The Best Stellar Aberration Model

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The M/M/+M® Model's Bearing estimator, also called the Best Stellar Aberration Model, will be compared to James Bradley's 1729 Falling Rain model and to the stellar aberration models described in Einstein's 1905 Special Relativity Theory (SRT) and the 2010 Astronomical Almanac (AA). The 1729 Falling Rain model, the 1905 SRT model and the 2010 AA model predict aberration values that closely agree with Bradley's Constant of Aberration (approximately 20.5 arc seconds) for a distant celestial body at a zenith point as viewed through a telescope travelling at Earth's orbital velocity. The Best Model is an adaptation of the 1905 SRT stellar aberration model described in Einstein's 1905 Electrodynamics publication. The Best Model is based upon a kinematical and dynamical form similar to those described in SRT with aberration prediction capabilities not described in SRT. The Best Model's aberration predictions are closer to the 2010 Astronomical Almanac's estimates than the Falling Rain or 1905 SRT models' estimates when the object is located at a sidereal angle other than 0º or 180º. The zenith and sidereal angles will be defined for each of the four models. The Best Model predicts that the aberration of a distant object near a zenith point will be closer to zero than 20.5″. The prediction of a Constant of Aberration value as large as 20.5″ for a zenith star is irrational and is contradicted by empirical observations. This conclusion has profound implications concerning the precise geometry and relationships between the kinematical and dynamical forms that are the cause of stellar aberration according to Einstein's 1905 Electrodynamics publication.

1. Introduction

The three leading stellar aberration models predict that a star which is at a zenith point directly overhead will not be seen at a 90° angle to a telescope's velocity vector. These three leading models estimate that the telescope must be pointed at a point that is 20.5″ (approx.) less than 90°. This value was first derived by James Bradley in 1729. He called this value the Constant of Aberration and he based it upon his Falling Rain Theory.

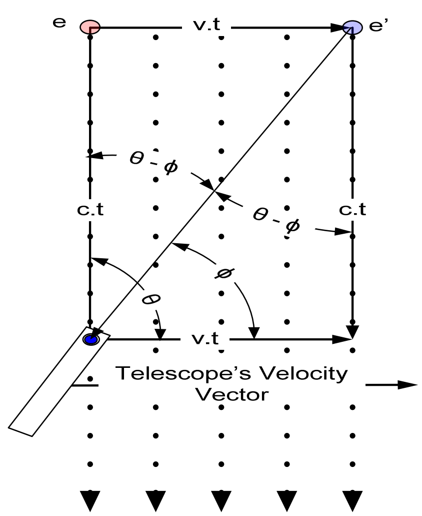

Fig. 1. Constant of Aberration from the Falling Rain Theory

Fig. 1 shows the geometry from which the Constant of Aberration was derived [1, 2]. Point *e* is the actual location of a star-that is perpendicular to the velocity vector. According to the Falling Rain Theory, point *e*′ is an image of the same star that will be seen by someone who is looking into a telescope that is moving at the Earth's orbital velocity. According to the Falling Rain Theory, the aberration value $\Phi - \theta = -\text{atan}(v/c) \approx -20.5''$ at Earth's orbital velocity v. The Best Stellar Aberration Model predicts that when a star is nearly perpendicular to the telescopes velocity vector its aberration will be closer to zero than -20.5″.

The Best Stellar Aberration Model is an integral part of the M/M/+M® Model Version 2 Release 0 (V2R0) software product that runs under Microsoft Excel®. The following section will compare the predictions of the Best Model against the predictions of the three leading stellar aberration models and will show that the three leading models erroneously assume that zenith stars closely approaching 90° with respects to the velocity vector will have a difference between the actual θ and observed Φ declination angles that will approach the value $\text{atan}(v/c)$.

