Gravitational Lensing in Empty Vacuum Space Does Not Take Place

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Findings show that the rays of star light are lensed primarily in the plasma rim of the sun and hardly in the vacuum space just slightly above the rim. The thin plasma atmosphere of the sun represents a clear example of an indirect interaction involving an interfering plasma medium between the gravitational field of the sun and the rays of light from the stars. Since the lower boundary of this vacuum space is only a fraction of a solar radius above the solar plasma rim, it is exposed to virtually the same gravitational field. The thin plasma atmosphere of the sun appears to represent an indirect interaction involving an interfering plasma medium between the gravitational field of the sun and the rays of star light. An application of Gauss' law clearly shows that, if the light bending rule of General Relativity were valid, then a light bending effect due to the gravitational field of the sun should be easily detectable with current technical mean in Astrophysics at analytical Gaussian spherical surfaces of several solar radii. More importantly, the very same light bending equation obtained by General Relativity was derived from classical assumptions of a minimum energy path of a light ray in the gravitational field of the sun and the rays of star light. An application of Gauss' law clearly shows that, if the light bending rule of General Relativity were valid, then a light bending effect due to the gravitational gradient field of the sun. An intense search of the star filled skies reveals a clear lack of lensing among the countless numbers of stars, where there are many candidates for gravitational lensing according to the assumptions of General Relativity. Assuming the validity of the light bending rule of General Relativity, the sky should be filled with images of Einstein rings. Moreover, events taking place at our galaxy center, a region known as Sagittarius A*, thought to contain a super massive black hole, is considered a most likely candidate for an observation of gravitational lensing. A lack of evidence for gravitational lensing is clearly revealed in the time resolved images of rapidly moving stellar objects orbiting Sagittarius A*.

Keywords: black hole, gravitational lensing, plasma atmosphere, galactic core, Gauss law, optical reciprocity.

1. Introduction

We shall examine clear evidence for gravitational lensing in our region of space near to us, starting with the nearest star to us, our sun. The light bending rule of General Relativity suggests that a direct interaction should take place between the gravitational field of the lensing mass and the rays of light from the stars, whether in a vacuum space or in a plasma atmosphere. If the gravitational lensing is observed only at the plasma rim of the sun, it is evident that the past century of the observed solar light bending was due to an indirect interaction between the gravitational field of the sun and the rays of star light. This argument is supported by a calculation which derives the very same light bending equation obtained by General Relativity. The equation was derived from the assumptions of a minimum energy path of light in a plasma atmosphere exposed to the gravitational gradient field of the sun. Appendix A gives a detail calculation and a derivation of this famous light bending equation. We shall take a closer look at the lower boundary of the vacuum space just above the plasma rim of the sun, the nearest star to us, just 8 light-minutes away. We shall examine the nearby stars in our own region of space, less than hundreds of light-years away. There are many cases in the star filled skies where the lenses and the light sources are by chance co-linearly aligned with the earth based observer, presenting vast opportunities for the observation of Einstein rings. We shall examine the collected images and the astrophysical data of the stars orbiting about Sagittarius A*, a region thought to contain a super massive black hole located at the center of our galaxy, the Milky Way, right in our own backyard, just 26,000 light-years away. Research convincingly reveals:

1. a lack of evidence for gravitational lensing in the vacuum space, just a fraction of a solar radius above the solar plasma rim; straightforwardly revealed by applying the Gauss' law of gravity to the solar mass
2. a lack of evidence for Einstein rings in a sky of countless numbers of stars, where the candidates for gravitational lenses and the light sources are by good chance co-linearly aligned with the earth based observer
3. a lack of evidence for gravitational lensing in the time resolved images of the stars orbiting about presumably a black hole at the site of Sagittarius A*

For nearly a century now, the effects of gravitational light bending have been observed primarily at the rim of the sun during events of solar eclipses. New findings clearly show that with a straightforward application of Gauss' law of gravity, an important fundamental of Mathematical Physics, the light bending rule of General Relativity apparently does not apply to the empty vacuum space above the rim of the sun.

