The Attractive Force between Two Parallel Currents Explained by Coulomb's Law

Jan Olof Jonson
stmarkstagen 50, SE-12342 Farsta, Stockholm, SWEDEN
e-mail: jajo8088@student.su.se

Experimental evidence questions the widely recognized concept of the Lorentz force. Specifically it fails to account for (1) the repulsive force within Ampère's bridge; (2) Ampère's early copper boat experiment, in which the boat moves away from the contact point where an electric current is applied to a basin filled with mercury; (3) the phase shift between the currents of a primary and secondary circuit in a typical transformer. The fundamental source of the fault is the 'Induction Law', extensively explored elsewhere by this author. This author has even begun to specify alternative conceptual methods to explain the above mentioned phenomena. The effort has succeeded by using Coulomb's Law, carefully taking into account the effects due firstly to the delay of action, and secondly the effects due to Special Relativity Theory (SRT). The Induction Law can successfully be replaced by using Coulomb's Law, as other papers by this author describe. In this paper it is rigorously explored how the attractive force between two electric currents can be explained using Coulomb's Law, thereby applying the effects due to delayed action and SRT. It is also indicated how the hypothetic-deductive practice by Ampère that lead to the definition of 'Ampère's Law' can be given a strict mathematical formulation. The two terms of Ampère's Law appear as the derivative of a serial expansion of the expressions attained using Coulomb's Law, delayed action and SRT altogether. A look into the efforts done by Ampère has been shown elsewhere to be the outcome of a 'guessing procedure' in the best hypothetical-deductive way, though not offering any idea of why his two terms appear. On the contrary, this paper offers a comprehensive understanding, based on a logical chain from a 'first cause', until the final result, where every step can be motivated.

1. Introduction

It is widely recognized that two parallel electric conductors are attracting each other, if the direction is the same. Here it will be shown by using a circuit with the basic structure of Ampère's bridge, how a theoretical result able to predict this can be derived. The model with two parallel conductors of infinite length of course does not correspond to a physical circuit. All currents must return to their origin. The idea of the two parallel conductors with infinite length is apparently an approximation, but a derivation of the formal basis for this would be beneficial for a strict treatment. However, if claiming this, it is by scientific argument also needed a physical circuit upon which the approximations are performed. As a matter of fact this author has already dealt with Ampère's bridge at several occasions and therefore a rigorous mathematical analysis already exists. It is rather simple to re-use these results with respect to this case. However, this author has also been giving good arguments for Coulomb's Law as a better one than Ampère's Law [1, 2, 3]. This will be repeated in this case. This author has succeeded in denouncing Ampère's Law in other papers.

2. Ampére's Law vs. Coulomb's Law

This section presents an analysis of the impact the branches have on the total force. For the convenience a set of Ampère's bridge is shown below in order to make the identification easier when solving the problem with two parallel currents.

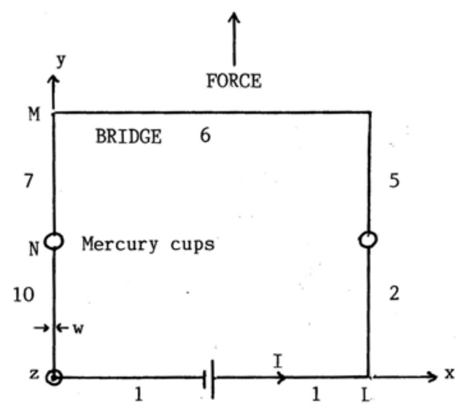

Fig. 1. Ampère's Bridge [1, 2, 3, 6]

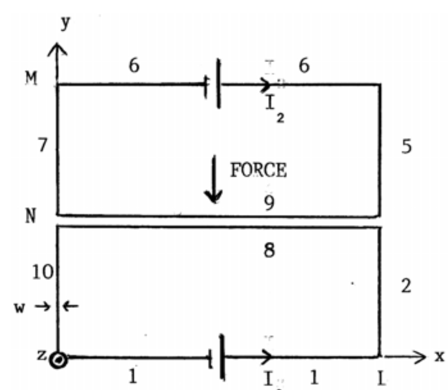

Fig. 2. The configuration according to Ampère's bridge used in order to realize a "two-infinite-parallel-conductor" circuit

When the distance between two of the branches is very small compared to the distances between other branches, it is apparent that the result will be dominated by the closely situated branches. What the contributions are have earlier been rigorously derived in another paper [1] and the expressions will be used straightforwardly, without giving all the details once again.

