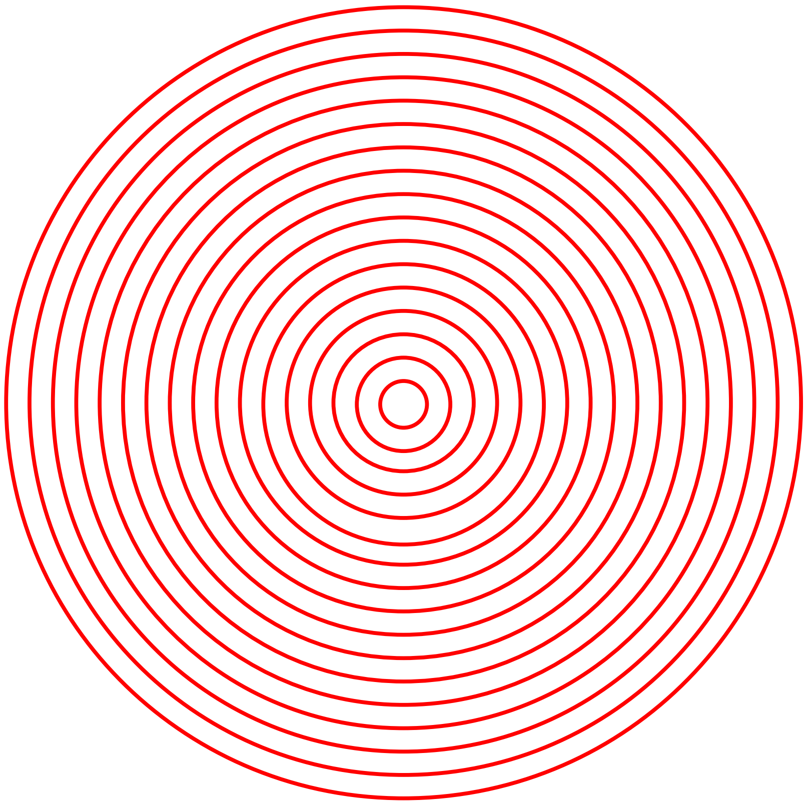
ELECTRODYNAMICS OF A PARTICLE

ACCELERATED TO THE SPEED OF LIGHT WITH CONSTANT MASS



Musa D. Abdullahi

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FAHIMTA

FAHIMTA PUBLISHING COMPANY

KATSINA, NIGERIA

Published by

FAHIMTA PUBLISHING COMPANY

Road L

Ƙofar Durɓi New Lay-out

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Katsina

Nigeria

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First Published 2006

ISBN 978 - 2869 - 01 - 5

DEDICATION

THIS BOOK IS DEDICATED

TO
THE EVER-GREEN MEMORY OF
MY LATE FATHER
\- ABDULLAHI SALIHU -

Who gave the time to implant in me the character of integrity
who imbued in me the spirits of independence and humility
who inculcated in me the virtues of patriotism and dignity
who impressed on me the habits of kindness, honesty
and hard-work, discipline, patience and sincerity
again instilled in me the practice of punctuality
and respect for the other person's opportunity

QUOTATIONS

I *"There is only one absolute good - 'knowledge'; there is only one absolute evil - 'ignorance'".*

Socrates of Athens (Greek Philosopher, 470 – 399 B.C.)

II *"The knowledge that has not come down to us is larger than the knowledge that has".*

Abu Zayd Abdul Rahman Ibn Khaldun (1332 – 1406)

III *"If a man will begin with certainties, he shall end in doubts; but if he will be content to begin with doubts, he shall end in certainties"*

Sir Francis Bacon (1561 – 1626), Advancement of Learning

IV *"I believe that there is no greater hatred in the whole world than that of ignorance for knowledge...I recant, yet it moves".*

Galileo Galilei (1564 – 1642). On motion of the Earth

V *"I am like a child playing with pebbles at the seashore while the great ocean of knowledge lies unfathomed before me…If I am able to see further than others, it is because I stand on the shoulders of giants…But to propose and prove by reason and experiment".*

Sir Isaac Newton (1646 – 1727)

VI *"When you can measure what you are speaking about and express it in numbers, you know what you are talking about".*

Sir William Thomson (Lord Kelvin, 1824 - 1907)

VII *"We haven't got the money, so we have got to think".*

Lord Ernest Rutherford (1871 – 1937). On research

VIII *"Nobody is sure of having taken the right road, me the least".*

Professor Albert Einstein (1879 – 1955). On relativity

IX *"I speak of freedom, I speak of knowledge, I speak of originality...Go beyond the veneer of knowledge...Drink deep from the fountain of knowledge...To restore the dignity of man".*

Dr. Nnamdi Azikiwe (Zik of Africa, 1904 - 1996)

X *"The human brain is the most powerful mechanism in the universe…Every big idea requires great imagination…The greatest idea is founded on simplicity…The greatest achievement is founded on sincerity…With the right minds together, everything is possible".*

News Media Networks. On education, science and discovery

XI *"Good thinking, good products; new thinking, new possibilities".*

The Japanese. In pursuit of technological excellence

XII *"He who seeks shall find" (Mai nema zai samu)* مَنْ جَدَّ وَ جَدَ

Motto of (Alma Mater) Barewa College, Zaria, Nigeria

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LIST OF SOME SYMBOLS

Vectors are indicated in **boldface** type and *scalars* in ordinary type

SYMBOL	DEFINITION OR DESCRIPTION
a, A	Acceleration constant; radius; amplitude; area; ampere
$\hat{\boldsymbol{a}}$	A unit vector (normal) perpendicular to a surface
b	Attenuation constant; radius of a spherical charge
$\boldsymbol{B}$	Magnetic flux intensity ($\boldsymbol{B} = \mu\boldsymbol{H}$)
c, C	Speed of light ($c=2.998\times10^8 m/s$); magnitude of $\boldsymbol{c}$; coulomb
$\boldsymbol{D}$	Electric flux intensity ($\boldsymbol{D} = \varepsilon\boldsymbol{E}$)
$-e$	Charge of an electron ($e = -1.602 \times 10^{-19} C$)
$\boldsymbol{E}$	Electric field intensity ($\boldsymbol{E} = E\hat{\boldsymbol{u}} = E\boldsymbol{c}/c$)
$\boldsymbol{E}_a$	Electrodynamic field due to an accelerated charge
E	Energy; magnitude of electric field
f, F	Frequency of revolution or oscillation; farad
$\boldsymbol{F}$	Accelerating force on a moving charged particle
$\boldsymbol{F}_E$	Electrostatic force of repulsion or attraction
$\boldsymbol{F}_G$	Gravitational force of attraction between bodies
$\boldsymbol{F}_o$	Electrostatic force on a stationary charged particle
G	Gravitational constant $\{G = 6.673 \times 10^{-11}\ Nm^2/(kg)^2\}$
h, H	Planck constant ($h = 6.626 \times 10^{-34}$ *Joule-sec.*); *h*enry
$\boldsymbol{H}$	Magnetic field intensity (magnitude of H)
$\boldsymbol{J}$	Electric current intensity ($\boldsymbol{E} = \rho\boldsymbol{J}$ *in a conductor*)
$\boldsymbol{k}$	Unit vector perpendicular to the plane of an orbit
K, kg	Kinetic energy; Electric charge; kilo-gram mass
$\boldsymbol{l}$, $\boldsymbol{L}$	Length of magnitude l; angular momentum
m, M	Metre; mass; mass of particle/body ($m_e=9.110 \times 10^{-31} kg$)
m_o, M_o	Classical mass or rest mass in relativistic mechanics
n, N	A number > 0; number of charges in a body
N_h	Number of orbits in the hydrogen atom
O	Centre of revolution or point of origin in space
p	A number (positive or negative) 0 to $\pm\infty$; pressure
P	Potential energy; electric potential; point in space; pressure
q, Q	A number > 0; electric charge (flux); exponential factor
r_o	Radius of circular revolution in classical mechanics
r, $\boldsymbol{r}$	Radius of revolution; distance from an origin ($\boldsymbol{r} = r\hat{\boldsymbol{u}}$)
R	Energy radiated; Rydberg constant; position; resistance
$\boldsymbol{R}_f$, R_p	Radiation reaction force (*difference* $\boldsymbol{F} - \boldsymbol{F}_o$)
R_p	Radiation power (*scalar product* $-\boldsymbol{v}.\ \boldsymbol{R}_f$)

