COMING FULL CIRCLE WITH QUANTUM HALL EXPLANATIONS

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The vast majority of attempts at describing how the two quantum Hall effects fit existing theory have started out by viewing the phonemena in a Copenhagen-Schroedinger perspective. In the course of time extraneous adaptations had to be made ranging from fractional charge, composite fermions all the way to a Chern-Simmons 3-forms invoking strings. Yet this step of entering the field now reveals a structural topology not conveyable by statistical Schrödinger methods. Ironically, the 1- and 2-form components of a physical 3-form used by Kiehn unify integer and fractional effects. More ironic is that this option had already been reported in ref.12 prior to the announced discovery of the fractional effect in 1982.

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The quantum Hall effect was discovered by von Klitzing *et al* [1] by injecting charge carriers in the 2-dimensional interaction layer of a Mosfet transistor at very low temperatures and very high magnetic fields. The QHE manifests itself as a striking (plateau) discreteness of the Hall impedance (ratio of Hall voltage over Hall current) $Z_H = (1/s)\ (h/e^2)$ with s an integer and h, e action- and charge quanta.

Thereafter, several teams at the Bell Telephone Laboratories [2,3,4] discovered an interesting variant by varying instead the applied magnetic field. Their findings showed two quantum numbers were needed to describe the appearance of this more frequently occurring plateau discreteness of Z_H. The plateau states of both quantum Hall observations jointly obeyed with great precision the empirical formula:

$$Z_H = (n/s)(h/e^2);\ n \text{ and } s \text{ integers} \quad (1)$$

The ratio n/s is naturally cited as a reduced fraction, e, h are the charge and action quanta. For $n=1$, it is said to be the integer effect and for $n>1$ it has been called the fractional effect.

The actual verification reveals a reproducibility and precision of the above formula ranking in the category of photoelectric- and Josephson effects. Their joint applications initiated a new era in the metrology of fundamental physical constants. Results of quantum Hall-Josephson observations appeared more consistent than using high precision QED data.

For future reference it is important to note that Eq.1 consists solely of numbers and fundamental physical constants, which means Z_H is a scalar, or pseudo scalar if you will. Eq.1 therefore can be regarded as a Diffeo(4) invariant; *i.e.*, a general relativistic invariant, whereas metric independence makes it a possible topological invariant. Last but not least, the observations collected by Eq.1 seem non-statistical.

In the process of discovering the fractional Hall effect, people at Bell Labs [2,3,4] made a crucial observation about the enumerator n as prevailingly odd. Since the experimentally observed ratio n/s is naturally cited in its reduced form, a prevalently odd n points at a prevalence of an even ***physical*** s'; *i.e.*, s' prior to ratio reduction. An even physical s' points at a boson formation, a condition for superconductivity, as indeed observed in the plateau states! This interpretive aspect accounts for the odd enumerator, as tying in with an observed superconductivity. It also shows experimentally how superconductivity is induced not only for the ground state $n=1$, but also for $n>1$; putting integer and fractional effects in the same ***two*** quantum number category.

Citing the austere mathematical nature of Eq.1 as an experimental end result as metric-independent and Diffeo(4) invariant is very remarkable. It should give reason for questions as to how it could relate to known well established, but mathematically less austere physical laws.

If Copenhagen doctrine is held as ruling supreme the Schrödinger equation is a first candidate that comes to mind and that is exactly what happened. Yet Schrödinger's equation is at best R(3) invariant and metric-dependent. Copenhagen holds its quantization to be statistical yet applicable to single systems.

From that point onward a principal physics' effort became using Copenhagen's statistical tool explaining a non-statistical phenomenon. This undertaking became a difficult uncertain endeavor. The Wikepedia reveals an explosion of publicity that shows how difficult it was.

First people felt that fractional charge was clear evidence of accommodating quark theories. Then when, despite many efforts, no evidence of actual fractional charge was forthcoming, the next idea was that that more flux could have the same effect as less charge. This led to the *composite fermion* by Jain [5]: *e.g.,* electrons with attached flux quanta.

The same result was obtainable by instead considering higher quantum states for the cyclotron charge carriers; an alternative rejected, because charge carriers above the ground state might not be expected to be compatible with the induced superconductivity observed in the plateau states. Together attached flux- and orbitally linked flux units gave values for n and s as required by Eq.1.

