

**A modified law of gravitation taking account of the
relative speeds of moving masses.
A preliminary study**

Bernard GUY
Ecole nationale supérieure des mines de Saint-Etienne
guy@emse.fr

Abstract

A modified law of gravitation is proposed which takes account of the relative speeds of the moving masses. In the case of the two-body problem, the acceleration is given by

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{|\vec{r}|^3}[\vec{r} - A\frac{d\vec{r}}{dt} \times (\frac{d\vec{r}}{dt} \times \vec{r})]$$

where \vec{r} is the radius vector to the source mass M, \times the vector product, G the constant of gravitation, A a new constant; we take it provisionally equal to 5.10^{-11} u.S.I. ($m^{-2}s^2$). The law simulates a "supplement" of mass with respect to the standard Newtonian law. Its application to several gravitation problems provides a good order of magnitude for the apparent supplement of mass associated with the motion of the Pioneer satellites, that of stars in galaxies or galaxies in galaxy clusters, for the same value of the additional parameter. The law equally simulates a lack of attraction, for the later stages as compared to the early stages, for a system of expanding masses, imitating a repulsive force. The order of magnitude of the corresponding energy fits with what is found in the literature for the "acceleration" of universe expansion. The conceptual framework in which the law is proposed is sketched out: it is based on the assertion of the fundamental link between the space and time concepts, and on a better symmetry of the physical laws with respect to these parameters. The study is preliminary, it simply establishes orders of magnitude for the expected effects, by an approximate approach of the two-body problem. In the near future it seems interesting to perform quantitative computer simulations so as to check whether the proposed law resists to further confrontations with observational data. If it did, it would avoid in the same time the use of dark matter and dark energy. The present work also gives clues to help reconsider the theory of relativity, in continuity with the modified law of gravitation, and its links with gravitation and electromagnetism.

Key words: gravitation, modified law, relative speeds, relativity, electromagnetism, Lorentz force, tri-dimensional time, Pioneer anomaly, dark matter, dark energy, acceleration of universe expansion, galaxy anomalies

1. Introduction: aim of the paper

The aim of this paper is to propose a new law of gravitation that takes account of the relative speeds of moving masses; a single additional parameter is required. The formulation is non-relativistic, but integrates in a conceptual framework consistent with the theory of relativity, although this theory should be revised in the new context. The first reason for this proposal stems from general considerations that are briefly outlined. The second reason, which is investigated more fully, is the possibility to examine a number of problems that are set today in physics, all of which relate to gravitation: anomalies of the Pioneer satellites, abnormal movements of the stars in galaxies and of the galaxies in clusters of galaxies, acceleration of universe expansion, calling upon the possible existence of dark matter and dark energy. In a first approximation, the new law explains the anomalies for the same value of the unique additional parameter (the numbers used in this paper are given as a rough guide and only aim at calculating orders of magnitude).

2. General framework

In the past years, the author conducted various researches on the concepts of space, time and motion (see Guy's works in the list of references). They lead to understand time and space in an associated way (they are the same "substance"), and to propose the use, in the equations of physics, of a temporal parameter having the meaning of a movement, with three components t_x , t_y , t_z . For example, one can figure out that time is marked by the position of the sun in the sky, or of by that of a photon in an atomic clock. This leads to a better symmetry between the space parameters x , y , z , and the time parameters t_x , t_y , t_z , in particular for the Lorentz relations of relativity (see Guy, op.cit.; see also Franco, 2006). The epistemological problems associated with this approach are discussed in the works of Guy, and we do not go back to them here. In particular, it is shown that, like in quantum mechanics, relativity is subjected to what we can call "uncertainty" and "incompleteness". In this framework, one is led to propose that any physical quantity has two faces: we will speak of a "spatial" face \vec{g} and a "temporal" face \vec{h} . A law of physics is expressed by equating the sum of the derivatives of one (spatial or temporal) face with respect to the variables of a given (spatial or temporal) type, to the derivatives of the other face with respect to the other variables. This is the archetype of a

physical law that symmetrically links changes in space and time parameters to changes of the two faces of a physical quantity; it contains the essence of the theory of relativity. One can state as a general principle that laws of physics are globally invariant by exchanging space and time parameters. This is written as follows, for the two types of possible relations:

