

The Theory of Mercury's Anomalous Precession

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Urbain Le Verrier published a preliminary paper in 1841 on the Theory of Mercury, and a definitive paper in 1859. He discovered a small unexplained shift in the perihelion of Mercury of 39" per century. The results were corrected in 1895 by Simon Newcomb, who increased the anomalous shift by about 10%. Albert Einstein, at the end of his 1916 paper on General Relativity, gave a specific solution for the perihelion shift which exactly matched the discrepancy. Dating from the 1947 Clemence review paper, that explanation and precise value have remained to the present time, being completely accepted by theoretical physicists as absolutely true. Modern numerical fittings of planetary orbits called Ephemerides contain linearized General Relativity corrections that cannot be turned off to see if discrepancies between observation and computation still exist of the magnitude necessary to support the General Relativity estimates of the differences.

The highly technical 1859 Le Verrier paper was written in French. The partial translation given here throws light on Le Verrier's analysis and thought processes, and points out that the masses he used for Earth and Mercury are quite different from present day values. A 1924 paper by a professor of Celestial Mechanics critiques both the Einstein and the Le Verrier analyses, and a 1993 paper gives a different and better fit to some of Le Verrier's data. Nonetheless, the effect of errors in planet masses seems to give new condition equations that do not change the perihelion discrepancy by a large amount. The question now is whether or not the excess shift of the perihelion of Mercury is real and has been properly explained in terms of General Relativity, or if there are other reasons for the observations. There are significant arguments that General Relativity has not been proven experimentally, and that it contains mathematical errors that invalidate its predictions. Vankov has analyzed Einstein's 1915 derivation and concludes that when an inconsistency is corrected, there is no perihelion shift at all!

1. Introduction

In his 1916 paper on General Relativity [1], Albert Einstein announced that he had explained the apparent anomalous shift in the perihelion of Mercury, discovered by the French astrophysicist, Urbain Le Verrier [2, 3] in 1859 and corrected slightly by Simon Newcomb [4, 5] in 1895. As the accompanying table shows, the total observed precession of 5600 arc seconds per century is made up mostly by a coordinate transformation, whereas the calculation of the effects of other planets remains at roughly the 1895 values, and Einstein's correction is taken as a real effect that exactly explains the difference.

Amount (arc-sec/century)	Cause
5025.6	Coordinate (due to <u>precession of equinoxes</u>)
531.4	Gravitational tugs of the other planets
0.0254	Oblateness of the sun (<u>quadrupole moment</u>)
42.98±0.04	General relativity
5600.0	Total
5599.7	Observed

Fig. 1. Sources of the Precession of Perihelion for Mercury

Fig. 1 is taken from a 1947 paper by Clemence [6], which is essentially the definitive analysis of all Mercury data taken up to that time. Clemence says, "It is at once evident that the effect can be detected most easily in the motion of Mercury. Indeed, Einstein's announcement of the general theory of relativity in its definitive form was immediately hailed by some astronomers as

explaining a previously unaccountable discrepancy between the observed and theoretical motions of this planet. Others were, however, intuitively opposed to relativity, and they directed attention to a small discrepancy yet remaining as evidence that the theory of relativity could not be correct: the relativists contended that the small remaining discrepancy was due to errors either in the observations or in the classical theory of the motion. In justice it should be said that the questions involved are not simple ones, but are complicated by three causes:

1. Observations of Mercury are among the most difficult in positional astronomy. They have to be made in the daytime, near noon, under unfavorable conditions of the atmosphere; and they are subject to large systematic and accidental errors arising both from this cause and from the shape of the visible disk of the planet.
2. The planet's path in Newtonian space is not an ellipse but an exceedingly complicated space-curve due to the disturbing effects of all of the other planets. The calculation of this curve is a difficult and laborious task, and significantly different results have been obtained by different scientists.
3. The observations cannot be made in the Newtonian frame of reference. They are referred to the moving equinox, that is, they are affected by the precession of the equinoxes, and the determination of the precessional motion is one of the most difficult problems of positional astronomy, if not the most difficult. In light of all these hazards it is not surprising that a difference of opinion could exist regarding the closeness of agreement between the observed and theoretical motions."

