The Gravitational Potential for a Moving Observer, Mercury's Perihelion, Photon Deflection and Time Delay of a Solar Grazing Photon

Curtis E. Renshaw
Tele-Consultants, Inc., 4080 McGinnis Ferry Road, Suite 902, Alpharetta, GA 30005
e-mail: crenshaw@teleinc.com

Utilizing the principle of equivalence and the radiation continuum model of EM radiation, it is demonstrated that Newton's gravitational potential applies only for static or slowly moving objects. The addition of velocity dependent terms, derivable from the principle of equivalence and the equivalence of matter and energy, produces the full dynamic gravitational potential. This dynamic potential is applied to the problem of Mercury's orbit and photon deflection, fully accounting both, and providing a result identical in form and value to that obtained utilizing the curved space-time of GRT.

1. Introduction

It has been shown in a previous paper that under a Galilean-Newtonian treatment of light, EM radiation may be treated as emanating from a source at all velocities from 0 to some upper value C, which is greater than c, and may be infinite. In this model, a photon may be viewed as an expanding spherical volume, with the leading edge of that volume expanding at a velocity of C. This is in contrast to the special relativistic model of a photon expanding in a spherical shell at a velocity of c, but with that value c dependent on the velocity of the observer, and not measured with respect to the source. As a result of modeling the photon as an expanding spherical volume, there can be found a component of this extended photon with any velocity one chooses between 0 and C. With respect to the source, all velocity components of the emitted continuum photon are at the same frequency, thus the wavelength increases proportionally with velocity. Any observer will be susceptible to that component with a velocity of c relative to the observer. For example, an observer moving away from the source with a velocity of v would be susceptible to that component which leaves the source with a velocity of $c + v$ with respect to the source, and thus has a velocity of c with respect to the observer. This model has been shown to allow a Galilean invariance of Maxwell's equations [4], and to support all experimentally verified Doppler shift results.

Newton's gravitational potential was developed by studying stationary or slowly moving objects ($v \ll c$). As such, we can consider this to be the "static" gravitational potential. In what follows, we will show that there is also a velocity dependent term, derivable from the RCM model of light. Once the full "dynamic" gravitational potential is developed, we will apply it to an analysis of Mercury's anomalous perihelion advance and the deflection of a solar grazing photon, and show that the form and results of the solutions are identical to those derived utilizing GRT.

2. The Gravitational Red-Shift

For an observer moving directly away from the source, with increasing distance represented as a positive velocity, the observer will be sensitive to that component of light which leaves the source at a velocity of $c + v$. In what follows, we will refer to the velocity of light measured with respect to the source as c', while the velocity of light observed is always c. The red-shifted frequency due to such motion is equal to the product of the source frequency times the ratio of c, the observed velocity, to the initial velocity component of light emitted with respect to the source c', or:

$$\upsilon' = \frac{c}{c'}\upsilon = \frac{c}{c+v}\upsilon = \left(1+\frac{v}{c}\right)^{-1}\upsilon . \tag{1}$$

Although it can be demonstrated that any given component of light increases its velocity as it leaves a gravitational field [1], the light also loses energy, as measured by a receiver (atom or clock, for example) located outside the field, accounting for the gravitational red-shift experienced by the observer. The Pound-Rebka-Snider experiment demonstrated that we can counter the effects of the gravitational red-shift by moving the observer toward the source at a certain velocity [5]. The velocity required is such that the increase in energy due to the motion induced Doppler blue-shift exactly counters the loss in measured energy due to the gravitational red-shift. If the height of the Pound-Rebka test apparatus is h, then the velocity required will be given by:

$$v = \frac{gh}{c} , \tag{2}$$

where g is the acceleration due to gravity near the surface of the earth.