2. Comparisons of Stellar Aberration Models

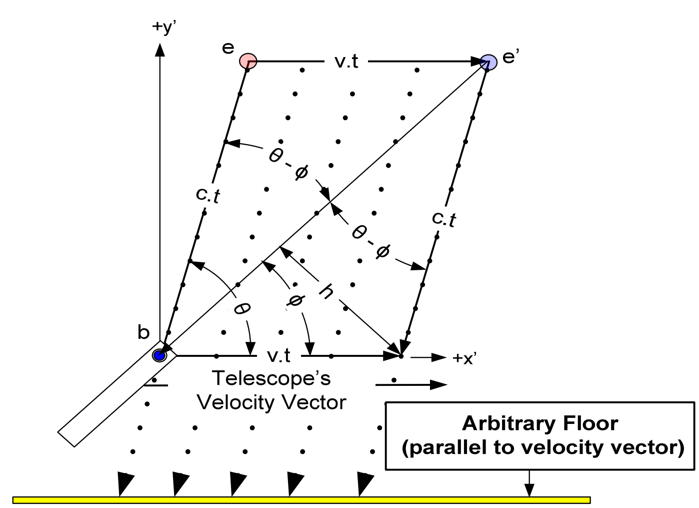

Fig. 2. The Falling Rain Model

The three leading stellar aberration estimators are: the Falling Rain model developed by James Bradley in 1729 [1, 2], the stellar aberration model described in Einstein's 1905 Special Relativity Theory (SRT) [3], and the stellar aberration model described in the 2010 Astronomical Almanac (AA) [4]. The derivation of the equations used by these three models as well as the Best Model will be described and the predictions of the Best Model will be compared against the predictions of the three leading models.

Fig. 2 describes the geometry of the Falling Rain model. The equation that estimates aberration in the Falling Rain model for any declination θ in the $x'y'$ plane is derived from Fig. 2:

$$\sin\Phi = \frac{h}{vt} \qquad \therefore h = vt\sin\Phi \tag{1}$$

$$\sin(\theta-\Phi) = \frac{h}{ct} = \frac{vt\sin\Phi}{ct} = \frac{v}{c}\sin\Phi \tag{2}$$

Finally

$$\begin{aligned} \theta-\Phi &= \operatorname{asin}\left(\frac{v}{c}\sin\Phi\right) \\ \Phi-\theta &= -\operatorname{asin}\left(\frac{v}{c}\sin\Phi\right) \end{aligned} \tag{3}$$

According to the Constant of Aberration (Fig. 1):
When $\theta = 90^{\mathrm{o}}$, v = 29.93 km/sec, c = 299792.458 km/sec

$$\begin{aligned} \Phi-\theta &= -\operatorname{atan}\left(\frac{v}{c}\right) = -\operatorname{atan}\left(\frac{29.93}{299792.458}\right) \\ &= -0.005720166° = -20.592598198'' \\ \Phi &= 90° - 0.005720166° = 89.994279834° \end{aligned} \tag{4}$$

According to the Falling Rain Theory (Fig. 2):

$$\begin{aligned} \Phi-\theta &= -\operatorname{asin}\left(\frac{v}{c}\sin\Phi\right) = -\operatorname{asin}\left(\frac{29.93}{299792.458}\cdot\sin 90°\right) \\ &= -0.005720166° = -20.592598198'' \end{aligned} \tag{5}$$

Thus, both Eqs. (4) and (5) return the Constant of Aberration value for a zenith star with $\theta = 90^{\mathrm{o}}$.

Fig. 2 illustrates the vector configuration during event 1 when the star's vectors from points e and e' with declinations θ and Φ respectively are in the $x'y'$ plane that contains the telescope's velocity vector and is perpendicular to the arbitrary floor shown in Fig. 2. This $x'y'$ plane will be called the 90º Boundary Plane.

When points e and e' are not in the $x'y'$ plane, the Boundary Plane will hinge on the velocity vector and will be tilted with a declination angle β less than 90º with respect to the floor (Fig. 3). At times other than during event 1, the two star vectors lie in a β Boundary Plane, and the Falling Rain model's equation becomes:

$$\Phi-\theta = -\operatorname{asin}\left(\frac{v}{c}\sin\Phi\right)\cdot\sin\beta\,. \tag{6}$$

2.1. Near Zenith Star Example (Fig. 2)

When $\beta = 90°$, $\Phi = 89.994277834^{\mathrm{o}}$, $v = 29.93$ km/sec, c = 299792.458 km/sec, then $\Phi-\theta$ = -0.005720166º = -20.592598198″ and $\theta = 89.999998^{\mathrm{o}}$.

2.2. Near Zenith Star Example when β<90° (Fig. 3)

When $\Phi = 89.999698523^{\mathrm{o}}$ and $\beta = 89.9998003°$, then $\Phi-\theta$ = -0.005720166º = -20.592598198″. Therefore, $\theta = \Phi + 0.005720166^{\mathrm{o}}$ = 90.005418689º.