Le Verrier became famous by discovering Neptune by its effect on the orbit of Uranus. In analyzing the orbit of Mercury, perhaps he was looking for a similar discovery. Since the original Le Verrier paper is written in French, some of the considerations that led him to his conclusions may have been missed by modern researchers. A partial translation of the paper given below (with some comments in square brackets and a comparison to modern planetary mass data) serves to point out how he did his analysis, and what influenced his conclusions.

The general flavor of Le Verrier's work is a complicated analytical fit to the observational Mercury data over about 50 years. This consisted of deriving, "Condition Equations," containing seven types of terms, some of which are periodic and others time-dependent, plus approximate perturbations of these terms to obtain equations that match the observations with minimal residuals. The final product is a series of Tables predicting when Mercury would be observed in the future. A byproduct of the analysis was the discovery that the contributions of the other planets left an unexplained residual shift of the perihelion by 39 arc-seconds/century.

Le Verrier explored possible reasons for this perihelion shift, and concluded that some of the planet masses were possibly incorrect, or that there was an unknown small planet that was the cause of the shift. Le Verrier's search for the hypothetical planet Vulcan was unsuccessful.

By comparing the masses Le Verrier used (scaled to the mass of the Sun) with the modern values, it appears that *the mass of the Earth he adopted is too small by about 6%, and the mass used for Mercury is much too large by a factor of two.* On the other hand, the other masses seem to be fairly accurate except for Mars which only has a small effect. But it leads to the question, could his unexplained perihelion shift be due to these errors?

There is a 1924 critique of Einstein's claim by Charles Lane Poor [7], a professor of Celestial Mechanics, who claims that Einstein's bending of space and time only distorted the problem and did not solve it, saying that his solution actually reduces to the Newtonian solution. Poor also claims that the motion of Mercury is made up of two "wobbles", and Le Verrier could not disentangle them with his approach.

There is a 1993 reanalysis of some of Le Verrier's data by Takeshi Inoue [8], wherein if a few bad points are omitted, a much better fit is obtained for the excess motion of the ascending node of Mercury's orbit. This suggests that modern computer analysis of old data may lead to quite different results than hand calculations made using approximate tabular interpolation methods of the past.

Finally, Vankov [9], in revisiting more recent analyses of the Mercury perihelion problem, cites critiques by Rana [10] and Pitjeva [11], stating, "The claim in the literature that the existence of the predicted anomalous effect is firmly confirmed *is not substantiated in rigorous terms of statistical theory.* In a long run (from Le Verrier to Einstein), there was a wide range of admissible values of 'the anomaly' (say, from 5 to 50 arc-seconds per century). Probably, striving for 'a true explanation' of the anomaly prevailed over work on determination of 'the true anomaly' value from astronomical observations, still of poor precision. Clemence stated that, given realistic uncertainties of input data and model parameters, the planetary Ephemerides can be, in prin-

ciple, adjusted by minimization of the anomaly gap in concordance with the General Relativity prediction. This was a solid result but still not a resolution of the problem because of a number of controversial issues remained unresolved." Vankov derives mathematical equations that can systematically test the accuracy of the perihelion observations.

Rydin [12] makes an argument that General Relativity rests upon weak experimental verification, citing that: 1) the Sloan Survey galaxy distribution data do not correspond to the General Relativity Big Bang solution; 2) the bending of light in the 1919 eclipse photographic measurements did not have sufficient accuracy to confirm the theory; and 3) The perihelion of Mercury measurements are suspect in accuracy. Furthermore, Rydin cites work by Heaston that indicates that the singularity in General Relativity was introduced by Einstein when he set $c = 1$, and cites mathematical work by Crothers that proves that the tensor equations used in General Relativity give a solution different from Schwarzschild's original solution which does not have a singularity at the origin.

