The Structure of Physical Mass: Introduction to Self-Momentum Theory

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In this paper, an original interpretation of the nature of physical mass is discussed. The well-known results of physics, to date, can be easily verified, if we assume that mass is not a scalar, but, in general cases, is a two-dimensional vector. The first part of this study shows that the inertial force is not only able to associate with the mass, but with equal physical quantities such as distance and frequency. Louis de Broglie's matter-wave hypothesis has been completed with the wave amplitude and frequency of the matter-wave. The introduction of these new concepts allows a simple calculation of the lepton masses. In the light of these new insights, by the generalization of Planck's radiation law, we have determined the ground states of the atomic masses of the periodic table. From a simple two-dimensional vector model of the physical mass, we introduced a special *mass-oscillator* concept, which is usable for a more precise foundation of the long understood *nuclear shell-model*. Supposing that the Newtonian gravity law is valid for the microscopic quantum particles, we have given a simple quantum mechanical model for gravity. We have investigated the gravity law between commensurable masses. From a theoretical aspect, we have shown, that the force of gravity must vanish between equal masses. In our earlier experiment, we have already observed the minimum exchange of the gravitational energy between equal masses.

1. Introduction

Mass is perhaps, the most important concept in physics. There are three reasons why this is so. Associated with mass, are the functions/attributes of inertia, weight and energy. A precise mathematical definition of mass began with Newton, whose fundamental law of dynamics is a proportionality factor. Newton assumed that the inertial mass and gravitational mass are strictly proportional. This principle is used in the mass balance measurements. Einstein's general theory of relativity (GRT) is also built on this principle, which was experimentally demonstrated, with high accuracy, for the first time in the world by the great Hungarian physicist, Roland Eötvös, with his special torsion pendulum.

This mass measure occurred, using simple balances based on the static method of every-day practice. But, in principle, there may be other ways to make this measurement, such as, when a mass is moving in a circular orbit and we have some kind of dynamometer (for example, spring load) to measure the 'centrifugal force', which is also proportional to the inertia of the mass.

In Newtonian dynamics, if the moving mass is in circular orbit, the acting centrifugal force (the negative of centripetal force) is the following:

$$F_{\text{inertia}} = mr\omega^2 . \tag{1.1}$$

If the inertial force is proportional to mass, it is valid, not just for the 'm' parameter, but also for the 'r' (in length) and 'ω2' (frequency-square) variables by the above equation! In theoretical physics, besides the mass dimension, both the length dimension and the frequency-square dimension can be legitimately used to express the inertia, among others. According to (1.1) the inertial force can be equivalently expressed with meter and Hz^2, as well in SI unit system, which is also proportional to them. In this present work, it has been shown that this new, unusual concept of the mass (inertia) leads to far-reaching consequences.

Another important mass definition can be obtained from Louis de Broglie's 'hypothesis' of the matter-waves:

$$\lambda = \frac{h}{p} = \frac{h}{mv} . \tag{1.2}$$

From the philosophical point of view, de Broglie's assumption is supported by a simple idea, namely that nature does not make a sharp distinction between light and matter. The wave nature of light is known, in some way, to be reflected in the material (mass) as well. According to the equation, the wavelength of the matter-wave is inversely proportional to the mass. De Broglie's equation was developed by connecting relativity and Planck's law of energy expression. The details would be superfluous to go into here (see literature).

Louis de Broglie's hypothesis (now experimentally established) taught us about the equivalent mass dimensions, as described above. The mysterious substance of the matter-wave could be refined by the definition of the matter-wave amplitude, frequency and wavelength with the help of appropriate dimensions of inertia, i.e. with the corresponding dimension of mass for the physical interpretation.

In summary, our statements:

- *Matter-wave amplitude is proportional to mass.*
- *Matter-wave frequency is proportional to the square root of mass.*
- *Matter-wave length is inversely proportional to mass.*

2. The Mass and Energy Relationship

The close relationship between mass and energy has been summarized in Einstein's paper published in 1905. This simple equation $E = mc^2$ remains known to this day, not only in physics but also for the public. The first direct practical application of this equation was the release of atomic energy, with the known positive and negative consequences. Einstein's relationship between energy and mass is one of the fundamental results in physics, the validity of which has been demonstrated in a number of ways for more than a hundred years! It cannot be seen at first glance, but equally important was the famous discovery of Planck's constant in 1900:

$$E = h\nu \equiv \hbar\omega \, . \tag{2.1}$$

Max Planck's discovery is considered the beginning of modern physics; the fundamental significance of this simple equation has also been proved, and the consequences revolutionized the whole of physics. It is preferable to write this equation in the following form:

$$\Delta E = h\nu = \hbar\omega \, . \tag{2.2}$$

This can give a direct contact with Einstein's mass-energy equation. In the most general case, the change of the mass is due to the electromagnetic radiation (emission and absorption). As it was detailed in the first section, the frequency-squared unit can be the dimension of the mass alternatively. Taking into account Einstein's equation, the rest energy of the mass can be written into an unusual form:

$$E = Mc^2 = \kappa\omega^2 \, . \tag{2.3}$$

Where there is an appropriate proportionality factor. In consequence of EM radiation, the mass (energy) of the body slightly changes according to Eq. (2.3):

$$\Delta E \cong 2\kappa\omega\Delta\omega = 2\kappa\omega^*\omega_0 = \hbar\omega^* \, . \tag{2.4}$$

Assuming that the frequency change cannot be arbitrarily small, only a small finite value, we can write:

$$\hbar = 2\kappa\omega_0 \; ; \quad \frac{\hbar}{2} = \kappa\omega_0 \, , \tag{2.5}$$

where ω_0 is the lowest frequency occurring in nature, that is, the lower limit of the mass-frequency. With the help of this simple consideration, we easily obtained the origin of Planck's constant:

$$\hbar = 2\kappa\omega_0 \; ; \quad \frac{\hbar}{2} = \kappa\omega_0 \, . \tag{2.6}$$

Applying the above result, Eq. (2.3) can be written:

$$E = mc^2 = \kappa\omega^2 \equiv \kappa\omega_0\left(\frac{\omega^2}{\omega_0}\right) = \hbar\omega_m \; ; \quad \omega_m \equiv \frac{\omega^2}{\omega_0} \, . \tag{2.7}$$

The energy (and the equivalent mass) appears to be linear in frequency, but this is only a formal result, the physical reality is that the energy (equivalent mass) has a frequency-squared dimension.