Fig. 4 describes the geometry of the 2010 Astronomical Almanac Model also called the 2010 AA Model. The equation that estimates aberration in the 2010 AA Model maps to the geometry in fig. 4 and is: $\delta'-\delta = \left(-\frac{v}{c}\sin\delta\right)\cdot\cos\alpha$. Where α is the angle between the velocity vector and the vertical projection of the vector from point e onto the celestial plane (Fig. 3).

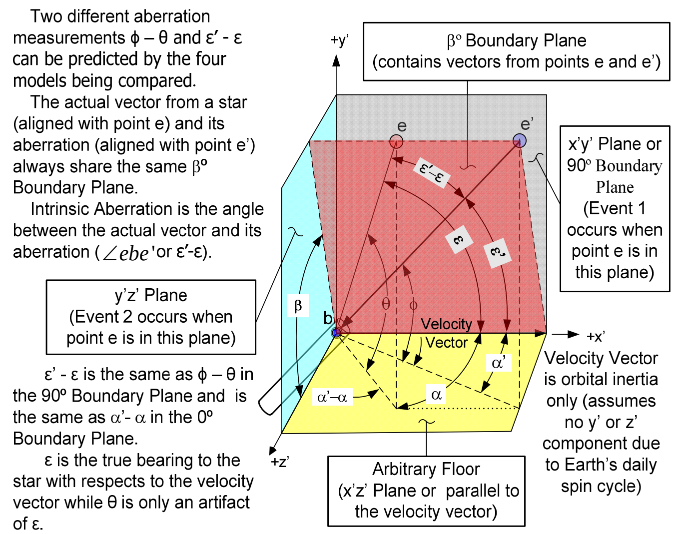

Fig. 3. Aberration Metrics Schematic Diagram

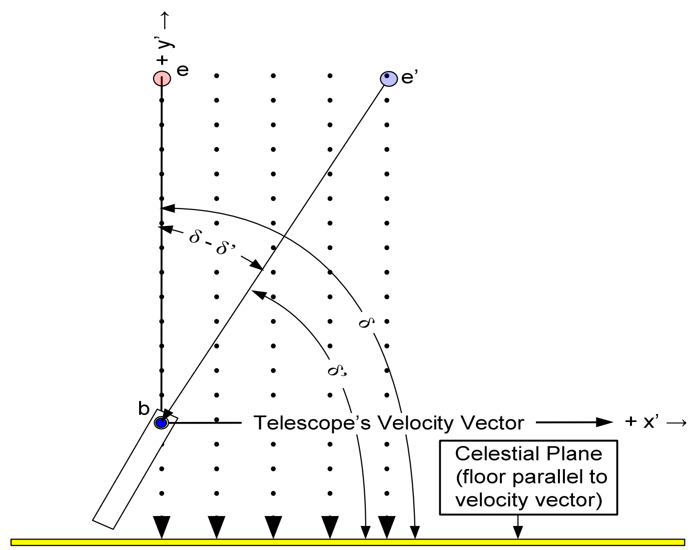

Fig. 4. 2010 Astronomical Almanac Model

The dynamic angle α will be called the sidereal angle. To facilitate comparison the Ecliptic Plane will be used as the floor for measuring declinations; therefore, δ becomes θ and δ' becomes Φ. Due to this translation to a different floor, zenith stars will be on ecliptic poles instead of celestial poles and the 2010 AA aberration equation becomes: $\Phi-\theta = \left(-\frac{v}{c}\sin\theta\right)\cdot\cos\alpha$.

2.3. Zenith Star Example (Fig. 4)

During event 1, when $\theta = 90^{\mathrm{o}}$, $\alpha = 0^{\mathrm{o}}$, v = 29.93 km/sec, c = 299792.458 km/sec, then $\Phi-\theta = \left(-\frac{29.93}{299792.458}\sin(90°)\right)\cdot\cos(0°)$

= -0.005720166° = -20.592598267″. Therefore: $\Phi = \theta = 90^{\circ} -$ 0.005720166° = 89.994279834°.

2.4. Near Zenith Star Example when α = 0° (Fig. 4)

When $\theta = 89.999998000^{\circ}$, then $\Phi = \theta =$ -0.005720166° = -20.592598267″.

