under LCT. This fact alone is giving us a certain degree of confidence that SRT and the Minkowski space-time are describing reality correctly since this metric line element signature is identical with the signature of a wave equation:

$$\frac{1}{c^2}\frac{\partial^2 \phi_n}{\partial t^2} - \frac{\partial^2 \phi_n}{\partial x^2} = 0 \tag{20}$$

The general solutions of Eq. (20) are equal to: $\phi_n = \phi_n\left(ct \pm x\right)$ and thus related to the propagation delay between cause and effect.

4. The Group Property of LCT

Since the Galilean Coordinate Transformations (GCT) form a group [2] under the simple velocity composition rule it is reasonable to expect that the LCT should also form a group since GCT should be a limiting case of LCT for $v << c$. However, it cannot be expected that the classical velocity addition will also hold for LCT. In this paragraph the LCT group characteristics will be confirmed and the velocity composition rule found.

The existence of the identity element is obvious by simply substituting $v = 0$ for the velocity into the transformation equations. The existence of the inverse element as a member of the group has already been established and used for finding the transform for the time coordinate. It is now only necessary to find whether the composition of two consecutive transformations belongs to the group resulting in the same transformation equations and what condition the corresponding velocities must satisfy. Assume we have two moving coordinate systems, primed and double primed, with different velocities relative to the laboratory coordinate system that have the following LCT transformations.

$$x = \frac{x' - v't'}{\sqrt{1 - v'^2 / c'^2}} \tag{21}$$

$$t = \frac{c'}{c} \frac{t' - x'v'/c'^2}{\sqrt{1 - v'^2 / c'^2}} \tag{22}$$

$$x = \frac{x'' - v''t''}{\sqrt{1 - v''^2 / c''^2}} \tag{23}$$

$$t = \frac{c''}{c} \frac{t'' - x''v''/c''^2}{\sqrt{1 - v''^2 / c''^2}} \tag{24}$$

By eliminating x and t from this system of equations and finding expression for the double prime coordinate x'' as function of the single primed coordinate the result is:

$$x'' = \frac{1 - \frac{v'v''}{c'c''}}{\sqrt{1 - \frac{v'^2}{c'^2}}\sqrt{1 - \frac{v''^2}{c''^2}}} \left(x' - t' \frac{v' - \frac{c'v''}{c''}}{1 - \frac{v'v''}{c'c''}} \right) \tag{25}$$

This result is simplified by introducing the formula for composition of velocities:

$$\frac{u'}{c'} = -\frac{\frac{v''}{c''} - \frac{v'}{c'}}{1 - \frac{v'v''}{c'c''}} \tag{26}$$

with the result:

$$x'' = \frac{x' - u't'}{\sqrt{1 - u'^2 / c'^2}} \tag{27}$$

Similarly, using the same velocity composition formula the transformation for the time coordinate becomes:

$$t'' = \frac{c'}{c''} \frac{t' - x'u'/c'^2}{\sqrt{1 - u'^2 / c'^2}} \tag{28}$$

The group property of LCT is therefore established without the necessity to postulate that the speed of light must be identical in moving and laboratory coordinate systems.

It is interesting to verify the group property of LCT by yet another way. It is possible to consider that the laboratory coordinate system was not originally at rest but was actually moving relative to another coordinate system with a velocity Δv. This means that for the primed and the double primed velocities according to Eq. (26) holds:

$$\left(\frac{v'}{c'}\right)_{new} = -\frac{\frac{v'}{c'} + \frac{\Delta v}{c}}{1 + \frac{v'}{c'}\frac{\Delta v}{c}} \tag{29}$$

$$\left(\frac{v''}{c''}\right)_{new} = -\frac{\frac{v''}{c''} + \frac{\Delta v}{c}}{1 + \frac{v''}{c''}\frac{\Delta v}{c}} \tag{30}$$

Substituting these values into Eq. (26) the result becomes as follows:

$$\left(\frac{u'}{c'}\right)_{new} = -\frac{\left(\frac{v''}{c''} - \frac{v'}{c'}\right) - \frac{\Delta v^2}{c^2}\left(\frac{v''}{c''} - \frac{v'}{c'}\right)}{\left(1 - \frac{v'}{c'}\frac{v''}{c''}\right) - \frac{\Delta v^2}{c^2}\left(1 - \frac{v'}{c'}\frac{v''}{c''}\right)} = \frac{u'}{c'} \tag{31}$$

The relative velocity of double primed coordinate system in reference to the single primed coordinate system does not change with respect to the motion status of the original laboratory coordinate system. It is therefore possible to select any coordinate system, moving or not, as a reference and evaluate motion of remaining objects relative to this system with the same results. This is the essence of relativity. However, the fact that the inertial motion cannot be internally detected within the particular coordinate system does not mean that there is no absolute reference frame. Perhaps other testing methods, not based on the inertial motion, can be developed to detect it. From Eq. (26) it is also important to realize that nothing can be added or subtracted from the speed of light. So, if it is considered that light moves in a medium that facilitates its propagation, the speed of light measured by an observer that moves relative to this medium will still be c.