Let us examine the black body radiation on a given frequency ω_s. The energy change of the body can be arbitrarily small or big, but the given frequency is limited. Knowing the elementary frequency introduced in (2.5), the black body mass (energy) cannot be changed any other way, such as Planck's finding in 1900:

$$\begin{aligned} &\Delta E \cong 2\kappa\omega\Delta\omega \equiv 2\kappa\omega^*\omega_0 \cong 2\kappa\left(n\omega_s\right)\omega_0 = n\hbar\omega_s \; ; \\ &\omega^* \cong n\omega_s ; \quad n = 0,1,2,3,\ldots \end{aligned} \tag{2.8}$$

3. The Structure of Mass

Several years ago, we put forward a hypothesis on the general structure of the physical mass, a strange theory of the particle's 'self-momentum'. In essence, the physical concept of particle's self-momentum is a natural extension of the theory of special relativity. Omitting the details of the theoretical reasons, we assumed that the central concept, of the four-momentum of relativity, is not only applicable to the external movement of the particles, but, in addition, there is a version which can be associated with the internal movement of particles (similar to the 'spin' concept of the particles). In the special theory of relativity the four-momentum have a highly symbolic equation:

$$E^2 = M^2c^4 + c^2p^2 , \tag{3.1}$$

Where E is the energy of an arbitrary body, M is the rest mass of the body and p is the traditional momentum of the body. The rest energy of the body can be obtained in case $p = 0$ according to basic Eq. (2.3):

$$E_0 = \pm Mc^2 . \tag{3.2}$$

It is customary to depict the speed of light as a dimensionless unity, by which the equations are simplified. By this condition Eq. (3.1) will be:

$$E^2 = M^2 + p^2 . \tag{3.3}$$

In this new theory of the physical mass, we propose that the internal structure of the mass, formally, is similar to the relativistic energy of a moving body:

$$M^2 = R^2 + S^2 , \tag{3.4}$$

M is the body's rest mass, which, can be measured experimentally, R and S are the base mass and self-momentum mass of the body, respectively.

According to the argument of the first section, the mass corresponds to the amplitude of the matter-wave, and, from the general study of the mechanical waves, the energy (the equivalent mass) is proportional to the square of the wave amplitude (the speed of light remains one unit):

$$E = M_E \sim M^2 = R_E^2 + S_E^2 . \tag{3.5}$$

For the interpretation of this surprising equation, it should be remarked that, for example, the gravitational energy is expressed also with mass-square quantity, and the Newtonian gravitational constant gives the appropriate dimension of the gravitational energy.

The mass structure defined with the Eq. (3.4) is supported from a completely different perspective by quantum mechanics. Indeed, the relativistic wave equations of various elementary particles can be written into a general form:

$$\mathbf{D_\mu D^\mu}\Psi = M^2\Psi , \tag{3.6}$$

where Ψ is the complex wave function elementary particle, $\mathbf{D_\mu}$ the typical four-momentum operator of the particle, which may include any potential of external force fields, and M is the observed (rest) mass of the particle. In the bound state of a particle (which is spatially localized), the wave function is normalized to unity:

$$\int_{\Omega} \Psi^* \Psi d\Omega = 1 . \quad (3.7)$$

The well-known interpretation of this condition, that the particle is staying in the space-time domain Ω, with a unit of probability, means, that is certainly within this limited range. Integrating the wave equation (3.6) for the space-time on the domain Ω we obtain:

$$\begin{aligned} &\int_{\Omega} \Psi^* \mathbf{D}_{\boldsymbol{\mu}} \mathbf{D}^{\boldsymbol{\mu}} \Psi d\Omega = M^2 \int_{\Omega} \Psi^* \Psi d\Omega \\ &= M^2 \int_{\Omega} \left[(\mathrm{Re}\,\Psi)^2 + (\mathrm{Im}\,\Psi)^2 \right] d\Omega = R^2 + S^2 = M^2 . \end{aligned} \quad (3.8)$$

From the attribute of the complex wave function, directly follows the introduced mass structure defined by Eq. (3.4). It is known, also, that the phase of quantum wave function of a free particle is indefinite. During the particles interaction, however, the phases of their wave functions come to be important. It means that for the mass parameters of the particles, where the wave phases are determined by *R* and *S*, only certain values are permitted in the interaction process.

4. Mass-Structure and Conservation Laws

The mass-structure Eq. (3.4) is illustrated with a right-angled triangle by Pythagoras' theorem:

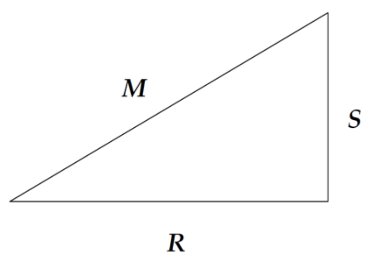

Fig. 4.1. The mass triangle: *M* = measured mass; *S* = self-momentum mass; *R* = basic-mass.

The consequence of the triangle inequality is that the measured (observed) mass of a particle can always be smaller than the masses of the components (*R, S*). This provides the stability of the particle, i.e., all experimentally observed particles are in bound state (or nearly bound state for the resonant type particles). A further consequence is that the quantum-mechanical description of a free particle, in reality, is never a plane wave, (which has been assumed generally,) but, is rather, the minimum of a two-particle bound state in which the phase of the wave function is strictly defined. It means that the component masses *R* and *S* of the particles are clearly determined and specific.

Regarding Einstein's basic equation of the mass-energy relation, the mass and energy conservation have been unified to a unique theorem. In the physical interactions, the mass can be changed, but the total energy can not. Knowing the mass triangle, it is obvious that the change of the rest mass of the particles is the consequence of the change of their component masses. By previous investigations, it has been shown, that in all interactions of the elementary particles the sum of the base-masses and the sum of the self-momentum masses remain independent:

$$\left(\sum R_i\right)_{IN} = \left(\sum R_j\right)_{OUT} ; \left(\sum S_i\right)_{IN} = \left(\sum S_j\right)_{OUT} . \quad (4.1)$$

5. Mass-Structure and EM Radiation

The mass change is typically the consequence of the electromagnetic radiation. In quantum electrodynamics, the electromagnetic field is a conglomeration of photons having different energies (frequencies). Photons, such as the atoms of light, can interact with the atoms, molecules, etc…, of the matter by emission and absorption processes. The objective of modern physics, since Planck's discovery in the year 1900, is the intensive study of the interaction between the material and the light (electromagnetic radiation). The more accurate knowledge in this field has been reached with the discovery of quantum mechanics in 1925-26. Actually, the fundamental question is how the interaction takes place in these processes, knowing the newly explored mass-structure of the particles.