s, S	Second; cycle in a revolution; spectral limit; position, area
t, T	Time; present time at a position *P*; period of revolution
u	Velocity of source of light relative to a frame of refernce
v	Velocity of a particle/body or observer rel. to a ref. frame
$\hat{\boldsymbol{u}}$	Unit vector in the radial or field direction
U	Intrinsic potential at the location of a charge
V, $\forall$	Volt; voltage; potential at a point in space; volume
w, W	Speed of light in a moving medium; energy; work
x, X	Unknown quantity; distance between body charges
y, Y	Unknown quantity; one of coordinates (x, y, z)
z, Z	Distance between two particles in space; a coordinate
z	Velocity of light relative to an observer $\{\boldsymbol{z} = \boldsymbol{c} + (\boldsymbol{u} - \boldsymbol{v})\}$
α *(alpha)*	Aberration angle; rotation factor in an aperiodic ellipse
β *(beta)*	Phase angle; Brewster angle
γ, Γ *(gamma)*	Ratio of relativistic and rest mass; gravitational field
δ, Δ *(delta)*	Small change or increment; fringe shift
ε *(epsilon)*	Permittivity of a medium ($\varepsilon_o = 1/36\pi \times 10^{-9}$ *F/m*)
ζ *(zeta)*	Relativistic mass or "submass"
η (eta)	Impacts per second; eccentricity of an ellipse
θ *(theta)*	Angle between accelerating force and velocity
ι *(iota)*	Angle of incidence ($\iota = \rho$)
κ *(kappa)*	*C*onstant of revolution in the bipolar model
λ, Λ *(lambda)*	Wavelength; extrinsic potential at a charge location
μ *(mu)*	Permeability ($\mu_o = 4\pi \times 10^{-7} H/m$); refractive index
ν *(nu)*	Wave number ($\nu = 1/\lambda$)
ξ *(xi)*	Mass of a "force particle" propagated at speed *c*
π *(pi)*	The angle between two points on a straight line
ρ *(rho)*	Angle of reflection ($\rho = \iota$); resistivity ($\rho = 1/\sigma$)
σ, Σ *(sigma)*	Conductivity; surface charge intensity; summation
τ *(tau)*	Refraction angle ($\sin\iota/\sin\tau = \mu$); retarded time ($\tau = t - r/c$)
υ (upsilon)	Critical angle; energy per unit volume
φ, Φ *(phi)*	Electric potential; angle of revolution in a closed ellipse
χ *(Chi)*	*C*onstant of revolution in the nuclear model
ψ. Ψ *(psi)*	Angle of revolution in central motion; magnetic flux
ω, Ω *(omega)*	Angular speed or angular frequency ($\omega = 2\pi f$)
∇ *(grad)*	Vector operator as "gradient of a scalar"
$\nabla.$ *(div)*	Scalar operator as "divergence of a vector"
$\nabla\times$ *(curl)*	Vector operator as "curl of a vector"

FOREWORD BY PROF. M.A. DANIYAN, BSc, MSc, PhD. NATIONAL UNIVERSITIES COMMISSION, ABUJA, NIGERIA

The special and general relativity theories, mainly formulated by the famous physicist Albert Einstein and the quantum theory devised by Max Planck, Niels Bohr and others, have gained such prominence in modern physics that nobody dares challenge them. However, these theories must be challenged, if found incompatible with one another or inconsistent with observations. This is the task taken up by Engr. Musa Abdullahi in this book of nine chapters for which I readily write the foreword.

In physics, we now deal in three systems of electrodynamics. There is classical electrodynamics for electrically charged particles moving at very small speeds compared to that of light, relativistic electrodynamics for particles moving at speeds near that of light and quantum electrodynamics for atomic particles moving at very high speeds. The author is of the view that there should be one consistent system of electrodynamics applicable to all particles up to the speed of light.

The author introduced a new system of electrodynamics, which he called ***radiational electrodynamics***, where a charged particle, like an electron, moving in an electrostatic field, radiates energy. A moving electron is subjected to *aberration of electric field*, due to an electrostatic force being propagated at the speed of light. The author showed that the force exerted by an electrostatic field, on a moving electron, depends on the magnitude (speed) and direction of its velocity. The electron is accelerated to the speed of light as maximum. Using Newton's second law of motion and extending Coulomb's law of electrostatics to a moving charged particle, he showed that an electron could be accelerated to the speed light with constant mass, contrary to relativistic electrodynamics.

The author is also of the view that *aberration of electric field* is the missing link in classical and relativistic electrodynamics. He contended that Larmor formula was an erroneous expression for radiation power, which misled physics to the Bohr's quantum theory of the hydrogen atom.

The author rejected the idea of constancy of the speed of light and accepted the Galilean-Newtonian relativity where the speed of light is a constant relative to the source only but, relative to an observer, the speed depends on motions of the observer and the source. On this basis, he derived an expression for the speed of light in a moving medium and gave a non-relativistic explanation of the result of Fizeau's experiment.

The introduction of two stable models of the hydrogen atom, a non nuclear one for the gaseous state and a nuclear one for the solid or liquid state, without recourse to quantum mechanics, is very interesting. Both models give rise to emission of radiation of discrete frequencies, as given by the Balmer-Rydberg formula, in accordance with observations.

The author derived the formula, $E = ½m(c^2 + v^2)$, as the total energy of a body of mass m moving with speed v, where $m = m_o$ the rest mass, is independent of speed. The simplicity of derivation of this equation, from basic electrical principles, compared to the relativistic formula $E = mc^2$ (where mass increases with speed as $m = \gamma m_o$), is a significant result.

A notable result is found in the non-relativistic rationalization of the apparent increase of the mass of an electron with its speed. The author showed that the expression for the radius of circular revolution of an electron, round a positively charged nucleus is γr_o, the same in relativistic electrodynamics and in *radiational electrodynamics*. This is larger than the radius r_o obtained in accordance with classical electrodynamics. He suggested that the increase in radius with speed, becoming infinitely large at the speed of light, which has the same effect as apparent increase of mass with speed, was misconstrued as increase of mass with speed.

A remarkable development is the unification of Coulomb's law of electrostatics with Newton's universal law of gravitation. The electrostatic forces of repulsion and attraction between the masses of two bodies, each containing an equal number of positive and negative electric charges, cancel out exactly. The gravitational forces, being proportional to the product of <u>square</u> of the charges, remain positive and attractive.

If the author's formula for radiation power; the unification of electrostatic and gravitational forces; the non-relativistic explanations of the results of Fizeau's, Roger's and Bertozzi's experiments; the rationalization of apparent increase of mass of a particle with its speed and the clarification for the origin of inertia of a body are proved to be correct, it would amount to a great scientific breakthrough.

I am pleased with this book titled, *"Electrodynamics of a particle accelerated to the speed of light with constant mass"*. It makes stimulating reading and provides good materials for teaching physics. I commend the author for providing some food for thought and recommend the new electrodynamics for study, critiquing and testing by physicists.

Professor M.A. Daniyan. **Abuja, Nigeria, August 2006**

PREFACE

"The modern physicist may rightly be proud of his spectacular achievements in science and technology. However, he should always be aware that the foundation of his imposing edifice, the basic notions, such as the concept of mass, are entangled with serious uncertainties and perplexing difficulties that have as yet not been resolved"

Max (Moshe) Jammer (1915 – Year), President of Association for the Advancement of Science in Israel

During the days as a secondary school student at Barewa College, Zaria, Nigeria, in the late 1950s, one had been enthralled by Galileo Galilei's principles of mechanics, Sir Isaac Newton's three laws of motion and law of universal gravitation and Charles Coulomb's law of electrostatics. Galilei's principle of relativity of motion was simple and straightforward. Newton's second law of motion was conceivable and employed for a body acted upon by a force causing acceleration. Newton's law for gravitational force of attraction between two masses, applicable to all bodies in the universe, was imaginable. Coulomb's law, for the electrostatic force of repulsion or attraction between stationary electric charges, was comprehensible.

On the other hand, right from the days as a student of physics at the University of Manchester in England, in the early 1960s, one had been uncomfortable with Albert Einstein's theory of special relativity and theory of general relativity and Max Planck's and Niels Bohr's quantum theories. One found inconceivable the principle of constancy of the speed of light, the relativity of space and time and the prediction of increase in mass with speed resulting in the mass of an electron becoming infinitely large at the speed of light, according to special relativity.

The idea of time being a fourth dimension of space, making a four-dimensional space-time continuum, and the concept of curving or warping of space-time, due to the presence of matter, to create gravitational force of attraction between masses, according to general relativity, were unimaginable. Bohr's quantum theory of the hydrogen atom was incomprehensible. Was one misplaced or out-of-date or being too incredulous or had one been naïve or have we deviated too far from the right road and got lost? Certainly, nature could not be so complicated and inconsistent, one had thought.

Throughout the undergraduate student days, one pondered on the theories of special and general relativity, quantum mechanics and Rutherford-Bohr's model of the hydrogen atom, regarded as great triumphs of the human mind and the greatest achievements of modern physics. This issue of greatness boiled down to a choice between expressing one's mind and suppressing it. Under some mental agonies and at the risk of jeopardizing one's scholarship, one opted to forego academic expediency and to express one's mind openly. One rejected the principle of constancy of the speed of light, questioned the prediction of increase of mass with speed, doubted the relativity of space and time, objected to Bohr's quantum mechanics, ventured to ask a few questions and made some propositions. For, as the philosopher and dialectician, Peter Abelard (1709 – 1142) said, *"By doubting we come to question, and in questioning, we perceive the truth"*.

Did the mass of an electron actually increase with speed, becoming infinitely large at the speed of light, as predicted by special relativity, while electric charge remained constant? If free space is empty, a void containing "nothing", how could "it" be warped by "time" which is also non-substantive? How could warping of space-time continuum, due to the presence of matter, create gravitational force of attraction between bodies, in accordance with general relativity? Can we consider space as a medium filled with emanations, like electric fields, from bodies?

Was Coulomb's law really independent of velocity of the electron, as supposed by classical and relativistic electrodynamics? If not, how did Coulomb's law apply to a moving electron accelerated by an electrostatic field? Was there something else, other than mass becoming infinitely large at the speed of light, which makes that speed as the maximum attainable by an electron, under the acceleration of an electrostatic field?

Back home in 1965, as a physics graduate and teacher, with the fact that electrons were easily accelerated, by an electric field, to the speed of light, in one's mind, one resolved to go back to the basic principles, as demonstrated by Galileo Galilei or formulated by Sir Isaac Newton or enunciated by Charles Coulomb, and do something to bring about an alternative electrodynamics applicable up to the speed of light, with mass of a moving electron remaining constant. At the same time, it is hoped, to provide some good and invigorating materials for teaching mathematics and physics; science subjects in which our students appear deficient.

