# Galactic Classification
# Quantum Gravity and Mass Spectra
# A Cosmological Mass Spectrum each Galaxy having a Quantized Black Hole Core Surface Area Described as under the s p d f g h i... Atomic Symmetry

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## 1 Abstract

There are two types of fundamental quantum gravitational mass amplitude states that are denoted by the subscripts $D$ and $P$. The $D$ amplitudes lead to Einstein's usual general relativity mass density functions. The $P$ amplitudes lead to Einstein's additional pressure mass densities, $3P/c^2$. Both of these densities appear in the stress energy momentum tensor of general relativity. Here they appear as solutions to a non-linear Schrödinger equation and carry three quantising parameters $(l_D, m)$ and $(l_P, m)$, The $l_D, l_P$ values are subsets of the usual electronic quantum variable $l$ which is here denoted by $l'$ to avoid confusion. The $m$ parameter is exactly the same as the electronic quantum theory $m$, there the $z$ component of angular momentum. In this paper, these parametric relations are briefly displayed followed by an account of the connection to the spherical harmonic functions symmetry system that is necessarily involved. Taken together, the two types of mass density can be integrated over configuration space to give quantised general relativity galactic masses in the form of cosmological mass spectra as was shown in previous papers. Here this aspect has been extended to ensure that every galaxy component of the spectra has a *quantised black hole core* with a consequent quantised surface area. This is achieved by replacing the original *free* core radius parameter $r_\epsilon$ with the appropriate Schwarzschild radius associated with the core mass. Explanations are given for the choices of two further, originally free, parameters, $t_b, \theta_0$. The main result from this paper is a quantum *classification scheme* for galaxies determined by the

form of their dark matter spherical geometry.

Keywords: Dust Universe, Dark Energy, Dark Matter,
Newton's Gravitation Constant, Einstein's Cosmological Constant,
Cosmological Mass Spectra, Quantised Gravity, Black Holes

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## 2 Introduction

This paper is a follow up of papers, [48], [49], [50], [52], [55] of similar titles and [56] on the problem of formulating the equation that describes the equilibrium of a gaseous material in a self gravitational equilibrium condition in the galaxy modelling context, [47], see also, appendix 2 of ([35]). In previous papers I have applied this new theory to examining the rotation curves for galactic star motions. That work established that the velocity curves for these quantized dark matter halos are decisively flat. That theory also implied a precise formula that can give many possible mass spectra each of which can give a discrete infinity of spectral lines determined by a quantum parameter $l$ with integral values, $1, 2, 3, \ldots, \infty$, starting at unity and extending up to integral $\infty$. Individual spectra are determined by three *free* parameters, $t_b, r_\epsilon, \theta_0$. In the previous paper, the theoretical structure radius only dependent theory was generalised into a three dimensional form by expressing it in terms of the standard spherical $(r, \theta, \phi)$ coordinates. The was achieved by showing how the many function of the spherical harmonics mathematical collection can be interleaved with the angular momentum solutions part of a non-linear three dimensional Schrödinger equation. The wave function solutions of this Schrödinger equation are then used to produce quantized mass values that can then be formed into a *feedback* potential, replacing the usual *external* potential $V(r)$ for the general three dimensional gravitational Schrödinger equation structure. Thus a quantum theory for galactic gravitational dynamical structure is produced of the same style as the Schrödinger dynamic theory of atomic states. The atomic *gravitational s p d f g h i...* states are examined in more detail in the rest of this paper. The main result introduced in this paper is an identification of one of the original free parameters of the mass spectra, $r_\epsilon$, with the radius of a black hole, $r_\epsilon \rightarrow r_S$, with the same contained mass as the core, $r_S$ being the *Schwarzschild* radius.

## 3 The Quantum Gravitational State Structure

The easiest way for the spherical symmetry quantum gravitational states, $l, m$ parameters, to be explained is via a table of their connections with the standard angular momentum parameters of atomic theory which I denote by $l', m$. The $m$, parameter is the same in both sets and can be called the z component of angular momentum in both sets but not called *the magnetic moment* of the system in the gravitational set because no electric charge is rotating necessarily in that case. The quantized gravity spherical harmonic function parameters $(l, m)$ relation with the usual atomic quantum angular momentum harmonic function parameters $(l', m)$ can take a mass density form, $D$, and and a pressure form, $P$, identified with Einstein's general relativity pressure term as follows,

$$
\begin{aligned}
D: \quad l' &= 2l - 1 && (3.1)\\
P: \quad l' &= 2(2l - 1) && (3.2)\\
l_D &= (l' + 1)/2 && (3.3)\\
l_P &= (l' + 2)/4 && (3.4)\\
-l' &\leq m_{D/P} \leq l'. && (3.5)
\end{aligned}
$$

Listed below are the relations between the $l'$ and the $l$ values starting at $l' = 0$ up to $l' = 20$.

$$
\begin{array}{llllll}
l' & , \; l_D & , \; l_P & ; m_D & ; m_P & (3.6)\\
0 & , \; 1/2, & 2/4 & ; 0 & ; 0 & (3.7)\\
1 & , \; 2/2, & 3/4 & ; \pm 1, 0 & ; 0 & (3.8)\\
2 & , \; 3/2, & 4/4 & ; 0 & ; \pm 2, \pm 1, 0 & (3.9)\\
3 & , \; 4/2, & 5/4 & ; \pm 3, \ldots, \pm 1, 0 & ; 0 & (3.10)\\
4 & , \; 5/2, & 6/4 & ; 0 & ; 0 & (3.11)\\
5 & , \; 6/2, & 7/4 & ; \pm 5, \ldots, \pm 1, 0 & ; 0 & (3.12)\\
6 & , \; 7/2, & 8/4 & ; 0 & ; \pm 6, \ldots, \pm 1, 0 & (3.13)\\
7 & , \; 8/2, & 9/4 & ; \pm 7, \ldots, \pm 1, 0 & ; 0 & (3.14)\\
8 & , \; 9/2, & 10/4 & ; 0 & ; 0 & (3.15)\\
9 & , \; 10/2, & 11/4 & ; \pm 9, \ldots, \pm 1, 0 & ; 0 & (3.16)\\
10 & , \; 11/2, & 12/4 & ; 0 & ; \pm 10, \ldots, \pm 1, 0 & (3.17)
\end{array}
$$

$$11 \quad , \ 12/2, \ 13/4 \ ; \pm 11, \ldots, \pm 1, 0; 0 \tag{3.18}$$
$$12 \quad , \ 13/2, \ 14/4 \ ; 0 \qquad ; 0 \tag{3.19}$$
$$13 \quad , \ 14/2, \ 15/4 \ ; \pm 13, \ldots, \pm 1, 0; 0 \tag{3.20}$$
$$14 \quad , \ 15/2, \ 16/4 \ ; 0 \qquad ; \pm 14, \ldots, \pm 1, 0 \tag{3.21}$$
$$15 \quad , \ 16/2, \ 17/4 \ ; \pm 15, \ldots, \pm 1, 0; 0 \tag{3.22}$$
$$16 \quad , \ 17/2, \ 18/4 \ ; 0 \qquad ; 0 \tag{3.23}$$
$$17 \quad , \ 18/2, \ 19/4 \ ; \pm 17, \ldots, \pm 1, 0; 0 \tag{3.24}$$
$$18 \quad , \ 19/2, \ 20/4 \ ; 0 \qquad ; \pm 14, \ldots, \pm 1, 0 \tag{3.25}$$
$$19 \quad , \ 20/2, \ 21/4 \ ; \pm 19, \ldots, \pm 1, 0; 0 \tag{3.26}$$
$$20 \quad , \ 21/2, \ 22/4 \ ; 0 \qquad ; 0 \tag{3.27}$$