In the following steps, we examine the photon emission of matter. By Planck's theory, the light emission of radiated electromagnetic energy cannot be continuous; it can only be realized in discrete portions (*quanta*). The non-continuous functions in mathematics are the series which are nothing more than arbitrary math functions within the specific domain of the natural numbers. It is assumed that a mass *M* can emit energies forming a sequence of numbers. Since the total radiated energy of the mass can be only a finite amount, the emitted energy series can only be monotonically decreasing (more precisely, strictly decreasing). By the simplest assumption, the mass loss of an object is the consequence of only one mass component decrease. Let it be the self-momentum mass. Knowing the mass triangle, the simplest mass spectra of a radiant object can be written in the form:

$$(M - W_n)^2 = (S - E_n)^2 + R^2. \quad (5.1)$$

In this equation, the mass and energy units are the same, because the speed of light is selected as unity. In this equation, the decrease of the self-momentum mass is equal to E_n, while the decrease of the rest mass is equal to the value W_n. From the property of the mass triangle follows:

$$W_n < E_n ; \quad n = 0,1,2,... \quad (5.2)$$

By the principle of Occam's razor, the electromagnetic radiation is so ordinary and common, that it cannot depend on many parameters therefore we can calculate the emitted energies with only a single math series. We reach the strict monotonic decrease in Eq. (5.2) by the following assumption:

$$W_n \equiv E_{n+1} < E_n ; \quad n = 0,1,2,... \quad (5.3)$$

This means that the simplest mass structure equation of the electromagnetic emission (photon emission) will be:

$$(M - E_{n+1})^2 = (S - E_n)^2 + R^2 ; \quad n = 0,1,2... ; \quad E_0 = S. \quad (5.4)$$

This is a simple recursive equation for the E_n energies. For the value E_0 has chosen *S*, following again the Occam's razor principle.

The main goal is to reduce the number of the oscillator's parameters to a minimum. Hereinafter, the particle having energy levels defined by Eq. (5.4) will be called the 'mass-oscillator'. For the physical interpretation of the mass-oscillator one can select the simple case where basic mass R is negligibly small, practically zero. In this case, the rest mass and the self-momentum mass are approximately equal, so S can be identified also with the same letter M:

$$\left(M-E_{n+1}\right)^2=\left(M-E_n\right)^2, \tag{5.5}$$

this implies:

$$M=\frac{E_n+E_{n+1}}{2}. \tag{5.6}$$

In the simplest case let the energy spectrum be a decreasing arithmetic series:

$$E_n=\left(n+1\right)\hbar\omega;\quad E_{n+1}=n\hbar\omega, \tag{5.7}$$

This defines a simple energy spectrum for the mass-oscillator:

$$M=M_n=\left(n+½\right)\hbar\omega=E'_n;\quad n=0,1,2,\ldots. \tag{5.8}$$

We obtain the known equation of quantized harmonic oscillator (QHO) for the energy, which provides the possible energies depending on the frequencies of the black body oscillators. To summarize above, the electromagnetic radiation spectrum of an arbitrary non-zero rest massive particle can be determined using the recursion Eq. (5.4) in general. The equation in the special limit $R=0$ case returns to the energy levels of QHO.

6. Radiation Theory of Atom Generation

These two simple assumptions; that mass can be expressed in frequency-square units and that mass has a two-dimensional vector structure, may cause a severe blow to the foundation of physics. In consequence of the new theoretical concepts important physical results have been reached. The consequences, however, are far from the over. In the following, a new generation theory of atoms will be shown, based on a certain generalization of Planck's black body radiation theory.

According to contemporary physics, all of the elements in the periodic table of elements (including isotopes) are synthesized in supernova stars at very high temperature, through a series of fusion processes. The energy-producing "nuclear fire" maintains the star in high temperature, while a huge energy radiates into space, mostly in the form of electromagnetic and neutrino radiation. At high temperatures, the neutrino radiation also follows the Maxwell-Boltzmann energy distribution, therefore the total radiation power in the elementary domain of the frequency must be the next:

$$dE\left(\omega,T\right)=\frac{\omega^2}{\pi^2c^3}\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}d\omega. \tag{6.1}$$

This expression is known as Planck's law of black body thermal radiation. In the development of quantum theory, including Einstein's photoelectric theory, it was clearly shown that the radiant electromagnetic energy is quantized, consists of multitude of photons (the particles of light). In the original expression of Eq. (6.1), the continuity of the radiation frequency remained questionable. Nevertheless, the electromagnetic and neutrino radiation at the extremely high temperature of supernovas, allows the emission of the binding energies from the atomic nuclei, thereby establishing the individual chemical elements.

The atoms are defined characteristically with the mass number 'A'; the radiation frequency may depend on the discrete number of the atomic mass. For this reason, Planck's law should be amended to allow discrete frequencies. The uncertainty of quantum mechanical energies ('natural line width') of the discrete atomic oscillators allows the integration of the radiation energy in a narrow frequency band in (6.1). Therefore, the radiant energy of the atom having mass number 'A' will be the next:

$$E\left(A\right)\sim\frac{\omega^4\left(A\right)}{e^{\hbar\omega(A)/kT}-1}. \tag{6.2}$$

In actual calculations, however, we need to know how the radiation frequency depends on the mass number of atoms, usually from the mass. In the present article, one of the fundamental statements is that the mass, alternatively, can be interpreted by frequency-square dimension. Since the masses of the atoms in a good approximation are proportional to their mass number, the squares of the atomic frequencies are proportional to their mass number:

$$\omega^2\left(A\right)\sim A. \tag{6.3}$$

Using the atomic synthesis model of supernovas, the math background will be the generalized radiation Eq. (6.2) in a simple rewritten form:

$$M\left(A\right)=c_1A-\frac{c_2A^2}{\exp c_3\sqrt{A}-1}, \tag{6.4}$$

where the constants 'c'-s can be determined from the fitting of the experimental values of atomic masses. This simple equation does not take into account the Z-number dependence of the atoms; however, at the outset, the first fitting results were very encouraging. Over the years, the accuracy of Eq. (6.4) has been gradually improved [2], and, what has been deemed the final neutral atom mass equation, was finally obtained. This is the Radiation Model of the atomic synthesis:

$$M\left(Z,A\right)=M_0\left[A-\frac{Q^5}{2}\times\frac{A^2-3.5A-6}{\left(1+Q\right)^{\sqrt{A-1}}-1}+Q\left(\frac{A-2Z}{A+4}\right)^2\right]. \tag{6.5}$$