3. Ampére's Law as an Explanation of Attractive Force between Two Electric Conductors

Ampère's Law predicts the force that a current will affect another one with. The formulas have earlier been given in [1, 2] and are as follows, on differential form:

$$\frac{d^6\vec{F}}{dx_1dx_2dy_1dy_2dz_1dz_2} = \vec{r}\left(-2\frac{\vec{J}_2 \bullet \vec{J}_1}{r^3} + 3\frac{\left(\vec{J}_2 \bullet \vec{r}\right)\left(\vec{J}_1 \bullet \vec{r}\right)}{r^5}\right) \quad (1)$$

$$d^2\vec{F} = I_2 I_1 \vec{r}\left(-2\frac{d\vec{s}_2 \bullet d\vec{s}_1}{r^3} + 3\frac{\left(d\vec{s}_2 \bullet \vec{r}\right)\left(d\vec{s}_1 \bullet \vec{r}\right)}{r^5}\right) \quad (2)$$

These formulas may be applied on an electric circuit, provided the geometry has been adequately defined. In the case that two currents come infinitesimally close to each other, the second formula has to be used (implying thus volume integrals to be used). In the other cases, line integrals will suffice. In the following sections it will be shown what force is expected to arise due to the different combinations of branches.

In this configuration, two of the branches, 8 and 9, are very close to each other, in mathematical terms the distance of order much less than the macroscopic boarders of the circuits, i.e. L, M and N. This makes the force between them to become the dominant term, and hence, the other contributions may be neglected.

3.1. Integral from Branch 1 to 6

This case excludes the effect due to the current source in branch 1 (please see chapter 2.3). The reason for including the analysis of the force between branch 1 and branch 6 is that they have the same geometrical positioning as branch 8 and 9. In this case the result is [1]:

$$d^2\vec{F}_{1\to 6} = I^2\left(2\sqrt{1+\left(\frac{L}{M}\right)^2} - \frac{2M}{\sqrt{L^2+M^2}}\right) \quad (6)$$

3.2. Integral from Branch 8 to 9

This case is similar to the preceding one (i.e. section 2.2.4, 'Integral from branch 1 to 6') in that two parallel conductors are regarded. The result will therefore be alike Eq. (6) above, but with other values of the variables. Since the distance between the two actual branches is only r_{12}, instead of M one should use r_{12}. Hence,

$$d^2\vec{F}_{8\to 9} = I^2\left(2\sqrt{1+\left(\frac{L}{r_{12}}\right)^2} - \frac{2M}{\sqrt{L^2+{r_{12}}^2}}\right) \quad (7)$$

4. The Force between Two Parallel Conductors

As appears from the discussion in the previous section, a circuit, where two parallel conductors are situated infinitesimally close to each others may in a mathematical sense be regarded, as if these two conductors were the only ones in the circuit. This is consistent with the way one is usually treating two parallel conductors. Usually, one uses the Lorentz force Law in order to derive the force, and the fact that Ampère's force Law for this special case arrives at the same result may have inferred physicists to believe that the two Laws are also formally identical. However, in the paper by this author being referred to [1] that treats Ampére's bridge, it is convincingly shown that this is not the case.