The above is a list showing how the *new* isothermal equilibrium gravitational states represented by the $l$ parameter are related to the usual quantum mechanical Schrödinger equation states for angular momentum, here denoted by $l'$ to avoid confusion. There does seem to be a technical difficulty associated with this list in that values for the $l$ parameter appear that are multiples of $1/2$ or of $1/4$ and many of which do not reduce to integers whereas only integral values have been found in the isothermal $l$-state theory. Among these values which reduce to multiples of $1/2$, there are those that could possible arise from spin which is so far not included in the isothermal equilibrium theory. However, there is no existing explanation for those which reduce to multiples of $1/4$. I cannot resolve the true significance of either of these cases at this juncture so I shall proceed with this work, temporally regarding these two cases as not of *immediately* physical significance. I shall, however work with the above list accepting the existence of *theoretical* gaps that possible may be filled some day. On the positive side, the mathematics of this structure does give *definite* values associated with these parametric gaps. The physical structure of this theory depends heavily on the mathematics of the *Spherical Harmonic functions*. I spell out some relevant essentials of the theory of these functions in the next subsection.

### 3.1 Spherical Harmonics

The gradient operator $\nabla$ in spherical polar coordinates takes the form below and when squared it becomes 3.29

$$\nabla = \frac{\mathbf{e_1}\,\partial}{\partial r} + \frac{\mathbf{e_2}}{r}\frac{\partial}{\partial \theta} + \frac{\mathbf{e_3}}{r\sin(\theta)}\frac{\partial}{\partial \phi} \tag{3.28}$$

$$\nabla^2 = \frac{\partial}{r^2\partial r}\left(\frac{r^2\partial}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\left(\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin(\theta)}\frac{\partial^2}{\partial\phi^2}\right) \tag{3.29}$$

$$= \frac{2\partial}{r\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\left(\frac{\cot(\theta)\partial}{\partial\theta} + \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^2(\theta)}\frac{\partial^2}{\partial\phi^2}\right) \tag{3.30}$$

$$= \frac{2\partial}{r\partial r} + \frac{\partial^2}{\partial r^2} - \frac{\hat{\mathrm{L}}}{r^2} \tag{3.31}$$

$$\hat{\mathrm{L}} = -\left(\frac{\cot(\theta)\partial}{\partial\theta} + \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^2(\theta)}\frac{\partial^2}{\partial\phi^2}\right) \tag{3.32}$$

$$\hat{\mathrm{L}}\psi_\lambda = \lambda\psi_\lambda \tag{3.33}$$

$$x = r\sin(\theta)\cos(\phi) \tag{3.34}$$

$$y = r\sin(\theta)\sin(\phi) \tag{3.35}$$

$$z = r\cos(\theta), \tag{3.36}$$

The spherical harmonic functions $Y(\theta,\phi)$ are the angular *only* factorial parts of solutions, $\psi(r,\theta,\phi)$, of the Laplace equation,

$$\nabla^2\psi = \frac{\partial}{r^2\partial r}\left(\frac{r^2\partial}{\partial r}\right)\psi \quad + \quad \frac{1}{r^2\sin(\theta)}\left(\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin(\theta)}\frac{\partial^2}{\partial\phi^2}\right)\psi = 0 \tag{3.37}$$

$$Y^e_{m,n}(\theta,\phi) = \cos(m\phi)\sin^m(\theta)T^m_{n-m}(\cos(\theta)) = \cos(m\phi)P^m_n(\cos(\theta)) \tag{3.38}$$

$$Y^o_{m,n}(\theta,\phi) = \sin(m\phi)\sin^m(\theta)T^m_{n-m}(\cos(\theta)) = \sin(m\phi)P^m_n(\cos(\theta)) \tag{3.39}$$

$$Y_{m,n}(\theta,\phi) = \exp(im\phi)\sin^m(\theta)T^m_{n-m}(\cos(\theta)) = \exp(im\phi)P^m_n(\cos(\theta)) = Y^e_{m,n}(\theta,\phi) + iY^o_{m,n}(\theta,\phi) \tag{3.40}$$

and they are even in $\phi$ of the form (3.38), odd in $\phi$ of the form (3.39) or in complex exponential form in $\phi$ as at (3.40). The functions $T^m_{n-m}(\cos(\theta))$

are called tesseral harmonics and can be defined as

$$T^m_{n-m}(x) = \frac{d^m P_n(x)}{dx^m} \quad (3.41)$$

$$x = \cos(\theta), \quad (3.42)$$

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n \quad (3.43)$$

where $P_n(x)$ is a Legendre polynomial conveniently defined by (3.43). The functions $P^m_n(\cos(\theta)) = \sin^m(\theta)T^m_{n-m}(\cos(\theta))$ above are the associate Legendre functions and are solutions to the Legendre equation (3.44). The $x$ above and below is used as an abbreviation for $\cos(\theta)$.

$$\frac{d}{dx}\left((1-x^2)\frac{dP}{dx}\right) + \left(l(l+1) - \frac{m^2}{1-x^2}\right)P = 0. \quad (3.44)$$

The associated Lagrange functions $P^m_n(\cos(\theta))$ which are solutions to the above equation can also be expressed, in terms of the Legendre polynomials $P_n(x)$ as

$$P^m_n(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m}, \ m \geq 0. \quad (3.45)$$

Then using (3.43), we infer that

$$P^m_n(x) = \frac{1}{2^n n!}(1-x^2)^{m/2}\frac{d^{n+m}}{dx^{n+m}}(x^2-1)^n. \quad (3.46)$$

In volume integrals in three space polar coordinates , the volume element, $dv$, is

$$dv = r^2 \sin(\theta) dr d\theta d\phi. \quad (3.47)$$

The mathematics of this system requires the evaluation of the total mass within a spatial volume given the mass density within that volume and this density involves the spherical harmonics contributing to that density as factors in the form $Y(\theta,\phi)Y^*(\theta,\phi)$. So evaluating total amounts of mass in a given volume involves integrals of the form

$$\int_0^\pi \int_0^{2\pi} Y(\theta,\phi)Y^*(\theta,\phi)\sin(\theta) d\theta d\phi. \quad (3.48)$$