2. Misapplied Fundamentals on Gravitational Lensing Concepts

An application of Gauss's law, applied to gravitation as well as to electromagnetism along with the principle of optical reciprocity clearly show that a co-linear alignment of the observer,
the lens and the source is unnecessary for an observation of a gravitational light bending effect, as predicted by the light bending rule of General Relativity. The gravitational effect at the surface of an analytical Gaussian sphere due to the presence of a point-like gravitating mass that is enclosed inside of the sphere depends only on the quantity of mass enclosed. The size or density of the enclosed mass particle is not important [12, 13]. The Gauss’ law of gravity (see, e.g., [1, 4]) is a Mathematical Physics tool that encloses a gravitating mass particle inside of an analytical Gaussian surface which applies directly to the gravitational field of the enclosed mass. An analogy to this principle encloses an electrically charged particle inside of a Gaussian surface in application to the electric field of the charged particle in the discipline of Electromagnetism [4]. The principle of optical reciprocity [2, 3] simply states that the light must take the very same minimum energy path or least time path, in either direction between the source and the observer. This fundamental principle is an essential tool for the understanding of complex lensing systems in Astronomy and Astrophysics [6]. We shall now correctly apply all of these well founded and proven fundamentals to these gravitational lensing problems.

![Diagram](image)

**Fig. 1.** Gauss’ Law applied to Equal Gravitating Masses of Different Radii Enclosed

### 2.1. Gauss Law Applied to a Point-Like Gravitating Mass

Any gravitational effect on a light ray due to the presence of a gravitating mass at the impact parameter \( R \) would theoretically depend on the amount of Mass \( M \) that is enclosed within the analytical Gaussian sphere of radius \( R \) as illustrated in Figure 1. Any gravitational effect that would be noted at the surface of the analytical Gaussian sphere should in principle be totally independent of the radius of the mass particle or the density of the mass that is enclosed within the Gaussian sphere. From Gauss’s Law, Eq. (2), equal masses of different radii will theoretically have equal gravitational effects at the surface of the Gaussian sphere. The light bending rule

\[
\delta \theta = \frac{4GM}{Rc^2}
\]  

(1)

of General Relativity is essentially a localized \( 1/R \) effect. Since we are dealing with astronomical distances, the impact parameter \( R \) is practically a localized effect only. The predominant gravitational effect would take place in the vicinity of the gravitating mass, namely the lens, maximizing at the point where the light ray is tangent to the Gaussian sphere of radius \( R \).

A very essential tool of Mathematical Physics known as Gauss’ law [1, 4],

\[
\int_{S} \vec{g} \cdot d\vec{A} = 4\pi GM
\]  

(2)

is applied directly to the gravitating masses where the gravitational field \( \vec{g} \) is a function only of the mass \( M \) enclosed by the spherical Gaussian surface \( S \) [1, 2, 4]. The gravitational flux at the surface of the analytical Gaussian sphere is totally independent of the radius \( R \) of the sphere. The idea here is that the gravitational field at this analytical Gaussian surface is only a function of the mass that it encloses [1, 4]. Any mass \( M \), regardless of the radius of the mass particle that is enclosed inside of the Gaussian spherical surface of radius \( R \) will contribute exactly the same gravitational potential at the Gaussian surface. In Figure 1, the gravitational field points inward towards the center of the mass. Its magnitude is \( g = \frac{GM}{R^2} \). In order to calculate the flux of the gravitational field out of the sphere of area \( A = 4\pi R^2 \), a minus sign is introduced. We then have the flux

\[
\Phi_g = -gA = -\left( \frac{GM}{R^2} \right) \left( 4\pi R^2 \right) = -4\pi GM
\]

Again, we note that the flux does not depend on the size of the sphere. It is straightforwardly seen that a direct application of Gauss’s law to the light bending rule, Eq. (1), coupled with the essential principle of optical reciprocity [3], removes any requirement for a co-linear alignment of the light source, the point-like gravitating mass particle (the lens) and the observer for observation of a gravitational lensing effect as suggested by General Relativity [12, 13].

