# Mass Varies in High Rotation Even at Non-Relativistic Velocities!

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This paper endeavors to show (within the realm of Newtonian mechanics [6]) that the mass of a gyroscope consisting of a thin cylindrical rotor and weightless shaft having a fixed pivot and steadily precessing (without nutation) in a horizontal plane appears to gain mass if the rotor is spinning compared to the same situation but with non-spinning rotor in that if the precession is stopped dead (not altering the rotor spin angular velocity), then the impact is considerably greater in the spinning rotor case than in the case where the gyroscope is constrained to move freely in the horizontal plane with the same angular velocity as the former precessional velocity just prior to impact. This is surprising because French [5] (in his discussion of a steadily precessing gyroscope) shows that the centrifugal force of a gyroscope in the same situation except that it is allowed to precess steadily, and is not stopped dead, exhibits no apparent change of mass within the realm of Newtonian mechanics in that the spinning rotor exhibits the same centrifugal force independent of spin, the angular velocity of the non-spinning gyroscope (for comparison's sake) which is constrained to move freely in a horizontal plane being the same. But, experimentally [2, 3], both in the impact case and also in the steadily precessing case which were just mentioned, these Newtonian results are **not,** in general, observed … even to first order! Also, it is shown that Newtonian mechanics is either discontinuous or paradoxical.

## 1. Introduction

Eric Laithwaite [3] and later Harvey Fiala [2] and others have shown experimentally that a steadily precessing gyroscope loses mass due to the rotor spinning both in the sense of reduced impact when precession is stopped dead and in the sense that the gyroscope exhibits considerable reduction in centrifugal force as it precesses. Harvey has a patent [3] for a space drive in which no mass is ejected, but rather the center of mass of the drive accelerates (after frictional forces are subtracted off) due to its internal forces – impossible in the realm of Newtonian, of course; but he has a working prototype! My results are based on Euler's equations of motion involving two angular velocities with the main coordinate system being fixed in the precessing gyroscope, a situation very similar to the situation already analyzed by G.E. Hay [1], but Hay's book is currently just back in print; however, Hay closely follows Synge and Griffith [4] here, and their book is a standard reference work.

## 2. The Mass of a Gyroscope

We know that for a gyroscope with a fixed pivot which is horizontally precessing (so that the angle $\theta$ that the gyro shaft makes the vertical is $\pi/2$) with a constant angular velocity $p_0$ and spinning with a constant angular velocity $s_0$, but not nutating, then the equation of motion is precisely [1, p. 94]

$$p_0 = \frac{m g l}{I_3 s_0}, \tag{1}$$

where $m$ is the mass of the solid thin disc flywheel (thin cylinder), $I_3$ is the moment of inertia of the flywheel about the rotor axis (where the shaft from the fixed pivot to the flywheel center is of length $l$), and $g$ is the acceleration due to gravity. So let's consider the following situation [1, pp. 90-94] (Hay's Example 3 and Figure 50 for a force diagram of gyro):

Let the precession $p = p_0\left(1 - \frac{t}{e}\right)$ for time $t$ between $t = 0$ and $t = e$, so that $p$ varies linearly between $p_0$ at $t = 0$ to zero at $t = e$, with $\frac{dp}{dt} = -\frac{p_0}{e}$, which is not a function of $t$. We examine the force on the gyroscope's thin solid disc rotor's center of mass necessary to simultaneously keep nutation to zero, to keep the gyroscope precessing horizontally, and finally to make the gyroscope's precession $p$ behave as just specified from $t = 0$ to $t = e$, with the precession $p = p_0$ and spin $s = s_0$ being as in the displayed equation above at $t = 0$.