In the course of studies and research in mechanics, electrostatics, electromagnetism and electrodynamics, *aberration of electric field* was found to be the missing link in classical and relativistic electrodynamics. In many years of research, by fits and starts, through trials and errors and after countless presentations, untold distractions and endless rejections, success was achieved, as presented in this book, with the title: ***"Electrodynamics of a particle accelerated to the speed of light with constant mass"***.

Radiational electrodynamics is the new electrodynamics that makes the accelerating force, exerted by an electrostatic field, on a moving electron, dependent on its velocity. This dependency is due to *aberration of electric field*, which results in a *radiation reaction force*. Work done against the *radiation reaction force* appears as radiation.

Aberration of electric field, due to motion, is a phenomenon similar to aberration of light discovered by the English astronomer, James Bradley, in 1728. "Aberration" (which means: *deviation from the normal or departure from the stationary*), is a phenomenon observed as a displacement of the direction of the electric field intensity (or light ray) from a source, as a result of motion of a body (or an observer).

Radiational electrodynamics actually involves an extension of Coulomb's law to a moving electron accelerated by an electrostatic field. Newton's second law: (force = mass × acceleration) remains valid, with constant mass, but the accelerating force, due to some kind of impacts on the electron, depends on the velocity of the electron. This aspect of force is treated in line with the concept of the Greek philosopher, Aristotle (350 BC) [1, 2], who maintained that force could only be communicated between bodies by impacts or pressure. An impact is due to a change in momentum as a result of one body impinging on another body.

Consider the force $q\boldsymbol{E}$ on a body of charge q in an electrostatic field of intensity $\boldsymbol{E}$, as due to the "impacts" of infinitesimal "particles" each of mass ξ moving at the velocity of light $\boldsymbol{c}$ in the direction of the field $\boldsymbol{E}$. A "particle" impinging on a stationary body, under a perfectly elastic collision, will recoil with velocity $-\boldsymbol{c}$ and change of momentum equal to $2\xi\boldsymbol{c}$. With η impacts per second, the rate of change of momentum, equal to the impressed force, is $2\eta\xi\boldsymbol{c} = q\boldsymbol{E}$. This gives $2\eta\xi = qE/\mathrm{c}$. A "particle", of mass ξ, striking a body moving in the same direction with velocity $\boldsymbol{v}$, will recoil with velocity $-(\boldsymbol{c} - 2\boldsymbol{v})$ and change of momentum equal to $2\xi(\boldsymbol{c} - \boldsymbol{v})$. The accelerating force, from η impacts per second, is $\boldsymbol{F} = 2\eta\xi(\boldsymbol{c} - \boldsymbol{v}) = qE/\mathrm{c}\ (\boldsymbol{c} - \boldsymbol{v})$. Newton's second law of motion, for a body of mass m moving with velocity $\boldsymbol{v}$ at time t, gives the accelerating force as:

$$\mathbf{F} = 2\eta\xi(\mathbf{c}-\mathbf{v}) = \frac{qE}{c}(\mathbf{c}-\mathbf{v}) = m\frac{d\mathbf{v}}{dt} \qquad \text{(i)}$$

Equation (i) is the basic expression of *radiational electrodynamics*.

In *radiational electrodynamics*, the difference between the accelerating force $\boldsymbol{F}$ on a moving charged particle and the force $q\boldsymbol{E}$ on a stationary charged particle, is the radiation reaction force $\boldsymbol{R}_f = qE/c(\boldsymbol{c} - \boldsymbol{v}) - q\boldsymbol{E}$. If $\boldsymbol{E}$, $\boldsymbol{v}$ and $\boldsymbol{c}$ are collinear, the radiation reaction force is obtained as $-(qv/c)\boldsymbol{E}$ and radiation power, scalar product $-\boldsymbol{v}.\boldsymbol{R}_f$, is qEv^2/c. Radiation power is zero (0) in circular motion where $\boldsymbol{v}$ and $\boldsymbol{R}_f$ are orthogonal. Work done against the radiation reaction force appears as light or heat. This aspect of radiation, missing in classical and relativistic electrodynamics, informed the choice of the name: *radiational electrodynamics*.

The book is composed in nine separate chapters, as papers 1 – 9, with five appendices. A good knowledge of mechanics, electrostatics, electromagnetism and electrodynamics and a grasp of vector algebra and calculus are required to follow the presentations made in the papers.

In the first paper, titled *"An alternative electrodynamics to the theory of special relativity"*, it is proposed that the speed of light c is an ultimate limit, not because mass of an electron increases with speed becoming infinitely large at the speed of light, but as a result of accelerating force, exerted by an electrostatic field on a moving electron, reducing to zero at that speed. While mass remains constant, the accelerating force on a moving electron becomes zero at the speed of light c and the electron continues to move at that speed as a limit. It is also shown that an electron of charge $-e$ and mass m, equal to the rest mass m_o, revolves in an electrostatic field of magnitude E due to a positively charged nucleus, in a circle of radius r given by:

$$r = \frac{mv^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \qquad \text{(ii)}$$

where $m = m_o$ and $r_o = m_o v^2/eE$ is the classical radius. This variation of radius of revolution, in equation (ii), has the same effect as increase of mass with speed, according to the relativistic mass-speed formula:

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_o \qquad \text{(iii)}$$

The second paper is titled: *"Revolution of a charged particle round a centre of force of attraction"*. Here, it is shown the revolution may be by an electron going round a positively charged stationary nucleus or the motion may be by two particles, of equal and opposite charges and the same mass, going round their centre of mass. Circular motion of an electron, round a centre of revolution, is shown to be without radiation and inherently stable, outside quantum mechanics. In the stable state, charged particles revolve in circular coplanar orbits round a centre of attraction. Radiation, at the frequency of revolution, occurs only when a charged particle is dislodged from the stable circular orbit. An excited particle revolves in an unclosed elliptic orbit with emission of radiation, in many cycles of revolution, before settling back into the stable circular orbit. Interactions between particles moving in different orbits, gives radiation of discrete frequencies as given by the Balmer-Rydberg formula for the spectrum of the hydrogen atom.

The third paper titled: *"A nuclear model of the hydrogen atom outside quantum mechanics"*, introduces a new nuclear model for the solid or liquid state of the hydrogen atom. It consists of a number N_h of coplanar orbits. In each orbit, a particle carrying the electronic charge $-e$ and a multiple nm of the electronic mass m, revolves round under the attraction of a nucleus of charge $+eN_h$. The number n (1, 2, 3… N_h) leads to quantization of the orbits. A particle revolves in the nth circular orbit, without radiating, or it moves in an unclosed (aperiodic) elliptic orbit, in many cycles of revolution, emitting radiation, before reverting back into the nth stable circular orbit.

The fourth paper titled: *"A non-nuclear model of the hydrogen atom"*, introduces a non-nuclear model for the atom of hydrogen gas. It consists of a number N_h of coplanar orbits. In each orbit, two particles carrying the electronic charges e and $-e$ and a multiple nm of the electronic mass m, revolve in the nth orbit, under mutual attraction, round their common centre of mass. If a particle is disturbed by being dislodged from the nth circular orbit, it moves in an unclosed elliptic orbit, with emission of radiation, before reverting back into the nth circular orbit.

The fifth paper comes under the title, *"On the speed of light in a moving medium"*. Here, monochromatic light emitted by a stationary source is incident normally on the plane surface of a medium of refractive index μ, moving with speed v in a vacuum, in the normal direction. The speed of transmission w, in the medium, is derived as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \qquad \text{(iv)}$$

where c is the speed of light in a vacuum. The speed of light c is an absolute constant relative to the source. This simple equation (iv) is used to give a non-relativistic explanation of the result of Fizeau's experiment, which measured the speed of light in moving water

In the sixth paper, titled, *"On the energy and mass of electric charges in a body"*, the mass m of an electric charge, expressed in terms of electrical quantities, is deduced as proportional to the square of the charge. It is then shown that the electrostatic energy of a body of mass M, containing a distribution of equal number of positive and negative electric charges, is $\frac{1}{2}Mc^2$. The total energy E of a mass M, moving at speed v, with kinetic energy $\frac{1}{2}Mv^2$, is then obtained as:

$$E = \frac{M}{2}\left(c^2 + v^2\right) \qquad \text{(v)}$$

Equation (v) is in contrast to the mass-energy equivalence law of Einstein's theory of special of relativity, which gives:

$$E = Mc^2 = \frac{M_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{(vi)}$$

where M_o is the rest mass and the relativistic mass M becomes infinitely large at the speed of light c. Equation (vi), with infinite mass at speed $v = c$, is the main issue in the fourth paper. An infinite mass, which is the mass of the whole universe, is not tenable at a point anywhere in space.

The seventh paper, under the title, *"A unification of electrostatic and gravitational forces"*, shows that the electrostatic forces of repulsion and attraction, between the masses of two neutral bodies, in accordance with Coulomb's law, cancel out exactly. The gravitational forces, being proportional to the product of the masses, also proportional to <u>the product</u> *of* <u>the sum of squares of the electric charges in one body</u> *and* <u>the sum of squares of the electric charges in the other body</u>, add up to constitute the force of attraction due to gravity, in accordance with Newton's universal law of gravitation. This result of *radiational electrodynamics*, making gravitation as electrical in nature, offers a simple explanation of the long-awaited unification of electrostatic and gravitational forces. It is an exciting discovery which should put general relativity to rest.