As a matter of fact, Schwarzschild himself disagreed with Einstein. In Abstracts from a Letter from K. Schwarzschild to A. Einstein dated 22 December, 1915, he says, "Honored Mr. Einstein; in order to be able to verify your gravitational theory, I have brought myself nearer to your work on the perihelion of Mercury, and occupied myself with the problem solved with the First Approximation. Thereby, I found myself in a state of great confusion. I found for the first approximation of the coefficient something other than your solution. It had beside your α yet a second term, and the problem was physically undetermined."

"From this I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result: It gave only a line element, which at the null point and only in the null point is singular. The equation of the orbit remains exactly as you obtained in the first approximation, only one must understand for x not $1/r$, but $1/R$, which is a difference of the order of 10^{-12} , so it has practically the same absolute validity."

"The difficulty with the First Approximation resolves itself thereby: so as α is given, so will the solution be divergent by continuation of the approximation. It is after all the clear meaning of your problem in the best order. It is an entirely wonderful thing, that from one so abstract an idea comes out such a conclusive clarification of the Mercury anomaly. As you see, it means that the friendly war with me, in which in spite of your considerable protective fire throughout the terrestrial distance, allowed this stroll in your fantasy land."

There is a valid question about the perihelion discrepancy of Mercury, *if it indeed exists.* Only a 0.7% standard deviation in the measured total precession of Mercury, or a similar error in the precession of the equinoxes would fully cover it, and these are admittedly "among the most difficult measurements in positional astronomy". Just the differences between Le Verrier's 1841 and 1859 values for planetary effects can explain one third of the discrepancy. The tortuous paths of the planets in the Solar System need to be analyzed more fully.

2. Partial Translation of Le Verrier’s 1859 Paper

(From page 19) “The formulas that we assemble here, with few exceptions, are based on the provisory values attributed to various elements in Chapter VII.”

“The masses of the planets, in particular, have for their expressions [which recognizes that they may have to be modified later]:

Planet	1859 Planet/Sun	2009 Planet/Sun	Old/New
Mercury, m	3.333E-7	1.66E-7	2.0
Venus, m'	2.4885E-6	2.45E-6	1.015
Earth, m''	2.8174E-6	3.0E-6	0.937
Mars, m'''	3.339E-7	3.09E-7	1.08
Jupiter, m''''	9.52381E-4	9.5E-4	1.005
Saturn, m'''''	2.84738E-4	2.84E-4	1.000

Fig. 2. Mass Ratios Relative to the Sun

“In comparing the masses of the Earth, Jupiter and Saturn to their volumes, one remarks that the densities of these planets are, to some degree, inverse to their mean distances from the Sun. This rule is not true for Venus and Uranus. With respect to Mercury, in deducing the density of this planet, and thus its mass, we suspect that the accepted radius is too low. One finds then that the mass of the planet should be a *two-millionth* of that of the Sun. I have here reduced this mass to a *three-millionth*, in consideration of the perturbations that have been found by the Comet Encke in its passage on the perihelion in 1838. But, according to Encke, the mass of Mercury should be even smaller. We conclude only that this mass is very small and that it cannot have any sensible influence on the calculation of the major axis of the orbit.” [In fact, the mass used for Mercury by Le Verrier was not reduced enough and is still twice the currently accepted value].

(From page 78) “One thus sees that the discussion of the observations of the passages of the planet past the Sun furnishes a precise relation between the eccentricity and the longitude of the perihelion; but only to determine one of these two elements, it shall be indispensable go back to employ the meridian observations.”

“The annual movement, $2.72e' + \omega' = +0.''392$, should fix in our attention: this quantity being essentially tied to *values admitted for the masses of the planets*. The secular variations of the eccentricity and the perihelion of Mercury have been calculated in attributing to the masses of the planets furnished by considerations outside the theory of Mercury, but which one has reason to believe are strongly exact [Some of the masses differ from present values, especially the Earth!]. One can thus hope that the discussion of the observations of Mercury simply confirm previous research. Now this is not negligible: we see here that the approximately 3-fold secular movement of the eccentricity, added to the secular movement of the perihelion, gives a sum in which the observations are greater by 39'' per century than those which result from calculation. The part of this sum, due to the action of Venus, is equal to 288'' [The previous given value was 280''.6] given by the calculation based on the solar mass fraction 2.4885 E-6: and in consequence, to make the theory agree with the observations of Mercury, one should augment the mass used for Venus by about a seventh [14%] of its value!”

