# Centron – One of the Simplest Particles of Classical Electrodynamics (CED)

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**Abstract:** The name "Centron" was given because of complete central symmetry of this solution. It consists of inside region and spherical boundary. Both are carry charge and energy. The boundary has chosen to be in a stedy equilibrium because we do not know how to get solution in the case of oscillating boundary. The total charge and energy (mass) of the Centron are constant while they have some oscillating parts if the surface is considered saparately. Centron radiates the longitudinal spherically symmetric dummy waves in vacuum at a stedy rate. Obviously, the spin (or magnetic moment) of Centron is zero.

## 1. Introduction

The conventional Classical Electrodynamics (CED) is the "field" theory when it comes to electromagnetic field and even field of density of current. But it remains a "global" theory when it comes to the Dynamics part because it relies on Newton's Dynamics of material point (certainly improved to the contemporary Relativistic Dynamics). An elementary particle is a complicated structure that consists of a boundary with fields inside and outside of this boundary. The model of material point is a very rough representation of a particle. It has an integrated ("global") charge, integrated ("global") energy (mass). It is not a "field" therefore we have a kind of inconsistency in conventional CED.

How to make the dynamical part of CED consistent with fields? The one way is to take many material points and avarage them as a density of mass not changing the dynamics law. This way we are getting a "fluid" dynamics. It is a "field" dynamics that works for macroscopic fluids but it does not work if we want to go inside the elementary particle. **Newton's Dynamics applicable only to an elementary particle as a whole.** It works for macroscopic fluids only because these fluids actually are not continuous. They are built of elementary particles. The fields of densities and velocityes are not real – they are statistical representations.

The other way is to extend CED is by abandoning the Newton's law and introducing a new field dynamics (Dynamics of Material Continuum, see [1] ). The present paper is a further demonstration of how the new Dynamics of Material Continuum works.

## 2. A Static Central Symmetric Part

We claim that in CED the electromagnetic potential $A^k$ is uniquely represent the physical reality. Here we assume that the particle has a static spherical boundary (we do not know yet how to find a solution in the case if the particle's boundary is not static). Basically we impose the static spherical boundary on the solution as it did Abraham when he described the electron's model. Abraham (and Pauncare) claimed some unknown nonelectromagnetic forces that hold the static sphere in position. Now we claim that the new dynamics can hold the static sphere in position. In our case the equations to be satisfied are (see the same article):

$$A^k{}_{|k} = 0 \quad (1) \qquad (A_{in}{}^{k|b}{}_{|b} - k_0^2 A_{in}{}^k)^{|i} - (A_{in}{}^{i|b}{}_{|b} - k_0^2 A_{in}{}^i)^{|k} = 0 \quad (2) \qquad A_{out}{}^{k|a}{}_{|a} = 0 \quad (3)$$

where vertical lines before indexes mean covariant/contravariant derivativws. Let us consider the solution:

$$A_{in}^0 = \alpha\left(R_0(x) - R_0(x_1) + x_1 R_1(x_1) + q\right), \quad 0 \le x \le x_1$$
$$A_{out}^0 = \alpha(x_1 R_1(x_1) + q)\frac{x_1}{x}, \quad x_1 \le x < \infty; \quad x = k_0 r \qquad (4)$$

Where $R_0(x)=\sin(x)/x$, $R_1(x)=\sin(x)/x^2-\cos(x)/x$ are the spherical Bessel functions, and $x_1$ correspond to the radius of the sphere. This solution starts at some constant value at x=0, it is continuous on the sphere, and it comes to zero at infinity. The calculated values for this solution are:

$$E_{in}^r = \alpha k_0 R_1(x), \quad 0 \le x \le x_1; \quad E_{out}^r = \alpha k_0 (x_1 R_1(x_1) + q)\frac{x_1}{x^2}, \quad x_1 \le x < \infty$$
$$j_{in}{}^0 = \frac{k_0^2 c}{4\pi}\alpha R_0(x); \quad Q = \frac{\alpha}{k_0} x_1 (x_1 R_1(x_1) + q); \quad Q_{surf} = \frac{\alpha}{k_0} x_1 q$$
$$\frac{8\pi}{k_0^2} T_{in}^{00} = \alpha^2 (R_1^2(x) - R_0^2(x)); \quad \frac{8\pi}{k_0^2} T_{out}^{00} = \frac{\alpha^2 x_1^2}{x^4}(x_1 R_1(x_1) + q)^2$$
$$\frac{8\pi}{k_0^2} T_{in}^{11} = -\alpha^2 (R_1^2(x) + R_0^2(x)); \quad \frac{8\pi}{k_0^2} T_{out}^{11} = -\frac{\alpha^2 x_1^2}{x^4}(x_1 R_1(x_1) + q)^2 \qquad (5)$$

(see the same artcle). It has a volume charge density inside the sphere and a serface charge density on the sphere (delta-function as you go along the radius). The constant q defines the serfice charge. As the result of a surface charge the electric field has a corresponding jump.