Not quite satisfied with the ad hoc nature of the existing state of affairs, people kept looking for a more fundamental background of that n/s ratio. A next item to accommodate this remarkable n/s discreteness became a Chern-Simons 3-form [6]. It had come up in the integrability classification of Pfaffian one-forms and theory of characteristic classes of geometric manifolds. The principal thrust of physical ideas now moves to quantum discreteness as topological structure. Here is a first sign of abandoning assessing macro order with a Copenhagen tool, i.e., breaking away from the past and turning a new leaf.

At this point pioneers of string theory start entering quantum Hall theory. Since string theory holds an esoteric geometric position in theorizing, permit me to introduce here a more physical intermediate.

R.M. Kiehn [7] has introduced three cyclic integrals that have the advantage of an already existing degree of applicability in physics; here is an overview:

Path & period (residue) integrals I^c,II^c,III^c

$$\oint_{c1} A = n\frac{\tilde{h}}{\tilde{e}} \leftarrow \oint_{c1} A = \sum \frac{\tilde{h}}{2\tilde{e}};\ \text{Aharonov-Bohm} \quad \text{Ic}$$

$$\oiint_{c2} \tilde{G} = s\ \tilde{e}\ ;\ \text{Gauss-Ampère};\ n,\ s,\ \text{integers} \quad \text{IIc}$$

$$\oiiint_{c3} A \wedge \tilde{G} = ns\tilde{h};\ \text{Kiehn's 3-form integral} \quad \text{IIIc}$$

An inspection immediately shows how the empirical formula ***for Z_H equals the flux over charge ratio of the Aharonov-Bohm path integral over the Gauss-Ampère period integral.***

A cyclic path integral is simultaneously a time- and a space closure doubling the flux period contribution h/2e. The applicability of period integration rests on c_1 and c_2 residing in field-free interiors, which for c_1 calls on electrons as having an *E,B* field-free interior.

Using overlapping arguments, Carver Mead [8] arrived at identical results. Mead and the present author [9] have so far remained alone in this minority pursuit. They have taking exception to using the Schrödinger process in dealing with the QHE. Both call in essence on a pre-statistical collective status of quantization, naturally calling on a two-quantum number description. Action quantization of the Schroedinger equation can't do it, neither the 3-form IIIc, but its components do!

Yet despite a considerable handwriting on the wall, these experiences show the AB integral is still held in question by Copenhagen doctrine by holding Schrödinger as a primary law statement. That primary position is the reason why a derivation of Schrödinger's equation is outstanding.

Copenhagen orthodoxy and undue emphasis on claiming the fractional effect as different in nature from the integer effect may have led the Bell teams to putting aside their crucial odd *n* observations. Yet, the obstinate reality is that those odd enumerator observations surprisingly match the dissenting expositions [8,9].

Mindful of Copenhagen's *nonclassica*l Ψ function claiming a non-adaptive, ever-present, statistical nature, it is difficult to see how Schrödinger assessments can render service in creating a superconducting order. The odd *n,* revealing Boson formation, can be taken as uncovering a ***classica**l* Bose-Einstein type statistics. The latter can condense into superconductive order.

To account for the quantum Hall effect, the [8,9] descriptions need to step outside the Copenhagen realm. The majority using the Schrödinger many-body technique of Laughlin [10] has to show how *composite fermions* cover fractional charge.

The processes [8,9] call for a, de facto, already accepted single system adaptation of quantization in the sense of Aharonov-Bohm, not a Schroedinger quantization. Ref.9 does so following ref.7 and ref.8 calls on earlier work by Barut [11] using collective notions.

The photoelectric effect does not permit Schroedinger treatment* and neither does QHE. Quantization laws [Ic,IIc] are strictly ***global***; their ***local*** counterparts are Maxwell field equations that no longer have explicit references to *h/e* and *e*.

Hamiltonians in a Schrödinger context deal with locally randomnized ***ensembles***. The AB and GA integrals strictly ***globally*** explore ***single systems.*** The period integrals follow from global conservation. For references to de Rham theory and the ramifications for Copenhagen doctrine see [9].

A preliminary outline of needed Copenhagen *reprogramming* and its impact on the now unified QHE and other prominent single system manifestations are discussed in ref.[9].

So ironically the two quantum number quantization is a compellingly simple end result for the QHE odyssey hiding behind a 3-form. More ironic is that the components of Kiehn's 3-form hold the key to that solutoion. Even more ironic is that this solution was published [12] prior to to the publication of the fractional quantum Hall effect.

