Mass Changes and Potential-Energy Changes Unified

In conventional SRT, mass is said to increase with speed according to the factor γ in the equation $m = \gamma \times m_o$. Also, the energy of m is expressed $E = mc^2$. It is customary to identify E in this latter equation as the combination of rest energy and kinetic energy. This note examines the implication of assuming E also includes a particle's potential energy, and the additional implication of 'potential mass' in all $m = E/c^2$. The Bohr Atom Theory is used for a test.

Energy Storage Between Two Charges

The force f_{12} between two charges can be expressed as the rate of change of energy with respect to distance:

$$f_{12} = \frac{dE_{12}}{dr} \tag{1}$$

Thus, Coulomb's Law may be considered as a simple ordinary differential equation of the first order [1]. For two charges of magnitude e its solution is:

$$E_{12} = \pm ke^2 \int_{\infty}^{r} \frac{dr}{r^2} = \pm \frac{ke^2}{r} + C \tag{2}$$

where $k = 1/4\pi\varepsilon_o$, r is the distance between the two charges, and C is a constant.

It is customary to set $C = 0$ and deal only with relative changes in E_{12}. This note examines the implication of a different choice for C. Conventional physics takes the rest mass of an electron or positron as constant and separate from the electric field. By contrast, this author chooses instead to relate the mass to the field through the parameter C. The proposed relationship is different for like versus unlike charges.

For *unlike* charges (say an electron and a positron, which have equal charge magnitude e but opposite sign), this author assumes $C_{(unlike)} = 2m_ec^2$. This assumption sets the potential mass of the electric field equal to the isolated rest masses of the two charged particles, in effect, making them "one and the same".

To analyze *like* charges (say two electrons), consider a scenario where a positive and a negative charge, both of magnitude e, are bound closely together, and relate to a third charge, also of charge e, at some distance. If the 'bound charges' were brought from a near infinite to a very near zero distance with respect to the third charge, the *unlike* charge relationship would lose an amount of energy equal to $2m_ec^2$ while the *like* charge relationship would gain $2m_ec^2$ of energy. This is because at any distance the bound-charge system experiences no electrical force with the third charge, and thus no change of total energy. The energies of each type of relationship (*like and unlike*) compliment each other. So at near infinite distance, in the *like* charge relationship the energy must be $E_{ee} = ke^2/r \rightarrow 0$, and then $C_{like} = 0$.

Continuing the development of formulae for the unlike-charge electron-positron system and using the above assumption:

$$E_{ep} = 2m_ec^2 - \frac{ke^2}{r} \tag{3}$$

The distance r obtained by setting $E_{ep} = 0$ in (3) is:

$$r = \frac{ke^2}{2m_e c^2}$$

The author represents this distance r by the symbol R_k. It is one-half times the commonly known electron radius, $r_e = ke^2/ m_e c^2$:

$$R_k \equiv \frac{ke^2}{2m_e c^2} \qquad (4)$$

After all the values are substituted in (4), $R_k = 1.4089697\text{x}10^{-15}$ m.

Also, the author defines the symnbol E_k to be C for *unlike* charge relationships:

$$E_k \equiv 2m_e c^2 \qquad (5)$$

After values are substituted into (5), $E_k = 1.6374529\text{x}10^{-13}$ J. Substituting (4) and (5) into Coulomb's Law for charges of magnitude e, we have force:

$$f_{12} = \frac{\pm E_k R_k}{r^2} \qquad (6)$$

and energy (3):

$$E_{ep} = E_k \left(1 - \frac{R_k}{r}\right) \qquad (7)$$

Our first observation of (7) is that energy E_{ep} is positive for all $r \geq R_k$. Also, the corresponding E_{ee} would be positive for all r. The author defines the energy, E_{ep} to be zero for $r < R_k$ (negative energy does not exist, except mathematically). Eq. (7) describes the stored potential energy in the electron-positron system. Dividing (7) by $2c^2$, the potential mass of one of the particles in the electron-positron system would then becomes:

$$m_o = m_e \left(1 - \frac{R_k}{r}\right) \qquad (8)$$

Eq. (8) shows that potential mass could vary as a function of r.

Table 1 collects the formulae thus far. These formulae are presented first so that new constants, E_k and R_k, could be used in the development below for support of the potential mass variation concept. For example, if the charged particle has a mass greater than m_e, like a proton, then its mass is increased or decreased according to $m_p - m_e R_k/r$.

Table 1. Static force, energy, and mass formulae for two particles with charge magnitude *e*.

relationship	force f_{12}	energy E_{12}	mass particle m_o
l ike	$\frac{-E_k R_k}{r^2}$	$E_k \frac{R_k}{r}$	$m_e \frac{R_k}{r}$ *
unlike ($r \geq R_k$)	$\frac{E_k R_k}{r^2}$	$E_k - E_k \frac{R_k}{r}$	$-m_e \frac{R_k}{r}$ *

* Increase or decrease in quantity from whatever it starts with at very large r.

Bohr Theory as Support for Mass Variation Concept

A computer model of the Bohr Atom was written using the formulae of Table 1, Bohr's assumption that the *total* angular momentum is nh, the formula for centrifugal force, relativity, and some Dirac factors (very minor modification to Bohr Theory). Table 2 gives the Bohr-atom computer-model results for light spectral frequency and mass variations between energy states $n = 1$ and $n = 20$. The measured wavelength is 914.039 and the unit of mass is the kg.

Table 2. Bohr-atom computer-model results.

Assumption	Computed Å	%error	'Mass' of photon (kg)	Mass loss of atom (kg)
Mass constant	914.024	0.0016	$2.418164855 \times 10^{-35}$	$(2.418358396 \times 10^{-35})$
Mass varies	914.0484	0.0010	$2.418100381 \times 10^{-35}$	$2.418100396 \times 10^{-35}$

There is little difference in the computed light wavelength between energy states $n = 1$ and $n = 20$ whether one assumes that the mass varies (due to potential energy changes), or that it is constant. However, Table 2 shows that in Bohr Atom Theory, the mass-constant concept is inconsistent with mass and energy conservation: the total mass of the atom actually increases when the photon is emitted. By constrast, the concept of mass variation due to potential energy changes is consistent with energy-mass conservation; i.e. the mass loss of the atom is equal to the mass-energy equivalent of the emitted photon. Thus, this analysis supports the mass-variation concept.

As the electron of the hydrogen atom falls from an excited state to a lower energy state, its potential mass is converted to kinetic mass. It has a higher speed. This potential mass loss is roughly equal to the kinetic mass it gains so the total mass of the electron remains nearly constant. The energy for the emitted light photon comes mainly from the potential mass loss of the proton. Also, we see in this analysis that potential mass of the electron can be augmented with kinetic mass.

Conclusions

1. The concept of mass variation with potential energy variation is not inconsistent with energy-mass conservation in Bohr Atom theory.
2. The concept of there being stored energy-mass between *unlike* charged particles at distances greater than R_k, including distances approaching infinity, is supported.
3. The static charge formulae of Table 1 are useful.

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References

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