Gyroscopic Paradox of Motion –Part II

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The cause of the several conundrums and paradoxes involving gyroscopic and centrifugal forces which were introduced in Part I are examined in greater detail in Part II. The merits of Newtonian Absolute Space are weighed against those of Machian Relative Space. Modification of a simple rotational device presents a more complex analytical challenge: its calculated conformance with local angular momentum conservation depends upon a basic assumption regarding the precise quantification of gyroscopic torque. Postulation of Machian interaction of the local system with distant matter to account for centrifugal and gyroscopic forces would require radical reassignment of components of the inertia tensor; any local discrepancy in angular momentum conservation would strongly indicate such interaction. An 'existence theorem' for this contingency is framed and examined, though it remains decidedly unproven. Another calculative paradox is introduced in the process; the paper concludes with a restatement of the fundamental issue in mechanics.

1. Introduction

In Part I, [1] certain features of gyroscopic behavior were introduced which suggested paradoxes in the laws of motion. Unanswered questions relating to Prof. Laithwaite's experiments (examined by other authors [2,3] as well) were also posed, particularly with regard to reported variance of measured centrifugal forces with those predicted by Newtonian mechanics. A more complex form of the 'revolving dipole' example will be employed to explore gyroscopic quantities more intimately: another paradox of motion is introduced in the development. While validation of a calculative discrepancy between gyroscopic torque values and the Third Law of Motion (TLM) is not expressly discovered herein, it shall also be noted that Mach's Principle is not incompatible with classical results –provided that distant matter is incorporated in the formulation of Angular Momentum Conservation (AMC). The fundamental problem of assignment of angular momentum (AM) values arises again in this case, especially regarding the various components of the inertia tensor.

2. Description of 'Dynamion' Device

Fig. 1 depicts a construction in which masses m_1 and m_2 form a dumbbell-shaped object which is free to spin about a horizontal axle **S** at a given distance, r_S, from the vertical z-axis. The **S** axle is 'welded' to a vertical support, which is itself 'welded' into a surrounding rigid hollow sphere of large mass. A counter-weight ($2m$) will centrifugally balance the system when placed on **S,** opposite the dipole. The entire system exists in an otherwise gravity-free environment. Thus when a 'turn' angular velocity ω_z is imposed on the entire system, and a 'spin' ω_S on the dipole itself about **S**, torques will manifest about **S** (centrifugal), about z (Eulerian), and about N (gyroscopic) --N being normal to **S** and the z axis. Due to the immense moment of inertia of the surrounding sphere, ω_z will not appreciably change in magnitude or inclination as the internal dipole spins, although exchanges in AM will occur. (Dotted line figure at center, where $r_S = 0$, represents original position of fixed dipole in Part I of this paper. It will be observed herein that both centrifugal and gyroscopic torque magnitudes are independent of r_S, but the action is more easily described with an extended r_S; R is the instantaneous distance between z and either point m.)

Before proceeding, an operative quantitative convention will be introduced, principally to facilitate the calculative process. Unless otherwise specified, all quantities will appear as *scalar* magnitudes, whereby the various vector directions will be identified with respective subscripts on the left side of the equations, and the magnitudes, in terms of particular standardized quantities, on the right: i.e., $(spin)L_x$ will refer to the magnitude of the spin angular momentum of the 'ponderable dipole' about the x-axis; $(gyr)T_y$ the gyroscopic torque component exerted about y. Also, all dynamical magnitudes will be expressed in terms of I_S, the constant moment of inertia of the dipole about **S**, wherein the 'turn' angular velocity ω_z again remains a virtual constant.