The double time component of the energy-momentum tensor is energy density and can be integrated over the volume giving the whole energy/mass of the static particle:

$$mc^2 = \frac{\alpha^2}{2k_0}\left(-x_1^2 R_0(x_1) R_1(x_1) + x_1 (x_1 R_1(x_1) + q)^2\right) \qquad (6)$$

At present we do not know how to calculate the energy of the serface of the particle directly. It is not included in (6) which is only the volume integral. But we will calculate it later.

The double radius component of the energy-momentum tensor is the flow of linear momentum because:

$$\frac{\partial}{c\partial t}\int_V T^{10}dV = -\int_\Sigma T^{11}d\Sigma_1 \tag{7}$$

where radius represented by index 1, and Σ is a closed surface (also see [1]). The conservation law (7) gives us the dynamics of the solution. Suppose Σ encloses some unit square piece of the particle boundary so that one side of Σ is inside the particle (at $x_1-\delta$) and the other side is in vacuum (at $x_1+\delta$, where δ is small). Then we have:

$$\frac{\partial}{c\partial t}\int_V T^{10}dV = -\int_\Sigma T^{11}d\Sigma_1 = T^{11}_{in}(x_1) - T^{11}_{out}(x_1) = \frac{\alpha^2 k_0^2}{8\pi}\left[\frac{q^2}{x_1^2} + \frac{2q}{x_1}R_1(x_1) - R_0^2(x_1)\right]$$

where the left side is the pressure (force on unit square) directed from inside of the particle and applied to the boundary surface. This pressure can be chosen to be zero (by choosing q and $x_1$) or not zero (in which case we assume that the surface of the particle has tension which compensates this pressure).

### 3. An Oscillating Central Symmetric Part

Above is the simplest (scalar) solution that pertains to the Ideal Particle (IP). We are unable to find a simple check of the stability of our solutions. Presumably the IP is unstable (with respect to a perturbation). The oscillating part is more complicated vector solution. But if we will get over it then we can hope that some day we will find the solution for the real electron. At first we need to state some general facts about how to build a vector solution out of a scalar solution. Now we need the 3 different variables that connected to the same radius: $x=k_0r$, $z=k'r$, $y=kr$, where $k=\omega c$, $k'^2=k_0^2+k^2$. We suppose that all functions depend on time in an oscillatory manner having the multiplier exp(-ikct). We are using the complex representation remembering that we have to take real parts at the end of the day. Suppose we have a scalar solution of a generalized Helmholtz equation:

$$\Phi(z,t) = \Phi(z)e^{-ikct}; \quad \Phi^{|a}{}_{|a} - k_0^2\Phi = 0; \quad \Phi^{|a}{}_{|a} = -\Delta\Phi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi; \quad \Delta\Phi + k'^2\Phi = 0 \tag{8}$$

Then we can consider a longitudinal vector: $\vec{L}(z,t) \equiv \frac{1}{k'}\nabla\Phi(z,t); \quad \Delta\vec{L} + k'^2\vec{L} = 0$

and build up a solution of (2) taking: $A^0 = \alpha\Phi; \quad \vec{A} = \frac{\alpha}{k'}\dot{\vec{L}}$ where dot means time derivative. This solution will also satisfy to the conservation law (1) (can be checked). The other way to get a solution of (2) is to take a dummy wave:

$$A^k(y,t) = G^{|k}; \quad G^{|a}{}_{|a} = 0; \quad G(y,t) = G(y)e^{-ikct}; \quad \Delta G + k^2 G = 0$$

