The Major Player in Special Relativity is the Metric Tensor

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This article argues that the real basis of Special Relativity is the Lorentz Metrics and illustrates this idea on the example of 1-dimensional geometry.

1. The Meaning of Special Relativity (SR)

Special Relativity is about how to describe a physical reality by using mathematical (numerical) construct known by the name "coordinate system". The numerical data in this system is not obtained by using the corresponding "frame" and measuring with meter-stick and clock. This data is assigned by us. Only this assignment is done so that any past/future possible experiments will be in numerical agreement with the predictions obtained from our description. So, we never "go out", build "frames", and we do not need any natural body that happen to be there and happen to be inertial. The coordinate system is always at our disposal – it belongs to the raum of "ideas". But the coordinate system as a mathematical idea needs to have a metric tensor to be complete. In this respect it is different from "frame". The "original" orthogonal and rectilinear coordinate system (t,x,y,z) or (x_0,x_1,x_2,x_3) needs to be assigned with a Lorentz Metrics η_{ik}:

$$\eta_{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \mathrm{diag}\begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix} \tag{1}$$

$$ds^2 = \eta_{ik} dx^i dx^i = dt^2 - dx^2 - dy^2 - dz^2$$

We assume $c = 1$. By η_{ik} we mean the particular (Lorentz) form of the second rank covariant metric tensor g_{ik}, which we consider in general tensor algebra. This "original" coordinate system gives us access to the physical "absolute" 4-space (space-time). By the term "original" I mean "taken by us at first (before any coordinate transformations). Also it means that the metrics has to be assigned. When we later make a coordinate transformation, the metrics in a transformed coordinates will be transformed from the "original" metrics. By the term "absolute" I mean that the 4-space is the same in whole world and cannot be influenced by matter or fields (notice that this statement denies any physical meaning behind GR). This particular form of metrics η_{ik} defines the "light cone" in every point of 4-D space. It gives us the constancy of the speed of light. It provides us with an inertial coordinate system. The act of taking η_{ik} for our original coordinate system is a very responsible act. It is because that after taking η_{ik} the whole SR is a consequence (no Einstein's postulates necessary – they are just consequences). The invariance of η_{ik} with respect to Lorentz Transformations is a consequence of (1). Everybody knows that (who knows tensor algebra). But it looks that nobody is willing to admit that the actual basis of SR is not the Einstein's postulates. The actual basis is Lorentz Metrics η_{ik}, and it follows from the Maxwell's Equations.

2. One-Dimensional Geometry (Number Axis)

To understand SR one needs to know Riemann n-D Geometry. I am going to illustrate the basic geometrical concepts on the example of One-Dimensional Geometry.

What does it mean "Geometry"? It means identifying some objects from physical reality by some unique mathematical constructions (in particular by numbers). The perfect example of Geometry would be the system of Social Security Numbers in USA. It is an example of one-dimensional discrete geometry. But let us consider one-dimensional continuous geometry. Suppose that we have some real variable x that changes continuously from -inf to +inf (it is a mathematical construct that belongs to the raum of Ideas). What can we offer on the physical side? We cannot offer a string made from matter because matter is discrete (contains of atoms). We have to assume some physical continuous line so that each physical point on this line corresponds to a definite real number x. It looks like one dimension of space. Did we make a discovery right here? Yes we did. The discovery of real numbers "led" to the discovery that space is continuous.

Suppose now that our one-dimensional space has a "metrics" property. Suppose that if a two points are given on the line then a distance between these points is definite. Suppose that the metrics is $g = 1$, which will mean that the distance between the 2 given points equals the difference of their coordinates: $ds = dx$. Let us now transform the coordinate arbitrary (but unique with unique inverse):

$$x' = x'(x), \quad x = x(x') \tag{2}$$

We have: $ds = \frac{dx}{dx'} dx' = g' dx'$. We have a new coordinate x' and a new metrics g' (which in general is not equal to 1) for the very same physical object. It is very important to understand that in Riemann Geometry we never change the physical objects. We only "play" with their mathematical description.

Now let us ask a 1000 dollar question: Can we find such a transformation $x' = x'(x)$ that is different from $x' = x$ and g' still equal to 1? The answer to this question is: only translation $x' = x + a$ (where a is a constant) can do that. Translation won't change the metrics in any dimensions. But even in 2 dimensions we have more possibilities. If we have the original metrics positive on x and on y – then a rotation by arbitrary angle won't change the metrics. If the original metrics has a different signs on x and on y – then a Lorentz transformation with arbitrary velocity won't change the original metrics. So far it is pure algebra. But may be again we made a discovery? Yes we did! Real physical space appears to cooperate with that. And it led to another discovery when we consider vectors and tensors. In n-dimensional space the transformations of coordinates are always arbitrary. But everything else transforms “accordingly”. Knowing the law of transformation allows us to classify other “mathematical objects”. An example: 3-D space and time do not constitute independent objects because their transformation interdependent. But both together constitute the mathematical object (4-D space) with a definite transformation law. This is in the raum of ideas (mathematics). But we project that in the raum of reality. It is a discovery. Vectors and tensors are also mathematical objects because they transform according to the definite transformation laws. So, they can represent reality.

Yes, some algebra in the raum of ideas led to discoveries in the raum of reality. But certainly, not any algebra can do that. Riemann Geometry is the mathematics that every physicist has to know [1].

References

[1] Ref?