$$\sum_i \frac{\partial g_i}{\partial t_i} + \sum_j \frac{\partial h^j}{\partial x^j} = 0 \quad \text{and} \quad \sum_i \frac{\partial g_i}{\partial x^i} + \sum_j \frac{\partial h^j}{\partial t_j} = 0 \quad (1)$$

where g and h are vectors with three components; x_i ($i = 1, 2, 3$) are the three space coordinates x , y and z , and $t_i = t_x, t_y, t_z$ the three time coordinates, in a way representing three other spatial coordinates. We have written indices at different heights to indicate that one can interpret relations (1) in a tensorial fashion (see Guy, *op. cit.*); we will not use these notations thereafter. Tsabary and Censor (2005) have also proposed relations similar to (1) that we can understand as very general conservation laws. One can go from writing (1) with three time parameters to a writing with one time parameter t verifying $t^2 = t_x^2 + t_y^2 + t_z^2$; this requires to specify the orientation of the direction along which the movement associated with time is defined. The calculations are explained in the works of Tsabary and Censor, and Franco. Such an approach accounts for certain forms of the Maxwell equations (see also Teli, 1984). The electric field E appears as the spatial face of the electromagnetic quantity: it is used for determining the force induced by static electrical charges, as a function of their mutual distances. The magnetic field B appears as the temporal face of the electromagnetic quantity; it is used for determining the force induced by moving charges, as a function of their relative speeds. From Maxwell equations one can derive the various electrostatics and electrodynamics laws, in particular the total force between two charges q and q' (distance r) with relative motion at speed v ; it involves two terms, one as a function only of positions, and the other as a function of both positions and speeds.

Thanks to the correspondence mentioned above between the scalar time and the three-dimensional time, one can reversely transform a scalar quantity to a vector quantity, and generalize the previous approach to couples combining a scalar and a vector (electric charge density and current J ; energy E and momentum p etc.). One can then express the general laws (1) for the corresponding couples. The useful mathematical derivations are given in the works of Franco, and Tsabary and Censor; these authors did not propose our general interpretation (see developments in Guy, *op. cit.*). In a coordinate system in motion relative to the starting

“rest” one, one can write new laws similar to relations (1) ruling the new coordinates $x', y', z', t'_x, t'_y, t'_z$, and the new quantities g' and h' . The laws (1) implicitly incorporate the relations connecting the old coordinates and quantities to the new ones. In our interpretation of the time as attached to a mobile, when one moves with respect to one coordinate system, interchange between the quantities and parameters associated to space and to time naturally occur; these express the change of the viewpoint on what is mobile and what is immobile. This is a way to understand the various equations of relativity; equations (1) express the relativistic invariance in general.

3. Proposal of a modified law of gravitation

Let us go to gravitation. The g field does not fulfil the relations (1). We need another field to balance it. Within the previous context, we are led to associate with the gravity field created by a distribution of masses considered as static and dependent on their mutual distances, an additional field, created by the relative motions of these masses, and dependent on their relative speeds. We will thus have the duality

$$\vec{g} = -\frac{GM}{|\vec{r}|^3}\vec{r} \quad \text{and} \quad \vec{h} = A\frac{GM}{|\vec{r}|^3}\left(\frac{d\vec{r}}{dt}\times\vec{r}\right) \quad (2)$$

for the fields created by the source mass M , located at the origin of coordinates, on a unit mass located at $r(x, y, z)$ (two-body problem), with speed $v = dr/dt$; \times is the vector product, G the universal gravitation constant: $G = 6.67 \cdot 10^{-11}$ u. S.I., A a constant to determine, its dimension is the inverse of a speed squared: $[L]^{-2}[T]^2$. Fields g and h shall be linked by the general laws (1). From them, we can get an equation of motion; the passage is similar to the derivation, from Maxwell equations (1), of the motion of a charge depending on the position and speed with respect to another charge. The correspondence between the vector and the scalar time (see cited authors) is then needed. The acceleration for the new law of gravitation reads:

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{|\vec{r}|^3}\left(\vec{r} - A\frac{d\vec{r}}{dt}\times\left(\frac{d\vec{r}}{dt}\times\vec{r}\right)\right) \quad (3)$$

One recognizes in the first term of the second member of eq. (3) the Newtonian law of gravitation. A term has been added that depends on the relative velocities and is the analogue of Lorentz force for moving charges.