From this relation, we can express the formula for the gravitational red-shift in the form of Eq. (1). If we consider an observer stationary in the gravitational field, the velocity component with respect to the source is c, and the shifted frequency is given by replacing v in Eq. (1) with the expression of (2):

$$\upsilon' = \frac{c}{c+gh/c}\upsilon = \frac{1}{1+gh/c^2}\upsilon . \tag{3}$$

As a check on the validity of the form of Eq. (3), we can derive the standard formula for the gravitational red-shift as follows, where $gh \ll c^2$, $g = GM/r^2$, and $1/R_1 - 1/R_2 \approx h/R^2$.

$$\frac{\upsilon'}{\upsilon} = \frac{1}{1 + gh/c^2} \approx 1 - \frac{gh}{c^2},$$

or $$\frac{\upsilon' - \upsilon}{\upsilon} = \frac{\Delta\upsilon}{\upsilon} \approx -\frac{gh}{c^2} \approx -\frac{GMh}{c^2R^2} \approx -\frac{GM}{c^2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{4}$$

3. The Gravitational Red-Shift for a Moving Observer

If we now consider an observer moving away from the source with a velocity v, the observer is sensitive to light leaving the source at a velocity of $c' = c + v$ (Fig. 1). For the moving observer carrying a receiver with him, the height, h, traversed by the light, depicted in Fig. 1, is derived by the following expressions:

$$h + \delta h = h + vt = (c + v)t, \qquad \text{or} \qquad t = \frac{h}{c'},$$

or $$h + vt = h\left(1 + \frac{v}{c}\right). \tag{5}$$

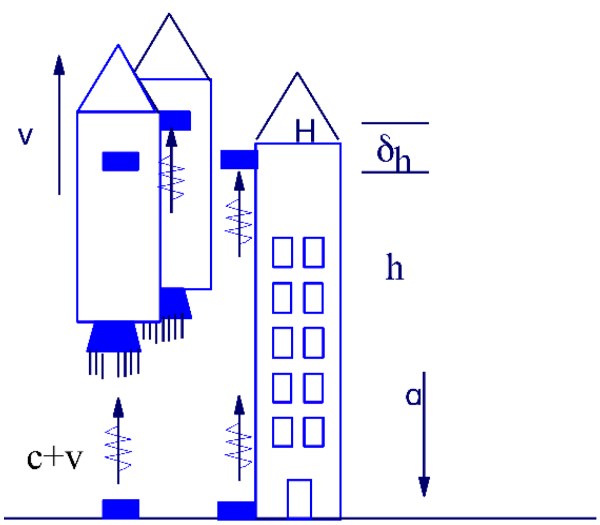

Fig. 1. Radial motion and gravitational Doppler

In what follows, we will use g' for the acceleration due to gravity as experienced by moving observers, and verify whether, for each type of motion (radial and transverse), $g' = g$. Using the proper values for the velocity of light with respect to the source, c', and the effective value for h in Eq. (3) yields:

$$\frac{\upsilon'}{\upsilon} = \frac{c}{c' + \frac{g'h(1+v/c)}{c'}} = \frac{c}{(c+v) + \frac{g'h(1+v/c)}{(c+v)}}$$
$$\approx \frac{1}{(1+v/c) + \frac{g'h}{c^2}} \approx \frac{1}{(1+v/c)} \cdot \frac{1}{1 + \frac{g'h}{c^2}}. \tag{6}$$

Thus, the total red-shift in received energy is equal to the expected motion-induced Doppler shift (given by the first term in the last expression) times the expected gravitational red-shift (given by the second term). From this we can deduce that radial motion has no effect on the measured gravitational red-shift or on the effective gravitational potential for such motion.

Next we consider an observer carrying a receiver and moving transversely to the gravitational field. As can be seen in Fig. 2, the effective height, h, does not change. Since we are interested in the component of light that strikes the receiver with apparent perpendicular incidence, the aberrated velocity of light with respect to the source, c', is given by:

$$c' = c\sqrt{1 + \frac{v^2}{c^2}}. \tag{7}$$

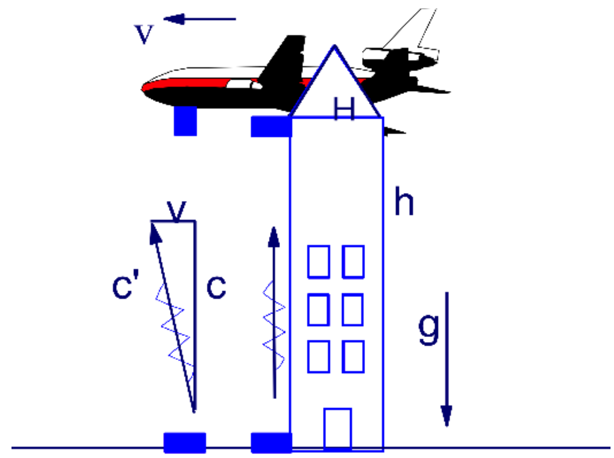

Fig. 2. Transverse motion and gravitational Doppler

The reason we are interested in light that appears to be perpendicularly incident to the receiver is that we are going to develop the equivalent gravitational relation, and we are interested in that force that appears in the reference frame of the test particle to be along the line joining the gravitational source and the test particle. In other words, all gravitational force is directed along this line. The gravitational force vector always appears radial, with no transverse components. Thus, the total equation for the change in received energy is given by:

$$\frac{\upsilon'}{\upsilon} = \frac{c}{c' + \frac{g'h}{c'}} = \frac{c}{c\sqrt{1+v^2/c^2} + \frac{g'h}{c\sqrt{1+v^2/c^2}}}$$
$$= \frac{1}{\sqrt{1+v^2/c^2} + \frac{g'h}{c^2\sqrt{1+v^2/c^2}}} = \frac{1}{\sqrt{1+v^2/c^2}} \cdot \frac{1}{\frac{g'h}{c^2(1+v^2/c^2)}}. \tag{8}$$

In this case, the change in energy is equal to the product of the transverse Doppler shift for apparent perpendicular incidence, given by the first term, times the gravitational red-shift, given by the second term. However, if the above equation were correct, unlike the case of radial motion, the gravitational red-shift in energy observed in the moving system would be different from that measured at the same point in the field by a stationary observer.

Imagine two observers in free-fall in empty space, far removed from any gravitational fields. Allow one observer to have an initial velocity, v, with respect to the other observer. Further allow the moving observer to pass very near the stationary observer, at the same instant that each observes the light from a distant source. If each observer records the frequency of light received from a distant source, then, after accounting for the Doppler shift experienced by the moving observer, they will each obtain the same frequency for that light. Now, the principle of equivalence tells us that the same experiment performed by these same two observers in free-fall in a strong gravitational field should obtain the same results as they did in free-fall far removed from any gravitational fields. Thus, the observed shift in frequency due to the gravitational field alone (after backing out motion induced Doppler effects) should be the same for both observers. In order for this to be the case, Eqs. (3) and the right hand side of Eq. (8) must produce the same result--each observer will measure the same value for the gravitational red-shift. From this, the following relation becomes immediately apparent:

$$g'\left(1+\frac{v^2}{c^2}\right)^{-1}=g\,,\qquad \text{or}\qquad g'=g\left(1+\frac{v^2}{c^2}\right). \tag{9}$$

If we now replace this transverse motion v with $r\dot{\theta}$, and append the dynamic adjustment of Eq. (9) to Newton's static potential, we arrive at the dynamic gravitational potential for a moving observer:

$$V=\frac{Gm_2m_1}{r}\left(1+\frac{\left(r\dot{\theta}\right)^2}{c^2}\right). \tag{10}$$

One might argue that the principle of equivalence doesn't actually go as far as to imply that a free fall observer with an initial velocity should see the same gravitational red-shift at a particular point in space as does a stationary observer, so we will present a gedanken experiment that illustrates that if such is not the case, a contradiction will occur. It has been demonstrated experimentally that clocks in a gravitational field slow down according to a relation identical in form (though inverted) to that for the gravitational red-shift. In other words, the amount of slowing depends only on the strength of, and thus the position in, any particular gravitational field. It has also been demonstrated that a clock placed in motion slows down according to a well-defined formula dependent only on the velocity of the clock relative to its rest frame.