4.1. A Non-relativistic Analysis

This case is a classical one being taught within undergraduate courses. The most common way of explaining the attractive force between two current carrying conductors is using the so-called F=BiL rule, based upon a usage of the Lorentz force. Now it can be easily be tested, whether Ampère's Law is an alternative, using the calculations in this very paper. There is a suitable formula for the part of Ampère's bridge, containing two parallel conductors, Eq. (7) above. The most convenient circuit demonstrating the force was described by Neumann [4].

The mentioned equation may be applied using instead of M the radial distance between two parallel conductors, r_{12}. When the distance between the conductors is very small, infinitesimal analysis allows for neglecting terms that becomes very small in comparison to the 'main term'. The term that remains within Eq. (7) is the $(L/M)^2$ term within the left square root. This gives

$$d^2\vec{F}_{8\to 9} \to I^2\left(2\frac{L}{r_{12}}\right) \quad (8)$$

Realizing that there is needed a minus sign if putting the directions of the two currents the same, the force between the currents becomes negative,

$$d^2\vec{F}_{8\to 9} \to -I^2\left(2\frac{L}{r_{12}}\right) \quad (9)$$

which indicates a negative, hence attractive force between the currents, in good accordance with experience. This seems to imply that the 'cross product Lorentz's Law' is unnecessary.

Ampére's Law also has the benefit of being able to explain the repulsive force in the mercury basin between the electric poles and a copper boat described by Ampère [5], which the Lorentz' force Law is unable to.

5. Coulomb's Law as an Explanation of Attractive Force between Two Electric Conductors

5.1. A Paradox

It may also be mentioned that Coulomb's Law cannot account for the attractive force. The force namely becomes repulsive between two parallel conductors, as appears when looking at the equation, here repeated for convenience. As described in [1, ch. 5.2], Coulomb's Law is represented by the 'b-term' (times one-third). The treatment in this paper obeys the original paper on Ampère's bridge by this author [3], but it has the benefit of having included all the necessary, though tedious, steps. This is intended at saving the time of the readers.

$$d^2\bar{F}_{1\to 6,b} = I^2\left(-2\sqrt{1+\left(\frac{L}{M}\right)^2}+4-\frac{2M}{\sqrt{L^2+M^2}}\right) \tag{10}$$

Making the same infinitesimal analysis as with respect to Eq. (7) above, namely gives a minus sign in front of the $(L/M)^2$ term within the left square root and after having changed sign, in analogy with above, the final sign with respect to two parallel current flowing in the same direction becomes positive, hence a repulsive force. This would seem to prove the unfeasibility to use Coulomb's Law in the actual experimental situation, and, therefore disproving the whole theory. At the disproval of a theory in one sole case namely implies that the theory is false. However, it must be kept in mind that thus far the Special Relativity Theory (SRT) has not been applied. The mere fact that the electron velocities are extremely low compared to the speed of light makes one easily a victim of the temptation not to investigate the effects of the SRT at all.

As will be seen in the following section, the usage of the SRT will infer another result to arise, one that satisfies the sign requirement (i.e. an attractive force between the two parallel currents).

5.2. The Solution via Special Relativity Theory (SRT)

What has been left to be done is a relativistic analysis. If a non-relativistic approach is insufficient in order to explain a physical phenomenon involving velocities, it is reasonable to se to what extent an application of the Special Relativity Theory (SRT) will change the result. The need is in this case felt acute, since it does not exist any explanation to Ampère's Law. Efforts to relate Ampère's Law to the Lorentz force have been made, but have been disproved [2]. In the preceding section it was shown that that Coulomb's Law did not succeed using non-relativistic methods. So far this constitutes a huge problem : Simultaneously it is impossible both to derive Ampère's Law and to use Coulomb's Law. Ampère's Law is rather arbitrarily defined by Ampère as an ad hoc definition and Coulomb's Law predicts the wrong sign of the force. Admittedly, Keele [6, 7] has made an effort to use apply the Special relativity theory(SRT) on Coulomb's Law, but his analysis has appeared to be incomplete with respect to retarded action and to the usage of the SRT [8]. The thus far best solution will appear if taking into account all known facts concerning geometry, retarded action and the SRT. It appeared to be rather complicated to do in reality, but if doing every part in a very strict mathematical way, success followed.