From Eq. (2) the flux of the gravitational potential through the surface would be the same for all enclosed mass particles of the same mass \( M \), regardless of the size of the mass particle. As a result, as illustrated in Figure 1, each mass particle will produce the very same gravitational light bending effect \( \delta \theta = \delta \theta_1 + \delta \theta_2 \), where \( \delta \theta_1 \) and \( \delta \theta_2 \) are the bending effects on the ray of light on approach and on receding the lens, respectively. This of course assumes the validity of Eq. (1). This symmetry requirement suggests that \( \delta \theta_1 = \delta \theta_2 \), and from Eq. (1) that \( \delta \theta = 2\delta \theta_1 = \frac{4GM}{Rc^2} \). It follows that \( \delta \theta_1 = \delta \theta_2 = \frac{2GM}{Rc^2} \). This says that the total contribution of the light bending effect due to the gravitating point-like mass particle on any given infinitely long light ray is theoretically divided equally at the impact parameter \( R \) separating the approaching segment and the receding segment of the optical path. A confirmation of this will be clearly seen later with application of the principle of reciprocity and a demonstration of a simple derivation of the equation of the Einstein ring, illustrating the symmetry requirement of General Relativity. From Gauss’s law, all that is needed here is the total mass of the particle that is enclosed inside of the analytical Gaussian sphere. The actual size of
the gravitating point-like mass particle is totally unimportant and need not be known.

2.2. Principle of Optical Reciprocity applied to the Lensed Light Ray

![Fundamental Principle of Optical Reciprocity Illustrated on a Lensed Light Ray](Fig. 2)

In any space, the principle of reciprocity, [2, 3] a very fundamental principle of optics, must hold as illustrated in Fig. 2. The principle simply states that any photon or wave of light moving on a preferred optical path, from the source to the observer, must take the very same optical path from a hypothetical laser gun of the observer back to the source. As a consequence of this fundamental principle, any additional sources placed along the same preferred optical path will all appear to the observer to be located at the very same image position of the original source. As a consequence of this principle, all light emitting sources on a single preferred optical path will all appear to the observer to be located at the very same point, appearing as a single light emitting source. This scarcely mentioned fundamental principle of optics is directly applicable to the Astrophysics at the galactic center. The total gravitational light bending effect acting on the light ray is directly applicable to the Astrophysics at the galactic center.

![Symmetry Requirement of the Accumulative Lensing Effect](Fig. 3)

The astronomical distance $D_L$ is the distance from the observer to the lens, $D_{SL}$ is the distance from the lens to the source and $D_S$ is the distance from the observer to the source. Also again, it is important to note that this case is a simplified special case, where $D_L = D_{SL}$, presented in most academic textbooks. There is no requirement at all that the lens be positioned exactly at the midpoint for an observation of a theoretical Einstein ring. Of course, this again would assume the validity of the light bending rule of General Relativity. This is addressed in the next section dealing with the axis of symmetry of the lensed light ray for corresponding near and far observers. It is readily seen that the axis of symmetry for a given light ray is perpendicular to the line joining the source and the observer only for the special case where the lens is positioned exactly at the midpoint between the source and the observer. The star filled sky reveals a clear lack of gravitational lensing among the countless numbers of stars. There are many cases whereby the lens and the source are co-linearly aligned with the earth based observer, thereby presenting the opportunity for the observation of an Einstein ring. Assuming the validity of the light bending rule of General Relativity, the night sky should be filled with images of Einstein rings.

2.3. Symmetry Requirement Demonstrated on derivation of Einstein Ring Equation

From symmetry we have

$$\delta \theta_1 = \delta \theta_2 = \frac{2GM}{Rc^2}$$  \hspace{1cm} (4)

Again, the astronomical distance $D_L$ is the distance from the observer to the lens. Since we are dealing with very small angles, from Figure 3, the deflection of the light ray due to the gravitational effect on approach to the gravitating mass is simply

$$\delta \theta_1 = \frac{R}{D_L} = \frac{2GM}{Rc^2}$$, wherefrom $\frac{R^2}{D_L} = \frac{2GM}{c^2}$ and $\frac{R^2}{D_{SL}} = \frac{2GM}{D_{SL}c^2} = \delta \theta_1^2$.