In this expression, the Z-dependence of the atomic masses already has been given in the third term. The equation has been fitted to close to 2000 isotopes with the Monte Carlo method. The equation contains only one fitting parameter, namely the proportionality factor M_0, which is in atomic units:

$$M_0=1.003215\ldots\text{ au}. \tag{6.6}$$

The dimensionless number $Q=2/9$ frequently occurs in our theoretical works, the details can reach, for example, in [1]. The conventional method for calculating the relative error in nuclear physics is:

$$\delta\left(A,Z\right)=\frac{M_{\text{calculated}}-M_{\text{experiment}}}{Z\times M_H+\left(A-Z\right)\times M_n-M_{\text{exp}}}. \tag{6.7}$$

The model accuracy is characterized by the relative standard deviation of errors:

$$\sigma = \sqrt{\frac{\sum \delta^2(A,Z)}{n+1}} = 2.9 \times 10^{-2}\,, \tag{6.8}$$

where n is the number of the fitted atoms.

From the mathematical point of view, mass Eq. (6.5) is a simple continuous function, which can only broadly characterize the binding energies of atoms. But neither in theory, nor in practice, is there further opportunity to enhance the accuracy significantly.

The binding energies of individual atoms, including the ground state and excitation energies (masses) as well, can be more precisely calculated with the help of additional experimental atomic data, and by using the other physical (quantum mechanical) models of nuclear physics. It is important to emphasize that the relative percentage standard deviation of the present Radiation Model (~2.9%) is better than the traditional nuclear drop-model (~ 3.9%) of C.F. von Weizsäcker [3].

7. The Nuclear Shell Model

The nuclear shell model is a model of the atomic nucleus dating back to the 1930's, which uses the Pauli Exclusion Principle to describe the structure of the nucleus. The shell model is based on three-dimensional quantized oscillators of the component nucleons; the intended purpose was the interpretation of the so-called 'magic numbers'. As in the case of the noble gas atoms, the same outstanding chemical stability was observed at the level of the nuclei. In the periodic table of the elements, extremely stable isotopes exist (without any types of the radioactive decay), containing specific even numbers of protons and/or neutrons. The observed magic numbers are 2, 8, 20, 28, 50, 82 and 126. The developed 'deformed harmonic oscillator model', supplemented with the Pauli principle, was able to interpret the magic numbers. In this work, from the nuclear shell model theory, there is only one important aspect, which is, that the nucleus is built up from quantized harmonic oscillators formed by individual nucleons, in a good approximation of the physical reality.

We can suppose, that in the process of the nuclear synthesis in supernovas, all the nucleons independently from each other, radiate energy as individual (quantized) mass-oscillators, by the (5.4) definition. The masses of the nascent atoms can be easily calculated by a simple equation:

$$M(A) = AM_0 - \sum_{n=0}^{\infty} L_n(A)E_n \tag{7.1}$$

where M_0 is the mass of the nucleon before radiation, E_n is the radiant energy quantum defined by the (5.4) mass-oscillator, and L_n is the number of the emitted quantum related to E_n. It must be remarked, that this equation could be used later, not only for the ground state of the atoms (or more precisely the atomic nuclei), but for the calculation of the excitation energies as well.

The big problem is, at first glance, that the equation has too many 'unknown' variables. Fortunately, the number of unknowns is significantly reducible. The first thing that really simplifies the calculations is that the mass-oscillator energy levels decline, roughly exponentially, with the growth of the quantum numbers. Furthermore, by the simple hypothesis that the proton or neutron (as the simplest atomic nuclei), is obtained from the mass M_0 by decay, means that it cannot be substantially larger than the mass of the neutron. The ground state of atomic masses has been fitted into the above equation, with the simplification that it is sufficient to take account of the first four energy levels due to the exponential decline. The junction condition was that the mass M_0 must be minimized. The mass Eq. (7.1) can be written in the next form pulling out M_0:

$$M(A) = M_0\left(A - \sum_{n=0}^{\infty} L_n(A)\varepsilon_n\right), \tag{7.2}$$

where the quantities are the normalized energy levels of the mass-oscillator (5.4), they satisfy the next recursive equation:

$$\begin{gathered}\left(1-\varepsilon_{n+1}\right)^2 = \left(s-\varepsilon_n\right)^2 + r^2; \\ \left(n = 0,1,2,..;\ \varepsilon_0 = s;\ r^2 = 1-s^2\right).\end{gathered} \tag{7.3}$$

Already the first fitting tests have shown that the parameter 's' is very near to $Q = 2/9$. The dimension-less value occurred in the Radiation Model featured by Eq. (6.5):

$$s = \varepsilon_0 \cong Q = \frac{2}{9}. \tag{7.4}$$

The results of the Monte Carlo calculation are here:

$$M_0 = 1.0088857\text{...au};\quad Q = 0.2222010\text{...} \tag{7.5}$$

The mass M_0 ratio comparing to proton and neutron masses:

$$\frac{M_0}{M_p} = 1.001597\text{...};\quad \frac{M_0}{M_n} = 1.000218\text{...} \tag{7.6}$$

It can be said, that the mass M_0 practically is equal to the mass of neutron.

The first four energy values $E_n = M_0\varepsilon_n$:

$$\begin{aligned} E_0 &= 208.819 \text{ MeV} \\ E_1 &= 23.493 \text{ MeV} \\ E_2 &= 4.939 \text{ MeV} \\ E_3 &= 1.085 \text{ MeV}\end{aligned} \tag{7.7}$$

The ratios of the energy values:

$$\begin{aligned} E_0/E_1 &= 0.112506 \\ E_1/E_2 &= 0.210254 \\ E_2/E_3 &= 0.219699\end{aligned} \tag{7.8}$$

Eq. (7.2) has been fitted to near 2000 isotopes. It is important to notice that the ratio of the energy levels does not have an exactly constant rate, i.e. the exponential behavior of the energies is only an approximation. In the mass Eq. (7.2), the smallest radiant energy quanta E_3, determines the maximum error of the fitting, which can be up to about 1 MeV. Of course, the fitting accuracy can easily increase by accounting for more energy levels. It is also important to mention, that the energy levels of the nucleon mass-oscillators are determined only by two parameters: Q and M_0. An easier calculation model would be difficult to find, because in a trial among calculations, the four energy levels were considered as independent fitting parameters, but we did not get more accurate results, than from the original mass-oscillator model. Table 7.1 shows some 'shell structures' among the familiar isotopes.