Now, suppose we have three, identical, synchronous clocks, far removed from a gravitational field. We hold one of these clocks in space, stationary with respect to a distant, gravitating body. We lower another of the clocks into the field and leave it there, at rest with respect to the stationary space clock, though far removed from it and deep within the field. The clock in the field will tick more slowly due to its presence in the field, according to the strength of the field at that point. Next we accelerate the third clock outside the field to some known initial velocity with respect to the stationary clocks outside and within the field. Once this velocity is reached, we allow this clock to pass very near the stationary clock outside the field and note that this clock is running slow due to its velocity as measured by that clock, and we measure the rate of slowing. We allow the moving clock to continue on with the same velocity until it passes the stationary clock inside the field. We note again that this clock is running slow due to its velocity with respect to that clock, even though both clocks have also slowed due to their presence in the gravitational field, and we measure the rate of slowing. We compare the rate of the moving clock as determined by each of the stationary clocks.

Since the moving clock was at the same point in or out of the gravitational field as were the stationary clocks at the time each measured the moving clock's rate, each of these clocks should have determined the same rate of motion induced slowing for that clock, since the gravitational slowing would be the same for the moving and the stationary clock. However, if the gravitational red-shift for the observer with an initial velocity is different than that for the stationary observer, then the gravitational effect on the moving clock will also not be to the same degree as that on the stationary clock. This contradicts the original assumption and analysis which showed that the slowing of a clock due to a gravitational field is due only to the strength of that field and the distance from the source of that field, not the velocity through that field as well. Therefore, the gravitational red-shift as measured by a moving observer at a certain point in the field must be the same as that measured by a stationary observer, and we see that Newton's static gravitational potential must be modified to reflect the effects of motion through a gravitational field as is done in Eq. (10).

Since we wish to consider the case of planetary orbits and solar grazing photons, we obtain the following relations regarding angular momentum, l:

$$l=mr^2\dot{\theta}\,,\quad \text{or}\quad r\dot{\theta}=\frac{l}{mr}\,,\quad \text{or}\quad \left(r\dot{\theta}\right)^2=\frac{l^2}{m^2r^2}. \tag{11}$$

Substituting Eq. (11) into (10) yields (ignoring terms of higher order than $1/c^2$) a useful form of the dynamic gravitational potential for a moving observer:

$$V=\frac{Gm_2m_1}{r}\left(1+\frac{l^2}{m^2r^2c^2}\right). \tag{12}$$

We can now write directly the force due to the potential of Eq. (11), where Gm_1m_2 has been replaced by k [3]:

$$F=\frac{d}{dt}\left(\frac{dV}{d\dot{r}}\right)-\frac{dV}{dr}=-\frac{k}{r^2}\left(1+\frac{3l^2}{m^2r^2c^2}\right). \tag{13}$$

Utilizing a change of variables to $u = 1/r$, and applying the LaGrangian treatment of motion to Eq. (13) yields:

$$\ddot{u}+u=\frac{-km}{l^2}-\frac{3ku^2}{mc^2}. \tag{14}$$

This is, of course, the exact form derived under general relativity.

4. The Perihelion Shift of Mercury

Eq. (14) has the form of an elliptical orbit plus a small perturbation term. Thus we can choose a solution of the form:

$$u_0=-\frac{km}{l^2}\left(1+\varepsilon\cos\theta\right). \tag{15}$$

Eq. (15) shows that with each orbit of the planet, the perihelion will advance by an amount given by:

$$\delta = 2\pi \frac{3k^2}{l^2 c^2} = \frac{6\pi G m_2}{ac^2\left(1-\varepsilon^2\right)}. \tag{16}$$

Eq. (16) is of course the exact equation derived in relativity theory, and results in an anomalous perihelion advance for Mercury of 43″ per century, as confirmed by observation.

5. The Deflection of a Solar Grazing Photon

Beginning with the equation of motion derived from the dynamic gravitational potential, and considering a photon approaching from $\pm\infty$, we see that $l^2 = \pm\infty$, and we can neglect the first term on the right-hand side of Eq. (14). Thus, we can assume a homogenous solution of the form:

$$u_h = \frac{1}{R}\cos\theta\,, \tag{17}$$

and obtain:

$$\delta = \frac{R-x}{y} = \frac{2k}{m_1 c^2 R} = \frac{2GM}{c^2 R}. \tag{18}$$

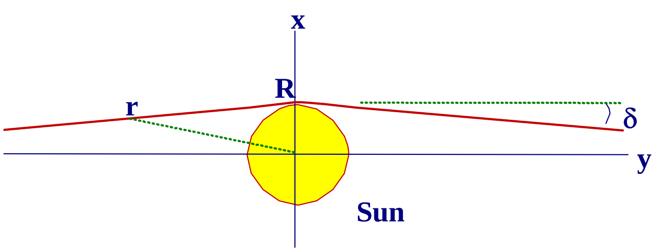

Fig 3. Deflection of a solar grazing photon.

