The Vicious Circle: Mathematics - Physics

Viktor N. Moroz
675 Tysens Lane, Apt. 5G, Staten Island, NY 10306
e-mail: vktrmrz7@gmail.com

We attempt to discuss one aspect of the relation between mathematics and physics. Well known contradictions in the foundations of mathematics, contradictions in conceptions and axioms give us a considerable amount of mathematics containing illogical and discrepant theories. Confidence in the mathematical accuracy and logicality gives physicists possibility to create abstract theories which are far from Nature. There are many areas of physics too: classical, relativity, quantum and other different kinds of alternative physics. Finally, there is the matter of "confirmations" of illogical theories by physical experiments, especially in quantum mechanical experiments. We discuss also the strengths and weaknesses of mathematics: creation of new notions and unification, and try to formulate problem of redefinition of the notions of mathematics and physics. In addition, we consider the necessity of the development of an "open physics project" with constant open discussion of physics' foundations and physical methodology. We argue that the initial notions are of space , substance, and time, and discuss a few open problems.

To see a World in a grain of sand,
And a Heaven in a wild flower,
Hold Infinity in the palm of your hand,
And Eternity in an hour.
-- **Auguries of Innocence**, William Blake.

1. Introduction

In a very candid book [1], Morris Kline explains the false opinion, that "Mathematics was regarded as the acme of exact reasoning, a body of truths in itself, and the truth about the design of nature." "It is now apparent that the concept of a universally accepted, infallible body of reasoning – the majestic mathematics of 1800 and the pride of man – is a grand illusion."[1, p. 5] This work was so candid that five years later, in 1985, he gave an apology in [2]; he tried to show that "mathematics has given us knowledge and mastery of major areas of our physical world", and "for many vital phenomena, mathematics provides the only knowledge we have. In fact, some sciences are made up solely of a collection of mathematical theories adorned with a few physical facts." This reversed opinion and many works by Frege, Russell, Brouwer, Hilbert, Cantor, Gödel lead us to call a spade a spade: mathematics does not have a logical foundation. There are four main sorts of approaches to mathematical foundations and many sub-approaches. The main approaches are: Logicism, Intuitionism, Formalism, and the Set-Theoretic ones. Those approaches differ from each other by differing systems of axioms and postulates, and have a similar goal of the construction of a consistent mathematics. In our opinion, they do not pay enough attention to discrepancies in the basic notions of infinity, infinitesimal, and zero. Here we consider such contradictions.

In [3] Eugene Wigner defines mathematics as "…mathematics is the science of skillful operation of concepts and rules invented just for this purpose. The principal emphasis is on the invention of concepts." Below we attempt to show the dangerousness of arbitrary invention, generation, and redefinition of notions in mathematics and physics. It is not enough to assert a property or the existence of some structure to avoid contradictions and illogicality.

The deep problems in foundations of mathematics and physics lead us to begin from the beginning, and to start the Open Physics Project. Below we discuss a few approaches to this project. In an epigraph, we see infinity and eternity as objects of poetry, but a shaky object should not be the object of science.

2. Infinity, Infinitesimal, Zero

The notions of infinity, infinitesimal, and zero have different ages, but their interior contradictions are well known. Sometimes those notions were completely rejected, but some times were accepted completely ignoring their discrepancies, and even worse – with the proclamation these as the epitome of the progress of mathematics (i.e. by Weyl – mathematics is the science about infinity). The importance of those notions cannot be overestimated, because we can find them in almost all mathematical and physical theory, and from the discrepancies of infinity, the infinitesimal, and the zero, there result inconsistencies in these theories. In short, we can refer to the contradiction of infinity by the term "finite infinity", the contradiction of the infinitesimal as "the notion which is equal and not equal of zero at the same time", and the contradiction of zero as "the declaration of the existence of the non-existent".

Mathematicians have noticed long ago the contradiction in the expression – "infinite number", because any number is a finite object, and consequently, infinity is not a number. The infinite sequence of steps and the infinitesimal can be seen in paradox due to Zeno of Elea, of Achilles and the tortoise, Dichotomy (ca. 490 BC – ca. 430 BC). If we will consider a finite number of steps of Achilles and tortoise, having finite distances between them, then Achilles will catch the tortoise in finite number of steps. But if we consider the division of this finite distance into an infinite number of parts, then we arrive at a contradiction: the steps become infinitesimal, but not zero, because an infinite sum of zeros equal zero, but an infinite sum of non-zero constant length steps equals an infinite distance, or we would accepted equivalence of part to whole in case of steps with variable length.