In the general case of a distribution of masses m_i , located at distances r_i to an arbitrary origin, each r_i vector will verify the following law:

$$\frac{d^2 \vec{r}_i}{dt^2} = - \sum_j \frac{G m_j}{|\vec{r}_{ij}|^3} (\vec{r}_{ij} - A \frac{d\vec{r}_{ij}}{dt} \times (\frac{d\vec{r}_{ij}}{dt} \times \vec{r}_{ij})) \quad (4)$$

where the summation involves the other masses m_j , with relative distances $r_{ij} = r_j - r_i$; the relative speeds are the time derivatives of the distances r_{ij} .

We temporarily assign to A constant the value $A = 5.10^{-11}$ u.S.I.

4. Notes on gravitation and electromagnetism

In the electromagnetic theory applied to the movements of elementary charges in vacuum, we would have $A = \epsilon_0 \mu_0 = 1/c^2$. When the theory is applied to the movements of/in macroscopic matter, a factor $\epsilon \mu$ is involved which may be different from its value "in vacuum", and larger by several orders of magnitude (up to 10^4 to 10^6 times). We do not comment on the distinction between the two theories, say in vacuum and in matter, involving the double duality (E, D) for the electric field on the one hand, and (H, B) for the magnetic field on the other hand; it has its practical use without necessarily having a fundamental meaning. For gravitation, we will also remain pragmatic, and will consider that we can assign two (or several?) values for A in the following situations: - ordinary matter in motion, on the one hand, and, by extension: - light photon movement, on the other hand. In effect, in addition to gravitation of moving masses, we can try to extrapolate the modified law for the motion of massless particles such as the photon: we will consider that we must then take $A = \epsilon_0 \mu_0 = 1/c^2$. Another way to do would be to re-write the law proposed in equations (3) or (4) by using relativistic formulations for the velocities of particles approaching or reaching the speed of light, like in

electromagnetism. The factor $\gamma = (1 - v^2/c^2)^{-1/2}$ would then appear, limiting the importance of the term containing velocities when $v = c$. We do not do it here.

Behind all this is hidden the meaning of gravitation and its possible links with electromagnetism. Some authors (e.g. Assis, 1992) have proposed that the first theory is a kind of macroscopic theory, averaging forces of only electromagnetic origin. In these circumstances, one can understand that the value of A parameter for macroscopic matter is different from $1/c^2$. The actual speeds of particles inside matter are indeed several orders of magnitude larger (for example, 10^3) than the speeds of the whole macroscopic matter. For the consistency of the equations, the value of A and that of the speeds would verify an equation such as:

$$A(\text{macroscopic velocity})^2 = (1/c^2)(\text{microscopic velocity})^2$$

because the speeds are involved by their square in the double vector product in equations (3) or (4). We can see that if $v_{\text{micro}}/v_{\text{macro}} = 10^3$, $A = 10^6/c^2$ and A parameter comes close to the factor 10^{-11} proposed above. These considerations are given at heuristic title. There is a factor of the same order of magnitude for the ratio of the actual speeds of electric charges to the macroscopic speeds of matter to which we may attach any particular field (E, B) for instance.

5. Introduction to the study of some problems

The previous laws (3) and (4) integrate within a relativistic approach, but their expression is classic. In any case, one will check that the second term of the second member of the laws (3) or (4) containing two vector products expresses an attractive force, regardless the relative positions of the vectors r and v (Fig. 1). The additional force therefore gives the impression of a supplement of mass or of gravity relative to the standard Newtonian case. In the case of a two-body system, where θ is the angle between the radius vector and the relative speed v , this mass M or gravity g supplement verifies:

$$\frac{\Delta M}{M} = \frac{\Delta g}{g} = Av^2 \sin^2 \theta \quad (5)$$

where the new force h has been projected onto the usual g gravity vector (parallel to radius vector r). The new force cannot be detected in cases where the radius and speed vectors are parallel, as is the case for example when one measures mass acceleration in a terrestrial laboratory. When the radius and speed vectors are perpendicular, $\theta = \pi/2$, $\sin\theta = 1$, and the mass or gravity supplement verifies:

$$\frac{\Delta M}{M} = \frac{\Delta g}{g} = Av^2 \quad (6)$$

Below in the text, we will discuss a number of situations that, according to the literature, cannot be interpreted within the standard Newtonian frame, and consider whether the modified law may apply. Our approach is preliminary, and for the examination of the problematic cases, we simply refer to a number of review works such as: Turyshev and al. (2005) for Pioneer anomalies; Blanchet and Combes (2009), Heyvaerts (2006), LeMeur (2009), Smolin (2007), for anomalies in the movement of stars and galaxies; Riess and Turner (2004) for dark energy, without entering the details of the discussions on each of these issues.

