# Division in the Veritable Number System

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The veritable number system is more versatile with respect to division than the real number system.

## 1. Introduction

My new book, **The Nature of Negative Numbers**, [1] is the fullest presentation to date of the veritable number system, a system which directly solves the mystery of $\sqrt{-1}$ . This article is not a reiteration of that, but goes deeper into the character of the division of veritable numbers than the book did.

Those who already have this book will find this article to be a good supplement; those who have not read this book may find herein a reason to obtain a copy.

In the real number system, the division of two negative numbers is a positive number, as is the multiplication of two negative numbers. Expressed in extreme symbolism, $-/-=-\cdot-$ . To get the full body of real signs from this equation, both sides would need to be multiplied by $-$, producing $-\cdot-/-=-\cdot-\cdot-$ . Dividing $-/-$ on the left and multiplying $-\cdot-$ on the right would yield $-+$ . Completing the real sign with $+-$ is easy.

In the veritable system, the division of two negative numbers is a negative number, as is the product of two negative numbers.

Veritable number system operations with positive and negative signs are separate and distinct from the arithmetical operations of addition, subtraction, multiplication, division, ratio, and roots. In this system, the arithmetical operation of addition is symbolized by the ampersand, "&"; the arithmetical operation of subtraction by " ~ "; multiplication and division, and ratios are represented in the same way as can be found in the real system; roots, by the radical symbol (√) only. The positive and negative veritable signs are symbolized by "+" and "– ", respectively; they represent direction without any alloy compounded with addition, subtraction, or the other standard operations of arithmetic. The separation of the fundamental spatial attribute of direction from combination and reduction provides the veritable system with many advantages over the real system. This is discussed in **The Nature of Negative Numbers**.

The present paper examines an aspect just touched upon in that book. This is whether the division of the pure arithmetical symbols, & or ~ , would in all cases operate in the same way as the impure symbols found in the real system.

Before that can be undertaken, a further, but inextricably related use of the symbols, & and ~ must be discussed. In the example, (-2) & (-2) = -4, the combinatory & is employed strictly as a symbol for the operation of addition; but when the ampersand is placed just in front of a number without a break, as in &(-2), it signifies a *surplus,* in this instance, that of negative two. (-2). Similarly, in the example, (-2) ~ (-2) = ***0***, ~ means just subtraction and nothing else besides. It is separated from both the minuend (-2) and the subtrahend (2) by a space. But when there is no separation between it and a number, as in ~(-2), it signifies a *deficit* or deficiency.

The veritable deficit and the veritable surplus play a great role in the veritable number system; but here their use is restricted to the operations of division and ratio – operations which they have in common with the real number system.

In **The Nature of Negative Numbers**, the following veritable positive equation is considered: $(+dy)/\sim(+d\theta)=(+\sin\theta)$ . The + sign in each terms indicates the direction; for reasons that cannot be discussed here, there can be no standard equations involving mixed use of signs in the veritable system. (Throughout the rest of this article, all signs will be positive; they could just as well have been negative. Whether a sign is negative or positive makes no difference for what is under discussion here.)

Returning then to the problem before us, multiplying both sides by ~ or ~ $\cdot$ (+1) produces $\sim(+dy)/\sim(+d\theta)=\sim(+\sin\theta)$ .

The question is asked: Does the division of subtraction into subtraction produce $\&\left[(+dy)/(+d\theta)\right]$, a veritable surplus, as the comparable division of a negative sign by a negative sign would yield a positive in the real system? If it is worked strictly in analogy to - / - in the real system, it would have to mean that a deficit divided by a deficit is really a surplus.

But not in the veritable system. Since the right side of the equation, namely $\sim(+\sin\theta)$ is a subtraction, a veritable deficit, it would be a contradiction if the left side were an addition. In the veritable system, adding something can never be the same as subtracting something, nor vice versa. All that is meant is that the ratio of two fluxions, the "derivative," in other words, is simply a subtraction. Stated mathematically, $(+dy)/(+d\theta)=\sim(+\sin\theta)$ [2]. The unadorned expression on the left is equal to, in fact *is* a subtraction, i.e., is a deficit.

There are three neutralities known to exist in the veritable system —the neutrality of the zero in the midst of the number line [3], the neutrality of the absolute number with respect to sign, and the neutrality of a quantity that is neither in excess nor in deficiency, i.e., a *normality*.

The ratio on the left is expressed as a normality; this is what the division, ~ / ~ , is equal to, not &. The expression on the right, $\sim(+\sin\theta)$ shows that it is actually a deficit.

That is as far as the book goes. Let us go further.

## 2. Veritable Divisions and Ratios

Would the result be the same if, instead of being asked what is dy divided by dθ, both sides of the equation were pure numbers, as in ~(+10)/~(+2) = ~(+5)? If we proceeded just as before, ~(+10)/~(+2) = (+10)/(+2). This would mean that the left side, which is the same as a normal would be equal to a deficit. or (+10) /(+ 2) = ~(+5). This would mean that ~(+5) · (+2) = (+10), a plain impossibility. There is a difference between *receiving the formal instruction* to find the ratio between two fluxions or a derivative, and *simply* finding the quotient by dividing by pure numbers.