(From page 79) “On the other hand, the value adopted for the mass of Venus influences several things. It enters into the expressions for the secular variations for e, ω, θ, e' and ω' [That is, it affects Mercury and Earth]: it finds itself in the periodic perturbations of the longitude of Mercury and the longitude of the Sun. The preliminary discussion, of which we have written, shows that we cannot confront these actions one at a time...”

“These are the corrections which, being added to the values of the elements, beginning in the 2nd Section, furnish the data most precise on which we base our definitive tables on the movement of the planet. They contain the remarkable result, already signaled above, that is to say, the considerable value of the function $2.72e' + \omega'$: a value that seems incompatible with the magnitudes adopted up to now for the masses of the planets, and notably for the mass of Venus. This consequence of the discussion of the observations of Mercury and their comparison with theory being most grave from the viewpoint of the constitution of our planetary system, it shall be good to think about this and throw an attentive eye on the path already followed, to see if there is anything we can infer on the consequences of this and the significance we should attribute to it.”

(From pages 98-101) “The exactness of the observations that have been used being placed outside the cause, one can ask if from the masses of the perturbing planets being used, the secular movement of the perihelion and the eccentricity of the orbit of Mercury have been exactly deduced.”

“We must remark in this regard, that outside the determination given in the present work, we have been disposed to refer to a paper published in 1841 on the secular variations of the elements of the orbits of the planets, and having considered elements of the first and third orders. We have reviewed these two determinations, separately, as in the 1841 paper, with terms of various orders, and in consideration of the masses used in the present work.”

Planet	1841 Original	Correction	Total	1859 Total
Venus	287''	-6''	281''	280''.6
Earth	86	-3	83	83.6
Mars	3	0	3	2.6
Jupiter	158	-6	152	152.6
Saturn	8	0	8	7.2
Uranus	0		0	0.1
Total	542	-15	527	526.7

Fig. 3. Secular Movement of the Perihelion of Mercury

Planet	1841 Original	Correction	Total	1859 Total
Venus	1''.1	1''.7	2''.8	2''.8
Earth	0.3	0.8	1.1	1.1
Mars	0	0	0	0
Jupiter	-0.7	3.5	4.2	4.2

Fig. 4. Secular Movement of the Eccentricity of Mercury

“One sees that there is an agreement between the results, since in the second work we did not borrow anything from the first.”

"Now, how can one, in taking account of the uncertainties in the observations, consider these diverse observations as compatible between themselves?" [There follows a page of discussion]

(From page 102) "We do not insist on any of these considerations, awaiting we repeat, that having reviewed all the elements of the discussion, each can produce some knowledge of the cause, and then we adopt the conclusions which seem most sure. It then remains to us only, for the case where one believes that the mass of Venus cannot be augmented, to examine to which consequences one shall be led to by the necessity to make an increase in the movement of the perihelion of Mercury by the *action of unknown masses*. It remains, that we do not need to review the research of all the causes that could produce this result. We should be content to indicate those that appear most probable, in reason of our actual knowledge about the physical constitution of our planetary system."

"The troubling mass, if it exists, does not have any sensible effect on the movement of the Earth. We ignore that it has some effect on Venus; and in awaiting this point to be clarified, we admit that this action is insensitive or at least weak except on Mercury. In this hypothesis, the mass sought should find itself below the orbit of Mercury. If in addition one wants its orbit to not entangle with that of Mercury, it is required that the aphelion distance does not exceed *eight-tenths* the average distance of Mercury, that is to say *three-tenths* the average distance of the Earth to the Sun." [Hereafter comes a discussion of the hypothetical planet Vulcan which has never been observed. Le Verrier was famous for discovering Neptune from discrepancies in the orbit of Uranus, and postulated a similar situation for Mercury].