Solving this for the radius of the impact parameter of the light ray and thus the radius of the Einstein Ring expressed in units of radians we have

$$\delta \theta_1 = \frac{2GM}{\sqrt{D_{SL}c^2}}$$  \hspace{1cm} (5)

which is the radius of the Einstein ring in units of radians for a lens plane exactly midway between the source and the observer. This is a special case, where $D_L = D_{SL}$ (See Appendix B for the general case where $D_L \neq D_{SL}$.) Note that the gravitational bending effect on the light ray for the approach segment alone is exactly equal to the radius of the solved Einstein ring expressed in radians and is given as

$$\delta \theta_1 = \frac{2GM}{Rc^2}$$  \hspace{1cm} (6)

This effect is exactly one half of the total accumulative gravitational effect acting on the light ray for the approach and receding segments [12, 13]. This principle, an essential Mathematical Physics principle on lensing, is often totally missed by researchers attempting to deal with this topic. From symmetry requirement, the integral gravitational effect on a light ray upon approach to a gravitating mass positioned exactly at the midpoint of a line joining the source and the observer, must equal that of the integral gravitational effect on the light ray upon receding the gravitating mass.
as suggested by Eq. (4) and the laws of conservation of energy and of momentum [12].

This is a rarely covered fundamental on gravitational lensing in modern academic textbooks. The accumulative gravitational effect along the light ray must sum the total effects of gravity acting on the light ray for both the approach and receding segments of any ray of light passing by a point-like gravitating mass [12]. The total light bending effect is therefore

$$\delta \theta_1 = \delta \theta_2 = \frac{2GM}{Rc^2}$$

Regardless of one’s position on gravitational lensing of light, the fundamental principle of optical reciprocity must hold. This is a given. The principle of optical reciprocity simply states that any light ray or a photon of light must take the very same path, along the same minimum energy path, in either direction between the source and the observer as depicted in Fig. 2.

Using the light bending rule of General Relativity, it is here-with demonstrated that all observers of varying distances from a gravitating mass or lens should see an Einstein ring. In Reference [14], the relative position of the observer, the lens and the source was analyzed. The placement of the observer relative to the lens showed that the near-field observer would see the largest, most lensed Einstein ring. The far-field observer would see the smallest, least lensed Einstein ring. The axis of symmetry always leans towards the near-field observer and away from the far-field observer. The axis of symmetry is perpendicular to the line joining the mid-field observer and the light source where the lens is exactly midway between the observer and the source. Only a mid-field observer would derive Eq. (6) which gives exactly the same value as that given by Eq. (5) for a simplified special case, where \(D_L = D_{SL} \), which is presented in most academic textbooks. Both the near-field and the far-field observers would also derive Einstein ring equations with coefficients corresponding to their unique geometries. Each observer has distinct sets of lensed light rays, each with their corresponding axis of symmetry. It is very important to note that all of the lensed light rays belong to the very same family of equations derived from the light bending rule (Eq. (1)) of General Relativity. Any light ray that is gravitationally bent by a point-like gravitating mass, as predicted by General Relativity, will always have an axis of symmetry which would be perpendicular to the line joining the source and the observer only when the lens is positioned exactly at the midpoint on the line joining the observer and the source. All observers will see, according to General Relativity, an Einstein ring according to the geometry of the lensed rays as a function of the position of the observer relative to the lens and the source. [12, 13] This essential key point is totally missed in many textbooks and lectures on this subject matter.

3. The Important Fundamentals Applied

3.1. The Fundamentals Applied to the Thin Plasma Rim of the Sun

Historically, the effect of light bending has been noted only at the solar rim, the thin plasma of the sun’s atmosphere due to a refraction of the light by the solar plasma only. This is a firm and well founded observational fact. In [14] the gravitational light bending effect as functions of various Gaussian surface radii impact concentric to the center of the sun was illustrated. The past century of astrophysical observations have revealed light bending only at the solar plasma rim. All solar lensing effects, as suggested by General Relativity are essentially 1/R effects.