Taking the complex version version for $Y(\theta,\phi) = Y_{m,n}(\theta,\phi)$ at (3.40) the integral above becomes

$$\begin{aligned} & \int_0^{\pi}\int_0^{2\pi} P_n^m(\cos(\theta))P_n^m(\cos(\theta))\sin(\theta)d\theta d\phi \\ = \;& 2\pi\int_{-1}^{+1}(P_n^m(x))^2dx = \frac{4\pi}{2n+1}\frac{(n+m)!}{(n-m)!}. \end{aligned} \tag{3.49}$$

The last stage of this integration at (3.49) takes about a page of algebra from the $x$ integration preceding it. However if we take either of the real versions of the $Y$, (3.38) or (3.39). The integral (3.50) or (3.51) which are the same for $m > 0$ because of (3.53).

$$\begin{aligned} & \int_0^{\pi}\int_0^{2\pi} Y_{m,n}^e(\cos(\theta))Y_{m,n}^e(\cos(\theta))\sin(\theta)d\theta d\phi \\ = \;& \int_0^{2\pi}\cos^2(m\phi)d\phi\int_{-1}^{+1}(P_n^m(x))^2dx = \frac{4\pi}{\epsilon_m(2n+1)}\frac{(n+m)!}{(n-m)!}. \quad (3.50) \\ & \int_0^{\pi}\int_0^{2\pi} Y_{m,n}^o(\cos(\theta))Y_{m,n}^o(\cos(\theta))\sin(\theta)d\theta d\phi \\ = \;& \int_0^{2\pi}\sin^2(m\phi)d\phi\int_{-1}^{+1}(P_n^m(x))^2dx = \frac{4\pi}{\epsilon_m(2n+1)}\frac{(n+m)!}{(n-m)!}, \quad (3.51) \\ = \;& N_{n,m}^2, \; \textit{say for } m \geq 0, \quad (3.52) \end{aligned}$$

where

$$\begin{aligned} \frac{1}{\epsilon_m} &= \frac{1}{2\pi}\int_0^{2\pi}\cos^2(m\phi)d\phi = \frac{1}{2\pi}\int_0^{2\pi}\sin^2(m\phi)d\phi, \quad m > 0 \quad (3.53) \\ &= \frac{1}{1}, \quad m = 0 \implies \epsilon_m = 1 \quad (3.54) \\ &= \frac{1}{2}, \quad m = 1,2,3\ldots \implies \epsilon_m = 2. \quad (3.55) \end{aligned}$$

However, if we take the complex version (3.40) for the spherical harmonics in the mass density functions there is no $\epsilon_m$ factor appearing and so there is no half factor necessary as at (3.55). An important and useful relation between the associated Legendre functions for positive and negative values for $m$ is

$$P_l^{-m}(x) = (-1)^m\frac{(l-m)!}{(l+m)!}P_l^m(x). \tag{3.56}$$

The integrated product expression (3.48) can be used to find what is called the normalization constant for the spherical harmonics by using it to define normalised spherical harmonics for which this integral would have the value unity

$$\int_0^{\pi}\int_0^{2\pi}\dot{Y}(\theta,\phi)\dot{Y}^*(\theta,\phi)\sin(\theta)d\theta d\phi = 1. \tag{3.57}$$

Consequently in the special case of the complex spherical harmonics a normalised version, denoted by the overhead dot, $\dot{Y}_{m,n}(\theta,\phi)$ for $Y_{m,n}(\theta,\phi)$ at (3.49) would take the form

$$\dot{Y}_{m,n}(\theta,\phi) = (-1)^m\left(\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}\right)^{1/2} Y_{m,n}(\theta,\phi), \tag{3.58}$$

if the negative option from taking the square root of $N^2_{m,n}$ is identified with odd values of $m$ as is the usual convention adopted. The normalised products are important in atomic quantum theory because there the concern is with probability density functions obtained from quantum state functions depending in various parameters, $l$ and $m$ for example, the integration over such products giving the total probability for finding a particle in some region in which it is certain to be found is represented by unity. In this theoretical work the important information is the total amount of mass in some region which is obtained from integrating over mass densities obtained from products of quantum states depending in this case on subsets of the $l$ parameter and again on $m$. However, whereas the atomic quantum states are normalized by taking the integrated products equal to one the gravitational states make great use of the same normalisation factors in the very inverse way of using the square of their values or equivalently the integrals of the not normalized state function products in terms of $l$ and $m$ to determine gravitational mass spectra in terms of $l$ and $m$. By taking the complex conjugate of the normalised version of $Y_{m,n}(\theta,\phi)$ at (3.40), we get

$$\dot{Y}^*_{m,n}(\theta,\phi) = (-1)^m \left(\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}\right)^{1/2} \exp(-im\phi)P_n^m(\cos(\theta)) \tag{3.59}$$

$$= \left(\frac{2n+1}{4\pi}\frac{(n+m)!}{(n-m)!}\right)^{1/2} \exp(-im\phi)P_n^{-m}(\cos(\theta)) \tag{3.60}$$

$$= (-1)^{-m}\dot{Y}_{-m,n}(\theta,\phi) \tag{3.61}$$

$$\dot{Y}_{-m,n}(\theta,\phi) = (-1)^{+m}\dot{Y}^*_{+m,n}(\theta,\phi). \tag{3.62}$$

At (3.60), formula (3.56) has been used to replace $P_n^m(x)$ with $P_n^{-m}(x)$ and in the last line above functions have been transposed across the equals sign to give a definition for the normalized $Y$ now for possible negative $m$ showing it to be equal to the complex conjugate of $Y$ for positive $m$ with the additional $(-1)^{+m}$ factor. Thus the normalisation constant given at (3.52) for the complex versions of the $Y$ generally can be taken to have the property

$$N'^{-2}_{n,m} = \left(\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}\right) \tag{3.63}$$

and the property below for the real versions of the $Y$

$$N^{-2}_{n,m} = \left(\frac{\epsilon_m(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}\right) \tag{3.64}$$

$$\epsilon_0 = 1,\ \epsilon_m = 2,\ (|m| = 1,2,3,\ldots,l'). \tag{3.65}$$

The information on the spherical harmonic functions given above is sufficient to explain the gravitational mass spectra functions and calculate the numerical mass spectra given also, of course, explicit mathematical realisations of these functions of $\theta$ and $\phi$ in terms of the $l$ and $m$ parameters and the radial dependence of the gravitational quantum wave functions. This aspect will be examined in the next section.