3. Assessment of Angular Momentum Values

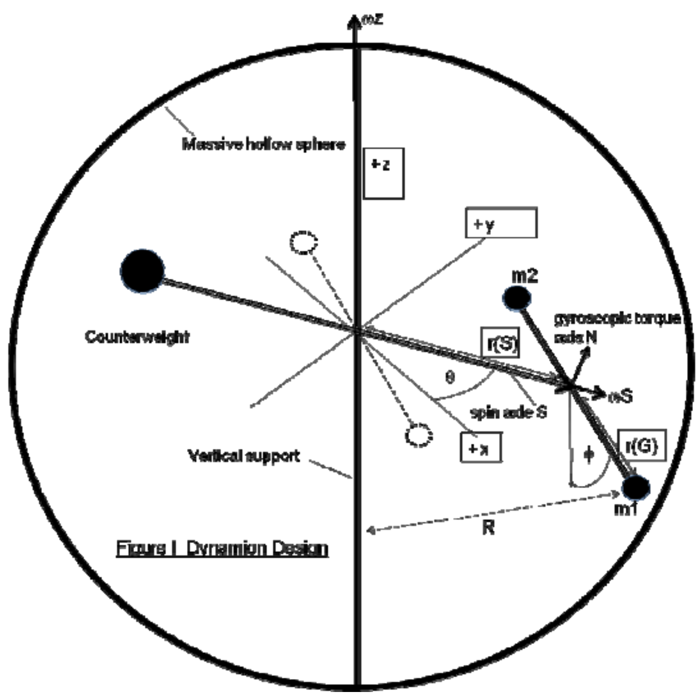

Fig. 1. Dynamion Design

Referring to Fig. 1, let the positions of the two 'point masses' m_1, m_2 be identified as follows:

$$\begin{aligned}(r_x)_1 &= x_1 = r_s(\cos\theta) - r_g(\sin\varphi\sin\theta)_1 \\ (r_y)_1 &= y_1 = r_s(\sin\theta)_1 + r_g(\sin\varphi\cos\theta)_1 \\ (r_z)_1 &= z_1 = -r_g(\cos\varphi)_1\end{aligned} \tag{1a}$$

$$\begin{aligned}(r_x)_2 &= x_2 = r_s(\cos\theta) - r_g(\sin\varphi\sin\theta)_2 \\ (r_y)_2 &= y_2 = r_s(\sin\theta)_2 + r_g(\sin\varphi\cos\theta)_2 \\ (r_z)_2 &= z_2 = -r_g(\cos\varphi)_2\end{aligned} \tag{1b}$$

AM values may be found by taking the first derivatives of these quantities with respect to time. For each point mass, the AM may be expressed in terms of instantaneous momentum crossing the various moments, and the results in terms of mass, length and linear velocity (from generic $r\omega$) are found to be:

$$\begin{aligned}L_x &= m\left[\left(r_y \times v_z\right) + \left(r_z \times v_y\right)\right] = m\left(r_y v_z - r_z v_y\right) \\ L_y &= m\left[\left(r_z \times v_x\right) + \left(r_x \times v_z\right)\right] = m\left(r_z v_x - r_x v_z\right) \\ L_z &= m\left[\left(r_x \times v_y\right) + \left(r_y \times v_x\right)\right] = m\left(r_x v_y - r_y v_x\right)\end{aligned} \tag{2}$$

(This analysis will henceforth concentrate upon the relevant quantities about x and y exclusively; those about z, though pertinent to other discussions, will be omitted here.) Inspection of the situation reveals that symmetry will serve either to cancel or to combine the effects for the two masses; thus, the moment of inertia, $I_S = 2mr_G{}^2$, for the dipole itself about **S**, and this may be employed in all inertia-involving terms. The AM magnitudes about the x, y axes --now expressed in angular terms-- become:

$$\begin{aligned}L_x &= +I_S\omega_S\cos\theta - I_S\omega_z(\sin\varphi\cos\varphi)\sin\theta \\ L_y &= +I_S\omega_S\sin\theta + I_S\omega_z(\sin\varphi\cos\varphi)\cos\theta\end{aligned} \tag{3}$$

The first term on the right side of each equation will be identified as the 'spin angular momentum' $(\text{spin})L$ about each axis at a given position, since it is determined by the direction that the axle **S** points at a given instant. The second term will be identified as the 'cinematic angular momentum' $(\text{cin})L$, since it is determined from the apparent projected motion of the masses, as sighted parallel to the respective coordinate axes at a given instant. The latter may in fact be identified with some of the off-diagonal terms of the inertia tensor, since they quantify, essentially, the respective products of inertia (I_{xz}, I_{yz}) at a given instant, multiplied by the angular velocity (ω_z), and the spin AM about **S** would technically be written $(I_{SS}\omega_S)$; but these formalities will not be necessary for the present purpose.