Let us consider the well known explicit redefinition of infinity by Cantor: he defines the number of elements of infinite set as omega or aleph-null, and operates with them as numbers. This generalization – redefinition of a non-number as number by Cantor, Hilbert refers to as a "mathematical paradise".

Solutions of the Dichotomy paradox often are expressed as a limit of infinite sum of inverse powers of two. If we accept this, then we accept an illogical result: on infinity this sum, half would equal to the whole. This contradiction we find in limit theory and mathematical analysis as the equivalence of the part to the whole. In set theory, the equivalence of a part of a set to the whole is used to give the definition of an infinite set [4]. It is completely illogical: from this we get the consequences – the part is more than itself, and the whole is less itself, and so we have lost the equivalency itself for the part and the whole (part equal to part, and whole equal to whole).

The notion of zero has three main meanings: 1) the physical – nothing, empty space, not existent something; 2) the geometrical – dimensionless point, which does not have any parts according to Euclid's Elements; 3) mathematical – digit, number. The physical meaning of zero conflicts with geometrical and mathematical ones, where objects of zero size claim to be existent objects. We can see in one the algebra's axiom three contradictions: a) declaration of the existence of an element zero; b) we can add zero to another number – summing using empty space; c) in the binary operator – addition, we can use with one operand, because zero is empty space. One objection is that after the postulation of existence of the zero-element, cases b) and c) became valid. We can point that zero is involved in the manipulation of expressions, and exactly for the "automatic" calculation of empty space characterizations. For this goal of calculation, they were defined and this concludes our discussion concerning operations involving zero.

Frege gives an interesting definition of zero in [5, §74]:

> "Since nothing falls under the concept "not identical with itself", I define nought as follows: 0 is the Number which belongs to the concept "not identical with itself". … All that can be demanded of a concept from the point of view of logic and with an eye to rigor of proof is only that the limits to its application should be sharp, that it should be determined, with regard to every object whether it falls under that concept or not. But this demand is completely satisfied by concepts which, like "not identical with itself", contain a contradiction; for of every object we know that it does not fall under any such concept."

We see, that for the sake of logic and rigor of proof Frege involves concepts having contradictions, and it defines an object, which does not exist, because "we know that it does not fall under any such concept." We can find a lot contradiction and illogicality in mathematics and physics, but we should not accept them, we should draw the right conclusion. Discrepancy of notions of infinity, infinitesimal, and zero leads to the inconsistent notions of irrational numbers, continuity, and geometrical objects. Applications of these notions do not prove their consistency, and they can be estimated as approximations.

3. Redefinition of Notion

In the introduction we already cited a definition of mathematics by Wigner. Poincare defines mathematics as the science, which gives the same name to different things. It is usual to find the strength of mathematics in unification, generalization, abstraction, and idealization, but here we can lose the quality of an object. It is well known that each physical notion has two aspects: quantity and quality, quantity is expressed numerically, and quality is expressed by dimensionality. To operate on physical notions, we should know theirs quality, for example, we can add physical quantities having the same dimension. But this is not enough: we can write the sum of densities, but we do not have any physical process to double density, consequently, this sum does not make sense. We can multiply values different quality, for example, mass and speed, and get value third kind of quality, in this case – impulse. Some mathematical structures are defined as consisting of one kind of physical quantity. Group theory defines a binary operation on just one kind of object for both operands, and the result of the operation is of the same type of physical quantity. From the physical point of view, physical groups are a very special case, but to generalize their occurrence, the study of group theory was invented, including exotic objects such as strings and brane. The well known physical redefinition is: $h = c = 1$, along with mention of an imaginary system of reference. This is absurd from the points of view of logic, mathematics, and physics, namely, the equivalence of very small value h having one type of dimensionality, and the very big value c with having a different dimensionality, and the dimensionless unite. We have to emphasize importance of dimensionality: it defines quality of physical quantity and binds it with reality. If mathematics works with dimensionless numbers, then it is up to physics to validate the meaning of its equations by using the dimensions of its physical values.

Now we consider very important overlooked aspect of the mathematical object: dimensionless numbers. Dimensionless numbers have qualities (properties) too. We have the whole numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9 – call these, if you wish, Mathematics' atomic table – they are unique, each of them equal to itself and between different ones there exists rigorous inequality, and their different appearances correspond to different meanings. The last of the properties is one of strings so that $1 > 0.999\ldots$, because right and left sides this inequality has different appearance. If we would accepted equality $1 = 0.999\ldots$, then we will accept illogicality that on infinity $9 = 10$, again – part equal to whole. The rest of all mathematics consists of expressions – bigger numbers then 9 are expressions, and the next quality of numbers – rational – are expressions, with uniqueness of expressions guaranteed by the uniqueness of the nine digits. It is easy to see that mathematics is the science of the manipulation of symbolic expressions.