Someone may have noticed that, in our new nuclear model, the Pauli principle, which is usually important in the traditional shell model, was not taken into account. The success of the traditional shell model, in reproducing the magic numbers with the contribution of the Pauli principle, may be a coincidence. In the development of nuclear physics during the sixties and seventies of the last century, various cluster models appeared, in which the individual atomic nucleons were incorporated into smaller 'clusters' by different principles, reducing the difficulty of the many-body problem. These clusters could have been, in principle, fermions and bosons as well. So, the validation of the Pauli principle for the atomic nuclei was questionable, even in the past.

Atom	A	L_0	L_1	L_2	L_3	R. Error
H	2	0	0	0	3	-8.67×10^{-05}
H	3	0	0	2	0	-7.06×10^{-07}
He	4	0	1	1	2	-2.13×10^{-05}
B	10	0	3	0	0	-2.56×10^{-05}
C	12	0	4	1	0	-3.67×10^{-05}
N	14	0	4	3	3	-7.41×10^{-05}
O	16	0	5	3	4	-3.64×10^{-05}
Ca	40	1	6	3	1	-6.52×10^{-06}
Fe	56	2	4	2	2	-8.69×10^{-06}
Cu	68	3	0	0	1	-1.32×10^{-05}
Sr	94	4	0	4	1	-8.72×10^{-06}
Sn	120	5	1	3	1	-7.65×10^{-06}
Cs	130	5	5	0	1	-2.22×10^{-06}
Nd	154	6	3	3	1	-7.32×10^{-06}
Pt	174	7	0	0	3	-3.33×10^{-06}
Pb	208	8	3	0	2	-9.39×10^{-07}
Th	211	8	2	2	3	-2.99×10^{-06}
Pu	230	8	8	1	3	-4.21×10^{-07}
U	233	9	0	2	2	-9.80×10^{-07}
Pu	234	9	0	3	2	-5.75×10^{-07}
U	235	9	1	0	1	-1.08×10^{-06}
Pu	235	9	1	0	0	-2.60×10^{-07}
U	238	9	1	4	0	-2.34×10^{-08}

Table 7.1. 'Shell structure' of some familiar isotopes.

8. The Mass Structure of Leptons

According to the literature, two significant empirical relationships exist for the three leptons, in regards to their mass relations. The first is the Koide formula [4]:

$$\frac{m_e+m_\mu+m_\tau}{\left(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}\right)^2}=0.666659(10)\approx\frac{2}{3}\,. \tag{8.1}$$

Denominations: m_e = mass of the electron, m_μ = muon mass and m_τ = mass of the tau lepton. The current information indicates that the physical background of the Koide formula is unknown, so it is a purely theoretical relationship. Another important relationship exists between the lepton masses, which was published by G. Rosen [5]. According to the author, the lepton masses are determined with high precision with this equation:

$$M_k\cong M_0\left(1+\sqrt{2}\cos\alpha_k\right)^2, \tag{8.2}$$

where $M_0=313.85773$ MeV, $\alpha_k=2k\pi/3+Q$, and $Q=2/9$.

The lepton masses are obtained by selecting $k=1$ for the electron mass, $k=2$ for the muon mass, and finally $k=3$ for the tau mass. The results of the calculation given by the author (in MeV):

$$\begin{aligned}0.5109965\text{ MeV}&=m_e\left(1-4.70\times10^{-6}\right);\\105.65891\text{ MeV}&=m_\mu\left(1+5.09\times10^{-6}\right);\\1776.9764\text{ MeV}&=m_\tau\left(1-7.63\times10^{-6}\right).\end{aligned} \tag{8.3}$$

The model is able to reproduce the lepton masses to at least the relative accuracy of 10^{-5}, which for this simple equation is surprisingly good. Rosen gives a possible theoretical background for his lepton mass equation, but we will now try to give an alternative way to prove the physical validity of mass Eq. (8.2). In the first step, we rewrite (8.2) into a simple form:

$$M\sim\left(1+\sqrt{2}\cos\alpha\right)^2\equiv1+2\sqrt{2}\cos\alpha+2\cos^2\alpha\,. \tag{8.4}$$

In classical mechanics the wave energy is proportional to the square of amplitude, which can be expressed with equivalent mass. Eq. (8.4) can be written into an equivalent form:

$$\begin{aligned}M&\sim M_c^2-M_b^2=\left(c_1^2+c_2^2+2c_1c_2\cos\alpha\right)-\left(b_1^2+b_2^2-2b_1b_2\cos^2\alpha\right)\\&=M_a^2;\\b_1&=b_2=c_1=1;\;\;c_2=\sqrt{2}\,.\end{aligned} \tag{8.5}$$

This result can be expressed by drawing it into a right-angled triangle. The lepton masses are proportional to the square of the horizontal sides, M_a, of the three right triangles, which are defined with Pythagoras' theorem, for the three values, according to (8.2):

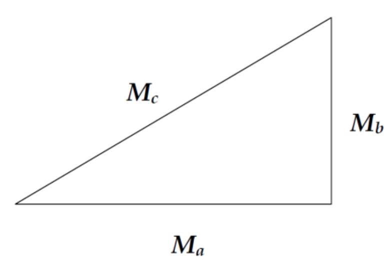

Fig. 8.1. The mass triangle.

The (8.5) form of Eq. (8.2) for the lepton masses calculation has special theoretical significance because it independently proves the theoretical model of the united electroweak interaction (sometimes mentioned as Glashow–Weinberg–Salam model). In this model, the so-called gauge-bosons play an important role. They are the short-lived $W^\pm$ and Z_0 particles (Weinberg bosons) which mediate energy in the weak interactions, for example in the decay of the neutron. This phenomenon, in earlier nuclear physics, was called the 'β^- decay' of the neutron, where β^- was the sign of the emitted electron.

The Feynman graph (Fig. 8.2.) of this process now has symbolic significance. In the entry of the graph, the neutron is signed with its three quark components, (the vertical axis shows the direction of the passage of time). In the first step the neutron decays into negative-charged W^- boson and the boson decays into electron and antineutrino.