3. 1924 Critique by Charles Lane Poor [7]

"Einstein deduces a law of motion for the planets about the Sun; and this law of motion apparently differs from Newton's law of motion by a single very minute term. And this little term seems to fit into a kink in Mercury's orbit, and to explain or account for certain observed motions of that planet. But, in deducing his law of motion, in traversing the complicated maze of mathematics, the relativist meets difficulty after difficulty, and somehow surmounts them all. For each new difficulty some new mathematical device is evolved, and many of these devices are so intricate and complicated that it is extremely difficult to follow them through all their ramifications. Some of these devices seem to be ordinary approximations, but are called by Einstein substitutions or transformations of coordinates. One such transformation; or approximation, which is used in many portions of the theory, involves the method of measuring the distance between two particles of matter. Instead of using the exact distance between the centers of such particles, the relativist adds a small, a very small factor to this distance; and, in his formulas, uses this modified distance as if it were the true distance between the bodies. In ordinary terms this would be called an approximation, and any result derived there-from would be termed approximate. But to the Einsteinian, such a procedure is a transformation and the result is called *exact*."

"Further, the relativity conception of time differs from that of classical mechanics. From the earliest days of scientific thought, time has been regarded as independent of everyone and every-

thing; the same for all portions of space, for all bodies, whether in motion or at rest: a minute is a minute the world around and everywhere in space. But this identity of time is denied by Einstein: according to the relativity theory time depends upon motion, and the internal of time, known to us as a minute, varies from place to place; it is different for a person at rest and for an aviator. Thus each planet has its own particular system of relativity, or "proper" time, and even this special time changes as the planet changes its speed in different portions of its orbit."

"Thus the formula, or law of motion, as deduced by the Einsteinians, does not represent the motion of a planet about the sun in the ordinary astronomical units of distance and of time. It represents that motion in a special system, in the relativity system of time or coordinates. Now it can readily be shown that this Einstein formula of motion, this formula which has *aptly* been called the essence of the relativity theory, can be derived directly from the Newtonian formula by merely changing the system in which the motion is measured, and by changing from the astronomical to the relativity method of measuring time. By omitting the special, or approximate systems of measurement, and using the ordinary astronomical measures of time and distance, the Einstein formula becomes identical with that of Newton: on the other hand, if in the Newtonian formula the relativity system of measurement be used, then this formula becomes identical with Einstein's. The two formulas thus apparently represent the same motion of a planet, but give that motion in different units; just as a stated amount of gold may be expressed as so many francs, or so many dollars."

"Yet this formula of motion, this formula which can be derived from that of Newton by a simple change in the method of measuring time or distance, is used by Einstein as proof conclusive of his theory. And this so-called proof depends upon certain minute and little understood motions of the planet Mercury. This planet is exceedingly difficult to observe; its motions are rapid and it is never far from the glare of the sun. Its path is not a simple curve: the large planets, - Venus, the Earth, and Jupiter - pull and haul at it, and under these pulls Mercury writhes and squirms along a difficult and tortuous path. Le Verrier calculated the effects of the various pulls of the six larger planets upon unfortunate Mercury, and found that these planets do not fully account for all the writhing of Mercury's orbit. He found a very slight discrepancy: he found that some other force, some other very minute pull, was affecting the motions of Mercury. And this discrepancy in the motions of Mercury consists of a combination of two small" wobbles," one of which is the celebrated motion of the perihelion, or rotation of the orbit in space. But the motion detected by Le Verrier is not a simple motion of the perihelion; it is a combination effect, a combination which Le Verrier himself could not disentangle into its separate parts. Within limits any value could be assigned to the perihelion motion, and to each such value there would be a definite wobble of the eccentricity. Le Verrier gave, however, 38 seconds of arc per century as the most probable motion of the perihelion, which corresponds to a very small change in the eccentricity. Some years later, in 1895, Simon Newcomb confirmed these general results of Le Verrier; but he made the motion of the perihelion slightly larger and the change in the eccentricity correspondingly smaller, and at the same time he found several other small discrepancies,

or wobbles, in the motions of Mercury and in the motions of other planets as well." [Poor suggests that the problem is highly nonlinear, and is not amenable to linear perturbation fixes].