# 4 Obtaining Gravity Mass Spectra

Below the data for the $D$ states above at (4.66) to (3.27) has been separated out in order to concentrate on the mass spectra associated with these states.

My objective is to obtain the numerical mass spectra structure *firstly* for the $D$ states in order to see if any conclusion can be formed about the physical significance of these states.

$$l' \quad , \quad l_D \; , \quad ; m_D \qquad ; \tag{4.66}$$
$$1 \quad , \quad 2/2, \quad ; \pm 1, 0 \qquad ; \tag{4.67}$$
$$3 \quad , \quad 4/2, \quad ; \pm 3, \ldots, \pm 1, 0 \; ; \tag{4.68}$$
$$5 \quad , \quad 6/2, \quad ; \pm 5, \ldots, \pm 1, 0 \; ; \tag{4.69}$$
$$7 \quad , \quad 8/2, \quad ; \pm 7, \ldots, \pm 1, 0 \; ; \tag{4.70}$$
$$9 \quad , \quad 10/2, \quad ; \pm 9, \ldots, \pm 1, 0 \; ; \tag{4.71}$$
$$11 \quad , \quad 12/2, \quad ; \pm 11, \ldots, \pm 1, 0; \tag{4.72}$$
$$13 \quad , \quad 14/2, \quad ; \pm 13, \ldots, \pm 1, 0; \tag{4.73}$$
$$15 \quad , \quad 16/2, \quad ; \pm 15, \ldots, \pm 1, 0; \tag{4.74}$$
$$17 \quad , \quad 18/2, \quad ; \pm 17, \ldots, \pm 1, 0; \tag{4.75}$$
$$19 \quad , \quad 20/2, \quad ; \pm 19, \ldots, \pm 1, 0; \tag{4.76}$$

The mass spectra generating function for these states is given at (4.82) in the form it has for the general polar spherical angular $l, m$ parameter dependence if the states chosen for representation are the complex versions with normalisation given at (3.63). Thus the mass spectra function for the $D$ and $P$ cases are respectively

$$M_{l,m,D}(r_\epsilon, \theta_0, t_b) = \frac{c^2 \Lambda s(t_b)}{G} \left( \frac{(2l-1)^{4l} 2l \theta_0^{2l} r_\epsilon^3}{3(4l-3)} \right) A(2l-1, m) \tag{4.77}$$

$$M_{l,m,P}(r_\epsilon, \theta_0, t_b) = \frac{c^2 \Lambda s(t_b)}{G} \left( \frac{(2l-1)^{8l-2}(4l-1)\theta_0^{4l-1} r_\epsilon^3}{(8l-5)} \right) \times A(4l-2, m), \tag{4.78}$$

where $s(t)$ is used to denote the dimensionless function, $\sinh^{-2}((3\Lambda)^{1/2} ct/2)$, of time $t$ from the general relativity dust universe model with $t_b$ the formation date of the mass accumulation and

$$A(l', m) = \frac{1}{\epsilon_m (2l'+1)} \left( \frac{(l'+m)!}{(l'-m)!} \right) = \frac{1}{\epsilon_m (2l'+1)} \left( \frac{\Gamma(l'+m+1)}{\Gamma(l'-m+1)} \right) \tag{4.79}$$

with the appropriate value for $l'$ for the $D$ and $P$ cases. In the $D$ case $l' = 2l - 1$. In the $P$ case $l' = 4l - 2$. The two cases being

$$A(2l-1, m) = \frac{1}{\epsilon_m(4l-1)} \left( \frac{\Gamma(2l+m)}{\Gamma(2l-m)} \right) \tag{4.80}$$

$$A(4l-2, m) = \frac{1}{\epsilon_m(8l-3)} \left( \frac{\Gamma(4l-1+m)}{\Gamma(4l-1-m)} \right). \tag{4.81}$$

Thus the mass spectrum for the $D$ case (4.82) can be expressed as a product as at (4.83)

$$M_{l,m,D}(r_\epsilon, \theta_0, t_b) = \frac{c^2 \Lambda s(t_b)}{G} \left( \frac{(2l-1)^{4l} 2l \theta_0^{2l} r_\epsilon^3}{3(4l-3)} \right) A'(2l-1, m) \tag{4.82}$$

$$M_{l,m,D}(r_\epsilon, \theta_0, t_b) = M_D(t_b, r_\epsilon) S_{l,m,D}(\theta_0) \tag{4.83}$$

$$M_D(t_b, r_\epsilon) = \frac{c^2 \Lambda s(t_b) r_\epsilon^3}{G} \tag{4.84}$$

$$S_{l,m,D}(\theta_0) = \left( \frac{(2l-1)^{4l} 2l \theta_0^{2l}}{3(4l-3)} \right) A'(2l-1, m) \tag{4.85}$$

where

$$A'(2l-1, m) = \frac{1}{(4l-1)} \left( \frac{(2l-1+m)!}{(2l-1-m)!} \right) = N_{l,m}^{\prime 2}/(4\pi). \tag{4.86}$$

The mass spectrum for the $P$ case can be expressed as a product as at (4.88)

$$M_{l,m,P}(r_\epsilon, \theta_0, t_b) = \frac{c^2 \Lambda s(t_b)}{G} \left( \frac{(2l-1)^{8l-2} (4l-1) \theta_0^{4l-1} r_\epsilon^3}{(8l-5)} \right) \times A(4l-2, m) \tag{4.87}$$

$$M_{l,m,P}(r_\epsilon, \theta_0, t_b) = M_P(t_b, r_\epsilon) S_{l,m,P}(\theta_0) \tag{4.88}$$

$$M_P(t_b, r_\epsilon) = \frac{c^2 \Lambda s(t_b) r_\epsilon^3}{G} = M_D(t_b, r_\epsilon) \tag{4.89}$$

$$S_{l,m,P}(\theta_0) = \left( \frac{(2l-1)^{8l-2} (4l-1) \theta_0^{4l-1}}{(8l-5)} \right) A(4l-2, m) \tag{4.90}$$

where

$$A(4l-2, m) = \frac{1}{\epsilon_m(8l-3)} \left( \frac{\Gamma(4l-1+m)}{\Gamma(4l-1-m)} \right). \tag{4.91}$$