This effect, however, disappears on rotating platforms where the Lorentz circumference length contraction is exactly compensated by the centrifugal force and the reaction of the disc atomic matter to it causing the circumference length dilation. The simple possible elastic disc expansion, however, is not considered in these derivations. This result leads to the Sagnac effect where the standard GCT can be used for the circumference velocities addition [3].

Finally, an important point to notice is that the formulas contain only the ratios of velocities to the local speed of light. It is therefore clear that time in each coordinate system cannot be determined without knowing the speed of light in that system. It thus seems that the speed is the primary physical quantity, not the time, and that the perception of time is determined by comparing the particular time measuring process speed or any sensing process speed to the local speed of light.

5. Application to Space-Times with a Gravitational Field

The fact that LCT can be obtained without the constancy of speed of light postulate allows now to use it in the space-times that have a gravitational field in them. A typical example is the space-time with the gravitational field of the centrally gravitating body. The general metric line element for this space-time is as follows:

$$ds^2 = g_{tt}(cdt)^2 - g_{rr}dr^2 - g_{\phi\phi}\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) \tag{32}$$

However, the coordinates used in this metric line element are referenced to the center of the gravitating body, so it is necessary to perform a transformation to apply LCT to this case. The simplest way is to place an observer at a certain distance from the center at the coordinates r_0 and t_0 and evaluate the motion of a test body that falls in the radial direction relative to this observer. The motion of the freely falling test body in the radial direction is described by the Lagrangian derived from the metric:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2 \tag{33}$$

The first integrals of equations of motion resulting from this Lagrangian are as follows:

$$g_{tt}\frac{dt}{d\tau}=1 \tag{34}$$

$$\left(\frac{dr}{d\tau}\right)^2=c^2-c^2g_{tt} \tag{35}$$

where $ds=cd\tau$ and where the condition $g_{rr}g_{tt}=1$ was also used. The time differential at the observer's place is obtained from the equation:

$$d\tau=dt_0\sqrt{g_{tt}} \tag{36}$$

The local radial speed of light at the observer's place $c_0=dr/dt_0$ is found by setting the metric line element ds in Eq. (32) to zero. This results in the following:

$$g_{tt}c^2=\frac{1}{g_{tt}}\left(\frac{dr}{dt_0}\right)^2\left(\frac{dt_0}{dt}\right)^2 \tag{37}$$

To proceed further it is necessary to find the relation between the time differentials of the stationary and the moving objects. This can be found from the relation $d\tau=dt_0\sqrt{g_{tt}}$ for the stationary object and $d\tau=dt\ g_{tt}$ for the moving object. The result is:

$$dt_0=dt\sqrt{g_{tt}} \tag{38}$$

After the substitution into Eq. (37) the result for the speed of light at the observer's location becomes:

$$c_0=c\sqrt{g_{tt}} \tag{39}$$

Using this result in Eq. (35) and defining the radial speed of the test body relative to the observer as: $v_0=dr/dt_0$ we can write:

$$\left(\frac{dr}{dt_0}\right)^2\frac{1}{g_{tt}c^2}=\frac{v_0^2}{c_0^2}=1-g_{tt} \tag{40}$$

For the moving object $(t\equiv t')$ in relation to the stationary observer when both are located in the gravitational field then follows from Eq. (38) and Eq. (40) that:

$$dt_0=dt'\sqrt{1-v_0^2/c_0^2} \tag{41}$$

In case that the observer is removed from the gravitational field the relation between the time differentials will be:

$$dt_0(\infty)=dt'\sqrt{g_{tt}}\sqrt{1-v_0^2/c_0^2} \tag{42}$$

Substituting for $\sqrt{g_{tt}}$ using Eq. (39), it becomes clear that for the time differential of the falling coordinate system in a radial direction in a gravitational field and the time differential of the stationary observer removed from the gravitational field it is:

$$dt'=\frac{c}{c_0}\frac{dt_0(\infty)}{\sqrt{1-v_0^2/c_0^2}} \tag{43}$$

This result is identical with the relation derived in Eq. (16). This also demonstrates the compatibility between the generalized LCT and the curved space-time metric approach in describing the geometry of the space-time with a gravitational field of a centrally gravitating body.