4. 1993 Reanalysis of Le Verrier's Data, by Takeshi Inoue [8]

"We have reexamined the twenty-one observations of second and third contacts during transits of Mercury across on the disk of the Sun - the same Le Verrier used himself (Le Verrier 1859) - to check the reality of the excess of the observed motion of the longitude of the ascending node of Mercury's orbit over the theoretically predicted one. To this, we numbered these observations chronologically from 1 to 13 for the transits on November and from 14 to 21 for those on May."

"The results are as follows:

1. An initial analysis showed that three of the observations (Nos. 2, 8 and 21) might have been erroneously recorded. If the signs of observations Nos. 2 and 8 are changed and if the value for the observation No. 21 is changed from $-1.''03$ to $-2.''52$, the sum of the squares of the residuals is reduced by almost 60%.
2. Leaving out these observations altogether, as well as observations Nos. 10, 17 and 18 - which leaves us 15 observations to analyze - the sum of the squares of the residuals is drastically reduced to 5% of its original value (but now, there are of course also fewer observations contributing to this sum), and we obtain an excess motion of $16.''7 \pm 3.''4$ (sec.) per century. This shows that the appropriate choice of observations will indeed produce an estimate for the excess motion of the node which exceeds its formal standard error by a factor of 5.
3. Eliminating eight additional observations and thus utilizing only the seven observations, viz. Nos. 1, 9, 13, 14, 16, 19 and 20, the least squares adjustment of the remaining seven condition equations in the six adjustment unknowns produces an estimate for the excess motion of the ascending node of Mercury's orbit. This time it gives $15.''2 \pm 0.''1$ per century, with the sum of the squares of the residuals now reduced to the order of 10^{-5} , even with only even summands contributing, clearly an unrealistic result in view of the precision of observations attained in Le Verrier's time."

"The only conclusion one can draw from the data is thus that they do not contribute to a decision as to whether the actual motion of the ascending node of the orbit of Mercury exceeds that predicted by the theory."

5. Revisit of Newer Analyses, by Anatoli Vankov [9]

"The situation in observational astronomy radically changed with the technological boost started in 1950s and continued since then. The initiative was launched to include the GR theory in the form of the Parameterized Post Newtonian (PPN) approximation. The exact GR N-body solution, which would have the Newtonian limit, does not exist.

In particular, it became possible to study errors in approximate solutions of N-body problems and the role of relativistic corrections there. As for the 'anomaly' problem, we are not say-

ing that much better clarity than at Clemence's time has been achieved.

In the 70s and later on, the N-body problem was theoretically formulated in the parameterized post Newtonian (PPN) formalisms at the level of Ephemerides calculations. The PPN approximation idea is to linearize the equations under weak-field conditions for approximate N-body solutions. Inevitably, the relativity essence such as a concept of proper-versus-improper quantities has to be sacrificed."

"Nowadays, however, the N-body problem has an exact computer-supported solution in the Newtonian model. The incorporation of the approximate PPN formalism into the Ephemerides systems took place in a process of continual 'fitting' of Ephemerides systems to observational databases. Overall, an optimistic picture is claimed that astronomical constants and planetary Ephemerides are adjusted to hundreds of thousands of different observations (astronomical, radiometric and others). Allegedly, any orbit can be accurately displayed (in a sense of 'fitting') in a desirable coordinate system and in time directed into past or future for a long enough period.

We put the word 'allegedly', because there is a difference between 'fitting' and 'evaluation of statistical significance of observational data.' It is important for researchers to have an opportunity to handle the parameters by their levels (to be able 'to turn' them 'on' and 'off', for example), to compare results of the Newtonian model with the one including relativistic corrections."

"The question arises how to determine the difference between 'calculation' C(T) and 'observation' O(T) for different instants T in view of significant fluctuation of angular motion. It follows that the fine structure in calculations and observations must be accurately accounted for in order to preserve true physical meaning of measured quantities."

A computed total precession of Mercury [10] for about an 8 year period is shown in Figure 5. This represents the difference between the total measured value and the precession of the equinoxes, or about $575''$ per century. From the tortuous path of the precession, it is difficult to see how any anomalous discrepancy can be extracted.