## 5 Replacing the Galactic Core with a Black Hole

$$\rho_{D,l,m}(r) = \frac{\rho(t_b)\theta_0^{2l}(2l-1)^{4l}}{\epsilon_m(4l-1)}\left(\frac{r}{r_\epsilon}\right)^{-4l}\left(\frac{\Gamma(2l+m)}{\Gamma(2l-m)}\right) \tag{5.1}$$

$$\rho_{D,l,m}(r_\epsilon) = \frac{\rho(t_b)\theta_0^{2l}(2l-1)^{4l}}{\epsilon_m(4l-1)}\left(\frac{\Gamma(2l+m)}{\Gamma(2l-m)}\right) \tag{5.2}$$

$$\frac{3P(r)}{c^2} = \rho_{P,l,m}(r) = \frac{3\rho(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}}{\epsilon_m(8l-3)}\left(\frac{r}{r_\epsilon}\right)^{2-8l}\left(\frac{\Gamma(4l-1+m)}{\Gamma(4l-1-m)}\right) \tag{5.3}$$

$$\rho_{P,l,m}(r_\epsilon) = \frac{3\rho(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}}{\epsilon_m(8l-3)}\left(\frac{\Gamma(4l-1+m)}{\Gamma(4l-1-m)}\right) \tag{5.4}$$

$$\rho(t) = (3/(8\pi G))(c/R_\Lambda)^2\sinh^{-2}(3ct/(2R_\Lambda)). \tag{5.5}$$

The two basic mass density functions $\rho_{D,l,m}(r)$ and $\rho_{P,l,m}(r)$ describe quantum gravity mass distribution states in isothermal equilibrium. In their most primitive form they were divergent at the radius distance variable, $r = 0$, origin. To rectify this I introduced the parameter $r_\epsilon$ within which distance the divergent part of the function $\rho_{D/P,l,m}(r)$ was replaced with the constant value $\rho_{D/P,l,m}(r_\epsilon)$ so that invariably the mass of the core, $M_C$, in the volume with radius distance less than $r_\epsilon$ would have the value $M_C = 4\pi\rho_{D/P,l,m}(r_\epsilon)r_\epsilon^3/3$ which depends on the values of quantum parameters $(l, m)$ but within which the $r_\epsilon$ quantity could be given any arbitrary value. This situation I exploited to obtain mass spectra, keeping a fixed value of $r_\epsilon$ for any specific spectrum whilst the detailed line structure was found by ranging over the $(l, m)$, parameters. However, it seems to me that this mathematical device that conveniently removes the infinity can be given a physical justification by using the consensual view that most, if not all, galaxies have a black hole at their centre. This is simply implemented by finding the value of $r_\epsilon$ that ensures $r_\epsilon$ is equal to the value of the *Schwarzschild* radius that is implied by the mass of the core for the state

$(D/P, l, m)$. This can be achieved as follows by taking,

$$
\begin{aligned}
M_C = \frac{c^2 r_\epsilon}{2G} &= 4\pi r_\epsilon^3 \rho_{D/P,l,m}(r_\epsilon)/3 && (5.6)\\
\frac{c^2}{2G} &= 4\pi r_\epsilon^2 \frac{\rho(t_b)\theta_0^{2l}(2l-1)^{4l}}{3\epsilon_m(4l-1)} \left(\frac{\Gamma(2l+m)}{\Gamma(2l-m)}\right) && (5.7)\\
\frac{c^2}{2G} &= 4\pi r_\epsilon^2 \frac{3\rho(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}}{3\epsilon_m(8l-3)} \left(\frac{\Gamma(4l-1+m)}{\Gamma(4l-1-m)}\right) && (5.8)
\end{aligned}
$$

so that $r_\epsilon$ becomes the *Scharzschild* radius, $r_{Sch}$ by the first equality above. The value of the second equality above can be found using formulae (5.2) or (5.4). This is spelt out in the second and third lines above for the two cases, $D$ and $P$. Thus there are obviously different values for $r_\epsilon$ for the $D$ and $P$ case. These two values are given below in terms of the $r_\epsilon^3$, the needed form, and with state suffix identified.

$$
\begin{aligned}
r^3_{\epsilon,D,l,m}(t_b,\theta_0) &= \left(\frac{3c^2\epsilon_m(4l-1)\Gamma(2l-m)}{8\pi G\rho(t_b)\theta_0^{2l}(2l-1)^{4l}\Gamma(2l+m)}\right)^{3/2} && (5.9)\\
r^3_{\epsilon,P,l,m}(t_b,\theta_0) &= \left(\frac{c^2\epsilon_m(8l-3)\Gamma(4l-1-m)}{8\pi G\rho(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}\Gamma(4l-1+m)}\right)^{3/2} && (5.10)
\end{aligned}
$$

These two values for $r_\epsilon$ can be used in the mass spectrum formulae to give *cosmological mass spectra* that spell out the masses of galaxies against the spherical harmonic parameters $(l, m)$ that all have a quantised surface area core black holes. The main emphasis on this last piece of work has been on using the *basic densities* to get the black hole version of the cosmological spectra. I shall now give a more detailed version of this step working with the original mass spectra mass spectra functions which effectively reinforces the results at (5.9) and (5.10).

If the total galactic mass up to radius $r$ is evaluated up to the parameter value $r = r_\epsilon$. That is the core mass is evaluated we get the total core mass $M_l(r_\epsilon)$ of a Galaxy expressed as

$$
M_{l,m}(r_\epsilon) = M_{D,l,m}(r_\epsilon) + M_{P,l,m}(r_\epsilon) + M_{DE}(r_\epsilon) \qquad (5.11)
$$

where

$$\begin{aligned}
M_{D,l,m}(r_\epsilon,\theta_0,t_b) &= \frac{c^2\Lambda s(t_b)\theta_0^{2l}r_\epsilon^3(2l-1)^{4l}A(2l-1,m)}{6G} && (5.12)\\
A(2l-1,m) &= \frac{1}{\epsilon_m(4l-1)}\left(\frac{(2l-1+m)!}{(2l-1-m)!}\right) \\
&= \frac{1}{\epsilon_m(4l-1)}\left(\frac{\Gamma(2l+m)}{\Gamma(2l-m)}\right) && (5.13)\\
M_{D,l,m}(r_\epsilon,\theta_0,t_b) &= \frac{c^2\Lambda s(t_b)\theta_0^{2l}r_\epsilon^3(2l-1)^{4l}\Gamma(2l+m)}{6G\epsilon_m(4l-1)\Gamma(2l-m)} && (5.14)\\
M_{P,l,m}(r_\epsilon,\theta_0,t_b) &= \frac{c^2\Lambda s(t_b)\theta_0^{4l-1}r_\epsilon^3(2l-1)^{8l-2}A(4l-2,m)}{2G} && (5.15)\\
A(4l-2,m) &= \frac{1}{\epsilon_m(8l-3)}\left(\frac{(4l-2+m)!}{(4l-2-m)!}\right) \\
&= \frac{1}{\epsilon_m(8l-3)}\left(\frac{\Gamma(4l-1+m)}{\Gamma(4l-1-m)}\right) && (5.16)\\
M_{P,l,m}(r_\epsilon,\theta_0,t_b) &= \frac{c^2\Lambda s(t_b)\theta_0^{4l-1}r_\epsilon^3(2l-1)^{8l-2}\Gamma(4l-1+m)}{2G\epsilon_m(8l-3)\Gamma(4l-1-m)} && (5.17)\\
M_{DE}(r_\epsilon) &= \frac{c^2\Lambda r_\epsilon^3}{3G}. && (5.18)
\end{aligned}$$