The eighth paper gives an explanation of the result of Roger's experiment with electrons revolving in a circle and Bertozzi's experiment with high-energy electrons moving in a linear accelerator, without recourse to special relativity. The results of both experiments are in agreement with *radiational electrodynamics* on the basis of accelerating force decreasing with speed, reducing to zero at the speed of light.

The ninth paper treats longitudinal and transverse waves created by oscillating atomic charged particles. While the longitudinal waves, between the oscillating particles, are absorbed by the particles, manifesting as heat, the transverse waves are emitted as light radiation.

In *radiational electrodynamics*, *mass* is not a fundamental quantity. The fundamental quantities are put as *Length (L)*, *Time* (*T*), *Electric Charge* or *Flux (Q) and Electric voltage (V)*. In this (Metre–Second–Coulomb–Volt) system of measurements, the dimension of *mass (M)* becomes $[L^{-2}T^{2}QV]$. There is no fractional exponent of the dimension of a fundamental quantity *(L,T,Q,V)* in the dimension of a derived quantity.

We are under no illusion that these nine papers on *radiational electrodynamics* would gain easy acceptance, far less from the physics establishment committed to the current theories. Often, physicists dwell on complicated mathematics rather than seek simple explanation of a physical phenomenon. Indeed, every phenomenon in nature has a simple explanation and a mathematical expression, subject to experimental verification. Experiments do not lie, but interpretation of the results might be wrong and, consequently, the mathematical expressions and theoretical explanation would be complicated if not misleading.

It is said, *"Great ideas changed the world, but the greatest idea is founded on simplicity"*. Simplicity, it seems, is seldom a criterion for the acceptance of a new idea. Nowadays, the more complicated a theory is the more likely to engage the academics and impress the students. However, a theory too complicated to be understood by an intelligent and conscientious student, is probably wrong. For a student, the grounds for belief in any scientific theory are not just its complexity or acceptance in the academia or the authorities of the masters but its simplicity and consistency in nature and natural sense. Nature, when unraveled, is found to be simple, consistent, beautiful and wonderful.

Attempts to get the papers published in reputable scientific journals, at home and abroad, had failed. Publishers tend to restrict their journals to one discipline. So, papers with interdisciplinary approach, even if closely related ones like Mathematics, Physics and Engineering, might not find a place for acceptance. Also, most referees and reviewers of papers would rather promote their specialties and protect their interests than consider submissions from outsiders, intruders, skeptics or dissenters. One referee rejected a paper from the author on the grounds that it would have amounted to "re-writing the books". Who is afraid of re-writing the books? Surely, if books had not been re-written, knowledge would have been stale and referees would have had no work to do! By the grace of God, learning will progress, often with new advances from unexpected quarters. We thank the Almighty God for giving us the power to produce this work. The assistance and encouragements from our students, friends and well-wishers, are gratefully acknowledged.

For some time now the author has resorted to communicating the results of these papers by way of speeches, lectures, newspaper articles, advertisements, letters and self-financed publications, as in this book, and through the Internet. The communications have been targeted mainly at students, the young generation that is enlightened and least blighted by habits and prejudices. There is a strong tendency among us, the older generation, to keep behaving and believing in the old ways. We forget that advances in science, which we now so vigorously defend, were made mainly by persons who dissented. They defied suppression by authority and dared to speak their minds, contrary to the accepted doctrines. This was what the greatest scientist of all time, Galileo Galilei of Pisa, did and he was punished for it and left to be disgraced and he suffered alone.

Let us end this preface with the words of the most celebrated physicist, Albert Einstein, man of the 20th century, who pleaded:

> *"I beg you please to overcome your aversion long enough in this instance to read this brief piece as if you had not yet formed any opinion of your own but had only just arrived as visitor from Mars."*

I thank the reader for his esteemed attention and hope that this book is found as stimulating as much as I enjoyed writing it. I take full responsibility and beg forgiveness for any mistake or presumption in the book and plead for corrections and amendments accordingly.

Musa D. Abdullahi **Minna, Nigeria**, April 2006

1. AN ALTERNATIVE ELECTRODYNAMICS TO THE THEORY OF SPECIAL RELATIVITY

Abstract

An electrostatic force is propagated at the velocity of light $\boldsymbol{c}$ and the velocity of transmission of the force, relative to an electron moving with velocity $\boldsymbol{v}$, is the vector $(\boldsymbol{c} - \boldsymbol{v})$. The electron can be accelerated to the speed of light c and no faster. The accelerating force $\boldsymbol{F}$ on an electron of charge $-e$ and mass m moving at time t with velocity $\boldsymbol{v}$ at an angle θ to $\boldsymbol{F}$, in an electrostatic field of intensity $\boldsymbol{E}$ and magnitude E, is proposed as given by the vector equation:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) = \frac{-e\mathbf{E}}{c}\sqrt{c^2 + v^2 - 2cv\{\cos(\theta - \alpha)\}} = m\frac{d\mathbf{v}}{dt}$$

where α is the small angle of aberration such that $\sin\alpha = (v/c)\sin\theta$, and $(\theta - \alpha)$ is the angle between $\boldsymbol{v}$ and $\boldsymbol{c}$. For $\theta = 0$ or π radians, there is rectilinear motion with emission of radiation but stable circular revolution if $\theta = \pi/2$ radians.

Keywords: Charge, force, mass, radiation, velocity.

1.1 Introduction

There are now three systems of electrodynamics. There is classical electrodynamics applicable to electrically charged particles moving at a speed much slower than that of light. Relativistic electrodynamics is for particles moving at a speed comparable to that of light. Quantum electrodynamics is for atomic particles moving at very high speeds. There should be one consistent system of electrodynamics applicable at all speeds up to that of light.

Classical electrodynamics is based on the second law of motion, originated by Galileo in 1638, according to Lenard [1], but enunciated by the great physicist, Newton [2]. The theory of special relativity was formulated in 1905 mainly by the celebrated physicist, Einstein [3, 4] and the quantum theory was initiated by the renowned physicist Planck [5]. Relativistic electrodynamics reduces to classical electrodynamics at low speeds but the relativity and quantum theories are incompatible at high speeds. Both the relativity and quantum theories, therefore, cannot be correct. One of the theories or both theories may be wrong. Indeed, special relativity is under attack by physicists: Beckmann [6] and Renshaw [7]. This paper introduces *radiational electrodynamics*, as a new electrodynamics, applicable to electrically charged particles moving at speeds up to that of light c, with mass of a particle as constant.

According to Newton's second law of motion, a particle can be accelerated by a force to a speed greater than that of light with its mass remaining constant. But no particle, not even the electron, the lightest particle known in nature, can be accelerated beyond the speed of light. The theory of special relativity explains this limitation by positing that the mass of a particle increases with its speed, becoming infinitely large when the speed reaches that of light. That since an infinite mass cannot be accelerated any faster by any finite force, the speed of light is the ultimate limit. The famous Einstein's mass-speed formula of special relativity is:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o \tag{1.1}$$

where m is the mass of a particle moving with speed v, relative to an observer, m_o is the rest mass (at $v = 0$) or classical mass, c is the speed of light and γ is a ratio depending on v. Equation (1.1), where m is a physical mass supposed to have weight due to gravitational attraction, becoming infinitely large at the speed of light, is the bone of contention here.

The proponents of special relativity just ignore the difficulty with equation (1.1) expressing *mass expansion*, which results in infinitely large masses at the speed of light c. They avoid the difficulty altogether by arguing that the speed never really reaches c, that particles moving at speed c have zero rest mass or are exempted from equation (1.1) or that the increase in mass has no weight as it is not affected by the force of gravity. It is not said which physical property, volume or density of a body, increases with speed, while its electric charge remains constant.

Special relativity makes electric charge a quantity independent of speed. It is shown by the author [8] that the mass of a body is proportional to the sum of squares of the constituent electric charges. So, mass should remain constant with the magnitude of the electric charges.

Moreover, the mass-speed formula is challenged by virtue of a *positional principle* stated to the effect that *"any material property that is* *independent of its position in space is also independent of its velocity in space".* Doing away with infinite masses, at the speed of light, would bring great relief to physicists all over the world.

The difficulty with infinite mass, at the speed of light *(v = c)* in equation (1.1), is the Achilles' heel of the theory of special relativity. Resolving this difficulty is the main aim of this paper. It is shown that for an electron of mass *m* and charge *–e* revolving with constant speed *v* in a circle of radius *r*, under the attraction of a radial electrostatic field of magnitude *E*, the quantity *"m"* in equation (1.1) is actually the ratio $\{(eE)/(v^2/r)\}$ of force *(eE)* on a stationary electron to the centripetal acceleration (v^2/r) in circular motion. The acceleration reduces to zero at the speed *c*, where *r* becomes infinitely large for motion in a circle of infinite radius or a straight line.

Also challenged are decrease of *length* with speed or *length contraction* and increase of *time* with speed or *time dilation*. Without *mass expansion* these other aspects of special relativity, *length contraction* and *time dilation*, disappear. The difficulty associated with *length contraction*, of moving bodies becoming smaller or even disappearing, is seldom discussed in special relativity. The problem with *time dilation*, of moving bodies aging less or even becoming ageless, is dismissed by special relativity as "clock paradox". However, velocity, a vector quantity, central in special relativity, remains unchanged, as absolute distance covered *divided by* absolute time taken, not contracted distance *divided by* dilated time.