It can be checked directly that dummy wave satisfies (2) and (1). So, we claim that the oscillating part of Centron is:

$$G_{in} = \delta_1 R_0(y); \quad G_{out} = \delta_2 h_0^+(y); \quad h_0^+(y) = -\frac{i}{y}e^{iy}; \quad h_1^+(y) = -(\frac{i}{y^2} + \frac{1}{y})e^{iy}$$
$$A_{in}^0 = \delta R_0(z) - \delta_1 ikR_0(y); \quad A_{out}^0 = -\delta_2 ikh_0^+(y)$$
$$A_{in}^r = i\frac{\delta k}{k'}R_1(z) + \delta_1 kR_1(y); \quad A_{out}^r = \delta_2 kh_1^+(y)$$
$$E_{in}^r = \frac{\delta k_0^2}{k'}R_1(z); \quad E_{out}^r = 0; \quad j_{in}^0 = \frac{k_0^2 c\delta}{4\pi}R_0(z); \quad j_{in}^r = i\frac{k_0^2 c\delta k}{4\pi k'}R_1(z) \tag{9}$$

The both components of potential have to be continuous on the sphere. That brings us 2 conditions at $r=r_1$:

$$\delta_2 = \delta y_1^2[\frac{1}{k}R_0(z_1)R_1(y_1) - \frac{1}{k'}R_1(z_1)R_0(y_1)]$$
$$\delta_1 = \delta_2 + i\delta y_1^2[\frac{1}{k}R_0(z_1)N_1(y_1) - \frac{1}{k'}R_1(z_1)N_0(y_1)] \tag{10}$$

where $N_0=-\cos(y)/y$, $N_1=-\cos(y)/y^2-\sin(y)/y$ are spherical Neumann functions. We see that the whole oscillating central symmetric part is self consistent with one arbitrary constant $\delta$. The calculated constants $\delta_2$ is real while $\delta_1$ is explicitly complex. A very important note: the whole oscillating solution would be impossible without dummy waves inside and in vacuum. From (10) it follows that given the $\delta$, the $\delta_1$ and $\delta_2$ are definite. The whole physical meaning of dummy waves is to participate in fulfilling the boundary conditions. This is an argument against those who would say that dummy waves do not exist because they do not carry any energy.

## 4. The Whole Solution and Results

Since the equation (2) is linear we can add the both static and oscillating solutions obtaining what we call Centron. In the outside region we have only static electric field and outgoing dummy waves. The amplitude of dummy waves is proportional to $k\delta_2$. In the inside region we have to take the real parts of the fields:

$$E_{in}^r = \alpha k_0 R_1(x) + \delta\frac{k_0^2}{k'}R_1(z)\cos(kct)$$
$$j_{in}^0 = \alpha\frac{k_0^2 c}{4\pi}R_0(x) + \delta\frac{k_0^2 c}{4\pi}R_0(z)\cos(kct); \quad j_{in}^r = \delta\frac{k_0^2 kc}{4\pi k'}R_1(z)\sin(kct) \tag{11}$$

We can find the charge of the surface as a total charge minus integral of the charge density over the volume. Using (11) we found:

$$Q_{surftot} = \frac{\alpha}{k_0} x_1 q - \frac{\delta k_0^2}{k'^3} z_1^2 R_1(z_1)\cos(kct) \tag{12}$$

Using (11) we can express the energy-momentum tensor:

$$\frac{8\pi}{k_0^2} T_{in}^{00}(r) = \alpha^2 [R_1^2(x) - R_0^2(x)] + 2\alpha\delta [\frac{k_0}{k'} R_1(x)R_1(z) - R_0(x)R_0(z)]\cos(kct)$$
$$+\delta^2 \left\{ [\frac{k_0^2}{k'^2} R_1^2(z) - R_0^2(z)]\cos^2(kct) - \frac{k^2}{k'^2} R_1^2(z)\sin^2(kct) \right\}; \quad \frac{8\pi}{k_0^2} T_{out}^{00}(r) = \frac{Q^2}{k_0^2 r^4}$$
$$\frac{4\pi}{k_0^2} T_{in}^{01}(r_1) = -\frac{\delta k}{k'} [\alpha R_0(x_1) + \delta R_0(z_1)\cos(kct)] R_1(z_1)\sin(kct)$$
$$\frac{8\pi}{k_0^2} T_{in}^{11}(r_1) = -\alpha^2 [R_0^2(x_1) + R_1^2(x_1)] - 2\alpha\delta [\frac{k_0}{k'} R_1(x_1)R_1(z_1) + R_0(x_1)R_0(z_1)]\cos(kct)$$
$$-\delta^2 \left\{ [\frac{k_0^2}{k'^2} R_1^2(z_1) + R_0^2(z_1)]\cos^2(kct) + \frac{k^2}{k'^2} R_1^2(z_1)\sin^2(kct) \right\}; \quad T_{out}^{11}(r_1) = -T_{out}^{00}(r_1) \tag{13}$$