This means that in equations (41.24), (41.25), and (41.26) in [1, p. 93] (which are from Euler's equations of motion on page 87, but the minus sign in (41.26) is a typo and should be a plus sign instead), we must add a force $F_1$ along the unit vector $\hat{\mathbf{i}}_1$ and a force $F_2$ along the unit vector $\hat{\mathbf{i}}_2$ (but no vector component along $\hat{\mathbf{i}}_3$ as such a force would not have a moment about that unit vector which is along the shaft and from the origin $O$) such that the three just mentioned equations from Hay become the next set of three displayed equations, respectively (where we multiply the new forces by $l$, the distance between the pivot and the thin disc rotor, to get their moment about $O$ … as the new forces are assumed to be acting at the center of mass of the rotor). The force $F_1$ will result in a torque along $\hat{\mathbf{i}}_2$ since $\hat{\mathbf{i}}_3 \times \hat{\mathbf{i}}_1 = \hat{\mathbf{i}}_2$, and force $F_2$ will result in a torque along $\hat{\mathbf{i}}_1$ since $\hat{\mathbf{i}}_3 \times \hat{\mathbf{i}}_2 = -\hat{\mathbf{i}}_1$. So thus we must reverse the roles of these two forces once we convert to the associated torques, and there will be a negative sign with $F_2$ in conversion to the associated torque demanded

by Euler's equations. [Note, while it appears that, in the second equation below, there is a missed algebraic sign, it is nevertheless actually correct … coming from the second of Euler's equations (41.25) for Hay's gyro.]

$$I_1 \frac{dp}{dt}\sin\theta = I_1 \frac{dp}{dt} = -F_2(t)\cdot l \tag{2}$$

$$\left[I_3 s_0 - (I_3 - I_1)p\cos\theta\right]p\sin\theta = mgl\sin\theta + F_1(t)l \tag{3}$$

=> $$I_3 s_0 p = mgl + F_1(t)l$$

$$I_3\left(\frac{ds}{dt} + \frac{dp}{dt}\cos\theta\right) = I_3\frac{ds}{dt} = 0 \tag{4}$$

Now, above we have used that there is no nutation at any time so that $d\theta/dt = 0$ identically, and we have used that $\theta = \pi/2$ so that $\cos\theta = 0$ and $\sin\theta = 1$. Finally, and very importantly, we have assumed $dm/dt = 0$ identically. (It is this assumption that gets us in trouble experimentally!) Next we use the above definition of the precession $p$ between $t = 0$ and $t = e$, and substitute into the equations above to get:

$$\frac{I_1 p_0}{el} = F_2 \tag{5}$$

$$I_3 s_0 p_0\left(1 - \frac{t}{e}\right) - mgl = F_1(t)\,l \tag{6}$$

$$\frac{ds}{dt} = 0 \tag{7}$$

The first of the two equations above says that $F_2$ is a constant function. We can compute the counter-momentum (linear momentum) added to the rotor's center of mass in order to reduce the precession angular velocity from $p_0$ to zero by integrating $F_2$ from $t = 0$ to $t = e$, and the result is evidently ($I_1 p_0 / l$) and is not a function of $e$, $s_0$, or $I_3$. Thus the constant torque $F_2 \cdot l$ needed to steadily reduce the gyro's precession from $p_0$ to zero in time $e$ is $-I_1 p_0$, where $I_1$ is not the moment of inertia of the solid disc of Hay's gyro about the shaft axis, but rather the moment of inertia of this flywheel about an axis perpendicular to the axis $\hat{\mathbf{i}}_3$ that the gyro shaft lies along (and also perpendicular to the other axis $\hat{\mathbf{i}}_2$ along which this force $F_2$ lies), namely, the moment of inertia $I_1$ about the axis $\hat{\mathbf{i}}_1$. Now, of course, the constant torque needed to reduce the (non-spinning) rotor mass $m$ having a moment arm of length $l$ and being free to rotate freely in the horizontal plane [but being constrained to move freely in this plane because the non-spinning gyroscope would not steadily precess in the horizontal plane in view of equation (1)] and having initial angular velocity $p_0$ at $t = 0$, and coming to a stop in time $e$ is $-ml^2 p_0/e$, and integrating this from time zero to time $e$ gives $-ml^2 p_0$ as the angular counter-momentum having to be added to the rotor center of mass to halt the non-spinning rotor (the rotor being on an arm of length $l$). But for Hay's solid cylindrical rotor gyro the reduction fraction of the two angular counter-momentums is