The next numbers with a different quality are negative numbers, which for the first time were utilized in India to calculate of a money debt. It is clear that quality "negative" is a man-made for man concept, which does not exists in reality. We do not have positive and negative charges; we have one-kind of and second kind of charges, and use negative numbers to "automate" calculation of the direction of the interaction of charges. The generali-

zation of the sum of several the same operands brings to us multiplication. From this, multiplication by unity and negative numbers does not make sense. The next level of abstract generalization is the postulation of multiplication as a second kind of operation, simply different from summation. After that we get the postulation of multiplication by unity and negative numbers. We have to note, that this increasing of the level of abstraction leading to new notions of quality, and the mixing of the notions of different level of abstraction involve us in implicit contradictions even in the abstract world of mathematics and hide the inconsistency problems in physics.

The next level of abstraction is the root operation, and the next - complex numbers, but this amounts to "pipe dreams" as Roger Penrose points out [6].

The zero plays a special role, as a digit, it is used to calculate empty space in mathematical expressions, to hide opposite objects, and to give birth to not-zero structures and physical objects.

Geometry plays a particular role in mathematics and physics. Geometry is an abstract science with objects obtained from the abstraction and idealization of properties of solid bodies. Geometry has given birth to a lot of different kinds of abstract objects and made simple use of the following objects: irrational, rational, complex numbers and so on. The unification of the geometry of a solid body on space yields us more problems than advantages. The main problem is: space does not interact with any body or substance; this is the main property of space. Ivchenkov has shown [7] that Eddington's observations were within measurement error bounds. We still do not have any physical observations of any interaction with space. Geometry has within itself an inconsistency; namely, there is a point as an object without parts, as a geometrical zero.

All of the above show the importance of the investigation of the consistency of initial notions and concepts. The next step is the redefinition of all old notions to obtain confidence in the consistency of the new notions as in the cases of the old ones. Let's consider the redefinition of physical notions.

One of forms of the first postulate of special relativity theory (SRT) – namely, independency velocity of light from any inertial system of reference, that is, the express redefinition of relative value of the velocity of light, namely, light velocity as a single and absolute value. This looks like "redefinition without definition", because SRT does not define notion of "absolute velocity". The definition of 4-velocity gives us a unite four-velocity by definition, and the 4-acceleration which always orthogonal to 4-velocity by definition [8].

The well known Heisenberg uncertainty principle states that certain pairs of physical values, such as energy and time, and coordinate and impulse, cannot be simultaneously measured with arbitrarily high precision [9]. This principle has an inconsistency, because impulse is function of the coordinates, but the energy is function of time. If in this principle, there were involved different quantities like a quasi-impulse which was independent of the coordinates, and then we could have at least two incommensurable notions of impulse, and two incommensurable notions of energy.

The very interesting generalization-redefinition of an arbitrary translation of vector as parallel motion in non-Euclidian geometry is one we can investigate in [6, 8]. It is convenient to follow Penrose [6], where we can see diagrams of the surface geometry of a sphere. In spherical geometry, the analogue of the straight line in Euclidian geometry is the sphere's surface meridian. These two are very different objects: the radius of curvature of meridian is finite, but radius of curvature of straight line is infinite. They are similar because they express the shortest distance between two points, one in Euclidean two-space and the other in the two-space of the sphere's surface. Parallel motion in Euclidian geometry preserves the direction of a vector, and when a vector is translated along a close path, the vector will coincide with itself. According to Penrose and others, in non-Euclidian geometry, parallel translation of a tangent vector along a close path need not bring superposition of vector with itself. Let's consider some objections.

First of all, Penrose draws a tangent vector in 3-dimensional space, and maintains that on the sphere surface lies the tail of the vector. But the tangent vector of spherical surface must belong to the sphere's parts, and it is part of meridian, because meridian is a straight line of its sphere. Secondly, Penrose did not define equivalence of direction on sphere. Well known, that all meridians on sphere intersect each other in two points. This means that we can keep track of the direction of the tangent vector in attempting the parallel translation, along one meridian only. In this case, we get the superposition of the vector on itself. If we try to move a vector out of the starting meridian, we will have to move the vector to another meridian, which intersects first one, and this vector will not coincide with itself at the point of intersection of the two meridians, because we have changed its direction. In Euclidian geometry we get similar result if will change the direction of vector, and this translation is not parallel. On this arbitrary-parallel translation was built tensor analysis.

