Does the Speed of Light Have to be Constant?

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Abstract: By a short discussion of a simple problem encountered in Einstein's original closed static model I will show that the speed of light does not need to be a constant to maintain the general thrust of Special Relativity.
Keywords: special relativity, speed of light.

Starting with the following equations which express the coordinates of events of one inertial observer in terms of another:

$$x' = f(v)(x - vt)$$

$$t' = g(v)t + \frac{x}{v}\left(\frac{1}{f(v)} - g(v)\right).$$

From elementary calculus we know that velocity transforms according to the following rule:

$$\frac{dx'}{dt'} = \frac{f(v)(\frac{dx}{dt} - v)}{g(v) + \frac{1}{v}(\frac{1}{f(v)} - g(v))\frac{dx}{dt}}.$$

Let c be the speed of light in the rest frame Σ. Then x=ct and x=-ct implies that

$$x' = c\, t'_{\rightarrow}$$

and

$$x' = -c\, t'_{\leftarrow}$$

respectfully, where

$$c_{\rightarrow} = \frac{f(v)(c - v)}{g(v) + \frac{c}{v}(\frac{1}{f(v)} - g(v))}$$

$$c_{\leftarrow} = \frac{f(v)(c + v)}{g(v) - \frac{c}{v}(\frac{1}{f(v)} - g(v))}.$$

Let $\tilde{t}$ be the time it takes a photon to circumnavigate the universe as measured by an observer in the absolute frame of reference. Let $\tilde{d'}$ be the distance around the universe

as measured in Σ'. And let $(0, \tilde{t})$ be the event in Σ where two opposing photon bursts return to their place of origin. This event (the great illumination) according to the coordinates of the nearby moving frame Σ' is

$$(x',t') = (-vf(v)\tilde{t}, \; g(v)\tilde{t}) \; .$$

This means that the distance in Σ' between the two coordinate origins at the time of the great illumination is the absolute value of

$$-vf(v)\tilde{t} \; .$$

When the photons arrive at $x' = -vf(v)\tilde{t}$, the two bursts are still traveling in opposite directions. To a local observer in the frame Σ', the photons moving away from $x'=0$ have already been there and have traveled the distance $\tilde{d}' + vf(v)\tilde{t}$.

The photons headed toward $x'=0$ have only traveled the distance $\tilde{d}' - vf(v)\tilde{t}$. Considering the elapsed time and position of the photons for this event, we see that

$$c\,g(v)\tilde{t} = \tilde{d}' - vf(v)\tilde{t}$$
$$\overset{\rightarrow}{}$$
$$c\,g(v)\tilde{t} = \tilde{d}' + vf(v)\tilde{t}$$
$$\overset{\leftarrow}{} \; .$$

Subtract one equation from another and get

$$\underset{\leftarrow}{c} - \underset{\rightarrow}{c} = \frac{2vf(v)}{g(v)} \; .$$

This is a constraint on the general form of our coordinate transformation:

$$\frac{f(v)(c+v)}{g(v) - \frac{c}{v}\left(\frac{1}{f(v)} - g(v)\right)} - \frac{f(v)(c-v)}{g(v) + \frac{c}{v}\left(\frac{1}{f(v)} - g(v)\right)} = \frac{2vf(v)}{g(v)} \; .$$

The identity implies that

$$\frac{1}{f(v)} = g(v) \; .$$

This simplifies things a bit giving us as the final form of the transformation equations:

$$x' = f(v)(x - vt)$$

where we now have

$$f(v)(c-v)\tilde{t} = \tilde{d}' - vf(v)\tilde{t}$$

and

$$f(v)(c+v)\tilde{t} = \tilde{d}' + vf(v)\tilde{t} \; .$$

Therefore $\tilde{d}' = f(v)c\tilde{t} \Rightarrow \tilde{d} = c\tilde{t}$ is the distance around the universe in the absolute frame of reference. Therefore $\tilde{d}' = f(v)\tilde{d}$ is the distance around the universe in our moving frame. We have assumed that the postulates of special relativity apply in the greatest extent possible.

Consider now the two-way speed of light in the moving frame Σ'.

$$\underset{\rightarrow}{c} = f^2(v)(c-v)$$

$$\underset{\leftarrow}{c} = f^2(v)(c + v)$$

From a fixed point in Σ', let a photon propagate a distance L and then return:

$$L = f^2(v)(c - v)\breve{t}$$
$$L = f^2(v)(c + v)\hat{t}$$

Let the average speed

$$\frac{2L}{\breve{t} + \hat{t}} = c$$

Then we have

$$\frac{2}{\dfrac{1}{f^2(v)(c - v)} + \dfrac{1}{f^2(v)(c + v)}} = c$$

Consequently,

$$f(v) = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

Rewritting the equations with the traditional gamma:

$$x' = \gamma(v)(x - vt)$$
$$t' = t / \gamma(v)$$

Relativity in the spatially closed universe differs from that of ordinary Relativity. On the cosmological scale, SR has embedded in it an absolute time order and an absolute frame of reference. Locally, the two theories are absolutely indistinguishable. However, in an open universe that keeps expanding SR as traditional formulated remains correct both globally and locally, except with the exception of the questionable scale where the non-local nature of quantum theory and wave functions begins to come into play and again one encounters as a past article of mine mentioned[1] a return to a preferred frame of sorts or in the case of external curvature effects encountered under General Relativity.

An absolute frame of reference allows for motion faster than light if we were to admit one within our cosmological modeling. We then need to invoke a generalization of SR for superluminal velocities along the following lines:

$$x' = \tilde{\gamma}(v)(x - vt)$$
$$t' = t / \tilde{\gamma}(v)$$
$$\tilde{\gamma}(v) = \tilde{\gamma}(-v)$$
$$\tilde{\gamma}(v) = \gamma(v - 2wc)$$

if

$$(2w-1)c < v < (2w+1)c$$

then

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The w=1,2,3,…[2] is a warp factor of sorts that indexes different states of motion. This is actually akin to my own mention that one can have Lorentz Invariance hold in all frames irrespective of the local velocity of light without a violation of the general principles of relativity. There is nothing in the assumption of spatial homogeneity that disallows arbitrary coordinate systems and nonlinear transformations between different inertial frames of reference. Linearity in the coordinate transformation equations is not required to maintain at least the heart of Relativity. In fact, as often is mentioned, General Relativity allows for certain conditions under which this is true.

REFERENCES
1.) Hoiland P., Non-local to Local Space-time Map, and The Hidden Ether of General Relativity, and Non-Orientation of Space-Time Proves M-Theories Compacted or Embedded Regions, Journal of Theoretics.
2.) Borrowed from some of the ideas expressed in Modern Relativity by David Wait, http://www.geocities.com/zcphysicsms/ .

Received December 2003

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