In Eq. (18), δ is the tangent of the angle of deflection on one side of the sun, as seen in Fig. 3. Since the value of δ is very small, the tangent of the angle is almost identical to the angle itself, expressed in radians. The total deflection of the photon then, in radians, is 2δ, or:

$$\text{total deflection} = \frac{4GM}{c^2 R}. \tag{19}$$

Eq. (19) is the exact value obtained in general relativity theory and results in 1.75″ of deflection as confirmed by observation.

6. The Time Delay of a Solar Grazing Photon

We have seen in previous papers [1,2], that moving clocks and clocks in a gravitational field slow down, not due to the effects of special relativity nor to the space-time curvature of general relativity, but due only to the principle of equivalence and the conservation of energy. However, some might argue that there has been a further "test" of the effect of gravity on time, namely the measurement of the time-delay of a round-trip, solar-grazing radar beacon performed by Shapiro in the 1960's. In this test, Shapiro bounced a radar pulse off Mars at superior conjunction (a feat in itself for the time), and compared the measured round-trip travel time of this pulse with the expected round-trip time of a signal traveling at c for the entire trip, as determined from highly accurate planetary ephemerides. Shapiro had predicted this time-delay long before being technologically able to make such a measurement. While general relativity can be used to correctly obtain the magnitude of this delay, it is not the only explanation. As we did earlier with the analysis of clocks in motion and of clocks in a gravitational field, this paper will derive the same result without invoking the space-time curvature of general relativity.

6.1. Photon Velocity in a Gravitational Field

We begin by considering the Principle of Equivalence. Recall that, by this principle, if one is in free-fall (not being accelerated by a rocket or held in place by a floor), one cannot tell the difference between floating freely in space or falling toward a gravitating body such as the sun. No experiment one might perform could provide any knowledge about which state one is in, as the results of all experiments would be identical in both cases. This is also true of a photon in free-fall.

Now, a photon, upon entering a gravitational field, or well, acquires excess energy, compared with the energy of the surrounding gravitational field, much the way a ball dropped from a tower gains energy as it heads toward the ground. We can express the energy of this photon as:

$$E = \frac{hc}{\lambda}, \qquad \text{or} \qquad E = mc^2. \tag{20}$$

Thus, an increase in energy can be viewed as an increase in effective mass, or, conversely, as a shortening or "bunching up" of the wavelength components. A visual representation of this is when smoothly flowing traffic suddenly comes upon slowing due to rubbernecking a stalled vehicle across the road. The traffic slows, and the cars which had a comfortable, even spacing now become bunched up as they pass through this area. Upon leaving the congestion, the cars once again resume their original, spread out configuration and speed.

Now, due to the principle of equivalence and its presence in the gravitational field, the photon's velocity must slow down proportional to its decreased wavelength. For the photon, it travels in its frame of reference a distance l in a time given by l/c. If l decreases by $1/\alpha$ to l/α, then the photon's velocity will slow to c/α as well, so that it now travels the distance l/α in a time given by $(l/\alpha)/(c/\alpha)$, or l/c, as before. If we let the increased energy of the photon in the gravitational well (as compared with the energy of the surrounding field) be represented by αE, then we can rewrite Eq. (20):

$$\alpha E = \frac{\alpha hc}{\lambda} = \frac{hc}{\lambda/\alpha}. \tag{21}$$

From Eq. (21), we can again derive the new speed of this photon, realizing again that, due to the principle of equivalence, the photon's frequency must remain unchanged:

$$\nu = \frac{c}{\lambda} = \frac{c/\alpha}{\lambda/\alpha} \qquad \Rightarrow \qquad c' = \frac{c}{\alpha}. \tag{22}$$

Thus, the photon must slow down inversely proportional to the increase in energy it would otherwise gain during its fall through the gravitational field. We could also have simply stated that, since the velocity of a photon is equal to its frequency times its wavelength, the new velocity must be given by:

$$c' = \frac{\nu\lambda}{\alpha} = \frac{c}{\alpha}. \tag{23}$$

Either approach yields the same value for the new velocity of the photon. We must now turn our attention to deriving the value of the factor α .