4. Calculation of Torques

The second time derivative of Eq. (1) will yield rates of change of AM for the dipole about the several axes. Though not in themselves to be identified as applied torques, these quantities will indicate the magnitudes of necessary reactive torques against the system as a whole. These rates about x, y will then become:

$$\begin{aligned}\frac{d}{dt}L_x &= \left[-I_S\omega_z^2(\cos\varphi\sin\varphi)\cos\theta + I_S\frac{d\omega_S}{dt}\cos\theta\right] \\ &\quad \left[-I_S\omega_{sS}\omega_z\left(\cos^2\varphi - \sin^2\varphi\right)\sin\theta - I_S\omega_{Ss}\omega_z\sin\theta\right] \\ \frac{d}{dt}L_y &= \left[-I_S\omega_z^2(\cos\varphi\sin\varphi)\sin\theta + I_S\frac{d\omega_S}{dt}\sin\theta\right] \\ &\quad \left[+I_S\omega_S\omega_z\left(\cos^2\varphi - \sin^2\varphi\right)\cos\theta + I_S\omega_S\omega_z\cos\theta\right]\end{aligned} \tag{4}$$

The term involving $d\omega_S/dt$ is to be identified with the centrifugally induced acceleration of the spin rate due to the action of the sweep of **S** about the z axis, and may be readily calculated from the vector resolution of this radial 'force' ($R\omega_z^2$) as it crosses r_G, the radius of gyration of the dipole about **S**. The resultant 'centrifugal torque' magnitude about **S** is found to be:

$$(\text{cen})T_S = \frac{d}{dt}L_S = +I_S\omega_z^2(\cos\varphi\sin\varphi) \tag{5}$$

The x and y components are:

$$\begin{aligned}(\text{cen})T_x &= \frac{d}{dt}L_S = +I_S\omega_z^2(\cos\varphi\sin\varphi)\cos\theta \\ (\text{cen})T_y &= \frac{d}{dt}L_S = +I_S\omega_z^2(\cos\varphi\sin\varphi)\sin\theta\end{aligned} \tag{5'}$$

Most noteworthy in this result is that these axial torque values are equal and opposite to the corresponding first term on the right of Eq. set (4): the rate of turning about the ***z***-axis of the momentary 'cinematic AM' evidently cancels the centrifugal-induced gain in spin AM. This leaves the remaining two terms of Eqs. (4) unanswered: AM conservation is not maintained by the motion of the dipole itself, but must be sought elsewhere. The *negatives* of these latter terms must therefore precisely quantify the magnitude of the phenomenon known as 'gyroscopic torque': regrouping and condensing terms yields:

$$(\text{gyr})T_N = -2I_S\omega_S\omega_z\cos^2\varphi \tag{6}$$

for this torque magnitude about N. The x and y components are:

$$\begin{aligned}(\text{gyr})T_x &= +2I_S\omega_S\omega_z\cos^2\varphi(\sin\theta) \\ (\text{gyr})T_y &= -2I_S\omega_S\omega_z\cos^2\varphi(\cos\theta)\end{aligned} \tag{6'}$$

(If individual mass elements of a gyroscope flywheel are integrated as per Eq. (6), the familiar constant torque of uniform precession is obtained.) The variation of gyroscopic torque with φ might indeed arouse suspicion: it could be presumed that $(\text{gyr})T$ from any precessing body would be constant for given spin AM, regardless of its shape. This contention is not supported by calculation: the variation results from *turning of the products of inertia* about z. Although $(\text{gyr})T$ and $(\text{cen})T$ do hint of a Machian reaction with surrounding matter, the equations as such indicate that AM is conserved in the local system itself—*provided that identifications between the physical components of the system with the mathematical terms of the equations are valid.* Nevertheless, another dynamical paradox can be described for the situation: Let the dipole possess a *unit* spin velocity and be in the horizontal orientation ($\varphi = \pi/2$) when **S** is aligned with the x axis. If the rotation velocity about z is arbitrarily great, it seems that *neither* centrifugal *nor* gyroscopic torque (both *zero* when $\varphi = \pi/2$) will be able to act appreciably through a single quadrant of $\Delta\theta$: When **S** becomes aligned with y , will the spin AM have changed direction without any local reaction? Calculative resolution is problematic, owing to the intricacy of the mathematical minutiae; regardless of these findings, the physical issue can only be settled by experiment.

