Energy Storage in Inductors and Ampère's Law

According to classical electrodynamics the energy stored in a current-carrying inductor 'is stored in the magnetic field'. It is the intent of this paper to argue that the energy stored in a simple solenoid coil is caused by the Ampère force between the current elements of the wire in the inductor. Ampère's Law [1] describes the force between two current elements without reference to a magnetic field. Similarly, the energy 'stored in the magnetic field' of an inductor may be described without reference to a magnetic field. This letter shows that inductance of a simple solenoid can be calculated using Ampère's Law and a computer.

Ampère's Law gives the force between two current elements as:

$$\mathrm{f}_{12} = -\hat{\mathrm{r}}\frac{\mu_o I_1 I_2 ds_1 ds_2}{4\pi r^2}\left(2\sin\theta_1\sin\theta_2\cos\eta - \cos\theta_1\cos\theta_2\right) \quad (1)$$

where $\hat{\boldsymbol{r}}$ is the unit vector in the direction of $\boldsymbol{r}$ joining the two current elements. The constant μ_o is permeability of free space. The I_1 and I_2 are the current magnitudes in the current elements, ds_1 and ds_2 are the lengths of the current-element vectors $\boldsymbol{ds_1}$ and $\boldsymbol{ds_2}$. The angles are: θ_1 = angle between $\boldsymbol{ds_1}$ and $\boldsymbol{r}$; θ_2 = angle between $\boldsymbol{ds_2}$ and $\boldsymbol{r}$; η = angle between the plane of $\boldsymbol{ds_1}$ with $\boldsymbol{r}$ and the plane of $\boldsymbol{ds_2}$ with $\boldsymbol{r}$. RMKS units are employed except where noted.

The energy stored in an inductor is given by:

$$E = L\,I^2/2 \quad (2)$$

Where E is the energy, L is the inductance, and I is the current flowing in the inductor. Eq. (2) may be derived by circuit theory or found in any book on classical electrodynamics.

The calculations below use free space and the simple one-turn solenoid inductor as shown in Fig. 1.

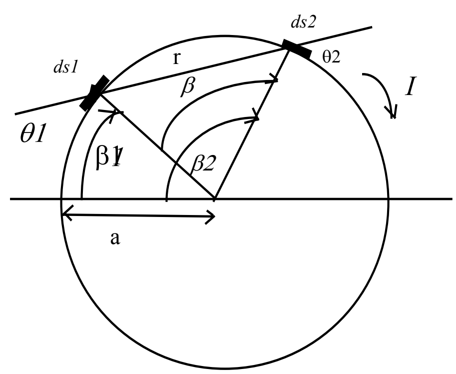

Figure 1. One-turn solenoid inductor.

The energy E_{12} stored between two current elements I_1ds_1 and I_2ds_2 is:

$$E_{12} = -\int_{\infty}^{r}\frac{\mathrm{f}\left(I_1, I_2, \theta_1, \theta_2, \eta, ds_1, ds_2\right)}{r^2}dr \quad (3)$$

If both differential lengths ds_1 and ds_2 are imagined to move outward with r to keep angle variables constant, then all the variables in the $f(\ldots)$ function are constant and only r varies. The $f(\ldots)$ function can then be moved outside the integral sign, and the result of the integration is then:

$$E_{12} = \frac{\text{- f(...)}}{r} \qquad (4)$$

Inserting Eq. (1) in (4) we get:

$$E_{12} = -\frac{\mu_o I_1 I_2 ds_1 ds_2}{4\pi r}\left(2\sin\theta_1\sin\theta_2\cos\eta - \cos\theta_1\cos\theta_2\right) \qquad (5)$$

From Fig. 1 we know $\beta = \beta_2 - \beta_1$, $r = 2\text{a}\sin(\beta/2)$, $\theta_1 = -\beta/2$, $\theta_2 = \beta/2$. To set up (4) for computer integration, we futher set $d\beta_1 = d\beta_2 = d\beta = 1$ degree so

$$ds_1 = ds_2 = ds = 2\pi\, a\, d\beta/360 = \pi\, a\, d\beta/180 \qquad (6)$$

For the present case, $I_1 = I_2 = I$. Substituting (6) into (5), we get:

$$E_{12} = -\frac{I^2}{2}\frac{\pi\mu_o a}{4(180)(180)}\left[\frac{2\sin(-\beta/2)\sin(\beta/2) - \cos(-\beta/2)\cos(\beta/2)}{\left|\sin(\beta/2)\right|}\right](d\beta)^2$$

$$= -\frac{I^2}{2}K\,\text{f}(\beta)(d\beta)^2 \qquad (7)$$

where K is a constant. Then the total energy E_t stored in the solenoid is:

$$E_t = -\frac{I^2}{2}K\sum_{\beta_1=0}^{358}\sum_{\beta_2=\beta_1+1}^{359}\text{f}(\beta) \qquad (8)$$

where the summation limits prevents counting of infinite self contributions and eliminates recounting of identical contributions.