Usually, physicists employ vector quantities, having magnitudes and directions, indicated in ***boldface*** type. Velocity, for example, is a vector quantity denoted by ***c, u, v, w*** or ***z*** and speed, its magnitude, is a scalar quantity shown in ordinary type as *c, u, v, w* or *z*. A useful way of denoting a vector is as a product, $\mathbf{c} = c\hat{\mathbf{u}}$, where $\hat{\mathbf{u}}$, a unit vector in the direction of ***c***, varies with direction only. An electrostatic field of intensity ***E*** and magnitude *E* may be written as $\mathbf{E} = E\hat{\mathbf{u}} = E\mathbf{c}/c$, where **c** is the velocity of light with which an electrical effect is propagated in a vacuum or empty space.

The velocity of light ***c*** in vacuum, relative to its source, is a constant independent of velocity of the source. In other words, light takes on the velocity of its source. The magnitude of velocity of light, in a vacuum, relative to its source, is a universal constant *c*, equal *to* 2.998×10^8 *metres per second*, as obtained with electromagnetic waves.

1.1.1 Maxwell's equations of electromagnetic waves

In 1873, James Clerk Maxwell [9], in his ground-breaking treaties, showed that electromagnetic waves were propagated in a vacuum at constant speed, equal to that of light c, given by:

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 2.998 \times 10^8 \, \textit{metres per second} \tag{1.2}$$

where μ_o is the permeability and ε_o the permittivity of a vacuum. Here, c is the absolute speed of light, the magnitude of velocity of light relative to the source. The absolute speed of light c, given by equation (1.2), is a significant principle in physics. Another issue on the speed of light is the result of Michelson-Morley experiment.

1.1.2 Michelson-Morley experiment

The Americans, physicist Albert Michelson and chemist Edward Morley in 1887 [10], conducted an experiment to detect the *ether*, the luminiferous medium in which light is supposed to be propagated, like sound waves in the air. This, the most delicate and accurate experiment ever performed by physicists, demonstrated that the *ether* did not exist and that empty space contained *nothing*.

According to a cardinal principle of the theory of special relativity, the speed of light c, relative to an observer, is an absolute constant independent of speed of the source or the observer. The result of Michelson-Morley experiment was misinterpreted, in 1905, to lend support to the supposed invariance of the speed of light according to special relativity. It was claimed that the experiment proved the constancy of the speed of light from a source, relative to an observer. In fact, the constancy of the speed of light, relative to an observer, was not proven by this experiment or any other observation. The correct situation is given by the Galilean-Newtonian relativity of classical mechanics.

1.1.3 Galilean-Newtonian relativity

A basic principle of physics is the Galilean-Newtonian relativity. According to this principle, the relative velocity of light $\boldsymbol{z}$ emitted by a moving source, as measured by an observer moving with velocity $\boldsymbol{v}$ relative to a frame of reference, is given, in magnitude and direction, by the vector equation:

$$\mathbf{z} = \mathbf{c} + (\mathbf{u} - \mathbf{v}) \tag{1.3}$$

where $\boldsymbol{c}$ is the velocity of light relative to its source and $\boldsymbol{u}$ is the velocity of the source relative to the observer's frame of reference. The velocity of light $\boldsymbol{c}$, being a constant, relative to its source, is an experimental fact. In equation (1.3), the relative velocity $\boldsymbol{z}$ can be less or greater than $\boldsymbol{c}$ depending on the magnitudes and directions of the velocities $\boldsymbol{u}$ and $\boldsymbol{v}$. The theory of special relativity makes the relative velocity $\boldsymbol{z}$ a constant equal to the velocity of light $\boldsymbol{c}$, irrespective of the magnitudes of $\boldsymbol{u}$ and $\boldsymbol{v}$. The velocity $\boldsymbol{z}$ would be equal to the velocity of light $\boldsymbol{c}$ only if the magnitude of $\boldsymbol{c}$ were infinite. The magnitude of velocity of light, relative to the source, although fantastically high, is a finite universal constant c equal to *2.998* $\times$ 10^8 *metres per second.*

The Galilean-Newtonian relativity, as expressed in equation (1.3), is one of the most significant principles in physics, but now relegated to the background in favour of Einstein's theory of special of relativity. Equation (1.3) was employed by the author [11] to deduce an expression for the speed of light w in a medium moving with speed v in the direction of the light ray emitted by a stationary source. The speed w of transmitted light incident normally on the plane surface of a medium of refractive index μ, which is moving with speed v in a vacuum, is simply obtained as as given by the equation:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \tag{1.4}$$

Equation (1.4) was used, without recourse to the theory of special relativity, to explain the result of an experiment performed by Fizeau in 1851 and repeated by Michelson and Morley in 1886 [12], to measure the speed of light in moving water. According to special relativity, the speed of light, relative to a medium moving in a vacuum, is c and the speed of light in a moving medium is in accordance with Einstein's *velocity addition rule*. Another issue, connected with relative velocity of light, is Doppler Effect.

1.1.4 Doppler Effect

Doppler Effect, described by the Austrian physicist Christian Doppler in 1842, pertains to change in frequency of a light wave (or sound wave) due to motion of the source and/or the observer. Further information on Doppler is given by Stoll [13]. The Effect clearly demonstrates the relativity of velocity of light on the source of light moving with velocity $\boldsymbol{u}$ and/or the observer moving with velocity $\boldsymbol{v}$. For the observer moving in the same direction away from the source (***c***, ***u*** and ***v*** are collinear), equation (1.3) gives z as the magnitude of the relative velocity and the frequency f_m (Doppler frequency) measured by the moving observer, is:

$$f_m = \frac{fz}{c} = \frac{f\{c+(u-v)\}}{c} = \frac{c+(u-v)}{\lambda} \tag{1.5}$$

where f is the frequency and λ the wavelength measured by the observer moving with the same velocity as the source, i.e. ($\boldsymbol{u} = \boldsymbol{v}$). The wavelength of emission by the source remains unchanged as $\lambda = c/f$. For the observer moving towards the source, v in equation (1.5), should be replaced by $-v$.

Equation (1.5) shows that if the speed z were absolutely equal to c, speed of light in vacuum, there would have been no change in frequency and no Doppler Effect. This Effect is an every-day occurrence where equation (1.5) applies to sound waves as well and can easily be verified with $c = s$ as the speed of soumd in air. The next issue is Larmor formula for radiation power of electrons accelerated by an electrostatic field.

1.1.5 Larmor formula of classical electrodynamicsaa

Larmor formula of classical electrodynamics, described by Griffith [14], gives the radiation power R_p of an accelerated electron as proportional to the square of its acceleration. For an electron revolving with speed v in a circle of radius r with centripetal acceleration of magnitude v^2/r, Larmor classical formula gives $R_p = (e^2/6\pi\varepsilon_o r^2)v^4/c^3$, where ε_o is the permittivity of vacuum. Special relativity adopted this formula [14] and gives radiation power $R = \gamma^4 R_p$, where γ is defined in equation (1.1). The factor γ^4 means that the radiation power increases explosively as the speed v approaches that of light c.

According to Larmor formula, the hydrogen atom, consisting of an electron revolving round a positively charged nucleus, would radiate energy as it accelerates and spirals inward to collide with the nucleus, leading to the collapse of the atom. But atoms are the most stable particles known in nature. Use of this erroneous formula was most unfortunate as it led physics astray early in the 20th century. It required the brilliant hypotheses of Niels Bohr's [15] quantum mechanics to stabilize the Rutherford's [16] nuclear model of the hydrogen atom.

1.1.6 Rutherford's nuclear model of the hydrogen atom

If Larmor formula is accepted then Rutherford's nuclear model of the hydrogen atom cannot stand and Bohr's quantum mechanics had to be contrived to stabilize it. It would have been much easier to discard Larmor formula since the nuclear model of the hydrogen atom is inherently stable without the need for the quantum theory. This is so because circular motion of an electron does not involve any change of potential or kinetic energy and, therefore, no radiation occurs. Radiation takes place only when the electron is dislodged from the stable circular orbit, resulting in a change of potential or kinetic energy. The excited electron revolves in an unclosed elliptic orbit of decreasing eccentricity, before reverting back, after many cycles of revolution, into the stable circular orbit. It is like an oscillating loop settling down into a circular ring and rolling along.

On the basis of stability of circular motion of an electron, a new model of the hydrogen atom was developed [17]. The formula derived by Balmer in 1885, generalised by Rydberg in 1889, and described by Bitter [18], for discrete frequencies of radiation from the hydrogen atom, is deduced without recourse to Bohr's quantum mechanics. The frequency of emitted radiation is related to the frequency of circular revolution of the electron; something which quantum mechanics failed to do. The next issue is Bertozzi's experiment with regards to rectilinear motion of electrons,.

1.1.7 Bertozzi's experiment

A most remarkable demonstration of the existence of a universal limiting speed, equal to the speed of light c, was in an experiment by William Bertozzi [19] of the Massachusetts Institute of Technology. The experiment showed that electrons accelerated through energies of 15 MeV or over, attain, for all practical purposes, the speed of light c. Bertozzi measured the heat energy J developed when a stream of accelerated electrons hit an aluminium target at the end of their flight path, in a linear accelerator. He found the heat energy released to be nearly equal to the potential energy P lost, to give $P = J = K$. Bertozzi identified J as solely due to the kinetic energy K lost by the electrons, on the assumption that the force on a moving electron is $-e\boldsymbol{E}$, independent of its speed and always equal to the accelerating force $\boldsymbol{F}$.