5.3. Four Contributions the Force between Conductors

Since each conductor has both immobile positive ions and moving electrons, there will appear four separate force terms due to the four ways the charges of the first conductor may interact with charges of the second conductor. In order to make the properties of the forces easily conceivable, the analysis is restricted to the simplistic case with two parallel conductors. This means that the angle with which a distance vector crosses both the conductors, ϕ and ψ, are equal, even though the simplification is not immediately conducted. In this case the so-called Standard Configuration of the SRT may be applied, with one system K, which is stationary with respect to the positive ions of the two conductors, hence the whole circuitry, and K′ following the movement of the electrons of the first conductor, along the x_1 axis, and with velocity v_1. The velocity of the electrons of the second conductor is assumed to be v_2. In this analysis only the simplistic case with equal velocities, $v_1 = v_2$, will be regarded, when comes *t*.

These definitions make it possible to compare the effects of the electrons of respective conductor on each other.

The focus in the analysis is on the force perpendicular to the currents, which is also the force that acts repelling or attracting between them.

The fundamental assumption that is made here is that Coulomb's Law is the only cause behind the force between electric charges, which implies that the very idea of magnetic fields is abandoned. It is necessary to emphasize that, since it is often claimed that Coulomb's Law is consistent with the existence of specific magnetic force terms within the expression of the total electromagnetic force.

What brings about a change of the shape of the expression of the force compared with the original Coulomb's Law is

1. The effects of the SRT (i.e. Lorentz transformation and derived expressions)
2. Delay effects concerning the retarded observation of fields being generated by charges

When taking into account the Special relativity theory, it is extremely important to very strictly define to which coordinate system the variables are referring to. Primed or unprimed variables will indicate whichever system is used.

5.4. The Force between Positive Ions of a Conductor

Since no velocities are involved, when deriving the force that the positive ions give rise to, the force can easily be attained by writing down plain Coulomb's Law for the case, when the charges are regarded as thin linear elements.

$$d^2F_{y,p1\to p2} \cong k\frac{\rho_1\rho_2\Delta x_1\Delta x_2}{r^3}y \tag{11}$$

5.5. The Force between the Electrons of a Conductor

In order to make the calculation of the force as easy as possible, it is favorable to regard the electrons of either system as being at rest. For convenience the *K′* system is chosen.

Being situated at the electrons of the second conductor, the combined effects of respectively retardation and Lorentz transformation will give the result:

$$d^2F_{y,e1\to e2} = \frac{k}{r^3}y\rho_1'\left(1-\frac{v_1}{c}\cos\theta\right)\rho_2'\left(1-\frac{v_2}{c}\cos\psi\right)dx_1dx_2 \tag{12}$$

where
$$\rho_1' = \rho_1\gamma(v_1) \tag{13}$$

and
$$\rho_2' = \rho_2\gamma(v_2) \tag{14}$$

These expressions will be used throughout the analysis.

When analyzing currents in conducting wires, it may be assumed that the velocities v_1 and v_2 are equal, say v.

5.6. The Force from the Electrons of the First Conductor to the Positive Ions of the Second Conductor

In this case the delay factor will be due to the electrons of the first conductor, when regarding this situation from the K system. This must be multiplied to the force as felt by the electrons in K' moving with velocity v_2 with respect to K

$$F_{y,e1\to p2} = -\frac{k}{r^2}\rho_1{}'dx_1(1-\frac{v_1}{c}\cos\theta)\rho_2 dx_2 \tag{15}$$

5.7. The Force from Positive Ions of the First Conductor to the Electrons of the Second Conductor

In this case the delay factor will be due to the electrons of the second conductor, when regarding this situation from the K system. Accordingly, the contribution to the total y force will in this case be

$$F_{y,p1\to e2} = -\frac{k}{r^2}\rho_1 dx_1\left(1-\frac{v_2}{c}\cos\theta\right)y\rho_2{}'dx_2 \tag{16}$$

5.8. The Sum of the Four Contributions to the Force between Two Conductors

Summing all four contributions, assuming also the simplest case with equal velocities, gives after some boring steps.