If a hydrogen atom is carried into a gravitational well, it will generate or absorb energy at a lower frequency than it would outside the field. Thus, if we build a clock based on the frequency absorbed or emitted by this atom, it will run more slowly than an identical clock not inside the field. The rate of slowing of this clock will be the inverse of the equation used to derive the gravitational blue-shift for a photon falling into a gravitational well. If ν' represents the frequency generated by the clock inside the gravitational field, and ν the frequency generated far removed from that field, we have the following equation for the clock slowing:

$$\nu' = \left(1 + \frac{GM}{c^2 R}\right)^{-1} \nu \ . \tag{24}$$

If this clock measured the frequency of a photon generated far outside the field, the measured frequency would be blue shifted by the inverse of the above factor. This is graphic evidence that the frequency of the photon in free-fall has not changed. We know that the clock in the gravitational field runs slowly by the relation in Eq. (24), because this has been tested many times with actual cesium clocks. We also know that the blue-shift of a photon in free fall as measured by these same clocks is exactly the inverse of Eq. (24), as the Pound-Rebka-Snyder experiment (1960, for example) has shown. Now, if the photon itself were actually blue-shifted, the measured shift would be twice that given by the inverse of (24). First, the photon would be blue shifted by this amount, and this blue shifted frequency would appear to be shifted even higher when measured with the slow running clock. But the effect appears only once, and it is due only to the clock slowing--the frequency of the photon, due to the principle of equivalence, has remained unchanged.

So now we have the new velocity of a photon in free-fall through a gravitational field. Combining Eqs. (24) and (23), we get:

$$c' = c\left(1 + \frac{GM}{c^2 R}\right)^{-1} \tag{25}$$

However, Eq. (25) must be modified slightly. The velocity of the photon is constantly changing during its trip, in a manner always proportional to $1/r$, r being the distance from the sun at any given point. We therefore replace R, the grazing radius of the sun, with r, the instantaneous distance from the sun at any time along the photon's journey:

$$c' = c\left(1 + \frac{GM}{c^2 r}\right)^{-1} \ . \tag{26}$$

This is still not the entire story. Eq. (26) represents the velocity of the photon as measured locally by a clock traveling with the photon (in the vicinity of the sun). However, we are interested in the value of this velocity as measured using an Earth based clock. Clearly, a velocity measured with a standard clock will be less than the same velocity measured with a gravitationally slowed clock. Taking the Earth based clock to be our standard, we see that the velocity as measured by that clock will be even slower than the velocity as measured by a local clock carried with the photon. The additional scaling of the velocity will be identical to the scaling already provided in (26). The effect appears twice. Once due to the actual slowing of the photon in its own reference frame measured by its own clocks, and again to translate this velocity to the velocity as measured by an Earth based clock. The velocity as measured by an Earth based clock, omitting terms of order higher than $1/c^2$, is then given by:

$$c'_e = c\left(1 + \frac{GM}{c^2 r}\right)^{-1}\left(1 + \frac{GM}{c^2 r}\right)^{-1} \approx c\left(1 + \frac{2GM}{c^2 r}\right)^{-1} . \tag{27}$$

6.2. The Derivation of the Time Delay

From Eq. (27), the following relations can be derived, where T is the total time of the photon's travel from 0 to d, as measured by an Earth based clock:

$$T = \int_0^T dt \ , \tag{28}$$

$$dt = \frac{ds}{c'_e} = \frac{1}{c\left(1 + \frac{2GM}{c^2 r}\right)^{-1}} ds = \frac{1}{c}\left(1 + \frac{2GM}{c^2 r}\right) ds \ , \tag{29}$$

$$T = \int_0^T dt = \frac{1}{c}\int_0^d \left(1 + \frac{2GM}{c^2 r}\right) ds \ , \tag{30}$$

$$ds = \left(r^2 - R^2\right)^{-1/2} r dr \ , \tag{31}$$

$$T = \frac{1}{c}\int_R^{\sqrt{d^2+R^2}} \left(1 + \frac{2GM}{c^2 r}\right)\left(r^2 - R^2\right)^{-1/2} r dr \ . \tag{32}$$