Bertozzi might have made a mistake in equating the heat energy J with the kinetic energy K of the electrons. The energy equation should have been $P = J = K + R$. Here, R was the energy radiated. Radiation, propagated at the speed of light, also caused heating effect upon falling or impinging at the same point or on the same target as the accelerated electrons. For rectilinear motion, the accelerating force ***F***, a vector in the **x**-direction, is given by:

$$\mathbf{F} = \frac{dK}{d\mathbf{x}} = \frac{dP}{d\mathbf{x}} - \frac{dR}{d\mathbf{x}} \tag{1.6}$$

In equation (1.6), $\boldsymbol{F} = dK/d\boldsymbol{x}$ is the accelerating force, $dP/d\boldsymbol{x}$ the electrostatic (impressed) force and $-dR/d\boldsymbol{x}$ is the radiation reaction force as a result of an electron moving along an electrostatic field. The radiation reaction force is a result of aberration of electric field.

1.1.8 Aberration of electric field

Figure 1.1 depicts an electron, moving at a point P with velocity $\boldsymbol{v}$, in an electrostatic field $\boldsymbol{E}$ due to a stationary source charge $+Q$ at an origin O. For motion at an angle θ to the accelerating force $\boldsymbol{F}$, the electron is subjected to aberration of electric field. This phenomenon is similar to aberration of light discovered by the English astronomer James Bradley in 1728 [20]. In aberration of electric field, as in aberration of light, the direction of the electrostatic field, indicated by the velocity vector $\boldsymbol{c}$ (see Figure 1.1), appears shifted by an aberration angle α, from the instantaneous line ***PO***, such that:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{1.7}$$

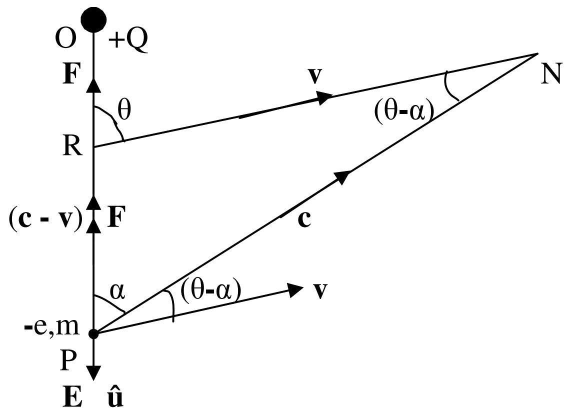

Figure 1.1. Vector diagram depicts angle of aberration α as a result of an electron of charge $-e$ and mass m moving, at a point P, with velocity $\boldsymbol{v}$, at an angle θ to the accelerating force $\boldsymbol{F}$. The unit vector $\hat{\boldsymbol{u}}$ is in the direction of the electrostatic field of intensity $\boldsymbol{E}$ due to a stationary source charge $+Q$ at the origin O.

where the speeds v and c are the magnitudes of the velocities $\boldsymbol{v}$ and $\boldsymbol{c}$ respectively. Equation (1.7) was first derived by the English astronomer, James Bradley.

The result of aberration of electric field is that the accelerating force on a moving electron depends on the velocity of the electron. If the accelerating force reduces to zero at the speed of light c, that speed becomes the ultimate limit. This aspect, which is missing in classical and relativistic electrodynamics, is used in the formulation of *radiational electrodynamics*.

1.2 Equations of motion in radiational electrodynamics

The force exerted on an electron, moving with velocity $\boldsymbol{v}$. by an electrostatic field is transmitted at the velocity of light $\boldsymbol{c}$ relative to the source charge and with velocity $\boldsymbol{c} - \boldsymbol{v}$ relative to the electron. The electron can be accelerated to the velocity of light $\boldsymbol{c}$ and no faster. In Figure 1.1 the electron can be accelerated in the direction of the force with $\theta = 0$ or it can be decelerated against the force with $\theta = \pi$ radians or it can revolve in a circle with $\theta = \pi/2$ radians.

The accelerating force $\boldsymbol{F}$ (see Figure 1.1), on an electron of charge $-e$ and mass m moving at time t with velocity $\boldsymbol{v}$ and acceleration $(d\boldsymbol{v}/dt)$, in an electrostatic field of magnitude E and intensity $\boldsymbol{E} = E\hat{\boldsymbol{u}}$, in the direction of unit vector $\hat{\boldsymbol{u}}$, is proposed as given by the vector equation and Newton's second law of motion, thus:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{1.8}$$

where $\boldsymbol{c}$ is the velocity of light at aberration angle α to the accelerating force $\boldsymbol{F}$ and $(\boldsymbol{c} - \boldsymbol{v})$ is the relative velocity of transmission of the force with respect to the moving electron. The simple idea behind a limiting velocity $\boldsymbol{c}$ is that the electrostatic force or "electrical punches" propagated at velocity of light $\boldsymbol{c}$, cannot "catch up" and "impact" on an electron also moving with velocity $\boldsymbol{v} = \boldsymbol{c}$. Equation (1.8) may be regarded as an extension or amendment of Coulomb's law of electrostatic force between two electric charges, taking into consideration the relative velocity between the electric charges.

Equation (1.7) links the angle θ with the aberration angle α (Figure 1.1). Equation (1.8) is the basic expression of *radiational electrodynamics*. Expanding equation (1.8), by taking the *modulus* of the vector $(\boldsymbol{c} - \boldsymbol{v})$, with respect to the angles θ and α, gives:

$$\mathbf{F} = \frac{-eE}{c}\sqrt{c^2 + v^2 - 2cv\left\{\cos\left(\theta - \alpha\right)\right\}}\,\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt} \tag{1.9}$$

1.2.1 Equations of rectilinear motion

For an accelerated electron where $\theta = 0$, equations (1.7) and (1.9) give the accelerating force $\boldsymbol{F}$, in rectilinear motion, as:

$$\mathbf{F} = -eE\left(1 - \frac{v}{c}\right)\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} \tag{1.10}$$

The solution of equation (1.10) for an electron accelerated by a uniform field of magnitude E, from zero initial speed, is:

$$\frac{v}{c} = 1 - \exp\left(-\frac{at}{c}\right) \tag{1.11}$$

where $a = eE/m = eE/m_o$, is the acceleration constant. Figure 1.2.C1 is a graph of v/c against at/c according to equation (1.11).

For a decelerated electron where $\theta = \pi$ radians, equations (1.7) and (1.9) give the decelerating force $\boldsymbol{F}$ as:

$$\mathbf{F} = -eE\left(1 + \frac{v}{c}\right)\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \tag{1.12}$$

Solving (1.12) for an electron decelerated from speed c, gives:

$$\frac{v}{c} = 2\exp\left(-\frac{at}{c}\right) - 1 \tag{1.13}$$

Figure 1.2.C2 is a plot of v/c against at/c according to equation (1.13).

Figure 1.2 shows a graph of v/c against at/c for an electron accelerated from zero initial speed, or an electron decelerated from speed of light c, by a uniform field: the solid lines, (A1) & (A2) according to classical electrodynamics, the dashed curve (B1) and line (B2) according to relativistic electrodynamics and the dotted curves (C1) and (C2) according to equations (1.11) and (1.13).

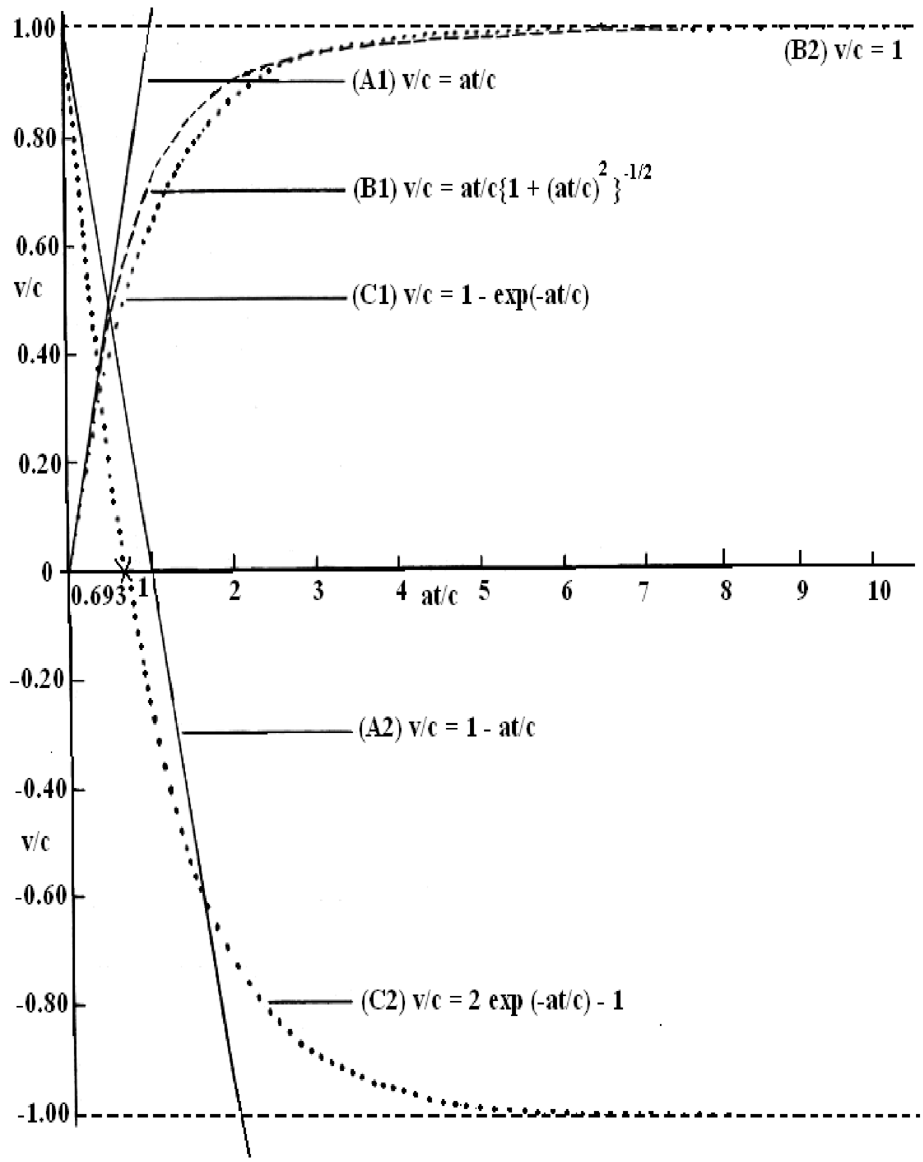

Figure 1.2. *v/c (speed in units of c)* against *at/c (time in units of c/a)* for an electron of charge *–e* and mass $m = m_o$ accelerated from zero initial speed or decelerated from the speed of light *c*, by a uniform electrostatic field of magnitude *E*, where $a = eE/m$; the lines (A1) and (A2) according to classical electrodynamics, the dashed curve (B1) and line (B2) according to relativistic electrodynamics and the dotted curves (C1) and (C2) according to equations 1.11 and 1.13 of *radiational electrodynamics* presented here..