6. The trajectories of the "Pioneer" satellites

Let us apply the previous orders of magnitude calculations to the motion of the Pioneer satellites. We do not envisage the details of the movements of the two satellites Pioneer 10 and 11 in their process of leaving the solar system. We will only refer to the summaries established by the authors: for distances of the order of tens of billions of kilometres = 10^{13} m, anomalous Δg 's of the order of $8.74 \cdot 10^{-10} \text{ m/s}^2$ are observed. For such distance g verifies: $g = GM/R^2 = 6.67 \cdot 10^{-11} \cdot 2 \cdot 10^{30} \cdot 10^{-26} = 1.3 \cdot 10^{-6}$ where the value of the sun mass has been taken for M . So:

$$\frac{\Delta g}{g} = 6.7 \cdot 10^{-4} \quad (7)$$

In order to compare this value to the prediction with the modified law of gravitation (equation (5) above), we need an order of magnitude of the relative speeds of the satellites to the sun, as well as of the angle between the radius and speed vectors. Authors say that speeds are of the

order of tens of kilometres per second, and the trajectories are a moderate angle with respect to radius vector, some 20° . Let us take $v = 10^4$ m/s and $v^2 = 10^8$ m²/s². One has $\sin 20^\circ = 3.4 \cdot 10^{-1}$ and $\sin^2 20^\circ = 11.6 \cdot 10^{-2}$. So:

$$Av^2 \sin^2 \theta = 5 \cdot 10^{-11} 10^8 11.6 \cdot 10^{-2} = 5.8 \cdot 10^{-4} \quad (8)$$

By comparing (7) and (8) we see that the proposed law gives a good order of magnitude for the observed gravity anomaly, with the chosen value of A constant. Subsequently we may refine these predictions with the precise knowledge of the angles of the trajectories of the different satellites, their speeds and positions, so as to calculate the values of g and of $\frac{\Delta g}{g}$ by Newton's law, and compare with the effect predicted by the $Av^2 \sin^2 \theta$ factor. One expects that:

$$\frac{1}{v^2 \sin^2 \theta} \frac{\Delta g}{g} = cste = A$$

Considering the amount of data available today on the movements of many satellites (including other than Pioneer satellites), many tests are possible in order to bracket the value of A more precisely. One observes that the $\Delta g/g$ value attached to Pioneer 10 is slightly larger than that attached to Pioneer 11, but the trajectory is more inclined. At first glance, this fits the previous laws (5) and followings, but, to fully discuss the matter, accurate calculations taking into account the variations of all the data with time would be needed.

7. The rotation of stars in galaxies

The movements of the stars in galaxies have been studied for nearly one century. The authors have established that the speeds of peripheral stars are the order of 200 to 250 kilometres per second. Taking into account the supposed mass of stars and stellar bodies and its distribution, depending on the distance to the centre of the galaxies, these speeds are not explained by standard Newton's law. All happens as if there was a missing invisible mass, and this mass was estimated the same order of magnitude as, or a little more than, the visible mass. With the same way of writing as before, we will say that $\frac{\Delta M}{M} = \frac{\Delta g}{g}$ is expected to be of the order of

unity. We are in a situation where the speed and radius vectors are perpendicular. If we apply the prediction given by relation (6) to this problem, we have $v^2 = 4 \text{ to } 6 \cdot 10^{10}$, so:

$$Av^2 = 5 \cdot 10^{-11} \cdot 4 \cdot 10^{10} = 2 \cdot 10^0$$

So we find back a factor of the order of unity. Therefore the proposed law allows us to obtain the right order of magnitude for the "missing" mass as announced by the authors in the literature, for the same order of magnitude of A constant that was chosen for Pioneer satellites.