Computing E_t for our one-turn solenoid with a radius of 0.254m:

$$E_t = \frac{I^2}{2}2.239\times 10^{\ 6} \qquad (9)$$

The value of L computed from (9) compares favorably with the measured value; *i.e.* within 2% of *2.205* micro henries.

Certain limitations apply to the computer technique of calculating the integrals piecemeal. One is the size of the $d\beta$ increment. Changing this size can change the result of the calculation, so (9) represents an approximation, however, a good approximation. It appears that an appropriately chosen element size can compensate for the internal energy storage effects of the current element. A current element size about 6.6 times the diameter of the wire was chosen for most of the computations.

That L for a multi-turn inductor varies approximately as the square of the number of turns can be reasoned quiet readily from (9). If the current were doubled, it would be like adding another turn. But the current is the same through all turns and E_t increases 4 times; thus L increases as the square of the number of turns, 2 turns in this case. Also, (7) shows that the stored energy in the one-turn solenoid, and thus its inductance, is a direct function of its radius a, or its circumference 2πa. Inductance measurements do not support inductance being proportional to the square of the number of turns. However, this is of secondary importance to the purposes of this paper. The inductance computed and measured for one-turn inductances are of primary importance.

In 1845,F. E. Neumann [2] arrived at the concept of electromagnetic potential. He developed a formula for the potential energy of two closed circuits using Ampère’s Law for force, and distance between current elements to derive the potential in units of energy. According to Dr. Peter Graneau many precise inductance calculations are based on Neumann's mutual inductance formula for which Neumann is best remembered.

The Ampère force on every current element in a one-turn solenoid is directed outward radial from the center. The recent explosion of "Godzilla", the powerful magnet created at Los Alamos National Laboratory, is testimony to the outward repulsive forces on wires in coils when huge currents are passed through them.

Ampère's Law explains the forces between current elements without reference to a magnetic field. And this letter shows the mechanism for energy storage in an inductor that does not refer to the magnetic field. Therefore, it appears the 'magnetic field' is a mathematical construct, which some texts acknowledge. A special kind of electric field is implicated with the Neumann potential.

Experimental Results

The measured inductance values were determined from the formula $L=1/C(2\pi f_o)^2$. The resonant peak of the voltage of the resonant circuit was detected with a scope and the frequency f_o measured with a highly accurate frequency counter. The capacitors used in the experiment had a measured tolerance of 2%. These measured values are compared with values computed from Eq. (8). The results are summarized in Table 1:

Table 1. Inductance of One-Turn Coils (micro henries)

shape	size	measured	computed	Δ%	d_s size (mm)
circle	r = 25.4 cm	2.205	2.239	1.5	4.4
square	side= 47 cm	2.563	2.592	1.1	4
rectangle	81cm x 14.2 cm	2.298	2.385	3.8	4
triangle	47 cm/side	1.704	1.688	-1.0	4

Conclusions

1. The potential energy set up by the Ampère force between current elements is the mechanism for energy storage in inductors. The use of Ampère's Law, without reference to a "magnetic field", explains energy storage in inductors
2. Eq. (5) is the general formula for energy storage between current elements and may be applied to all shapes.
3. Inductance and energy storage in coils may be calculated with the aid of a computer provided the current element size is taken into consideration. How inductance varies with the number of turns is a complication that this paper does not develop.
4. Because the energy storage appears to be in the "electric field" between current elements, which are analogous to individual charges, the one-turn inductor can be

used as a model for studying how multiple charges relate to each other and how energy is stored between them.

5. Energy associated with two current elements is added directly as a scalar and not as a vector as shown by (8).
6. The angle dependence of Ampère's Law is verified by the inductor experiment.

Acknowledgment

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References

[1] J. Keele, 'Theoretical Derivation of Ampère's Law', Galilean Electrodynamics, March/April 2002.
[2] P. Graneau, Ampère-Neumann Electrodynamics of Metals, p.23 (Nonantum, MA Hadronic Press, 1985).

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