$$\frac{-I_1 p_0}{-ml^2 p_0} = \frac{I_1}{ml^2}\,. \tag{8}$$

This would seem to correspond to one minus Harvey Fiala's "quality" [2] and should then be equal to ½ as the outside radius of the solid cylindrical rotor is $r$ and the inside radius is zero. But the moment of inertia of a thin solid cylinder about a diameter is known to be $\frac{mr^2}{4}$, and so by the parallel axis theorem, the moment of inertia about the axis $\hat{\mathbf{i}}_1$ (namely $I_1$) is then simply

$$\frac{mr^2}{4} + ml^2 = I_1 \tag{9}$$

However, then Harvey's quality is

$$1 - [1 + r^2/(4l^2)] = -r^2/(4l^2) \tag{10}$$

which, however, cannot be ½ as it is negative …

Note that this argument applies for **any** spin $s_0$ that does not vanish, but this result (8) is independent of this spin. However, if this spin **is** zero, then (10) must be 1 – 1 = 0 … since we are thus in the non-spinning rotor case, whence the impact mass ratio must be unity. This means that either there is a physical discontinuity at spin zero, or else there is an antinomy in Newtonian mechanics … in that we get **two distinct results** by calculating in two different ways, but nevertheless correctly following the rules of Newtonian mechanics in both calculations. We expect that there **is** a paradox in Newtonian mechanics exhibited here.

The second equation is less interesting. We can compute the angular momentum that the force $F_1(t)$ imparts to the flywheel in stopping its nutation during the time from $t = 0$ to $t = e$ (recall it acts at the center of mass of the flywheel) by integrating it times $l$ from $t = 0$ to $t = e$, and then we evidently obtain (in view of the first displayed equation at the top):

$$\left[I_3 s_0 p_0\left(t - \frac{t^2}{2e}\right) - mglt\right]_{t=0}^{e} = \frac{I_3 s_0 p_0 e}{2} - mgle = -\frac{I_3 mgle}{2} \tag{11}$$

[which is only a function of the rotor's weight $mg$, the moment of inertia $I_3$, and also $e$, or (alternatively) the rotor's initial precession velocity $p_0$, the initial spin angular velocity $s_0$, the length of the moment arm (shaft) of length $l$, and also $e$], so that as $e \to 0$, the counter-momentum needed to maintain the nutation velocity at zero itself approaches zero. Note that $F_1(0) = 0$, and $F_1(e) = -mg$, which is the weight of the rotor as expected.

Finally, we have $s = s_0$ between $t = 0$ and $t = e$ as $s$ is constant because its time derivative vanishes.

Thus, letting $e \to 0$ through positive values, we get the impact that stopping the gyro precession dead imparts, and it is not what one would expect nor what is observed by Eric Laithwaite in his 1974 Christmas lecture to the children of the Royal Society [3] as he showed a reduction in angular momentum upon stopping the precession dead but not altering the rotor spin angular velocity; however, our computation here shows an increase in angular momentum compared to a gyro with a fixed rotor mov-

ing in the horizontal plane with the spinning gyro's precession angular velocity!

In view of the fact that [1] has only just come back into print, it might be mentioned that Hay references and follows [4] rather closely, and Euler's equations of motion are just their equations (12.403) on page 352. The key vectors are defined as follows: $\hat{\mathbf{i}}_3$ is a unit vector in the plane of the paper and the gyro shaft (assumed infinitely thin and mass-less) of the gyro lies along it with the its fixed pivot being at the rectangular coordinate system's origin *O*, and the thin cylindrical rotor spins about its axis of symmetry which also lies along this unit vector at a length *l* from the pivot to the rotor. The unit vector $\hat{\mathbf{i}}_1$ also lies in the plane of the paper pointing upwards but at a right angle to $\hat{\mathbf{i}}_3$, and $\hat{\mathbf{i}}_2$ completes the triad and so is horizontal (considering the plane of the paper as vertical). Finally, the unit vector j is in the plane of the paper and vertical, and $\hat{\mathbf{i}}_3$ makes an angle θ with it (the tilt angle). Then we have that the moment of the gravitational force on the rotor