Using serial expansion of the expression gives

$$d^2F_{y,total} \cong \frac{k}{r^3}y\rho_1\rho_2\left(\frac{v}{c}\right)^2(\cos\phi\cos\psi-1)dx_1 dx_2 \tag{17}$$

Since the two parallel conductors were assumed to be of infinite length, the integration of the mixed cosine term will, being basically an odd function, putting thereby the origin of the axes in the middle, cancel. Hence

$$d^2F_{y,total} \cong -\frac{k}{r^3}y\rho_1\rho_2\left(\frac{v}{c}\right)^2 dx_1 dx_2 \tag{18}$$

and, after integration, a result equal to Eq. (9) above.

This results are in accordance with the attractive force felt between two parallel currents.

6. The Induction Law Disproved

Fundamental to rejecting the usability of the Lorentz force is to disprove the very basis. One example was given when investigating the 'Induction Law' [9, 10]. After a thorough analysis of an ideal transformer circuit, it appeared that factually, the phase shift displayed on an oscilloscope screen, or shown by any other means, is 90 degrees. Turning thereafter to Coulomb's Law, the phase shift was correctly predicted. This gives an advantage to the latter assumption, of course.

7. Ampère's Copper Boat

Ampère performed an experiment consisting of a double basin filled with mercury, upon which a catamaran made of copper was attached, an electric current attached at one end [5, 11]. The catamaran shaped boat floats along the direction of the current, in spite of the fact that the Lorentz force predicts no force in this direction. As has been shown by this author, Coulomb's Law does [3].

8. Conclusion

It has been possible to show that the force between two parallel currents is due to the combined effects of retardation and the Lorentz transformation.

In the above analysis, which is preliminary, secondary effects of the Lorentz transformation have been deliberately omitted, as the Lorentz transformation of the angle for example.

What has effect on the result is the first order terms of respective velocity.

It is interesting also to remark that the above analysis fits also with earlier made analysis of Ampère's bridge by this author [1, 3]. In that case $v_2 = v_1$ and therefore the Lorentz term disappears and only the retardation terms remain.

In the above case, the currents are held parallel to each other, in order to make an analysis of the most famous case of force between two currents. It may easily be observed that if the angle between the conductors would be brought to approach 90 degrees, the result will be zero due to the fact that there is no Lorents transformation in this direction. In this sense, too, the result is completely in accordance with experience.

9. Units (SI)

x_1, y_1, z_1 x_2, y_2, z_2	Cartesian variables of each respective branch
I	current
I_1, I_2	current, with indices, due to the respective number of circuit
$\bar{F}$	force (vector)
$\bar{J}_1$, $\bar{J}_2$	current density (vector)
$d\bar{s}_1$, $d\bar{s}_2$	infinitesimal element of the respective conductor
r, $\bar{r}$	distance between the two current elements (magnitude and vector)
r_{12}	distance between two parallel conductors when regarded as small compared to the other distances of the circuit
$\bar{F}_{10\to 5}$	force due to the effect of one branch upon another (here 10 and 5 respectively)
$\bar{F}_{2\to 7,b}$	force due to the effect of one branch upon another (here 2 and 7 respectively), the effect of the 'second (i.e. 'b') term'
$d^2F_{y,-to+}$	y component of the force due to the effect of one branch upon another (relativistic analysis; the force from electrons of the first conductor on the positive ions of the other)
ρ_1, ρ_2	charge density of the first and second conductor respectively
$\gamma(v_1)$, $\gamma(v_2)$	Lorentz ('gamma') factor of respective current due to the velocity of the electrons

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