Thus, we have at least two problems: the consistency of initial notions, and the consistency of redefined notions. We can see hierarchical relations between notions, and mixing notions of different level leads to inconsistency. The hierarchy of notions can be used to build physical and mathematical concepts.

4. Open Physics Project

These problems with physics and mathematics are not news. It is easy to find many different kinds of open projects, including the physics domain. Here we attempt to discuss different approaches to different tasks.

The idea of the Open Physics Project had risen in 2006, at the time of reading Smolin's book [10]. Lee Smolin gives a convincing, classical definition of a closed community of contemporary physicists, in 7 items [10, p. 284]. Smolin sets forth the requirements of an open community too, in 6 items [10, pp. 301-302], with two main criteria: peer review, and "allegiance and continued adherence to the shared ethic". Between the lines of his book, we can see a third criterion: acceptance of Einstein's theory of relativity. Those criteria supplement the strong educational principle "Shut up and calculate", with the prohibition of discussing physical foundations, which are accepted by men who fall under the power of authority, but not logic. Under these conditions, an open community cannot be built. Consequently, the first task of the Open Physics Project (OPP) is the creation of an open community of physicists, with a new ethic, with its authority being

logic, and with constant open discussions of the foundations of physics and mathematics. The word "open" here should be understood more widely, than just public and free. In [11] we find the very interesting biographical fact about Michael Faraday: he calls himself as "nonmathematical philosopher", and hates it when somebody calls his as "physicist". From our point of view, this fact leads to next two open tasks: question "What is physics?" and the development of physical methodology to study Nature. The world known physicist – Michael Faraday – worked with physical objects; his methodology should be renewed in physical methodology. It appears that Maxwell stood against the way of giving birth to physics. He transferred physical methodology to mathematics, and the work with physical objects changes to manipulation by abstract objects.

Today we can find scientists attempting to prove the correctness of Maxwell's methodology by plausible sequences: correctness of math in application to physics, selection of axioms from the postulated correctness of Einstein's relativity theory, finding new ontology from the assumed correctness of abstract quantum theory, and so on. From our point of view, it is a completely illogical, non-science approach. We do not have any choice abut to begin from the beginning: create consistent hierarchy of physical and mathematical notions, and discover consistent correspondences between them. By way of illustration, consider one version of a hierarchy of physical notions.

First level has a small list of notions: Universe, Nature, and World. We do not need to resort to undefined notions to begin and then proceed; we already have notions which correspond to reality.

The next level includes substance, space, interaction, no interaction, dependency, and independency. The notion of matter we propose leave for the philosophical body-mind problem. We accept the independency of reality from our mind. The 3-dimesional substantial body moves in the 3-dimentional space. Substance does not interact with space. Two substantial bodies can or cannot interact with each other. That there are no interacting bodies or independent bodies or bodies which are interacting which can be ignored, this is the core of Galileo's principle and Newton's first law. Bodies can be independent because of the inverse square of distance law: interaction can disappear and emerge again – this is base of probalistic behavior of moving bodies. Substantial bodies may have internal structure or sublevels having the notions: substance, space, interactions (atom or molecular level).

The next level has the notions time, distance, velocity, energy, mass, and so on. It is important to note that time is the man-made notion to describe motion of body in space, and cannot be mixed with space and substance.

The next level may include emergent, system notions like temperature, entropy and so on, which bind with the internal structure of a body and do not make sense, for example, for one molecule.

The shining example of the mixing of notions of different levels of abstraction is proposed by Hawking to discover new spatial dimension by holography [12]. It looks like 2-dimensional photo plate generates 3-dimensional image, but in reality 3-dimensional micro object, photo plate, generates 3-dimensioanl macro image.

Now let's look at a few open problems which seem well understood because they have a mathematical description.