We note that the total effect of the sun’s gravity on a ray of light on approach and on receding are virtually equal where the relation \(\delta \theta_1 = \delta \theta_2 \) still holds even though the Earth based observer is relatively close to the sun [12]. Remarkably, as it may seem, however, historically the solar light bending effect has been observed only at the solar rim, namely, the light refracting plasma of the solar atmosphere; a well confirmed observational fact. It is widely taught in many Physics lectures that the solar light bending effect noted at the solar rim is done primarily to maximize the effect for detection and that gravitational lensing is most sensitive at the solar rim. This erroneous teaching has also contributed to misapplication of the important fundamentals as well. We note again, that the thickness of the thin plasma shell of the sun, frequently referred to as the solar rim, is very negligible in comparison to the solar radius \(R\).

Assuming the validity of the light bending rule of General Relativity, the current technical means of the astronomical techniques should have easily allowed observations of solar light bending of stellar light rays at different solar radii of analytical Gaussian surfaces, namely at the radius of 2R, 3R and even at 4R, where R is one solar radius, as illustrated Reference [14]. For instance, at the analytical Gaussian surface of radius 2R, the predicted light bending effect of General Relativity would have been an easily detectable effect of one half the effect of 1.75 arcsec noted at the solar rim; at the surface of radius 3R, an effect of one third the effect at the solar rim, etc., etc. The equatorial radius \(R\) of the sun is approximately 695,000 km. The thickness of the solar rim has been recorded to be less than 20,000 km; less than 3 percent of the solar radius \(R\). From this, we can easily see that a gravitational lensing effect in vacuum space several solar radii above the solar plasma rim should be a very noticeable effect.

3.2. The Fundamentals applied to the Orbit of S2 about Sagittarius A*

The past decades of intense observations using modern astrophysical techniques in Astrophysics alone reveal an obvious lack of evidence for lensing effects on collected emissions from stellar sources orbiting about Sagittarius A*, believed to be a super massive black hole located at the galactic center of our Milky Way. This is most obviously revealed in the time resolved images collected since 1992 on the rapidly moving stars orbiting about Sagittarius A*. [7–11] The space in the immediate vicinity of a black hole is by definition an extremely good vacuum. The evidence for this is clearly seen in the highly elliptical orbital paths of the stars orbiting about the galactic core mass. The presence of material media near the galactic core mass would conceivably perturb the motion of the stellar object s16 which has been observed to move with a good fraction of the velocity of light. The presence of any media other than a good vacuum would have caused the fast moving stellar object s16 to rapidly disintegrate. Astro-
physical observations reveal that S16 has a velocity approaching 3% of the velocity of light when passing to within a periapsis distance corresponding to 60 astronomical units from Sagittarius A*, perceived to be a massive black hole. This gives solid evidence that the space in this region has to be, without a doubt, an extremely good vacuum.

Application of the light bending rule of Eq. (4) together with considerations of Gauss’s Law and the principle of optical reciprocity to the data of the observed orbit as worked out by Dowdye in [14], it is clear that some gravitational lensing effect should be detectable in the time resolved images of the orbit of S2 given the current level of today’s observational means. To demonstrate this, we shall assume a worst case in the two possible choices for inclination of \( i = \pm 46 \) given for of the orbit of S2. It is assumed that the projection of the distances between the S2 star and the galactic center mass, indicated in arcsec, represents an instantaneous impact parameter for the light rays coming from the S2 light source, passing by the galactic center mass, and then arriving at the earth based point of observation. The instantaneous impact parameter is used to calculate the worst case expected lensing based on Eq. (6) as a function of the position \((r, \theta)\) of S2 in the orbit.

We assume that the elliptical orbit of S2 has the form

\[
r(\theta) = \frac{a(1-e^2)}{1+e \cos(\theta + \theta_p)} \tag{9}
\]

where \( a = 0.119 \) arcsec is the semi-major axis, \( e = 0.87 \) is the eccentricity, \( \theta \) is the angle of polar coordinates and \( \theta_p \) is the phase angle to orient the orbit according to the best-fit data. Using the best fit observed \( r \) given in the first column of Table 2 and expressing \( r(\theta) \) in units of meters, the radial lengths would serve as a set of instantaneous impact parameters along the orbit of S2, namely the nearest point of approach of the light ray to the galactic center mass, since the orbit is a projection of the actual elliptical orbit of S2 with an inclination of \( i = \pm 46 \). Plugging this result into Eq. (4) by setting \( R = r(\theta) \) we have

\[
\delta \theta_1 = \frac{2GM}{Rc^2} = \frac{2}{a(1-e^2)e^2} \tag{10}
\]

where the semi-major axis \( a \) is expressed in meters. The predicted lensing \( \delta \theta_1 \) is expressed in arcsec. Eq. (10) is the predicted image of the actual orbital path of S2 given by Eq. (9), an entirely different configuration as illustrated by Dowdye in Reference [12], where the theoretical fit to the observed orbit of S2 about Sagittarius A* and the predicted lensing is compared.