In our lepton mass calculation model, the neutron and proton are commonly represented by the mass $M(\text{nucleon})$:

$$M(\text{nucleon}) \sim M_c^2 = c_1^2 + c_2^2 + 2c_1c_2\cos\alpha = 1 + 2 + 2\sqrt{2}\cos\alpha \ . \quad (8.6)$$

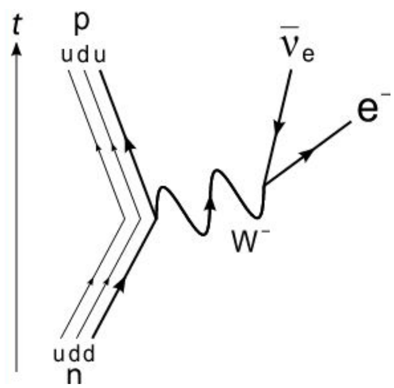

Fig. 8.2. The Feynman graph of neutron decay

In the expression, the 1+2 = 3 figure clearly indicates the three quarks of the nucleon (the resolution to 2+1 refers to the two types of quarks). The third term is proportional to the self-momentum mass of the nucleon, providing its stability. The side M_b of the mass triangle (Fig. 8.1.) clearly represents the Weinberg boson:

$$M(W) \sim M_b^2 = b_1^2 + b_2^2 - 2b_1b_2\cos^2\alpha = 1 + 1 - 2\cos^2\alpha \ . \quad (8.7)$$

The 1+1 figure indicates the quark/anti-quark pair, which generally forms the meson (boson). Finally, the common expression of the three lepton masses by mean (8.5) is:

$$M(\text{lepton}) \sim M_a^2 = M_c^2 - M_b^2 \ . \quad (8.8)$$

The proportionality factor in all three cases is the value given in (8.2):

$$M_0 = 313.85773 \text{ MeV} \ . \quad (8.9)$$

In the Feynman graph, the antineutrino particle also appears which, is not seen directly in our model, in explicit form. The missing antineutrino is emitted by the Weinberg boson reducing its self-momentum mass through the cosine-square term in (8.7). It must be mentioned, that this beta-decay model of the neutron is valid for the electron emission only. For the heavier leptons (muon and tau particles) the mass of a single neutron is not enough for this process. In those cases, a heavier nuclei environment is needed, which contains enough self-momentum mass (i.e. the sum of the self-momentums of the component nucleons).

9. Interference Theory of Physical Interactions

The main goal of physics is the endless research of physical interactions, primarily the fundamental interactions. The simplest interaction in physics may be the light interference, in which two light beams encounter one another at one point in space. The math model of light interference has long been known:

$$I \sim A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta \ , \quad (9.1)$$

where *I*, is the intensity of the light in a given point, A_1 and A_2 are the amplitude of the interacting coherent light beams and the resulting amplitude-square A^2 is proportional to the intensity of the light, which depends on the selected point of observation. The well-known interference image is determined by the phase factor δ, which is the function of the space.

Experimentally, Davisson and Germer first observed the interference of the matter waves for the electrons in 1927, following the idea of de Broglie. Since then, for a number of particles (atoms), the wave property of matter has also been demonstrated in similar interference experiments.

One of the basic statements, of the present paper, is that the matter wave amplitude is proportional to the amount of the mass. Therefore, the matter wave intensity (energy) of a particle is determined by the mass square of the particle. Taking account of the supposed mass structure of the particles, the matter wave energy E_w must be the next (M_w is the equivalent mass):

$$E_w \sim M^2 = R^2 + S^2 \sim M_w \ . \quad (9.2)$$

The generalization of the interference type of physical interaction provides a good opportunity for understanding the generation of the elementary particles.

Using, again, the Occam's razor principle, the existence of the elementary particles due to the self-interaction of the matter waves by a special interference:

$$M(\text{particle}) \sim M^2 = m_1^2 + m_2^2 + 2m_1m_2\cos\alpha \ . \quad (9.3)$$

In this particle model, the phase alpha of the interactive matter waves does not depend on the space but on the energy state of the particle (ground sate or excited states).

A good example, for the existence of this model of the particles, is the mass triangle defined by (8.5) in the previous section. The squares of the sides of right-angled mass triangle can be interpreted as really existing particles. In addition, we can define clearly the mass structure of the particles. Generally, the basic mass of a particle has a constant value, which is equal to the sum of the masses of the interactive particles before the interaction:

$$m_1^2 + m_2^2 \equiv R^2 = \text{const} \ . \quad (9.4)$$

and, the square of the self-momentum mass is responsible for the stability of the elementary particle:

$$2m_1m_2\cos\alpha \equiv S^2 \ . \quad (9.5)$$

10. The Origin of the Gravity

Now, our results and assumptions, so far, will be applied to gravity. For Isaac Newton and his con-temporaries the gravitational force was the number one problem, but here we focus on the energy aspects of gravity. The concept of gravitational energy appeared only much later, after Newton.

It is important to note that during the formation of the stars and planets, a huge gravitational energy is dissipated. This is an important physical condition of the gravitational binding of any massive objects in the Universe. The solid-state of the matter decisively provides the chemical bond; the gravitational binding is negligible compared to it. The gravitational binding energy of a solid-state is characterized by its gravitational self-energy, which is defined as:

$$E_c = -G\int_{\Omega} \frac{m}{r} dm \,. \tag{10.1}$$

The integration should be carried out for the entire volume of the body. A simple example is the self-energy of a homogeneous sphere having mass M:

$$E_c = -\frac{3}{5} G \frac{M^2}{R} \,. \tag{10.2}$$

It is generally true that the gravitational self-energy is proportional to the mass square of an arbitrary body. The negative sign refers to the binding energy of the body, which is dissipated in the form of heat in the mass condensation process. The 3 / 5 factor is valid for a homogeneous sphere, but in the general case, the constant pre-factor of G depends on the body shape and its mass density distribution. The gravitational self-energy of the bodies having any shape and mass density distribution can be written into a general form:

$$E_c = -\frac{\alpha G M^2}{R} = -\beta M^2 \,. \tag{10.3}$$

The opposite of the self-energy is the dissipation energy, which is also referred to as the separation energy of the body. It means how much energy needs to be given to the finite volume of the body to make an 'infinite' sized dust cloud. Let there be two masses (m and M) and calculate how much energy is dissipated during the merger of the two masses:

$$E_d = \beta\left(m + M\right)^2 - \beta m^2 - \beta M^2 = 2\beta m M = 2\alpha G \frac{mM}{r} \,. \tag{10.4}$$