$$\tilde{\mathbf{G}} = l\,\hat{\mathbf{i}}_3 \times (-mg)\,\hat{\mathbf{j}} = -mgl\,\hat{\mathbf{i}}_2\,, \tag{12}$$

because evidently $\hat{\mathbf{j}} = \hat{\mathbf{i}}_1 \sin\theta + \hat{\mathbf{i}}_3 \cos\theta$, and *m* and *g* are rotor mass and Newton's acceleration due to gravity at the earth's surface, respectively. Further, the angular velocity of the gyro is

$$\tilde{\boldsymbol{\omega}} = s\hat{\mathbf{i}}_3 + p\hat{\mathbf{j}} = p\sin\theta\,\hat{\mathbf{i}}_1 + (s + p\cos\theta)\hat{\mathbf{i}}_3\,, \tag{13}$$

where *p* and *s* are the precession and spin angular velocities, respectively. Then the angular velocity of the coordinate axes (assuming they rotate with the gyro shaft about the pivot) is given by

$$\tilde{\boldsymbol{\Omega}} = p\hat{\mathbf{j}} = p\left(\hat{\mathbf{i}}_1 \sin\theta + \hat{\mathbf{i}}_3 \cos\theta\right). \tag{13}$$

Finally, the coordinate axes associated with $\hat{\mathbf{i}}_1$, $\hat{\mathbf{i}}_2$, and $\hat{\mathbf{i}}_3$ are principle axes of inertia at *O*. And we have the moments of inertia $I_1 = I_2$, while all products of inertia vanish (the former are Synge and Griffith's [4] *A* and *B*, respectively, in their above mentioned Euler equations).

## 3. Conclusion

Evidently, the hypothesis that mass does not vary in non-relativistic but highly rotational velocities, must be revised, as it fails to correspond to experimental evidence and, in fact, is not (in general) even correct to first order. This has been observed by earlier authors in connection to the Dean drive (Google: Davis Mechanics), but they introduced the time derivative of the acceleration explicitly which causes endless problems such as runaway solutions (not corresponding to reality) in even the simplest cases … as this is bad mathematics, albeit good physics. [Also, Prof. R. Santilli has kindly informed the author via email that the haydron mechanics people (led by himself) have long since concluded that mass is not an invariant even in non-relativistic velocity situations.] The author's approach, however, involves the introduction of this derivative *implicitly* making the (non-constant) mass a function of this [7], and it turns out that this derivative apparently enters into the determination of the particle mass both in the matter of the angle between this derivative and the acceleration vector and also its magnitude does appear to the first power only as does the magnitude of the acceleration also. In fact, the formula is

$$m_r = m\left[1 - c\left\{a\frac{da}{dt}\cos^2(\alpha)\right\}\right], \text{ nearly,}$$

where *c* (not the velocity of light in a vacuum) is a non-zero constant to be determined experimentally, where $m_r$ is the *r*-mass (that is, the measured mass) of a point particle, where *m* is the rest mass of the point particle, and where *a, da/dt,* and $\alpha$ are the magnitude of the acceleration vector, the magnitude of the vector time derivative of the acceleration vector, and the angle between the acceleration vector and the vector time derivative of the acceleration vector (belonging the point particle), respectively. This formula is valid to **third** order in cos(α) and probably to **all** orders.

Finally, note that we can define an effective mass of the rotor here by setting it equal to

$$m\left(\frac{I_1}{m\,l^2}\right) = \frac{I_1}{l^2}\,, \tag{14}$$

and then it can be said that the rotor impacts the dead stop mechanism with the same impact as a point mass (spinning or non-spinning since it is a point mass and has no extension and hence no way of having its rotation exhibited) having mass equal to this effective mass and moving with velocity $p_0 l$, the (constant) precession angular velocity of the rotor center of mass before precession deceleration. In closing, we mention that Chapter 2 of [6] shows that Newton reduced all types of forces in his theory to impact force, and so our using impact above would seem to be right in line with his methodology.

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