In Table 1, some selected points of the observed orbit of S2 and the corresponding predicted lensing of the orbit, based on the light bending rule of General Relativity, are tabulated. It is clear from these calculations that the predicted magnitude of the lensing effect, which is orders of magnitude greater than the observed radial separation between the S2 source and the position of the galactic center mass, should be a very noticeable effect. To date, clear evidence of a gravitational lensing effect based on the light bending rules of General Relativity is yet to be revealed in the time resolved images of the stellar objects orbiting about Sagittarius A*, a region under intense astrophysical observations since 1992.

4. Conclusion

Historically, gravitational light bending has been observed primarily at the thin plasma rim of the sun. A direct interaction between the sun’s gravity and the rays of star light in the space free of solar plasma rim is yet to be observed. The same light bending equation is derived from assumption of a minimum energy path of light rays in the solar plasma rim; an effect totally independent of frequency. Modern Astronomy and Astrophysics should easily permit detection of lensing events among the large numbers of stellar objects in the star filled skies. With this in mind, the entire celestial sky should be filled with Einstein rings. From the fundamentals, it is clearly seen that the event taking place at the galactic center of the Milk Way, starring us right in the face. The evidence is clear in the everyday cosmological appearance.

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<th>( \theta ) (degrees)</th>
<th>Observed ( r ) (arcsec)</th>
<th>Observed ( r ) (light-days)</th>
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Table 1. Data of the Elliptical Orbit Observed and the Predicted Lensing of the Orbit

References

A calculation for the bending of light rays in the thin plasma rim of the sun is carried out in detail here. The calculation is based entirely on a conservation of energy concept considering the gradient of the gravitational field of the sun acting directly on the rapidly moving ionized material particles of the thin plasma atmosphere of the sun. The calculation considers a minimum energy path for rays of light. The results is found to be totally independent of frequency. The rapidly moving ionized particles of the solar plasma is assumed to be bounded by the gravitational potential of the sun given by

$$\phi_{\text{min}} = \frac{G M}{r} \int \frac{dr}{r} = \frac{G M}{R} \quad \text{(A1)}$$

It may be assumed that the plasma particles of the ionized solar rim move with random velocities such that their kinetic energies are as dictated by $\frac{1}{2} m v^2 = \frac{1}{2} k T + \phi m$, where $m$ is the mean mass of the plasma particles of temperature $T$, $v$ is the velocity of the plasma particle bounded by the gravitational potential $\phi$. The velocity $v$ of the moving ions may be assigned an upper bound of $v = \sqrt{\frac{2 G M}{R}}$, the escape velocity of the solar gravity at the surface of the sun. The solar plasma particles bounded by gravity in the solar rim may be considered as a dynamic lens under the intense gravitational gradient field of the sun. It is theoretically shown here, and in detail in [13], that a minimum energy path for light rays propagating in the solar plasma rim, subjected to the gradient of the gravitational field of the sun, yields the mathematical results of $\frac{2 G M}{Rc^2}$.