From these results and also according to Eq. (18) it is interesting to find that for the time differential of the falling observer in a radial direction and the time differential of the laboratory coordinate system removed from the gravitational filed the LCT exactly compensates the effect of gravity:

$$dt_0(\infty)=\frac{c_0}{c}\frac{dt'}{\sqrt{1-v_0^2/c_0^2}}=dt' \tag{44}$$

The gravitational field is removing the inverse time transformation symmetry of LCT, while the ct product group transformation symmetry is maintained. Except for the Doppler Effect when measuring frequencies the time differentials are identical. The Doppler Effect needs to be added, since during the derivation of the Lorentz coordinate transformation formulas the varying signal propagation delay from the moving object to the observer was not included. However, to measure the effect resulting from Eq. (44) alone it is necessary to understand in detail how the particular time interval generating device is constructed and affected by the gravity and velocity. In general, it seems that the free fall does always compensate for all the effects of gravity and no other effect except the Doppler shift can be observed. This finding has a significant consequence in cosmology where, if assumed that the receding galaxies are in a free fall falling away from Earth, no effect caused by the cosmological gravitational potential can be detected as was found elsewhere [4]. The time rate everywhere in the entire Universe on the large scale appears to be the same. This confirms again the complete symmetry (relativity) between the observer looking at Earth from the depths of the Universe and an observer on Earth looking back into the far distances even if the Universe is not flat.

From the metric in Eq. (32) also follows equation for the radial speed of light c_r relative to the centrally gravitating mass coordinate system. By setting the metric line element ds to zero this becomes:

$$c_r=cg_{tt} \tag{45}$$

By using this result together with Eq. (34) in Eq. (35) and defining the radial speed of a test body as $v_r=dr/dt$ it follows that:

$$\frac{v_r^2}{c_r^2}=1-g_{tt} \tag{46}$$

Finally, by comparing Eq. (46) and Eq. (40) it is clear that the following equality must hold:

$$\frac{v_0^2}{c_0^2}=\frac{v_r^2}{c_r^2} \tag{47}$$

This is confirming the previously used relation between velocities of generalized LCT.

From Eq. (35) it is also possible to derive the expressions for forces governing the free fall of a test body in the gravitation field of the centrally gravitating mass. By differentiating Eq. (35) with respect to τ the result is:

$$\frac{d^2r}{d\tau^2}=-\frac{c^2}{2}\frac{\partial g_{tt}}{\partial\phi_n}\frac{\partial\phi_n}{\partial r} \tag{48}$$

Multiplying Eq. (48) by the rest mass m_0 of the test body and realizing that $dt_0(\infty)=d\tau$ in Eq. (42), it is possible to write:

$$\frac{d}{dt}\left(\frac{m_0v}{\sqrt{g_{tt}}\sqrt{1-v^2/c^2}}\right)=-\frac{c^2}{2}\frac{\partial g_{tt}}{\partial\phi_n}\frac{\partial\phi_n}{\partial r}m_0\sqrt{g_{tt}}\sqrt{1-v^2/c^2} \tag{49}$$

where the index standing by the velocities was due to the validity of Eq. (47) omitted. From this result it is then clear that for the inertial and the gravitational mass of the test body it must hold the following:

$$m_i = \frac{m_0}{\sqrt{g_{tt}}\sqrt{1-v^2/c^2}} \tag{50}$$

$$m_g = m_0\sqrt{g_{tt}}\sqrt{1-v^2/c^2} \tag{51}$$

This conclusion is a direct consequence of the generalized LCT and the curved space-time metric that describes the free fall motion of a test body as is defined by the Lagrangian in Eq. (33).

Finally from Eq. (48) it also directly follows that since the left hand side is a contravariant geometrical object the right hand side must also be a contravariant geometrical object, it is thus necessary knowing that the gradient of a potential is a covariant quantity that the following relation must hold:

$$\frac{c^2}{2}\frac{\partial g_{tt}}{\partial \phi_n} = g^{rr} = g_{tt} \tag{52}$$

The solution of this equation is simple to find assuming the flat space at infinity with the result:

$$g_{tt} = e^{2\phi_n/c^2} \tag{53}$$

where ϕ_n is the Newton gravitational potential [5].