Now we want to remember that the static spherical surface was imposed on the solution at the very beginning. Therefore we do not want any oscillating pressure applied to the surface from inside of the particle. Looking on double radius component of energy-momentum tensor we see that we can satisfy that if we reqire:

$$\frac{k_0}{k'} R_1(x_1)R_1(z_1) + R_0(x_1)R_0(z_1) = 0; \quad [\frac{k_0^2}{k'^2} R_1^2(z_1) + R_0^2(z_1)] = \frac{k^2}{k'^2} R_1^2(z_1) \tag{14}$$

The $k_0$ is given because it is the constant of the theory. We can choose r1 and k so that (14) will be satifyed. The transcendent equations (14) have planty of descrete roots. After that only the constant pressure will remain. After satisfying (14) the constant pressure that is applied from inside of the particle to the particle surface will be:

$$p = \frac{k_0^2}{8\pi} \left[ \alpha^2 R_0^2(x_1) + \delta^2 \frac{k^2}{k'^2} R_1^2(z_1) - \frac{\alpha^2}{x_1^2}(q^2 + 2qx_1R_1(x_1)) \right] \tag{15}$$

We can clearly see that the surface charge q can help to make it zero if nesessary. This constant pressure has to be neutralized by the serfice tension. We can connect this pressure to the constant part of the serface energy. Suppose the spherical boundary had a little expansion in radius dr. That means that the pressure p made some work expanding serface. The expansion of the serface is: dS=8πrdr. The total energy of the surface will increase proportional to that. Comparing these things we can find that the constant part of the serface energy has to be: $2\pi(r_1)^3p$.

The integration of the energy density over the volume gives:

$$mc^2 = \frac{Q^2}{2r_1} - \frac{\alpha}{2k_0} x_1^2 R_0(x_1) R_1(x_1) - \frac{\alpha\delta k_0}{k'^2} x_1 z_1 R_0(x_1) R_1(z_1)$$
$$+ \frac{\delta^2 k_0^2}{4k'^3}\left[\frac{k_0^2 - k'^2}{k'^2} z_1^3 (R_0^2(z_1) - R_1^2(z_1)) + \frac{k'^2 - 3k_0^2}{k'^2} z_1^2 R_0(z_1) R_1(z_1)\right] - SE$$
$$SE = -\frac{\delta}{k'} x_1^2 R_1(z_1)\left[2\alpha R_0(x_1)\sin^2(kct/2) + \frac{\delta}{2} R_0(z_1)\sin^2(kct)\right] \qquad (16)$$

This is only volume integral and the energy of the surface is not included. Here SE is just a notation of the oscillating part. The time-radius component of the energy-momentum tensor defines the flow of energy from the inside of the particle to the surface. We can integrate it over time and find how much energy was aquired by the surface during some time. We did the integration and it appears coincide with SE (as it should be expected due to the conservation of energy): $4\pi r_1^2 \int_0^t T_{in}^{01} dt = SE$. Now we can find the Total Energy of the Surface (TES):

$$TES = \frac{x_1^3}{4k_0}\left[\alpha^2 R_0^2(x_1) + \delta^2 \frac{k^2}{k'^2} R_1^2(z_1) - \frac{\alpha^2}{x_1^2}(q^2 + 2qx_1 R_1(x_1))\right] -$$
$$-\frac{\delta}{k'} x_1^2 R_1(z_1)\left[2\alpha R_0(x_1)\sin^2(kct/2) + \frac{\delta}{2} R_0(z_1)\sin^2(kct)\right] \qquad (17)$$

From (16) and (17) one can figure out what will be the total (not oscillating) mass of the Centron.

Notice about the constants. The $k_0$ is given because it is the constant of the theory. The k and $r_1$ can be chosen (in the case of non oscillating boundary) from the infinite set of the solutions of (14). The α, q, and δ can be chosen arbitrary subject to total charge, total mass, and amount of outgoing dummy radiation.

**References**

[1]. http://www.ptep-online.com/index_files/2008/PP-12-02.PDF