2.5. Near Zenith Star Example when α > 0° (Fig. 3)

When $\theta = 89.999998000^{\circ}$ and $\alpha = 86.998931578^{\circ}$ then $\Phi = \theta =$ -0.000299477° = -1.078116795″. Therefore, $\Phi = 89.999998000^{\circ}$ - 0.000299477° = 89.999698523° which is the same as the Φ value in the near zenith star example for the Falling Rain model when $\Phi = 89.999698523^{\circ}$, $\beta = 89.9998003^{\circ}$ and $\theta = 90.005418689^{\circ}$.

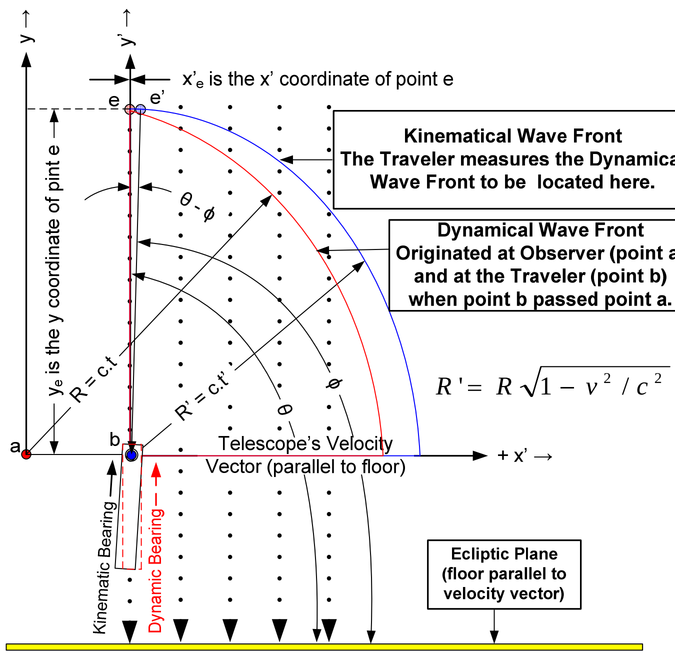

Fig. 5. The Best Model

Fig. 5 describes the geometry of the M/M/+M® Bearing estimator also called the Best Stellar Aberration Model. The equation that estimates aberration in the Best Model and maps to the geometry in Fig. 5 is:

$$\tan\theta = y_e / x'_e \tag{7}$$

$$\theta = \arctan\left(\frac{ct'\sin\phi}{\sqrt{c^2t^2 - c^2t'^2\sin^2\phi} - vt}\right) \tag{8}$$

$$\phi - \theta = \phi - \arctan\left(\frac{ct'\sin\phi}{\sqrt{c^2t^2 - c^2t'^2\sin^2\phi} - vt}\right) \tag{9}$$

2.6. Zenith Star Example during Event 1 (Fig. 5)

When $\Phi = 90^{\circ}$, v = 29.93 km/sec, c = 299792.458 km/sec, t'=1 sec′, t=1.000000005 then $\theta = 90^{\circ}$ and $\Phi = \theta = 90^{\circ}$ - 90° = 0°.

2.7. Near Zenith Star Example during Event 1 (Fig. 5)

When $\Phi = 89.999848723^{\circ}$ then $\theta = 89.999998^{\circ}$ and $\Phi = \theta =$ -0.000149277° = -0.537397142″.

The Best Model predicts that there is no aberration for a star located at a zenith point. A star at a zenith point will be seen in the same direction as the point where the dynamical wave front (red circular section with Radius R) and the kinematical wave front (blue circular section with Radius $R' = R\sqrt{1-\left(v^2/c^2\right)}$) intersect. Therefore, the points e and e′ will be aligned with the intersection point and Φ will be equal to θ during event 2 for a zenith star $\Phi = \theta = 90°$ and the vector from point e′ as well as the vector from point e will be in the $y'z'$ plane (Fig. 3).

The 1905 SRT Model is similar to the Best Model shown in fig. 5 except that the kinematical wave front is described as an ellipsoid of revolution with the axes $R\sqrt{1-\left(v^2/c^2\right)}$, R, R. Therefore, the y and z dimensions of a sphere at rest do not appear to be changed by motion while the x dimension appears to shorten by the ratio 1: $\sqrt{1-\left(v^2/c^2\right)}$.

The 1905 SRT Model assumes the Observer at point a is in motion in the Traveler's inertial frame which is reciprocal to the Best Model which assumes the Traveler at point b is in motion in the Observer's inertial frame.