Let us use the symbols, (+u), (+v), (+s), and (+t) to stand as variables for any positive veritable number, rational or irrational. It follows that ~(+u)/~(+v) = ~(+w) = (+u)/(+v) is not generally true.

Consider the opposite situation: ~(+u)/(+v) = (+w). Let us work with that. What this means is that a positive numerator in deficit is divided by a normal positive denominator, yielding another normal positive quantity. Multiplying both sides by [(+1)/~(+1)] makes ~(+u)/~(+v) = (+w)/~(+1).

Is this possible? To state it differently, can the normality, (+u) /(+v), ever be the same as (+w)/~(+1) ? Invariably, (+w) = (+w) /+1. It may be self-consistent to argue that (+1) times ~(+1) = ~(+1) is thinkable as a denominator in the abstract. But can it exist?

*It can, indeed*. If we consider concrete units, the fraction (+w)/ ~(+1) would mean that that whatever number of units +w was, it had to be w times as great as a unit in deficit. *The left side of the equation would be the same as that comparison*. What is the name of such a comparison in contrast to a division. That name is known everywhere. It is a *ratio*. Both sides together are in *proportion*.

(+w)/~(+1) differs from (+w) or (+w)/(+1) in that the latter is not restricted to being a ratio; it can be a full division. It also differs from ~(+w) or ~(+w)/(+1), the subject of the first problem. It would not be valid, however, as just division.

In the real system with its impure signs, w /-1 would have to be –w. It lacks the versatility of the veritable system.

Turning to the previous problem, ~(+u)/~(+v) = ~(+w) = (+u) /(+v) would be possible, if all that was asked was the proportions among these positive numbers.

What about ~(+u)/~(+v) = (+w)? Is this possible? If a deficiency is divided by a deficiency, what it would be emptied of is just that *lack*. Look at it this way: if you are owed $10 and then you divide that amount by an additional $2 you are owed, the quotient is just 5; the first debt is five times as great as the second. Dividing one deficit by another deficit does not result in a surplus. It follows that the plain, unadorned, normal (+w) would in that case be the answer.

The reason for the possible confusion is that in the real system, dividing a negative number by a negative number produces a positive number. But the real system is not pure. In the veritable system, arithmetical calculation is separated from the operation of the signs. The real system recognizes positive, negative, and absolute numbers. So does the veritable system. But with respect to computational operations, the veritable system recognizes signed quantities like (+w) and (-w) which are neither deficits nor surpluses, but simply normalities, sufficiencies, fulfillments, plenitudes, or the like. Unlike absolute numbers, these numbers have a sign.

This equation is a full normality. It is normal on both sides. In the first case, a normal ratio was shown to be the same as a deficit on the right side of the equation. Here, there is no disparity.

Now, consider ~(+u)/(+v) = (+s)/~(+t). There are two issues. The first, what is ~(+u) · ~(+t)? Let us put this in a concrete illustration. A deficit of (+5) times (+2) = a deficit of (+10) or ~(+5) · (+2) = ~(+10). This is not the same as ~(+5) · ~(+2). The former is the product of a deficit and its factor; the latter is a deficit of a positive number times the deficit of second positive number. What happens is that the deficit is emptied of its deficient character. It is just (+10). This result is simply a product as in standard Diophantine arithmetic.

The second issue is the meaning of (+u)(+t) = (+s)(+v). This is straightforward. It simply means that the (+u)(+t) is equal to (+s)(+v).

At this place in the investigation, let us replace (+u), (+v), (+s), and (+t) by "N," which stands for normality and by " ~ " to form two simple equations.

Is ~/N = ~ valid? This would mean that N · ~ = ~. It is valid. A deficit times a factor is, of course a deficit.

And N/~ = ~ ? This is also valid. It would mean that a deficit times a deficit is a normal. How different from the real system where a minus times a minus is a plus.

## 3. Rectilinear vs. Rectangular Multiplication

Consider & /N = & and N / & = &. The former is valid, since it means that a surplus times its factor is a surplus, while the latter is invalid, since it means that a surplus times a surplus is a normal. It *could* be a surplus. If you had a square block that was an extra 2" on one side and an extra 3" on the other, the total extra area would be 6 square inches.

Let us return now to the use of (+u), (+v), and (+w). What of ~(+u)/~(+v) = &(+w)? This is not possible at this level of abstraction using just numbers. The deficit of a deficit is only the plain condition of its removal, never a surplus. It is just a normality.

Consider &(+u)/&(+v) = &(+w). Is this the same as (+u)/(+v) = &(+w)? The latter states that a certain fraction has the same quantitative value as a certain surplus. It is valid, providing that it is not taken for an identity. The first equation does not mean that. It states that the division between two surpluses is a surplus.