The Schwarzschild radius $r_{S,M}$ associated with any mass $M$ is given by

$$r_{S,M} = \frac{2GM}{c^2} \qquad (5.19)$$

Thus we can write the three mass equations above as

$$\begin{aligned}
r_{S,D,l,m}(t_b,\theta_0,r_\epsilon) &= \frac{\Lambda s(t_b)\theta_0^{2l}r_\epsilon^3(2l-1)^{4l}\Gamma(2l+m)}{3\epsilon_m(4l-1)\Gamma(2l-m)} && (5.20)\\
r_{S,P,l,m}(t_b,\theta_0,r_\epsilon) &= \frac{\Lambda s(t_b)\theta_0^{4l-1}r_\epsilon^3(2l-1)^{8l-2}\Gamma(4l-1+m)}{\epsilon_m(8l-3)\Gamma(4l-1-m)} && (5.21)\\
r_{S,DE}(r_\epsilon) &= \frac{2\Lambda r_\epsilon^3}{3}. && (5.22)
\end{aligned}$$

It follows that the three possible mass accumulations above are black holes or not depends on the following list.

$$\frac{r_{S,D,l,m}(t_b,\theta_0,r_\epsilon)}{r_\epsilon} \quad \geq 1 \quad \textit{or not} \ < 1 \tag{5.23}$$

$$\frac{r_{S,P,l,m}(t_b,\theta_0,r_\epsilon)}{r_\epsilon} \quad \geq 1 \quad \textit{or not} \ < 1 \tag{5.24}$$

$$\frac{r_{S,DE}(r_\epsilon)}{r_\epsilon} \quad \geq 1 \quad \textit{or not} \ < 1, \tag{5.25}$$

where

$$M_{D,l.m}(r_\epsilon) = \frac{c^2 r_{S,D,l,m}(t_b,\theta_0,r_\epsilon)}{2G} \tag{5.26}$$

$$M_{P,l.m}(r_\epsilon) = \frac{c^2 r_{S,P,l,m}(t_b,\theta_0,r_\epsilon)}{2G} \tag{5.27}$$

$$M_{DE}(r_\epsilon) = \frac{c^2 r_{S,DE}(r_\epsilon)}{2G} \tag{5.28}$$

Thus the maximum value that $r_\epsilon$ can have, in the three cases taking $a_X(l,m)$ to be the surface area of the black hole is, for the mass accumulation involved to be a black hole in the $D$ case is given by (5.29) implying that the area of the black hole is quantised and given by (5.30) and the parameter $r_\epsilon$ given by (5.31). In the $P$ case for the mass accumulation involved to be a black hole, its quantised surface area is given by (5.33) implying that the parameter $r_\epsilon$ given by (5.34).

$$
\begin{aligned}
1 &= \frac{r_{S,D,l,m}(t_b,\theta_0,r_\epsilon)}{r_\epsilon} = \frac{\Lambda s(t_b)\theta_0^{2l}a_D(l,m)(2l-1)^{4l}\Gamma(2l+m)}{12\pi\epsilon_m(4l-1)\Gamma(2l-m)} && (5.29)\\
a_D(l,m) &= \frac{12\pi\epsilon_m(4l-1)\Gamma(2l-m)}{\Lambda s(t_b)\theta_0^{2l}(2l-1)^{4l}\Gamma(2l+m)} = 4\pi r_\epsilon^2 && (5.30)\\
r_{\epsilon,D}(l,m) &= \left(\frac{3\epsilon_m(4l-1)\Gamma(2l-m)}{\Lambda s(t_b)\theta_0^{2l}(2l-1)^{4l}\Gamma(2l+m)}\right)^{1/2} && (5.31)\\
1 &= \frac{r_{S,P,l,m}(t_b,\theta_0,r_\epsilon)}{r_\epsilon}\\
&= \frac{\Lambda s(t_b)\theta_0^{4l-1}a_P(l,m)(2l-1)^{8l-2}\Gamma(4l-1+m)}{4\pi\epsilon_m(8l-3)\Gamma(4l-1-m)} && (5.32)\\
a_P(l,m) &= \frac{4\pi\epsilon_m(8l-3)\Gamma(4l-1-m)}{\Lambda s(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}\Gamma(4l-1+m)} = 4\pi r_\epsilon^2 && (5.33)\\
r_{\epsilon,P}(l,m) &= \left(\frac{\epsilon_m(8l-3)\Gamma(4l-1-m)}{\Lambda s(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}\Gamma(4l-1+m)}\right)^{1/2} && (5.34)\\
1 &= \frac{r_{S,DE}(r_\epsilon)}{r_\epsilon} = \frac{2\Lambda r_\epsilon^2}{3} = \frac{\Lambda a_{DE}}{6\pi} \implies a_{DE} = 2\pi R_\Lambda^2 && (5.35)\\
1 &= \frac{r_{S,DE}(r_\epsilon)}{r_\epsilon} = \frac{2\Lambda r_\epsilon^2}{3} = 2\left(\frac{r_\epsilon}{R_\Lambda}\right)^2 \implies r_\epsilon = \frac{R_\Lambda}{\sqrt{2}} && (5.36)\\
a_X &= 4\pi r_{\epsilon,X}^2,\ \textit{according to case},\ X = D, P, DE. && (5.37)
\end{aligned}
$$

Thus the quantisation of the surface areas of the $D$ and $P$ mass spectrum central black hole cores is given by equations (5.30) and (5.33) respectively. Working from such a centre, for the core to be a black hole formed from *dark energy mass* it must occupy a sphere with the very large radius $\frac{R_\Lambda}{\sqrt{2}}$ given at (5.36) which is a very unlikely size for the core of any galaxy though still a possibility of theoretical interest.