In Eq. (32), we can obtain the effective path length of a photon traveling at c for a time T by multiplying through by c. Doing this, and breaking the integral into parts, yields the effective path length of the photon:

$$\begin{aligned} d' &= cT \\ &= \int_R^{\sqrt{d^2+R^2}} \left(r^2 - R^2\right)^{-1/2} r dr + \int_R^{\sqrt{d^2+R^2}} \frac{2GM}{c^2}\left(r^2 - R^2\right)^{-1/2} dr \end{aligned} \tag{33}$$

$$= \left(r^2 - R^2\right)^{1/2} + \frac{2GM}{c^2}\ln\left\{r^2 + \left(r^2 - R^2\right)^{1/2}\right\}\Big|_R^{\sqrt{d^2+R^2}} \tag{34}$$

$$\begin{aligned} &= d + \frac{2GM}{c^2}\left\{\ln\left(\sqrt{d^2 + R^2} + d\right) - \ln(R)\right\} \\ &= d + \frac{2GM}{c^2}\ln\left(\frac{\sqrt{d^2 + R^2} + d}{R}\right) \approx d + \frac{2GM}{c^2}\ln\left(\frac{2d}{R}\right). \end{aligned} \tag{35}$$

In Eq. (35), the last approximation holds for d>>R. The last term on the right-hand side of Eq. (35) represents the effective increase in path length, and is given by:

$$\Delta d = \frac{2GM}{c^2} \ln\left(\frac{2d}{R}\right). \tag{36}$$

For distances on the order of Earth or Mars, Δd is approximately 19 km. Thus, for a light signal traveling from Earth to Mars and back, with Mars at superior conjunction, the round trip increase in effective path length due to the time-delay of the photon is approximately 76 km, resulting in an increase in round trip travel time of 250 microseconds (out of a total trip on the order of thirty minutes). This result is equivalent to the results of relativity theory, and is confirmed (Shapiro, et al) to an accuracy of 0.1%.

7. Conclusion

Acceptance of Einstein's second postulate, combined with the principle of equivalence, requires the introduction of space-time curvature in the presence of massive objects. In this paper it has been demonstrated that a Galilean-Newtonian model of light results in a modification to Newton's static gravitational potential. Applying this modified potential to the case of Mercury's anomalous perihelion advance accounts for the entire 43″ per century, and provides a form equivalent to that of GRT. Thus this model will work equally well for any system to which it is applied, without resorting to the space-time curvature of GRT. The same potential, applied to the problem of a solar grazing photon, gives a solution identical in form and content to that of GRT as well. Finally, we showed that the same treatment applied to the time delay of light passing massive objects produces results identical to GRT. Such results lead to the conclusion that these effects are not attributable to massive objects curving space-time, but rather are just a natural consequence of the Principal of Equivalence applied in a Galilean-Newtonian framework.

References

[1] Curtis E. Renshaw, “The Effects of Motion and Gravity on Clocks”, *IEEE: Aerospace and Electronic Systems Magazine* **10** (10) (1995).

[2] Curtis E. Renshaw, “Moving Clocks, Reference Frames and the Twin Paradox”, *IEEE: Aerospace and Electronic Systems Magazine* **11** (1) (1996).

[3] J.B. Marion, **Classical Dynamics of Particles and Systems**, pp. 266-270 (Harcourt and Brace Publishers, New York, 1970).

[4] Curtis E. Renshaw, “The Radiation Continuum Model of Light and the Galilean Invariance of Maxwell's Equations”, *Galilean Electrodynamics* **7** (1) (1996).

[5] R. V. Pound, G. A. Rebka, Jr. and J. L. Snider, “Effect of gravity on gamma radiation”, *Phys. Review* **140**: B788-803 (1965).