8. Movement of galaxies in clusters of galaxies

The authors in the literature have also studied the motions of the galaxies in galaxy clusters. They have detected anomalies of the expected movements as compared to the standard law of gravitation. In these situations, they speak of a mass deficit equal to the one discussed in previous section, or reaching more than 10 times the visible mass. In the case of the galaxy clusters, the relative velocities of the galaxies reach values of 500 to 800 km/s. We can see that if the speed of the objects is multiplied by a factor 2 or 3 with respect to the previous case, the missing mass is multiplied by the square of this factor, so by a factor of 4 to 9. Then, while the preceding ratio $\frac{\Delta M}{M}$ equated a few units, it can now reach a factor equal to $2 \cdot 10^0 \cdot 5 = 10^1$, that is to say to tens of units (taking a multiplicative factor of 5 for the speed squared). This means that we can account now, thanks to the proposed law, for a "missing" invisible mass ten times larger than the visible mass, and it is always for the same value of A parameter as that accounting for the Pioneer and galaxy star anomalies. We can involve the corrective terms in $\sin^2\theta$ (because the motions of galaxies relative to the others may not verify the perpendicularity between the speed and radius vectors). We see that, with multiplicative factors of 10^1 for the speeds, and angular factors $\sin^2\theta$ of 10^{-1} , the good order of magnitude is kept for the estimated missing mass, as equal to 10 times the visible mass.

If our purpose is primarily to examine the qualitative behaviour and the orders of magnitude for the anomalies, we could show that we can derive from our approach the laws linking the the speeds of moving bodies to the distances to the centre of mass. This can be achieved by

writing the equations in the stationary rotating frame. When compared to the standard Newtonian case, one makes a link between the size and the mass of a galaxy appear (existence of a limit radius for a given mass). Similarly, the proposed -still classical- formalism may apply to compute the limit radius of black holes, as a function of their mass...

9. Accelerated expansion of the universe

Among the issues discussed in the literature, the so-called dark energy is responsible for the "accelerated" expansion of the universe. One should clarify first the meaning of the word "accelerate". In the case of the Big Bang model, galaxies and celestial bodies (supernovae etc.) go away from each other (expansion). The speed of their relative motion decreases in the course of time, because of the hampering by gravitation forces. "Acceleration of expansion", as authors say, actually corresponds to a decrease in the speed that is lower than expected within the standard Newton law. Thus, the objects appear more distant than expected. This effect allows one to speak of a repulsive force. The law we have proposed can contribute to this problem. It can be observed indeed that, for a system of bodies that move away from each other, the angles θ between the radius and the relative speed vectors will gradually decrease (Fig. 2). Without prejudging, from a quantitative viewpoint, of the cross and cumulated effects of the relative speeds of all moving masses (n-body problem for the proposed law), we can conclude that, by getting a parallelism between the radius and the relative velocity vectors, part of the initial attraction effect is cancelled. Therefore this simulates a repulsive force. In the two-body problem, we can write that, at the beginning of the evolution, there is an attraction to the origin, written as $g_I(r)$. Subsequently, the law reads $g_{II}(r)$, it is still attractive to the origin, but, all things being equal (for the same mass at same r), one observes that $g_{II} < g_I$. One can then write $g_{II} = g_I + g_{repulsive}$ where $g_{repulsive}$ is counted negatively. Speed contribution in g_I is equal to $AGMv^2/r^2$ and verifies the same law in $1/r^2$ corresponding to an apparent repulsive mass equal to AMv^2 (the values of the masses and of the initial speeds are unknown and only variations of speeds are observed).

This leads us to a semi-quantitative point of view, expressed in percentage as in the previous sections. We can say that the missing mass ΔM , counted in energy ΔE , and that seems to be missing later in the expanding mass system as compared to the normal initial attraction (the repulsive force) still fulfils a law of the type

$$\frac{\Delta E}{E} = \frac{\Delta M}{M} = \frac{\Delta g}{g} = Av^2 \sin^2 \theta$$

where v is the speed of the escaping object, counted from the origin where mass M is concentrated; θ is the angle between r and v . The velocities of the galaxies in the early stages of the expanding universe could reach a significant fraction of the speed of light, and attain or exceed 100 000 km/s, that is 10^8 m/s. The angles between r and v vectors are difficult to estimate, put a few degrees, with sine of the order of 10^{-1} to 10^{-2} and squared sine of the order of 10^{-2} to 10^{-4} . Take 10^{-3} . These small angles will then cancel. We see then that the factor $Av^2 \sin^2 \theta$ is equal to $5 \cdot 10^{-11} \cdot 10^{16} \cdot 10^{-3} = 5 \cdot 10^2$. In the end we can attain a 10^2 ratio for the apparent repulsive energy (talking of black or dark energy) with respect to the visible energy. This is in agreement with what authors propose in the literature, based on the escaping speed curves as a function of the distance to the observer (it is equivalent to the dependence on the time from the "original" explosion, because looking at far, is looking in the past). We may in the future perform more complete computer simulations and compare the curves $v(t)$ or $v(r)$ to those measured for the escape speeds of galaxies and celestial objects, according to the two assumptions (normal and modified Newton law).