Formerly, we have confirmed the gravitational dissipation energy harmonizing with the Newtonian law, but our deduction can rightly be criticized. In general, the shapes of interacting bodies are different from one another (i.e. they are not point-likes). The effective distance, R, is completely indeterminate, even before any arbitrary interaction. In principle, the gravitational interaction defined by (10.4), is in complete agreement with the interference theory of the physical interactions introduced in the above section; however this simple model is far from satisfactory. We must find a better way to interpret the gravitational interaction. From the experimental point of view, a new tendency in gravity research appeared in recent years. The object of experimental gravity changed to the study of the gravitational behavior of small masses. The multi-dimensional string theories assume that the Newtonian gravitational law is also valid for the micro world, at least for the elementary particles. It means that the classic gravity law is closely related to quantum mechanics; nevertheless, a generally accepted quantum gravity theory has not been found to date.

It seems that the interference theory of gravity, described above, is not sufficient to understand its origin. Additional theoretical consideration is necessary. There is, however, a little complicated version of interference in nature, which is more correctly known as coupled vibrations. The simplest example for the coupled vibration is the coupled pendulums. The experimental apparatus consists of two, nearly identical, simple pendulums, as well as a spring that provides a weak coupling force between the two. If one of the pendulums starts to swing, by a little push, we find that the moving pendulum slowly gives over its kinetic energy to the other pendulum, and then the process is reversed. The detailed math analysis shows, that if the frequencies of the pendulums are not precisely equal, two frequencies appear in the movement of the pendulums. The movement of both pendulums is proportional to the next expression (with a phase difference):

$$A(t) \sim \cos\left(\frac{\omega_a - \omega_b}{2} t\right) \cos\left(\frac{\omega_a + \omega_b}{2} t\right) . \tag{10.5}$$

The coupled phenomenon occurs in many areas of physics, known as beating. The first term in (10.5) gives the slow frequency of the periodic kinetic energy transfer between the pendulums ('beat envelop'). The higher self-frequencies of the pendulums are the same, determined by the second term of the equation; the 'carrier' frequency is the arithmetic mean of the two pendulum frequencies before the coupling. Similar phenomena occur, for example, when two organ pipes sound with nearly equal frequencies (this is the 'sound beat'). The air environment realizes the weak coupling in this case.

For the understanding of gravitation, an explanation similar to the frequency of the beating exists. Let there be given two point-like masses (m and M), which are approaching each other; a weak coupling appears between them by 'someway". By the basic statement of this paper, mass-frequencies are associated to both masses:

$$E_m = mc^2 = \kappa\omega^2 = \hbar\omega_m ;\; E_M = Mc^2 = \kappa\omega^2 = \hbar\omega_M \,. \tag{10.6}$$

Because of different mass-frequencies the frequency beating appears, establishing two additional energy levels expressing them directly with the two masses:

$$E_+ = \gamma\frac{M + m}{2} ; \quad E_- = \gamma\frac{M - m}{2} , \tag{10.7}$$

in this, γ is an appropriate proportionality factor.

According to the view of modern physics, gravity is valid not only for the macroscopic masses, but, theoretically, for the lightest elementary particles, for which quantum mechanics is also valid. Eq. (10.7) immediately suggests the idea of using the Bohr frequency condition. Applying it to the gravitational binding of two particles, the emitted (dissipated) energy is:

$$E_d = E_+ - E_- = \gamma m \,. \tag{10.8}$$

According to one of the statements of the present paper, the matter wave energy is proportional to its mass square. We can write the above expression in the following form:

$$E_d = \alpha\left(\frac{M + m}{2}\right)^2 - \alpha\left(\frac{M - m}{2}\right)^2 = \alpha m M = \frac{GmM}{R} \,. \tag{10.9}$$

We have the Newtonian gravity law applied to the dissipation energy, which is valid for all masses independently of their magnitude, according to contemporary physics. It turns out that quantum mechanics is staying in the background of gravity, regardless of the sizes of the interactive masses. From this deduction, it is obvious that resonant coupling (i.e. gravity), can occur between two masses, if the mass-frequencies differ from each other. It means that between two equal masses the gravity vanishes. At the turn of millennium, together with my colleague, Mr. L. Bodonyi, as well as independently from each other, we experimentally studied the gravity between equal (nearly equal) masses and experienced really minimum exchange of gravitational energy in these cases [6].

On the other hand, we got similar results in the quantum mechanical model of the hydrogen molecule. The chemical bond between two hydrogen atoms will only be realized if the energy of the two atoms differs slightly. This condition is fulfilled when the electron spins of the atom are in 'opposite direction'.

The actual theoretical and experimental analyses provide an opportunity to generalize an important physical principle:

> *Between two objects, which are the same in all physical parameters, attractive interaction cannot occur.*

Our statement seems trivial, but so far, no one has thought of this simple formulation of this physical fact. It would be difficult to find counterexamples to this claim (except the earlier misunderstood gravity law)!

11. Generalization of the Gravity Law

In consequence of the statement (10.10) the Newtonian law needs modification for the commensurable masses. The simplest change would be:

$$F = G^* \frac{mM}{r^2}\,; \quad G^* = \left(1 - \frac{m}{M}\right) G \tag{11.1}$$

In the case of the Earth-Moon's gravity, the *m/M* mass ratio is about *1/81*, the modified gravitational constant (starred) that is roughly one percent less than the theoretical value of *G*. In recent decades, the world's dominant gravitational constant experiments produced up to 1-2 per cent differences in extreme cases. Other nearby astronomical objects, with commensurable masses are not near enough to us, so we have no possibility of reliable control of the modified law of gravity, which would have a great theoretical significance.