The full mass spectra functions are

$$M_{l,m,D}(r_\epsilon,\theta_0,t_b) = \frac{c^2\Lambda s(t_b)}{G\epsilon_m(4l-1)}\left(\frac{(2l-1)^{4l}2l\theta_0^{2l}r_\epsilon^3\Gamma(2l+m)}{3(4l-3)\Gamma(2l-m)}\right) \tag{5.38}$$

$$\begin{aligned} M_{l,m,P}(r_\epsilon,\theta_0,t_b) &= \frac{c^2\Lambda s(t_b)}{G\epsilon_m(8l-3)} \\ &\times \left(\frac{(2l-1)^{8l-2}(4l-1)\theta_0^{4l-1}r_\epsilon^3\Gamma(4l-1+m)}{(8l-5)\Gamma(4l-1-m)}\right) \end{aligned} \tag{5.39}$$

The core masses below are each followed by the $r_\epsilon^3$ value which makes them black holes at the centre of the full spectral masses above.

$$M_{D,l,m}(r_\epsilon,\theta_0,t_b) = \frac{c^2\Lambda s(t_b)\theta_0^{2l}r_\epsilon^3(2l-1)^{4l}\Gamma(2l+m)}{6G\epsilon_m(4l-1)\Gamma(2l-m)} \tag{5.40}$$

$$r^3_{\epsilon,D}(l,m) = \left(\frac{3\epsilon_m(4l-1)\Gamma(2l-m)}{\Lambda s(t_b)\theta_0^{2l}(2l-1)^{4l}\Gamma(2l+m)}\right)^{3/2} \tag{5.41}$$

$$M_{P,l,m}(r_\epsilon,\theta_0,t_b) = \frac{c^2\Lambda s(t_b)\theta_0^{4l-1}r_\epsilon^3(2l-1)^{8l-2}\Gamma(4l-1+m)}{2G\epsilon_m(8l-3)\Gamma(4l-1-m)} \tag{5.42}$$

$$r^3_{\epsilon,P}(l,m) = \left(\frac{\epsilon_m(8l-3)\Gamma(4l-1-m)}{\Lambda s(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}\Gamma(4l-1+m)}\right)^{3/2}. \tag{5.43}$$

If we substitute the $r_\epsilon^3$ values into the full mass spectrum values at (5.44) and (5.48), we get the resultant values $M_{l,m,D}(\theta_0,t_b)$ and $M_{l,m,P}(\theta_0,t_b)$ at (5.47) and (5.51).

$$M_{r_\epsilon,l,m,D}(\theta_0,t_b) = \frac{c^2\Lambda s(t_b)\theta_0^{2l}r_\epsilon^3 2l(2l-1)^{4l}\Gamma(2l+m)}{3G\epsilon_m(4l-1)(4l-3)\Gamma(2l-m)} \tag{5.44}$$

$$\leftarrow r^3_{\epsilon,D}(l,m) = \left(\frac{\Lambda s(t_b)\theta_0^{2l}(2l-1)^{4l}\Gamma(2l+m)}{3\epsilon_m(4l-1)\Gamma(2l-m)}\right)^{-3/2} \tag{5.45}$$

$$M_{l,m,D}(\theta_0, t_b) = \frac{c^2(\Lambda s(t_b))^{-1/2}\theta_0^{-l}2l(2l-1)^{-2l}\Gamma(2l+m)^{-1/2}}{3^{-1/2}G\epsilon_m^{-1/2}(4l-1)^{-1/2}(4l-3)\Gamma(2l-m)^{-1/2}} \tag{5.46}$$

$$= \frac{3^{1/2}c^2\epsilon_m^{1/2}(4l-1)^{1/2}2l\Gamma(2l-m)^{1/2}}{G(\Lambda s(t_b))^{1/2}\theta_0^{l}(2l-1)^{2l}(4l-3)\Gamma(2l+m)^{1/2}} \tag{5.47}$$

$$M_{r_\epsilon,l,m,P}(\theta_0, t_b) = \frac{c^2\Lambda s(t_b)\theta_0^{4l-1}r_\epsilon^3(4l-1)(2l-1)^{8l-2}\Gamma(4l-1+m)}{G\epsilon_m(8l-3)(8l-5)\Gamma(4l-1-m)} \tag{5.48}$$

$$\leftarrow r_{\epsilon,P}^3(l,m) = \left(\frac{\Lambda s(t_b)\theta_0^{4l-1}(2l-1)^{8l-2}\Gamma(4l-1+m)}{\epsilon_m(8l-3)\Gamma(4l-1-m)}\right)^{-3/2} \tag{5.49}$$

$$M_{l,m,P}(\theta_0, t_b)$$

$$= \frac{c^2(\Lambda s(t_b))^{-1/2}\theta_0^{-2l+1/2}(4l-1)(2l-1)^{-4l+1}\Gamma(4l-1+m)^{-1/2}}{G\epsilon_m^{-1/2}(8l-3)^{-1/2}(8l-5)\Gamma(4l-1-m)^{-1/2}} \tag{5.50}$$

$$= \frac{c^2\epsilon_m^{1/2}(8l-3)^{1/2}(4l-1)\Gamma(4l-1-m)^{1/2}}{G(\Lambda s(t_b))^{1/2}\theta_0^{2l-1/2}(8l-5)(2l-1)^{4l-1}\Gamma(4l-1+m)^{1/2}} \tag{5.51}$$

## 6 Conclusions

In this paper, the cosmological mass spectra for galactic masses obtained in the previous paper ([56]) have been made more realistic by setting up the originally free parameter $r_\epsilon$ so that what was originally described as being the galactic core now becomes a definite quantised black hole. That leaves two free parameters, the galactic birth time $t_b$ and the dimensionless parameter $\theta_0$ to assign values to before the spectra can be calculate and displayed in detail. The obvious and very convenient way to decide the birth time parameter, $t_b$ possible values, is to leave it *arbitrary* for the following reasons. The factor $s(t_b)$ that appears in all the spectra formulae is *not* an *intrinsic* part of the formula. As things stand, it appears with fixed value multiplying every spectral mass. It simply scales the whole spectrum. Thus I suggest all spectra should be interpreted as *possible* sets of values associated with some arbitrary but definite evolutionary time $t_b$ and further that no mass member of such a set would have necessarily certainly occurred physically. The parameter $\theta_0$ is intrinsic, in that it determines the relative values between individual members of the spectrum. Thus its value is important in controlling the line spacing of the spectrum structure. I can see, at the moment, no theoretical way to determine this parameter. So the only alternative is to find it physically. The obvious way to approach it from that direction would be to have exact knowledge of the mass of a specific galaxy of known $(l, m)$ character and then evaluate $\theta_0$ from the appropriate mass spectrum function. However, at this time, the $(l, m)$ structure is not known for any galaxy and the actual galactic masses generally are only known very approximately. I think that this parameter probably has an optimum value that will turn out to be a fundamental scale constant. As neither of the usual options are available to deciding $\theta_0$ ,I have adjusted its value so that the central sub-value of the $l = 1$ state which has the least number of sub-options, actually only one, if we only take the integral options $(m = -1, 0, +1)$, to a specific mass value. This value $M_G$ is obtained from the formula

$$M_G = \frac{R_\Lambda c^2}{G} = \left(\frac{3c^4}{\Lambda G^2}\right)^{1/2} \approx 2.00789 \times 10^{53}\ kg \qquad (6.1)$$

which for *unknown* reasons give a number of kilograms which could be the actual mass of our universe.