5. A Specific Case (and a General Theorem)

The behavior of the device under various initial conditions can become exceedingly complicated, and general solution to equations of motion can approach the intractable; however, a relatively simple case and its ramifications will be addressed: First, if the spin rate is adjusted to result in $\omega_S = \omega_z$ when $\varphi = \pi/2$ (dipole horizontal), a general value for ω_S can be derived from Eq. (5), by which $d\omega_S/dt = (\sin\varphi\cos\varphi)\omega_z^2$. Hence:

$$\omega_S = \omega_z \sin\varphi \, , \tag{7}$$

Then, between any two angles (φ_a, φ_b), any $\Delta\theta\,(=\theta_b - \theta_a)$ can be determined from $d\theta = \omega_z dt$ and $d\varphi = \omega_z \sin\varphi\, dt$:

$$\Delta\theta = \int_a^b \frac{1}{\sin\varphi} d\varphi = \ln(\csc\varphi + \cot\varphi)_a - \ln(\csc\varphi + \cot\varphi)_b \tag{8}$$

These relations invite introduction of an 'existence theorem', whereby it might be claimed that: *There exists a* $\Delta\theta$ *, for which total AM is* not *conserved between positions* a *and* b *, for some particular assigned initial spin rate and dipole position.* For example, the values of ω_S and ω_z could be coordinated in a periodic process wherein $\varphi = 0$ when $\theta = 0$, and $\varphi = \pi/2$ when $\theta = \pi$. This situation might argue for a greater cumulative gyroscopic torque about the $-y$ axis than about the $+y$ axis *per revolution* about z , which, upon return of the dipole to its initial position, would yield a net *gain* in AM for the whole system. (On the other hand, a '*non-existence theorem'* might be more appropriate, if internal kinetic changes are to oppose such torque. That is, with $(gyr)T$ defined as the negative of $(d/dt)\left[(\text{spin})L + (\text{cin})L\right]$, it should be axiomatic that

$$\int_a^b (\text{gyr}) T_{x,y} dt + \Delta\left[(\text{spin})L + (\text{cin})L\right]_{x,y} = 0 \tag{9}$$

--which would affirm consistency in Newtonian mechanics.) Again, proof of either theorem is a purely experimental matter.

6. Conclusion

The aim of this paper has been to explore the possibility of a local defect in AMC for gyroscopic systems. Such a discovery would immediately confirm Mach's principle: TLM could only be preserved through reaction of distant matter to a local imbalance. However, if all paradoxes are found to be resolvable within Newtonian mechanics, the fundamental issue nevertheless remains: Mach's Principle still requires, in the absence of absolute space, that centrifugal and gyroscopic torques be a consequence of purely *relative* rotation between a local system and the surrounding universe. Therefore, these torques must yet again be *interactive* with distant matter, which must respond according to TLM. But this response will create an imbalance in the original rotational 'rest' frame: because Newtonian mechanics is now consistent in the local system, any AM acquired by distant matter will duplicate the AM formed in the off-diagonal components of the inertia tensor, when they are multiplied by local ω_z . If these torques are to be regarded as *real*, the off-diagonal terms must conversely be regarded as *fictitious.* This 'reversal of realities' poses some definite theoretical difficulties for a precise formulation of the necessary Machian field mechanism—a problem which will be addressed in Part III of this paper, in the context of a return to the experimental issues raised by Laithwaite.

References

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[3] H. Holushko, "Detecting Gyroscopic Thrust", *Proceedings of the NPA* **9**: This volume (2012).