It is shown that the moving ions acting as secondary sources within the plasma rim, moving with velocities not to accede the

velocity $v = \sqrt{\frac{2 G M}{R}}$, the frequency and wavelength of a light ray exposed to the plasma are:

$$v' = v_0 \left(1 - \frac{v^2}{c^2}\right) = v_0 \left(1 - \frac{2 G M}{Rc^2}\right) \quad \text{(A2)}$$

$$\lambda' = \lambda_0 \left(1 - \frac{v^2}{c^2}\right)^{-1} = \lambda_0 \left(1 - \frac{2 G M}{Rc^2}\right)^{-1} \quad \text{(A3)}$$

$$\lambda' \approx \lambda_0 \left(1 + \frac{2 G M}{Rc^2}\right) \quad \text{(A4)}$$

Thus, the energy $\epsilon$ per unit length of the light ray along the minimum energy path is. Consequently, the number of re-emitted waves per unit length along the photon path and thus the energy per unit length increases as $r$ increases. This translates to a downward, re-emitted path of the bent light ray, along a minimum energy path for the approaching segment of the light ray. If $\frac{d\epsilon}{dr} = +\epsilon_0 \frac{2 G M}{Rc^2}$ or $\frac{d\epsilon}{dr} = +\epsilon_0 \frac{2 G M}{Rc} \delta r$, then the re-emission of the light ray in the atmosphere of ions will occur such that the total energy along the minimum energy (conservation of energy) path for a given light ray would not change. If $\epsilon$ is the energy per unit length along the light ray and $\delta \epsilon$ is the change in energy in the direction of the gradient potential $\phi(r)$, then the angle of change during the approach segment of the light ray is

$$\delta \theta_{\text{app}} = \frac{\delta \epsilon_{\text{app}}}{\epsilon} = + \frac{\int R}{\int R} 2 G M \frac{dr}{r^2 c^2} = -\frac{2 G M}{Rc^2} \quad \text{(A6)}$$

and the path change for the receding segment of the light ray is

$$\delta \theta_{\text{rec}} = \frac{\delta \epsilon_{\text{rec}}}{\epsilon} = + \frac{\int R}{\int R} 2 G M \frac{dr}{r^2 c^2} = +\frac{2 G M}{Rc^2} \quad \text{(A7)}$$

The net change in the path of the light ray is

$$\delta \theta = \delta \theta_{\text{rec}} - \delta \theta_{\text{app}} = \frac{4 G M}{Rc^2} \quad \text{(A8)}$$

Appendix B: The Einstein Ring Equation; the General Case ($D_L \neq D_{SL}$)

The general case for the Einstein ring equation involves all values for the distances, whereby $D_L$ is the distance between the observer and the lens and $D_{SL}$ is the distance between the lens and the source. These are cases where $D_L \neq D_{SL}$. The general case for the radius of the Einstein ring in units of radians is

$$\delta \theta (\text{rad}) = \frac{D_{SL} 4 G M}{\sqrt{D_L + D_{SL}} D_L c^2} \quad \text{(B1)}$$
The radius of the Einstein ring at the image location the distance of $D_L + D_{SL}$ expressed in meters is

$$R(\text{meters}) = (D_L + D_{SL}) \delta \theta (\text{rad}) \quad \text{(B2)}$$

where $D_L$ and $D_{SL}$ are also expressed in meters. The impact parameter (IP) corresponding to the Einstein ring is the nearest point of approach of the light rays to the point-like lensing mass, when observed at a distance of $D_{SL}$ meters away from the observer, for the rays of light coming from the light source to the observer. Since this is a 3 dimensional problem, the impact parameter of the light rays that would produce an Einstein ring is also a ring itself. It is a virtual ring for purpose of the analysis of the problem. The impact parameter (IP) in meters is

$$R_{IP}(\text{meters}) = (D_L + D_{SL}) \delta \theta (\text{rad}) \quad \text{(B3)}$$

where $R_{IP}$ is the nearest point of approach of the gravitationally lensed light rays to the lensing star. It is that distance the lensed light rays will pass over the plasma rim of the lensing star, moving through the empty vacuum space well above the plasma rim of the lensing stars, moving along astronomical distances from the source to the observer. The radius of the predicted Einstein ring, according Eq. (B1) and the light bending rule of General Relativity, will be nearly 15 times the radius of a sun-like lensing star, the same mass and radius as the sun, when both are observed at the same distance $D_{SL} = 4$ light years away. Adjusting the parameter $D_{SL}$ would cause the radius of the Einstein ring to change. Increasing $D_{SL}$ would cause the image of the Einstein ring to increase in radius (an increase in magnification), assuming the validity of General Relativity. Setting $D_L = D_{SL}$ the Eq. (B1) becomes Eq. (5), the special case.