6. Conclusion

In this article it was shown that the Lorentz coordinate transformation can be generalized and derived from the three fundamental assumptions: Maxwell-Lorentz equations, the photon force equation, and the assumption that LCTs form a group. The postulate of the constancy of speed of light was not necessary. Furthermore it was shown that the primary physical quantity is the velocity rather than time and that time is a derived parameter that results from a comparison of a particular process speed to the local speed of light. The validity of the generalized LCT was confirmed by applying it to the curved space-time of the centrally gravitating body, which also allowed finding the dependence of the inertial and gravitational masses of the free falling test body on the metric coefficient g_{tt} and on velocity. Finally by applying the generalized LCT to the entire Universe it was found that the time on the large scale appears to run everywhere with the same rate.

References

[1] A. Einstein, *Annalen der Physik* **17**: 891 (1905).
[2] J. Rotman, **An Introduction to the Theory of Groups** (Springer-Verlag, 1994).
[3] J. Hynecek, *Phys. Essays* **22**: 179 (2009).
[4] J. Hynecek, *Phys. Essays* **22**: 325 (2009).
[5] J. Hynecek, *Phys. Essays* **20**: 313 (2007).

Appendix

The attractive force observed between the two charged non-conductive moving plates can be calculated by considering that in addition to the electrostatic force attracting the plates there is also a force based on the Biot-Savart law acting between the currents, which the moving plates now also represent. To calculate this force it is useful to first find the magnetic field H existing in the space between the plates. For the selected configuration the simplest way is to use the integral form of Maxwell's equation:

$$\oint \vec{H}\cdot d\vec{s} = I \tag{A1}$$

where the integration path and the current flow are illustrated in a drawing in Fig. 3. The magnetic field intensity between the plates up to their internal surfaces is then equal to:

$$H = \frac{Q}{L(v)W}v \tag{A2}$$

Similarly for the electric field from the Gauss law:

$$\oiint \vec{D}\cdot d\vec{S} = Q \tag{A3}$$

where the integrating surface S encloses one of the plates, it follows that:

$$E = \frac{Q}{\varepsilon_0 L(v)W} \tag{A4}$$

Both, the magnetic field as well as the electric field intensities are, of course, zero on the external surfaces of the plates, but through the plate's thickness increase linearly from zero to the full value found between the plates. This is the consequence of the original assumption that the embedded charge distribution within the plates' volume is uniform. The formula for the force is obtained from the Lorentz force equation:

$$\vec{F} = Q\left(\vec{E} + \vec{v}\times\vec{B}\right) \tag{A5}$$

which must be integrated over the plate's thickness z_p.

$$\left|\vec{F}\right| = \frac{Q}{\varepsilon_0 L(v)W}\left(1-\frac{v^2}{c^2}\right)\int_0^{z_p}\frac{z}{z_p}\frac{Q}{z_p}dz \tag{A6}$$

After completion of integration in Eq.A6 where the substitutions for the parameters: $Q = mq$, $\vec{D} = \varepsilon_0\vec{E}$, $\vec{B} = \mu_0\vec{H}$, and $\varepsilon_0\mu_0 = 1/c^2$ were also made, the result becomes:

$$F_q = \frac{m^2q^2}{2\varepsilon_0 L(v)W}\left(1-\frac{v^2}{c^2}\right) \tag{A7}$$

This formula is used in Eq. (5).

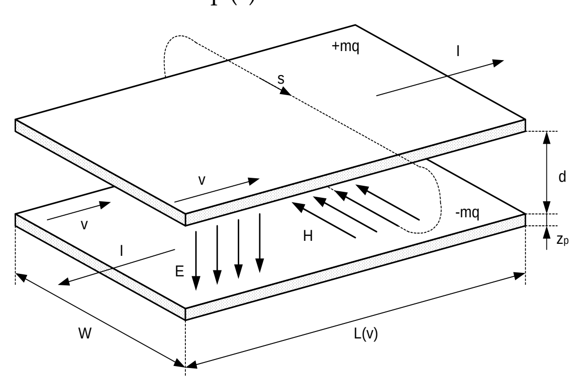

Fig. 3. Orientation of current I generated by the moving charged plates and of the resulting electric and magnetic fields. The magnetic field integration path s used in Eq. (A1) is also indicated.