First of all, there is the problem of electricity and magnetism. To see the inconsistency of Maxwell's theory, it is enough read his works [11, 13, 14, 15]. In short, Maxwell defines two kind of electricity – static and kinetic, and ignores observable fact that kinetic electricity – current and displacement, - is electrostatic neutral. In the well known equation $\mathbf{D} = \varepsilon \mathbf{E}$, $\mathbf{E}$ is function of electrostatic charges, and $\mathbf{D}$, as electrical displacement, is electrostatic neutral until molecular level is reached, as is the current too. In the equation of electrical induction, a variable magnetic field generates a so called vortex electrical field which is incommensurable with the electrical fields of electrostatic charges because of the conservation of charge law. It follows from the properties of electrical charges that it is not adequate to take charges as the medium – the smallest group of the same kind of charges is a big electrical bomb. The list of inconsistencies of Maxwell theory may be continued.

The next important open problem is the problem of interactions. Physics can be divided on two different parts: physics with local interactions, and physics with interaction at distance. The local interactions was based on the mathematical notion of continuity, and, as shown above, is inconsistent. The strongly criticized notion of interaction at distance was set forth using several names: entanglement, nonlocality, correlations, and so on. On the continuity notion was based many notions such as vortex, ether, field, manifold and others. Here we propose to develop the notion of interaction at distant as an observable phenomenon, and to consider all kind of radiation, including light, as interaction at distance of different types too. The velocity of interaction outside a moving object can be estimated as exchange of energy between bodies at a distance. So, the problem "What is light ?" is open too.

The most important open problem is the development of physical methodology, including mathematics as physical tool, but not as a generator of an abstract world. Specially, this methodology must be reflexive (meta- prefix for math means "self", but for physics it means "beyond") in meaning in order to study physics itself by physical methods. We have to study experiments of Thomson, Kauffman, Milliken, and the Wilson camera using the Faraday's methodology. There remains a long list of interesting experiments involving problems that are still open.

Thus, in spite of the well known opinion about the all importance of mathematics, we may obtain progress by utilizing experimentation with physical objects. A science becomes a true science, including mathematics and physics, when it follows logic, particularly, logic of Nature. To do that, to build up physics using good physical methodology, we propose to develop the Open Physics Project.

5. Conclusion

We have the worst of cases: a double crisis – a crisis of mathematics and a crisis of physics. This crisis feed each other and keeps each other "alive". We should separate them – physical for physics, and mathematical for mathematics. Mathematics works with abstract and symbolic objects, and needs elimination of its illogicality and its contradictions in both its concepts and its notions. Physics should work with physical objects, and needs the

development of physical methodology from the study of Nature. Both of them should follow the logic of Nature, not follow any logic involving abstract theory. It seems useful to collect different approaches along these lines in an Open Physics Project.

Acknowledgments

I'd like to thank Dennis Allen and Greg Volk for discussions and preparation this paper. Special thanks to Eugene Shklyar and Sergey Arteha for discussion and constant support of my works.

References

[1] M. Kline, **Mathematics: The Lost of Certainty** (Oxford University Press, 1980).

[2] M. Kline, **Mathematics and the Search for Knowledge** (Oxford University Press, 1985).

[3] E. Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Science", *Comm. Pure & App. Math.* **13** (1): 1-14 (Feb 1960).

[4] A. D. Aleksandrov, A. N. Kolmogorov, M. A. Lavrent'ev, Eds., **Mathematics: Its Content, Methods, and Meaning** (Dover, 1999).

[5] G. Frege, **The Foundations of Arithmetic** (Northwestern University Press, 1996).

[6] R. Penrose, **The Road to Reality: A Complete Guide to the Laws of the Universe.** (Alfred A. Knopf, 2004).

[7] G. Ivchenkov, **The Most Important Confirmation of GRT or What Did Lord Eddington Measure in 1919?** (in Russian), http://www.elibrary-antidogma.narod.ru/bibliography/eddington.htm.

[8] L. Landau & E. Lifshitz, **The Classical Theory of Fields** (Butterworth-Heinemann, 1975).

[9] W. Heisenberg**, The Physical Principles of The Quantum Theory** (Dover, 1949).

[10] L. Smolin, **The Trouble with Physics. The Rise of String Theory, the Fall of a Science, and What Comes Next** (Houghton Mifflin, 2006).

[11] J. C. Maxwell, **The Elementary Treatise on Electricity**, 2nd Ed. (Dover, 2005).

[12] S. Hawking, **The Universe in a Nutshell** (Bantam Books, 2001)

[13] J. C. Maxwell, **A Dynamical Theory of the Electromagnetic Field** (Wipf and Stock Pub., 1996).

[14] J. C. Maxwell, **A Treatise on Electricity and Magnetism, Vol 1** (Dover, 1954).

[15] J. C. Maxwell, **A Treatise on Electricity and Magnetism, Vol 2** (Dover, 1954).