10. Discussion

The foregoing must be considered as preliminary and provisional. A very important work is required to strengthen the presented approach and/or criticise it, in particular in the following directions.

10.1. Two body problem

Our brief discussion mostly dealt with the two-body problem. One should accurately examine within this framework the various situations explained by the standard Newton law and by general relativity (movements of rockets and satellites, planets etc.) and show that, for these examples, the modified law will not give significantly different results (no effect of a higher order of magnitude) than the standard laws, or that the effects are hidden in the uncertainties on the various parameters of the problem: masses, distances, speeds, value of the G constant,

etc. Perhaps this will require a re-evaluation of the standard situations; we are limited in these checkings by the resolution of the n-body problem (see next section).

10.2. N-body problem

The full understanding of the effects of the proposed law necessitates to go through a more general examination of the n-body problem. The resolution of this problem is already needed for the understanding of the movements of the planets in the solar system; this is still more wanted for a galaxy, to simulate as a “gas” of stars governed by laws involving relative velocities. The molecular simulation would be useful. Non-linear effects may arise: we may ask what is the effect of the integration of a distribution of masses with different relative speeds: can we average masses and relative speeds, by concentrating all the mass to the centre in a distribution with spherical symmetry for example, as we do for masses and distances within the Newtonian standard frame (Gauss theorem)? More advanced calculations will help answer this question.

An approximate reasoning shows that the effects predicted by the modified law of gravitation shall not exceed that predicted by standard Newton's law. And, for example in the case of the advance of the Mercury perihelion, one will get close to the corrections made by general relativity. Mercury perihelion advance, or any second order motion of a planet in the solar system, is indeed caused by the attraction by all the other planets and the sun, expressed as the sum of radius vectors weighted by the masses and distances as

$$\frac{d^2 \vec{r}_i}{dt^2} = - \sum_j \frac{G m_j}{|\vec{r}_{ij}|^3} \vec{r}_{ij} = \sum_j \alpha_{ij} \vec{r}_{ij}$$

With the modified law of gravitation, one will add to the combinations with the α_{ij} some other combinations with β_{ij} coefficients such that

$$\frac{d^2 \vec{r}_i}{dt^2} = - \sum_j \frac{G A v^2 \sin^2 \theta m_j}{|\vec{r}_{ij}|^3} \vec{r}_{ij} = \sum_j \beta_{ij} \vec{r}_{ij}$$

Coefficients β_{ij} are smaller than α_{ij} coefficients by a factor equal to $A v^2 \sin^2 \theta$, that may reach the value $A v^2$. That is to say, for planet speeds (relative to each other or relative to the sun) of

a few kilometres to a few tens of kilometres per second, this factor may exceed $5 \times 10^{-11} \cdot 10^8 = 5 \times 10^{-3}$. For Mercury movement relative to the sun ($v = 48 \text{ km.s}^{-1}$) the factor is equal to $A v^2 = 2.3 \times 10^9 \cdot 5 \times 10^{-11} = 10^{-1}$. It remains less than 1. This factor can be weighted by the angles between the radius vectors and the relative speeds of moving objects, that are not equal to a right angle (in particular for the movements of the planets with respect to the other planets). Taking account of a $\sin^2\theta$ factor equal to 10^{-1} to 10^{-2} , the ratios of the β_{ij} to the α_{ij} are of the order of 10^{-2} to 10^{-3} . Such calculations should be made for every pair of interacting masses. In the case of the advance of 40 seconds per century for Mercury perihelion, to be accounted for by coefficients β_{ij} , as compared to the 5600 seconds per century (coefficients α_{ij}), the ratio to be predicted is equal to $40/5600 = 7 \times 10^{-3}$. This is close to the above numbers. We therefore see at first glance that the new law does not go outside the known effects, and is able to get close to the magnitude of second order effects accounted for by general relativity.