Is the first equation even valid? To be valid, it would have to mean that &/& is &. This would mean that & · & = *&*. (Granting that neither the multiplier nor the multiplicand were equal to unity), that would imply that the product *&* was greater than the other ampersands in a way that was not a mere matter of straightforward magnitude. This will be explained more completely in the next paragraph. The division &/& *could* be an N. An example will suffice: suppose the task were to divide &7 by &2. The quotient would be 3.5, not &3.5. Why? As stated before, (& · N) = &. Let N = 3.5. Then (&2 · 3.5) = &7.

At this place, the reader might wonder, when is & the product of & · N and when is it the product of & · &? The answer lies in two of the three types of multiplication identified in **The Nature of Negative Numbers**. The first is rectilinear multiplication in which N serves as a multiplier, a factor, an extender. The second is rectangular multiplication in which the product is an area or some other conceptually appropriate existence, the unit of which is compounded out of the units of its factors. I was tempted to call this $\&^2$, except that & is not a number, but a designation that the quantity to which it refers is a surplus; which would invite confusion. I decided instead just to place the rectangular *&* in italics.

In **The Nature of Negative Numbers**, the following question is first posed and then answered:

> "Suppose that the mathematicians who had first obtained the square root of a minus number in their equations had interpreted it as a deficit instead of a change in direction. Would they have encountered the same insuperable difficulty? Yes. Suppose a real deficit like $\sqrt{\sim 4}$ . Does this have a square root? A person might think it is ~ 2. But this cannot be, since by the laws of arithmetic, the product of a deficit of two times a deficit of two, can only be an addition of four. Any real number use of the deficit would be subject to the same limitation as its square root.
>
> "The same would be the case with a veritable deficit of ~ 2. In the veritable system, ~ means subtraction and *only* subtraction. The veritable system uses the subtraction sign, absent of any contamination from the alien concept of direction."

But that is the case only when rectilinear multiplication is considered. The answer is different with rectangular multiplication: Suppose that I had purchased a board of lumber 12′ long and 4′ wide and had paid the amount down that was the equivalent of a 9′ x 3′ board, then I would owe the equivalent of a three square foot board. This would be ~ · ~ = ~ or ~ / ~ = ~. That is forbidden by the real law of signs.

## 4. Mixed Ratios

Going on, &(+u)/&(+v) = ~(+w) is generally invalid. The truth is that subtraction and addition are metaphysically different and their results are not parallel in all cases. Deficit is basically the former and surplus, basically the latter. It could, however be used in a proportion.

&(+u)/(+v) = (+s)/&(+t) is *not* generally valid. The numerator of the first fraction is a surplus and so is the denominator of the other. Using the other symbolization, &/N = N/&. The former fraction is equal to a &, and the latter is not.

Only a comparison of the magnitudes of positive variables is possible. Note that these would not be absolute values, since it concerns positive numbers.

What of a mixed ratio, say &(+u)/~(+v)? The comparable ratio in the real system, +u/-v, if it were equal to anything except itself, would equal –w.

This is not the way it is in the veritable system. &(+u)/~(+v) could not mean (+w). Standing by themselves, &/~ are not divisible, one into the other; surplus and deficit cannot be formed into a coherent result.

However, the ratio &(+w)/~(+1+) can exist. For example, suppose "&" referred to something in surfeit and "~," a debt. Then dividing the superabundant quantity by the quantity borrowed would be equal to (+w) if (+u) were in the same units as (+w) and (+v) as the same unit in the denominator on the right side The two would match. The fundamental units of a ratio do not have to be the same. For example, in the standard mathematical expression for measuring velocity, d/t, displacement is not the same as duration. All that is required is that there be an arithmetical compatibility, even in the presence of an incommensurability, (as with a division involving an irrational magnitude).

In the example before us, this would mean that the surplus was (+w) times as great as the deficit. &(+w)/(+1) would have the slightly different meaning that the ratio on the left was a surplus (+w) times as great as positive unity.

Consider the inverse: ~(+u)/&(+v) = ?. The answer could be not be ~(+w), but ~(+w)/&(+1) would be correct as a proportion (providing, of course, that (+u) were quantitatively larger than (+v)). Both numerators are deficits and both denominators, surpluses.

## 5. Conclusion

I believe that what has been covered in this article provides a sufficient foundation for the inquirer who also has possession of **The Nature of Negative Numbers** to go forward.

The symbols of combination and difference when considered in their purity, afford far more subtlety of expression than the real alloys which compound the sign (+) with arithmetical summing, and the sign (–) with arithmetical subtracting. The same goes for their extensions: multiplication, division, ratio, and root.

This allows for the precise recognition of more possibilities in the interpretation of matter in motion than is possible with the establishments' real system. A physics ignorant of the veritable number system courts desuetude.

## References

[1] Peter F. Erickson, **The Nature of Negative Numbers**, (Austin, TX: BookPros, 2011).

[2] Peter F. Erickson, **Absolute Space, Absolute Time, & Absolute Motion**, p. 146-153 (Philadelphia, PA: Xlibris, 2006). Refutes the idea that a derivative is not a ratio.

[3] The zero of neutrality is represented by "0". The other zero, utter zero, is represented by 0. It is not neutral, since it signifies an absence of being.