The 1905 SRT Model is unlike the two other leading models in that the moving Observer assumption results in a positive (≈ +20.5″) aberration estimate for zenith stars and near zenith stars while the 2010 AA Model and the Falling Rain model assume a negative (≈ -20.5″) value. A. Einstein [3] provides an aberration equation derived from Doppler's principle of aberration:

$$\cos\phi' = \frac{\cos\phi - v/c}{1-\left(v/c\right)\cos\phi} \tag{10}$$

$$\phi' = \arccos\left(\frac{\cos\phi - v/c}{1-\left(v/c\right)\cos\phi}\right) \tag{11}$$

$$\phi' - \phi = \arccos\left(\frac{\cos\phi - v/c}{1-\left(v/c\right)\cos\phi}\right) - \phi \tag{12}$$

2.8. Zenith Star Example - 1905 SRT Model when α = 0°

When Φ = 90°, v =29.93 km/sec and c = 299792.458 km/sec, then Φ' – Φ = 90.005720166° - 90° = 005720166° = +20.592598301″ which is close to the Constant of Aberration value -20.592598198″ except with the wrong sign.

The 1905 SRT aberration equation assumes the Observer is traveling in the Traveler's inertial frame (assumes reciprocity); consequently, it only yields positive ($\Phi' > \Phi$) aberration estimates. Accordingly, the 1905 SRT Model cannot be compared with the 0 or negative ($\theta \geq \Phi$) aberration estimates of other models. Also, the 1905 SRT Model only estimates aberration when $\alpha = 0°$ (only during event 1).

The exact relationship between the kinematical ellipsoid of revolution wave front and the dynamical spherical wave front is not lucidly described by A. Einstein [3]. Therefore, it is not clear how his aberration equation can be derived from the geometric relationships between the kinematical and dynamical wave fronts as the Best equation is derived from these geometric relationships as shown in Fig. 5.

Table 1 is an output of a Best Model simulation with estimates of aberration metrics from the four stellar aberration models. A value for θ and a target value for one additional variable (α, α', $\alpha' - \alpha$, $\Phi - \theta$, β, ε, etc.) are required inputs for a simulation. The models' estimates that come closest to the target will be in Table 1 at the end of the simulation.

Table 1 contains the results from the simulation: WHAT are the aberration metrics for a star IF it has a declination $\theta =$ 89.999998° and a sidereal target of $\alpha \approx 87°$? The results of this simulation show that the 2010 AA Model's estimate of aberration $\Phi = \theta =$ -1.078116795″. Table 1 also shows that Best Model's estimate of $\Phi = \theta =$ -0.117939034″ that is close to the 2010 AA estimate while the Falling Rain model's estimate is close to the Constant of Aberration (about -20.5″). Since the 2010 AA Model's estimates of aberration for near zenith stars when α is in the range 87° to 90° closely agree with actual telescopic observations, the Falling Rain estimates are extremely inaccurate for near zenith star when α approaches 90°.

The 2010 AA Model's estimate for aberration $\Phi - \theta$ when $\theta =$ 90° and $\alpha = 0°$ in the zenith star example (Fig. 4) was: $\Phi - \theta =$ -20.592598267″. Table 1 shows that another star only 0.0075″ away at $\theta = 89.999998°$ and $\alpha \approx 87°$ will have aberration $\Phi - \theta =$ -1.078116795″ according to the 2010 AA Model's estimates.

Table 1
Comparison of Falling Rain, 2010 AA, 1905 SRT & *Best* Models

θ (degrees) = 89.999998000	v (km/sec.) = 29.930000	c (km/sec.) = 299792.458	α (degrees) = 86.998931579
89.999998000	29.93	299792.458	86.998931579
β (degrees) = 89.999998003	ε (degrees) = 89.999999895	ε' (degrees) = 89.999965396	α' (degrees) = 3.303271324
Constant of Aberration = -atan(v/c) = -0.005720166 degrees	-20.592598198 arc sec	θ (degrees) = 90.000000000	ϕ (degrees) = 89.994279834
Falling Rain Model			
ϕ - θ = -asin((v/c).sin(ϕ)).sin(β) = -0.005720166 degrees	-20.592598301 arc sec	θ (degrees) = 90.005418689	ϕ (degrees) = 89.999698523
		ε'-ε (arc sec) =	-20.592598301
2010 AA Model			
ϕ - θ = (- v/c).sin(θ).cos(α) = -0.000299477 degrees	-1.078116795 arc sec	θ (degrees) = 89.999998000	ϕ (degrees) = 89.999698523
		ε'-ε (arc sec) =	-1.078116795
Best Model			
θ = (atan(((t'.c).sin(ϕ))/(((c.t)^2 - ((t'.(t'.c)).sin(ϕ)^2)$^{0.5}$-(t.v))))			
θ = 89.999998000		θ (degrees) = 89.999998000	ϕ (degrees) = 89.999848723
Aberration = ϕ - θ = -0.000032761 degrees	-0.117939034 arc sec	ε'-ε (arc sec) =	-0.124198740
1905 SRT Model			
ϕ' =acos(cos(ϕ)-(v/c))/(1-(cos(ϕ).(v/c))) ϕ' = Undefined		ϕ (degrees) 89.999998000	ϕ' (degrees) = Undefined
Aberration = ϕ' - ϕ = ε' - ε = Undefined degrees	Undefined arc sec	The 1905 SRT Model does not define aberration for sidereal angles other than alpha = 0	