10.3. Movements of light photons

Can we apply equation (3) laws to massless light photons? Is inertial and gravitational mass equality, as the basic hypothesis of general relativity, not another way to offer laws that do not depend on masses, in the limit valid for massless bodies? We can temporarily try to test our law for the photons; we then believe useful to simply take $A = 1/c^2$ as in vacuum electromagnetism. A calculation of this type in the Newtonian frame has already been made in the literature; we know that, for the deflection of light by the sun, one then gets half of the observed deviation α (Rougé, 2002). The introduction of the additional term depending on speeds causes a $\Delta\alpha/\alpha$ equal to 1. In the end, one finds then a good order of magnitude for the total deflection of light by the sun (this approximate reasoning amounts to varying the speed of light and should be re-written in a relativistic manner). Deflection of light by galaxies could be studied the same way, and the complete calculation of the influence of their elongated geometry on the movement of light should be performed (one cannot take a spherical symmetry like in the case of the sun). We know that the observed deflection effect is greater than that predicted by general relativity. It can be said that insofar as general relativity is an extension of Newton's law, and as we added a term to it, it can be expected that the effects of the modified law can in some cases be greater than those provided by the existing theories (see next section).

10. 4. The predictions of general relativity

All this leads us to ask the question: why doesn't general relativity (not better than standard gravitation) explain the various effects that we have discussed? This is another way to ask the question of the relations of general relativity with the proposed law. This last one is implicitly based on the relativity theory but in a modified formalism: it is twice three-dimensional and not four-dimensional (time has the meaning of a movement) and it gives symmetric roles to time and space. Relativity theory must be reconsidered on this basis. In this context, the role given to the metric and to the conservation of the ds^2 interval must be redesigned and articulated with the choice of the specific direction of the time-defining movement, and allowing one to change from three dimensional time to scalar time (on this point, see Guy 2010b). The question of the isotropy of the propagation of light (same speed in both directions in a particular location) is connected: this property is clearly not fulfilled in the vicinity of a massive body such as a black hole (centripetal and centrifugal speeds are different, while the postulation of the relativity theory does not differentiate them; see also Elbaz, 1984). We there have clues for the re-consideration of the theory of relativity and the discussion of a large number of questions raised therein (Guy, op. cit.).

The link between gravitation and space and time measurements can be discussed in this context. This link seems as related to context: space and time units are a way to measure the energy that humans must consume to move around on our earth. The efforts made by leg muscles to make a one metre step, or made by the heart muscle to beat one second, propelling the blood to the top of the body, directly depend on the gravity field. Man would define different values of the metre and the second on a planet with a different gravity. Moreover, a microscopic being who would essentially be subject to the electromagnetic forces, instead of gravitation, would define units of space and time in relation to these forces. Once space and time standards are defined through the gravitation field in an equation such as (3), they can be used in the equations that govern other forces, electromagnetic forces for example. Time and space are not a priori substances to discover in nature, they are constructed thanks to the comparison of the movements related to the various forces observed (Guy, op.cit.); as a consequence one can undo the link, which appears fundamental in general relativity, between space and time units and gravitation. Space and time standards can be defined thanks to the movements linked to any force. As a result, gravitation itself loses its privileged status and is

on the same footing as other interactions, and this opens up new perspectives (related to quantum mechanics for example).

10. 5. Other laws proposed in the literature

Various laws have been proposed in the literature to explain the anomalies that we discussed. We cannot review all of them. Especially the so-called "Mond" theory (Milgrom, 1983) caught the attention of the scientific community. It should be noted that, if this last theory does explain the movements of the stars in galaxies, it seems less adapted to account for the movements of the galaxies in galaxy clusters, or the movements of the Pioneer satellites. It does not discuss phenomena related to dark energy, and finally does not appears as related to a conceptual approach: it looks as an ad hoc proposal, even if it is powerful for a number of cases (see also Bruneton, 2007). Among many other theories, string theory does not seem to respond to the various questions raised here (Smolin, 2006).

In the more or less recent past, going back to the end of the 19th century, gravity laws containing terms depending on velocities, and more or less inspired by electromagnetism, have been proposed (for example, Assis, 1989, Heaviside, 1893, Jefimenko, 1994, Ragusa, 1992,...). These laws allow one to discuss in a Newtonian fashion issues related to the movement of the planet perihelion and to the deflection of light by massive stars. Many of these laws have been forgotten because of the later development of relativity. These laws are not strictly of the form we proposed here, and do not distinguish between the movements of the individual particles and that of macroscopic matter as we did (cf. the two electromagnetic theories "in vacuum" and "in matter"). The law proposed by Gruffat (2004) is the same as we proposed here but suffers from the previous limitations; this last author insisted on the use of relative velocities in electromagnetism.