The (11.1) generalized gravity force law, is the simplest choice for equal masses, when the gravity vanishes. In our quantum mechanical model of gravity, the gravitational constant is a random factor that surely increases with the mass difference (i.e. the difference of mass-frequencies), but not necessarily proportionately. The gravitational constant, as a random factor, depends on the mass ratio, of which we only know, that it has maximum value at small mass ratios, but, in the case of comparable sized masses, it gradually decreases and theoretically becomes zero at equal masses. Thus, the modified Newtonian gravitational law in general forms is:

$$\begin{gathered} F = G^* \frac{mM}{r^2} \quad \text{with} \quad G^* = \left(1 - \frac{m}{M}\right) G\,; \\ \lim_{m\to 0} G^* \frac{m}{M} = G \quad \text{and} \quad \lim_{m\to M} G^* \frac{m}{M} = 0\,. \end{gathered} \tag{11.2}$$

In the history of gravity, of course, the force formula law of gravity, originally developed by Newton, was firmly accepted. Within our Solar System, due to fortunate coincidences, gravity is observed only between small and large gravitational mass (the masses of the planets are significantly different from each other). These observations confirmed that, the gravitational constant has a constant value at all times, which is seemingly a universal property of the gravity law. For control of the validity of the modified gravity law, only the Earth-Moon system could be considered.

We examined Kepler's third law for the Earth-Moon system, in both conventional and modified gravitational law cases. The application of Kepler's third law to the two-body problem, using the conventional Newton's law, gives as follows:

$$\frac{4\pi^2 a^3}{T^2} = \mu; \quad \mu = G\left(M + m\right) = GM\left(1 + \frac{m}{M}\right). \tag{11.3}$$

Here *M* is obviously the mass of the Earth, and *m* is the mass of the Moon. The validity of the gravity law can be demonstrated by the next ratio, which theoretically must be unit:

$$\frac{\mu}{4\pi^2 a^3 / T^2} \approx 1\,. \tag{11.4}$$

The calculation has a 'suspiciously' good outcome:

$$\frac{\mu}{4\pi^2 a^3 / T^2} = 0.9999992... \tag{11.5}$$

This is understandable, since the Moon/Earth mass ratio was calculated according to Newton's law in the related literatures, which presume its unquestionable validity. In the case of the modified gravity law regarding the (11.1) modification, the calculation gives the next result:

$$\frac{\mu^*}{4\pi^2 a^3 / T^2} = 0.9876992...; \quad \mu^* = \mu\left(1 - \frac{m}{M}\right) \tag{11.6}$$

This result meets our expectation; the error by (11.6) is less than two percent, which can also relate to the inaccuracy of the *G*. However, it is important to remark, that the above calculations have been performed using, not the value *G*, but with Kepler's constant of the Earth measured earlier many times by artificial moons. Of course, there is no reason to doubt the accuracy of satellite measurements. It is possible that the simplest choice (11.1) for the generalization of the Newtonian gravity law is not the best. The essential criterion (11.2) meets also with the next choice for the modification of the gravity law:

$$\mu^* = \mu\left(1 - \frac{m^2}{M^2}\right) \quad \Rightarrow \quad \frac{\mu^*}{4\pi^2 a^3 / T^2} = 0.9998479..., \tag{11.6}$$

This leads, of course, to an acceptable result for Kepler's third law. Unfortunately, we can conclude that the only opportunity for a nearby double massive Earth-Moon gravitational system is not usable for measurement, to determine whether Newton's original or the modified Newtonian force law is closer to the physical reality. The reason simply is that the Moon's mass is much smaller than the mass of the Earth, at least in terms of our study.

Nevertheless, outside of our solar system, at very large distances, there are plenty of double stars with masses comparable, or nearly equal. They seem to contradict our new theory of gravity. The component masses of symmetrical binary stars are considered practically equal, but they are bound by strong gravity according to observations. The calculations, of course, have been and are still happening according to the traditional presumption of the validity of Newton's laws. However, the formation of the symmetric double stars is normally not considered, although theoretically, this is the most important phase of the gravitational binding. Probably, there is no other way that the symmetric binary stars can be generated from the splits of relatively big stars (proto-stars). In the condensation process of a huge amount of mass, the diameter of a newly forming proto-star continuously decreases; its rotational speed grows increasingly, in parallel with, its gravitational binding energy. At one moment, the star splits due to the increasing centrifugal force. At this moment the gravitational self-energy will be:

$$E_c = -\alpha_M G \frac{M^2}{R_M}\bigg|_{\substack{IN \\ state}} = -\alpha_1 G \frac{m_1^2}{R_1} - \alpha_2 G \frac{m_2^2}{R_2} - G\frac{m_1 m_2}{r}\bigg|_{\substack{OUT \\ state}}, \quad (11.7)$$

where $M = m_1 + m_2$.

This consideration formally confirms the usability of the Newtonian gravity law for symmetric binary stars, but not according to the original idea of the gravity, which is that the gravitational force instantly appears between two masses at all times and in all circumstances, without regard to their mass-ratios.

12. Conclusion

In the present work, we have completed Louis de Broglie's hypothesis of the matter waves by providing proportional amplitude and frequency to the mass. On this basis, we gave an easily understandable physical background for the Planck constant, which may be taught even in secondary schools.

For the purpose of a unified physical description of light and matter, we introduced a two-dimensional version of mass, which we named 'mass-structure'. The observed (experimental) mass of a particle corresponds to the hypotenuse of a right triangle, which is called the 'mass-triangle'. The shanks of the mass-triangle are the base mass and the self-momentum mass, which are independently conservable in physical interactions. This is especially true, in electromagnetic interactions where only the self-momentum mass can change.

We have generalized the black body radiation with the frequency-square interpretation of the mass. By this result, we have modeled the formation of neutral atoms in the high-temperature supernovas (Radiation Model). For the ground states of the atomic masses, this simple model produces relative accuracy below three percent, with just one fitting parameter for the entire Periodic Table. The traditional liquid drop model, itself, contains five fitting parameters and the accuracy is less than the Radiation Model.

A thought-provoking result, is, that the masses of the elementary particles are proportional to the square of the matter wave amplitudes. By this statement, we have successfully understood the origin of the masses of the three known lepton particles, in accordance with the Weinberg-Salam theory.

The successful reversing of gravity into quantum mechanical principles makes questionable the need for the research of quantum gravity. Additionally, by this method we have theoretically confirmed the validity of the Bodonyi-Sarkadi experiment [6] that, between two masses having the same or similar values, the gravity drastically reduces. This principle has been generalized to all types of physical interactions.

Following the example of gravity, a general binding mechanism has been proposed for the electrically neutral particles using the quantum mechanical aspects of the introduced mass-oscillator model of the elementary particles. The known charge independence of nuclear forces, under the nuclear interaction, can be based on a similar mechanism; probably even down to quark levels, making the QCD theory of the strong interaction unnecessary.

This paper can be reached by the following Internet link [7].

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