This large difference (≈19.5″) in estimated aberration between two stars only 0.0075″ apart becomes even larger as θ and α approach 90° and the two stars appear to come together. When stars become true zenith stars directly above with $\theta = 90°$ the 2010 AA Model estimates $\Phi - \theta =$ -20.592598267″ when $\alpha = 0°$. Three months later when $\alpha = 90°$ they are still true zenith stars directly above with $\theta = 90°$, but the 2010 AA Model estimates that Φ - θ = 0″. This is a contradiction because a zenith star with $\alpha = 0°$ cannot be differentiated from a zenith star with $\alpha = 90°$ since one sidereal angle cannot be distinguished from another for a zenith star. Therefore, its aberration must be 0″ every day of a year. The source of this contradiction is the invalid assumption of a Constant of Aberration value near -20.59″ for a zenith star.

The Best Model estimates that when $\theta = 89.999998°$, then $\Phi = \theta = 0''$ during event 1 and $\Phi = \theta =$ -0.117939034″ near the end of the three month cycle as shown in Table 1. Also, this change becomes smaller as θ approaches 90° instead of larger. The Best Model estimates that there is no change in aberration for a true zenith star (when θ reaches 90°) because aberration for a true zenith star is 0° at all times during a year.

Given the founding premise that a change in the magnitude v or angle ε between a telescope's velocity vector and the vector from the star through point e is the cause of a change in aberration as assumed by all four aberration models, then the angle between the star vector from point e and the vector from point e' denoted as $\angle ebe'$ or $\varepsilon' - \varepsilon$ is a true measure of aberration that changes only in response to a change in v or ε. Therefore, $\varepsilon' - \varepsilon$ is the intrinsic aberration measurement for all stars that depends only upon v and ε. The 2010 AA Model can not accurately estimate intrinsic aberration $\varepsilon' - \varepsilon$, and cannot accurately estimate declination aberration $\Phi - \theta$, since these calculations depend upon the value of v, θ and α instead of v and ε. The Best Model does not contradict the founding premise because its estimates for $\varepsilon' - \varepsilon$ are the same when v and the target value for ε are the same regardless of the value of θ.

3. Conclusion

The M/M/+M® V2 Model's Bearing estimator (or Best Model) is the most accurate estimator of stellar aberration as viewed from a platform traveling between stars or in orbit around a star at any velocity as well as for estimating aberration as seen from a platform orbiting the Sun at Earth's Orbital Velocity. The Best Model is an improved SRT Model with the capability of estimating aberration values as appropriate for star vectors at any declination (θ or Φ) and at any sidereal angle (α).

The comparisons of the estimates of the Best Model against those of the three leading stellar aberration models have show that the three leading models erroneously assume that stars closely approaching 90° with respects to the velocity vector will have a difference between the actual (θ) and observed (Φ) declination angles approaching the value $\pm \arctan v/c$. This assumption has resulted in the development of models with large errors at Earth's orbital velocity that become increasingly inaccurate for estimating aberration from platforms travelling outside Earth's atmosphere at velocities greater than Earth's orbital velocity.

This conclusion has important implications concerning the geometry of the kinematical and dynamical forms that are the cause of stellar aberration as well as philosophical and scientific implications regarding the nature of space-time.

References

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[3] A. Einstein, "On the Electrodynamics of Moving Bodies" (1905), in **The Principle of Relativity**, pp. 35-65 (Dover, NY, 1923).

[4] **Astronomical Almanac**, p. B28 (U.S. Government Printing Office, Washington, DC, 2010).