I have included in the last section of this paper a print out of the galactic mass spectrum based on the above mass with *minimum detailed* refinement, $X = 1$. That is to say, taking mass line separation to be the largest interval for the $l$ and $m$ parameters, 1. Only sufficient of this print out is given to fill one page for its beginning and a second page for its ending, in fact just a sample. The central part of the spectrum has not been included but the formula for printing it all out has been given. The formula for generating the printing out the whole table is adaptable, $X = 1$, to disregarding the possible line separation refinements $X = 1/2$ or $X = 1/4$, if that is preferred. The main result of obtained in this paper is essentially a system for classifying galaxies in terms of their spherical harmonic structure and their mass values. The unrefined version of the system involving the parameter $X = 1$ has been firmly established. The refined version involving $X = 1/2, 1/4$ is open to some uncertainty. The print out program follows

## 7 Spectral Table Print Out Program

```
MD[l_, m_, theta0_, tb_] :=
  (3^(1/2) c^2 epsilon[m]^(1/2) (4 l - 1)^(1/2) 2
     l Gamma[2 l - m]^(1/2)) / (G (Lambda s[tb])^(1/2)
     theta0^(l) (2 l - 1)^(2 l) (4 l - 3) Gamma[2 l + m]^(1/2))
MP[l_, m_, theta0_, tb_] := (c^2 epsilon[m]^(1/2)
     (8 l - 3)^(1/2) (4 l - 1) Gamma[4 l - 1 - m]^(1/2)) /
   (G (Lambda s[tb])^(1/2) theta0^(2 l - 1/2) (8 l - 5)
     (2 l - 1)^(4 l - 1) Gamma[4 l - 1 + m]^(1/2))
epsilon[0] := 1
Do[epsilon[p_] := 2, {p, 1/4, +50.25, 1/4}]
Do[epsilon[p_] := 2, {p, -50.25, -1/4, 1/4}]
s[t_] := Sinh[(3 Lambda)^(1/2) c t/2]^(-2)
c := 299 792 458
Lambda :=  1.35×10^(-52)
RLambda := (3/Lambda)^(1/2)
tc := (2 RLambda/(3 c)) ArcSinh[2^(-1/2)]
X = 1
tb = tc
theta0 = 2.97845
 Table[{l, m, (MD[l, m, theta0, tb] + MP[l, m, theta0, tb])},
  {l, 1, 18, X}, {m, -l, +l, X}]
```

## 8 Spectral Table Print Out Beginning

$$
\begin{aligned}
&\{\{\{1, -1, 4.53785\times10^{53}\},\\
&\{1, 0, 2.00789\times10^{53}\}, \{1, 1, 1.85718\times10^{53}\}\},\\
&\{\{2, -2, 6.58827\times10^{51}\}, \{2, -1, 2.06611\times10^{51}\},\\
&\{2, 0, 4.201\times10^{50}\}, \{2, 1, 1.71147\times10^{50}\},\\
&\{2, 2, 5.40563\times10^{49}\}\}, \{\{3, -3, 1.52704\times10^{50}\},\\
&\{3, -2, 3.11687\times10^{49}\}, \{3, -1, 5.89017\times10^{48}\},\\
&\{3, 0, 7.60407\times10^{47}\}, \{3, 1, 1.96335\times10^{47}\},\\
&\{3, 2, 3.71037\times10^{46}\}, \{3, 3, 7.57374\times10^{45}\}\},\\
&\{\{4, -4, 2.72076\times10^{48}\}, \{4, -3, 4.10169\times10^{47}\},\\
&\{4, -2, 5.80067\times10^{46}\}, \{4, -1, 7.89371\times10^{45}\},\\
&\{4, 0, 7.45886\times10^{44}\}, \{4, 1, 1.40959\times10^{44}\},\\
&\{4, 2, 1.91821\times10^{43}\}, \{4, 3, 2.71276\times10^{42}\},\\
&\{4, 4, 4.08964\times10^{41}\}\}, \{\{5, -5, 3.79656\times10^{46}\},\\
&\{5, -4, 4.53776\times10^{45}\}, \{5, -3, 5.138\times10^{44}\},\\
&\{5, -2, 5.60602\times10^{43}\}, \{5, -1, 5.97604\times10^{42}\},\\
&\{5, 0, 4.45427\times10^{41}\}, \{5, 1, 6.64004\times10^{40}\},\\
&\{5, 2, 7.07831\times10^{39}\}, \{5, 3, 7.72307\times10^{38}\},\\
&\{5, 4, 8.74465\times10^{37}\}, \{5, 5, 1.04519\times10^{37}\}\},\\
&\{\{6, -6, 4.32371\times10^{44}\}, \{6, -5, 4.28111\times10^{43}\},
\end{aligned}
$$

$\{17, 16, 1.35057\times10^{-30}\}$, $\{17, 17, 4.63241\times10^{-32}\}\}$,
$\{\{18, -18, 2.33064\times10^{17}\}$,
$\{18, -17, 7.54574\times10^{15}\}$, $\{18, -16, 2.40062\times10^{14}\}$,
$\{18, -15, 7.51663\times10^{12}\}$, $\{18, -14, 2.31968\times10^{11}\}$,
$\{18, -13, 7.06511\times10^{9}\}$, $\{18, -12, 2.12635\times10^{8}\}$,
$\{18, -11, 6.33111\times10^{6}\}$, {18, -10, 186694.},
{18, -9, 5458.06}, {18, -8, 158.354},
{18, -7, 4.5637}, {18, -6, 0.130765},
{18, -5, 0.00372855}, {18, -4, 0.000105884},
$\{18, -3, 2.99725\times10^{-6}\}$, $\{18, -2, 8.46396\times10^{-8}\}$,
$\{18, -1, 2.38635\times10^{-9}\}$, $\{18, 0, 4.75371\times10^{-11}\}$,
$\{18, 1, 1.89392\times10^{-12}\}$, $\{18, 2, 5.33977\times10^{-14}\}$,
$\{18, 3, 1.5079\times10^{-15}\}$, $\{18, 4, 4.26841\times10^{-17}\}$,
$\{18, 5, 1.21215\times10^{-18}\}$, $\{18, 6, 3.45623\times10^{-20}\}$,
$\{18, 7, 9.90329\times10^{-22}\}$, $\{18, 8, 2.85408\times10^{-23}\}$,
$\{18, 9, 8.28053\times10^{-25}\}$, $\{18, 10, 2.42083\times10^{-26}\}$,
$\{18, 11, 7.13865\times10^{-28}\}$, $\{18, 12, 2.1255\times10^{-29}\}$,
$\{18, 13, 6.39701\times10^{-31}\}$, $\{18, 14, 1.94835\times10^{-32}\}$,
$\{18, 15, 6.01275\times10^{-34}\}$, $\{18, 16, 1.88266\times10^{-35}\}$,
$\{18, 17, 5.98955\times10^{-37}\}$, $\{18, 18, 1.93919\times10^{-38}\}\}\}$

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