1.2.2 Equations of circular motion

For $\theta = \pi/2$ radians, motion is in a circle of radius r with constant speed v and acceleration $(-v^2/r)\hat{\mathbf{u}}$. Equations (1.7) and (1.9), with mass $m = m_o$ and noting that $\cos(\pi/2 - \alpha) = \sin\alpha = v/c$, give:

$$\mathbf{F} = -eE\sqrt{1-\frac{v^2}{c^2}}\,\hat{\mathbf{u}} = -m_o\frac{v^2}{r}\hat{\mathbf{u}} \qquad (1.14)$$

$$eE = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} = \varsigma\frac{v^2}{r}$$

$$\varsigma = \frac{eEr}{v^2} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (1.15)$$

Equation (1.1) for "m" and equation (1.15) for ζ are identical but obtained from two different points of view. In equation (1.1), the quantity "m" increases with speed v, becoming infinitely large at speed c. In equation (1.15), mass m remains constant at the rest mass m_o, and $\zeta = \{(eE)/(v^2/r)\}$ is the ratio of magnitude F_o, equal to the electrostatic force (eE) on a stationary electron, to the centripetal acceleration (v^2/r) in circular motion. This quantity ζ (not to be mistaken for physical mass $m = m_o$) may become infinitely large at the speed of light c, without any difficulty. At the speed of light, that is $(v = c)$, the electron moves in a circle of infinite radius, a straight line, to make "m" or ζ also infinite. The quantity ζ in equation (1.15) may be referred to as "*submass*".

In classical electrodynamics, the radius r of circular revolution for an electron of charge –e and mass m, in an electrostatic field of magnitude E due to a positively charged nucleus, is:

$$r = \frac{mv^2}{eE} = \frac{m_o v^2}{eE} = r_o \qquad (1.16)$$

where $m = m_o$, the classical mass or rest mass, is a constant and r_o is the classical radius of revolution.

In relativistic electrodynamics, where mass m varies with speed in accordance with equation (1.1), the radius of revolution becomes:

$$r = \frac{mv^2}{eE} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{1.17}$$

In *radiational electrodynamics*, where $m = m_o$ is a constant, the radius of revolution, obtained from equation (1.14), is:

$$r = \frac{mv^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{1.18}$$

Relativistic electrodynamics and *radiational electrodynamics* give the same expression for radius of revolution as $r = \gamma r_o$.

1.3 Radiation reaction force and radiation power

The difference between the accelerating force $\boldsymbol{F}$ on a moving electron (equation 1.8) and the electrostatic force $-e\boldsymbol{E}$ on a stationary electron, is the radiation reaction force $\boldsymbol{R}_f = \boldsymbol{F} - (-e\boldsymbol{E})$, that is always present when a charged particle is accelerated by an electrostatic field. This is analogous to a frictional force, which always opposes motion. Radiation reaction force $\boldsymbol{R}_f$ is missing in classical and relativistic electrodynamics and it makes all the difference. The radiation force, $-\boldsymbol{R}_f$, gives the direction of emitted radiation from an accelerated charged particle. For rectilinear motion, with $\theta = 0$ (Figure 1.1), equation (8) gives $\boldsymbol{R}_f$, in the direction of unit vector $\hat{\boldsymbol{u}}$, as:

$$\mathbf{R}_f = -\frac{eE}{c}(c-v)\hat{\mathbf{u}} + eE\hat{\mathbf{u}} = \frac{eEv}{c}\hat{\mathbf{u}} = -\frac{eE}{c}\mathbf{v} \tag{1.19}$$

In rectilinear motion, with $\theta = \pi$ radians, $\boldsymbol{R}_f = -(eEv/c)\hat{\boldsymbol{u}} = -(eE\boldsymbol{v}/c)$.

Radiation power $R_p = -\boldsymbol{v}.\boldsymbol{R}_f$, the scalar product of $\boldsymbol{R}_f$ and velocity $\mathbf{v}$. The scalar product is obtained, with reference to Figure 1.1, as:

$$R_p = -\mathbf{v}\cdot\mathbf{R}_f = -\mathbf{v}\cdot\left\{\frac{eE}{c}(\mathbf{c}-\mathbf{v}) + e\mathbf{E}\right\}$$

$$R_p = eEv\left\{\cos\theta - \cos(\theta-\alpha) + \frac{v}{c}\right\} \tag{1.20}$$

For rectilinear motion with $\theta = 0$ or π radians, equations (1.7) and (1.20) give the radiation power R_p as:

$$R_p = -\mathbf{v}.\mathbf{R}_f = eE\frac{v^2}{c} \tag{1.21}$$

Positive radiation power, as given by equation (1.21), means that energy is radiated in accelerated and decelerated motions. Note the difference between this equation and Larmor formula in classical electrodynamics.

In circular motion, where $\boldsymbol{v}$ is orthogonal to $\boldsymbol{E}$ and $\boldsymbol{R_f}$, the radiation power R_p (scalar product of $\boldsymbol{v}$ and $\boldsymbol{R_f}$) is zero, as can be ascertained from equations (1.7) and (1.17) with $\theta = \pi/2$ radians and $cos(\theta - \alpha) = sin\alpha = v/c$. Equation (1.20) is significant in *radiational electrodynamics* [13]. It makes circular motion of an electron, round a centre of revolution, as in Rutherford's nuclear model of the hydrogen atom, without radiation and stable, without recourse to Bohr's quantum theory. This result is used by the author [17, 21] to develop two new models of the hydrogen atom, which are inherently stable, outside Bohr's quantum electrodynamics.

Equations (1.19), (1.20) and (1.21) are the radiation formulas of *radiational electrodynamics*. These equations are in contrast to those of classical electrodynamics [14] where the radiation force is proportional to the rate of change of acceleration and the radiation power is proportional to the square of acceleration.

1.4 Mass-energy equivalence formula

The author [8] showed that the electrostatic energy of a particle of mass *m,* equal to the energy content of the mass, is $W = m/2\mu_o\varepsilon_o = ½mc^2$, where $c = (\mu_o\varepsilon_o)^{-1/2}$, is the speed of light in a vacuum and μ_o *is* the permeability and ε_o the permittivity of a vacuum [9]. The kinetic energy of a particle of mass m moving with speed v, is $K = ½mv^2$, so that the total energy content E, is:

$$E = W + K = \frac{m}{2}\left(c^2 + v^2\right) \tag{1.22}$$

This is in contrast to Einstein's most famous formula of special relativity, the mass-energy equivalence formula, that gives the total energy content of a body of mass m and rest mass m_o moving with speed v, relative to an observer, as:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.23}$$

1.5 Conclusion

In special relativity, Einstein's influence is so overwhelming that difficulties of the theory as in mass expansion, and contradictions as in Doppler Effect, are ignored or avoided altogether. Challenge of Einstein is considered, by the establishment physicists, as sacrilegious. Physicists might have been dazzled by Einstein's "brilliance" and the public amazed by adulation of a "genius" who toppled Galileo Galilei, dethroned Isaac Newton and overturned natural sense. However, now disproving some aspects of special relativity and putting general relativity to test, should not, in any way, detract from Einstein's stature and ingenuity as man of the 20th Century.

Einstein filled a gap that existed in knowledge during his time. He brilliantly answered the question why an electron cannot be accelerated beyond the speed of light by positing that mass increases with speed, becoming infinitely large at the speed of light. This position is apparently plausible as an infinite mass cannot be accelerated any faster by a finite force. Now, with the amendment of Coulomb's law for electrostatic force, giving rise to the *force-velocity formula of electrodynamics*, we have the ultimate speed without infinite mass. The speed of light c is the terminal speed for a charged paricle under the acceleration of an electrostatic field.

In *radiational electrodynamics*, there is no increase of *mass* with speed or *mass expansion*. The mass m of a moving electron remains constant as the rest mass m_o and it is the accelerating force that becomes zero at the speed of light c. Relativistic and *radiational electrodynamics* give the same expression (equations 1.17 and 1.18) for the radius of revolution of an electron round a positively charged nucleus. This explains the cause of misconception or delusion connected with increase of mass with speed. In this regard, the relativistic mass-energy and mass-velocity formula expressed in equation (1.23), the bone of contention here, is shown to be, but yet to be proved, wrong.