Ref.[12] entitled "Physical Dimensions and Covariance" was pending with *Foundation of Physics*: Its publication though had been delayed when editor Yourgrau passed away. His successor, editor van der Merwe offered me an opportunity for further remarks on the subject in the fall of 1981. I gratefully accepted that offer and inserted on page 194 of ref.[12] the here-cited passage related to the QHE:

(Quote) *Experimentalists deserve the credit for having made the "global" voice of nature more audible than the local voice of expediency. I am referring to the many experiments on quantum-interferometry. These experiments were made with a minimum of guidance from accepted mainstream theoretical ideas. Central among these discoveries are the quantization of flux and more recently the quantization of the* ***Hall impedance****. Precision measurements are more and more homing in on extremely accurate measurements of the quanta of charge and action and their ratios h/e flux and h/e^2 impedance.* ***Their dimensional and numerical correspondence to the residues of the period integrals [Ic, IIc, IIIc] and the absolute dimension of the constitutive tensor(16) can hardly be argued an idle coincidence.***(End of quote)

* This fact became already apparent during a 1927 seminar by Schrödinger in Munich when Heisenberg queried him on exactly that point.

I am truly sorry if the interjected passage did not elicit a public discussion of its potential. Yet, it could not all have been my shortcomings. An overly Copenhagen-dominated frame of mind has manifested itself throughout this history of quantum Hall phenomena.

Peer-reviewed media have shown an almost outrageous preference for far out unchecked ideas**,** whereas simple moves, merely aiming at having available mathematical-physics speak louder, seemed studiously avoided. A minority added mostly detail, such as: 1) The period integral quantization is Diffeo(4) invariant and metric-free, thus extending viability from macro to micro domain; 2) meeting period integral conditions some charge carriers require field-free interiors; 3) an energy gap, which is not necessarily a ground state gap, suffices for superconductivity.

Epilogue on Reductionism

Reductionism is the art of reducing a given phenomenon to a set of what are believed to be simpler and more basic phenomena. It is a cornerstone of reasoning in mathematics and in the exact sciences in general. Seen from this angle the story of the quantum Hall effect presents itself as an imperfect reduction. From the beginning, the event of a simple and precise empiric ratio of integers reducing to an integer itself was taken out of proportion.

While physical transparency was still shrouded in theory, the discoverers somehow insisted on basic physical distinctions between integer s/n and fractional s/n.

Whenever a chain of reduction-based reasoning runs into trouble, the choice and decision as to what is basic is likely at fault. In the QHE case the cause of the longtime trouble was overrating the *basics* ranking of Schroedinger's equation; not an unlikely happening in judging a gift from heaven: Weil's *favor of fortune*.

Invariance criteria help ranking QHE effect as more basic than Schrödinger. Aharonov-Bohm, Gauss-Ampère and QHE have a same invariance ranking, yet QHE is compounded, AB and GA are not. Integer effects simply follow charge injection and fractional effect flux injection.

Finally realizing that QHE is as basic as AB and GA, vast majorities have been constructing approximations showing how Copenhagen assessments are asymptotically related to the QHE law. That was already known in the light of simpter implications. Only an adapting classical statistics can clarify the dynamic relations between Schrödinger and AB integral. The erstwhile Schrödinger priority now belongs, through derivation, to the AB integral. Nonclassical statistics had made us mistake the effect (Schrödinger) for cause.

References

1. K. von Klitzing *et al,* Phys. Rev. Lett., **45**-6,194 (1980)
2. D. C. Tsui *et al*, Phys. Rev. Lett.**48,** 1559(1982)
3. H. L. Stoermer *et al*, Phys. Rev. Lett.**50**, 1953 (1983).
4. R. Willet *et al,* Phys. Rev. Lett.**59**, 1776 (1987)
5. J. K. Jain, Composite Fermions (Cambridge UniverityPress 2007)
6. B. Skoric and A. M. M. Pruisken, Nuclear Physics B **559**-3, 637(1999)
7. R. M. Kiehn, J. Math. Phys**. 18**, 614 (1977)
8. C. A. Mead*, Collective Electrodynamics* (MIT Press, 1999) p.71
9. E. J. Post, *Quantum Reprogramming* (Kluwer, 1995; Springer 2005) pp 131and 147
10. R. Laughlin, Phys. Rev. Lett.**50**, 1395 (1983)
11. A. O Barut, Zeitschrift für Naturforschung **44**a, 1051 (1989)
12. E. J. Post, Found. of Phys., **12**-2, 168 (1982)