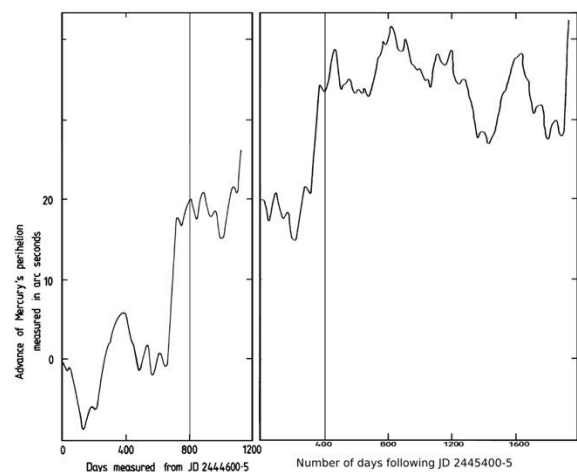


Fig. 5. Total Precession of Mercury Using 1 Day Time Steps

The conclusion of Rana's 1987 work [9] is, "The low precision of the geocentric angular data having an error of $1''$ are incapable of giving the rate of motion of the perihelion of Mercury to better

that 3" per century. Hence *the determinations apparently good to 0.3" per century are spurious*. Using too low a rejection level, only those data that happen to agree with the initially assumed value of the rate of motion of the perihelion contribute, and one will always obtain a very small correction to the initially assumed value. So *all the existing observational estimates*, namely, those made by Clemence, Morrison and Ward, and the JPL groups, *are suspect*.' The author noted that the Ephemerides uncertainties must be increased substantially. A similar opinion was expressed by Pitjeva [11] in 2005." *There is a significant difference between precision (by fitting) and accuracy (by evaluating errors)!* The author derives equations that can be used to evaluate the accuracy of the data.

In a new work, Vankov [13] analyses Einstein's 1915 paper where the General Relativity precession was derived using low order approximations. He finds that Einstein's proffered solution is inconsistent with Einstein's original formulation by an amount that exactly cancels the anomalous precession. Hence, there is no General Relativity effect at all! In his 1915 letter to Einstein, Schwarzschild pointed out that Einstein had made an error, and the formulation was indeterminate.

6. Conclusion

It is not clear that you can calculate the precise effect of other planets on the orbit of Mercury without solving simultaneously the gravitation equations for the Sun and at least five other planets, which is a highly nonlinear problem. It can be done numerically, but the solution evolves with time, and errors in data and initial conditions accumulate. Le Verrier did it with perturbation theory, and in 1859 he had to do the calculations by hand using tables and interpolation, a formidable task. Adding a General Relativity movement contribution, and then doing a fit to existing datasets is not the same thing, because the fit is self-correcting, leading to precision and not accuracy.

In the Solar System, where everything is in motion and all positions change strongly with time, you should get different orbit effects on each planet when all the planets are aligned and when they are quite out of phase. That is why I think that the Mercury discrepancy is not necessarily real, because the perturbing effects are ever changing with time, so there is no constant answer for each planet. Errors in the masses of some planets as large as 8% did not seem to make much difference to the final fit for Le Verrier.

But from a pure accuracy standpoint, Einstein's theoretical correction is only 8% of the total average contribution from all the planets, and could be statistical error instead of a real effect. 8% of 530" is +42.4". When the total measurement of the movement of the perihelion of 5600" is considered, an uncertainty of 0.7% would cover the discrepancy. Furthermore, Le Verrier's

own evaluations for the secular movement of the Perihelion of Mercury changed by about 3% between 1841 and 1859.

Finally, one would think that the measured average values like 5600" would have changed over the years, partly because in the 1800s mechanical devices would have been difficult to control accurately and mechanical measurements would have been imprecise, then in the early 1900s some sort of feedback control and measurement would have come into use, and then in the computer age digital measurement and computer control would have taken over. Just the changes in measured values over the years would have given an estimate of the standard deviation of the experimental results. Those differences do not seem to be available for analysis.

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