10.6. Perspectives outside physics

Outside physics, the understanding of a profound link between the time and space concepts brings up many interesting perspectives, because time has a role in all disciplines (philosophy, social and human sciences, cognitive science and psychology, etc.). This is not the place to talk about it here. One may for instance refer to Guy (2009b) for a discussion on the time paradoxes and on the formation of social groups in anthropology and sociology.

11. Conclusion

In conclusion, we have proposed in an exploratory way, a new law of gravitation which possesses the following qualities:

- it can predict the right qualitative behaviour for several phenomena corresponding to various contemporary problems in gravitation physics, resulting in the same time for different situations: - in an apparent mass supplement, or: - in a repulsive force for the later stages of a system of expanding masses;
- it can provide good orders of magnitude: - for mass or gravity "supplements" associated with the motion of the Pioneer satellites, of stars in galaxies or galaxies in clusters of galaxies, and:
 - for the value of the repulsive energy in universe expansion; all these predictions are obtained with the same value of the additional parameter. If our modified law withstands future tests, it may avoid the use of dark matter and dark energy;
- it has not an ad hoc character and is part of a coherent conceptual framework; and
- it opens ways of research for re-considering the theory of relativity, in continuity with the modified law of gravitation, and its links with gravitation and electromagnetism.

These reasons encourage us to perform further tests of the proposed law.

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FIGURES

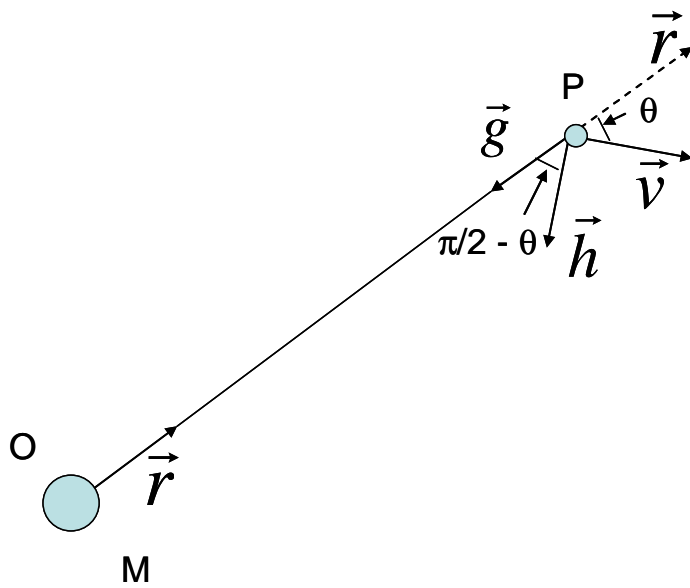


Figure 1

Forces acting at point P on a unit mass moving at speed v , as a result of the attraction by a mass M located at origin O (distance r). The angle between the radius r and the speed v vectors is θ . The standard gravitational g attraction is directed toward O . The new force h is perpendicular to v and located on g side.

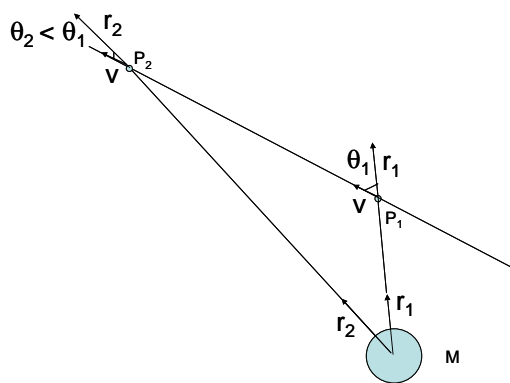


Figure 2

Motion of a unit mass at point P away from a mass M located at the origin (two-body problem); r (radius vector) and v (relative velocity) become parallel. So θ_2 at time t_2 for point P_2 is smaller than θ_1 at time t_1 , for point P_1 ; with time, θ goes to zero. This causes a reduction of the attractive force linked to speed, which, by comparing the later evolution to the early evolution, can be interpreted as a repulsive force (actual angles have the opposite sign, which does not change the reasoning).