The relativistic mass *"m"* in equation (1.23), being equated with the physical mass m of an electron, is a very expensive case of mistaken identity. This is a good example of Beckmann's *correspondence theory* [6] whereby the correct result is produced mathematically but it does not correspond with physical reality. For example, equation (1.23) leads to the speed of light as the ultimate limit but for the wrong reason. The easoning that mass m becomes infinitely large at the speed of light $v = c$.

is wrong Two other contentious issues in special relativity are *length contraction* and *time dilation*.

This paper regards *mass*, l*ength* and *time* as absolute quantities, independent of motion or position of the observer. *Mass expansion*, *length contraction* and *time dilation* are completely rejected. The contentious issue remaining is Larmor formula.

Larmor formula, an erroneous formula for radiation power of accelerated electrons, influenced physics early in the 20th century. It required Bohr's quantum theory, devised to prevent radiation and stabilize the Rutherford's nuclear model of the hydrogen atom.

Relativistic electrodynamics fails for electrons decelerated from the speed of light c. In *radiational electrodynamics*, an electron is readily accelerated to the speed of light c, through a potential energy of 15 MeV or higher. An electron accelerated to the speed of light c, is easily stopped by a decelerating field and then accelerated backwards to reach an ultimate speed $-c$ (curve C2, in contrast to lines A2 and B2, in Figure 1.2). An experiment may be performed to test this result by having a narrow pulse of electrons, accelerated to the speed of light (or almost to the speed of light) c, made to enter a decelerating field. The electrons being stopped at all and turned back in their track, invalidates special relativity. This is the litmus test of validity of *radiational electrodynamics*, as advanced in this paper.

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2. REVOLUTION OF A CHARGED PARTICLE ROUND A CENTRE OF FORCE OF ATTRACTION

Abstract

A charged particle of mass *nm* revolves in an orbit through angle ψ in time t, at a distance r from a centre of force of attraction, with constant angular momentum $n\boldsymbol{L}$, given by:

$$n\mathbf{L} = nmr^2 \frac{d\psi}{dt}\mathbf{k}$$

where n is an integer greater than 0, m is the electronic mass, $\boldsymbol{k}$ is a unit vector perpendicular to the plane of the orbit. The equation of the nth orbit of motion is:

$$\frac{1}{r} = \frac{A}{n}\exp\left(-b\psi\right)\cos\left(\alpha\psi + \beta\right) + \frac{1}{nr_1}$$

where A and β are determined from the initial conditions, b and α are constants and r_1 is the radius of the first stable orbit. An excited particle revolves in an unclosed ellipse, with emission of radiation at the frequency of revolution, before settling down, after many cycles of ψ, into a stable circular orbit of radius nr_1. A particle of mass *nm* carrying the electronic charge $-e$ revolves in a unipolar orbit, round a positively charged nucleus or two particles of the same mass *nm* and charges e and $-e$ revolve, in a bipolar orbit, round a centre of mass. The number n leads to quantization of the orbits without recourse to Bohr's quantum mechanics.

Keywords: Angular momentum, force, orbit, revolution, radiation.

2.1 Introduction

Revolution of a body, round a centre of force of attraction, is the most common motion in the universe. This comes with revolution of planets round the Sun, binary stars round their centre of mass, a moon round a planet or an electron round the nucleus of an atom.

The German astronomer, Johannes Kepler [1, 2] early in the 17th century, formulated three laws, named after him, concerning the motions of planets. Kepler based his laws on astronomical data painstakingly collected in 30 years of observations by the Danish astronomer Tycho Brahe [3], to whom he was an assistant. Kepler’s proposals broke with a centuries-old belief based on the Ptolemaic system advanced by the Alexandrian astronomer Ptolemy [4], in the 2nd century AD, and the Copernican system put forward by the Polish astronomer, Nicolaus Copernicus [5], in the 16th century.

The Ptolemaic cosmology postulated a geocentric universe in which the Earth was stationary and motionless at the centre of several concentric rotating spheres, which bore (in order of distance away from the earth) the Moon, the planets and the stars. The major premises of the Copernican system are that the Earth rotates daily on its axis and revolves yearly round the Sun and that the planets also circle the Sun. Copernicus's heliocentric theories of planetary motion had the advantage of accounting for the daily and yearly motions of the Sun and stars and it neatly explained the observed motions of the planets. However, the reigning dogma in the 16^{th} century, that of the Roman Catholic Church was in favour of the Ptolemaic system and it abhorred the Copernican theory.

The Copernican theory had some modifications and various degrees of acceptance in the 16^{th} and 17^{th} centuries. The most famous Copernicans were the Italian physicist Galileo Galilei [6] and his contemporary, the astronomer Johannes Kepler [1, 2].

By December 1609 Galileo [7] had built a telescope of 20 times magnification, with which he discovered the four moons circling Jupiter. This showed that at least some heavenly bodies move around a centre other than the Earth. By December 1910 Galileo [7] had observed the phases of Venus, which could be explained if Venus was sometimes nearer the Earth and sometimes farther away from the Earth, following a motion round the Sun.

In 1616, Copernican books were subjected to censorship by the Church [8]. Galileo was instructed to no longer hold or defend the opinion that the Earth moved. He failed to conform to the ruling of the Church and after the publication of his book titled *Dialogue on the Two Chief World Systems* [9], he was accused of heresy, compelled to recant his beliefs and then confined to house arrest. Galileo’s *Dialogue* was ordered to be burned and his ideas banned.

The ideas contained in the *Dialogue* could not be suppressed. Galileo’s reputation continued to grow in Italy and abroad, especially so after his final work. Galileo's final and greatest work is the book titled *Discourses Concerning Two New Sciences*, published in 1638. It reviews and refines his earlier studies of motion and, in general, the principles of mechanics. The book opened a road that was to lead Sir Isaac Newton [10] to the law of universal gravitation, which linked the planetary laws discovered by astronomer Kepler with Galileo's mathematical physics.

Kepler [1, 2] stamped the final seal of validity on the Copernican planetary system in three laws, viz.:

(i) *The paths of the planets are ellipses with the sun as one focus.*

(ii) *The line drawn from the sun to the planet sweeps over equal areas in equal time.*

(iii) *The square of the periods of revolution (T) of the different planets are proportional to the cube of their respective mean distances (r) from the sun* ($T^2 \propto r^3$)

The import of Kepler's first law is that there is no dissipation of energy in the revolution of a planet, in a closed orbit, round the Sun. Any change in the kinetic energy of a planet is equal to the change in its potential energy. The second law means that a planet revolves round the Sun with constant angular momentum, which is the case if there is no force perpendicular to the radius vector. From the third law ($T^2 \propto r^3$) and the relationship between the centripetal force and speed ($F \propto v^2/r$), it can be deduced that the force of attraction F on a planet is inversely proportional to the square of its distance r from the Sun ($F \propto 1/r^2$), as discovered by Newton around 1687 [10].

Kepler's laws played an important part in the work of the English astronomer, mathematician, and physicist, Sir Isaac Newton [10]. The laws are significant for the understanding of the orbital paths of the moon, the natural satellite of the Earth, and the paths of the artificial satellites launched from space stations.

While the orbital path of a satellite is a closed ellipse, the orbit of an electrically charged particle, round a central force of attraction, is an unclosed ellipse or a closed circle. A charged particle revolves in an unclosed orbit with emission or absorption of radiation. The energy that is radiated is the difference between change in kinetic energy and change in potential energy. Revolution in a circular orbit is without radiation and inherently stable as there is no change in the kinetic energy and potential energy of a revolving particle. Revolution in a circular orbit is the perfect motion as it involves no change in the status of energy.

The purpose of this paper is to derive equations of the orbit of motion of a charged particle revolving round a centre of force of attraction. The equations are used to show that the orbit of a charged particle is an unclosed (aperiodic) ellipse where it moves with constant angular momentum *nL*, *n* being an integer. The discrete masses, *nm*, (*m* being the electronic mass) of revolving particles lead to quantisation of the orbits. A particle revolves with emission or absorption of radiation of discrete frequencies, in many cycles of revolution, before settling into the stable circular orbit.

2.2 Unipolar motion under a central force

Consider a particle of charge $-e$ and mass *nm* at a point *P* and time *t* revolving round, anticlockwise, in an angle ψ and with velocity $\boldsymbol{v}$ in an orbit under the attraction of a positive charge *Q* fixed at origin *O*, as shown in Figure 2.1. Here, *n* is an integer greater than *0*, $-e$ is the electronic charge and *m* the electronic mass. The particle at *P* executes unipolar motion under a central force at *O*. In unipolar revolution, a charged particle, as the one and only pole of the orbit, revolves round a stationary centre under a force of attraction. The orbit of motion is an unclosed (aperiodic) ellipse with emission or absorption of radiation or a closed circle without radiation.

2.2.1 Velocity and acceleration in unipolar central motion

In Figure 2.1, the radius vector ***OP*** makes an angle ψ with the ***OX*** axis in space. The position vector $\boldsymbol{r}$ of the point *P*, in the direction of unit vector $\hat{\boldsymbol{u}}$, (radial direction) and the velocity $\boldsymbol{v}$ at time *t*, are respectively given by the vector equations:

$$\mathbf{r} = r\hat{\mathbf{u}} \tag{2.1}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\hat{\mathbf{u}}}{dt} \tag{2.2}$$

For orbital motion in the *X*-*Y* plane of the Cartesian coordinates, ***dû/dt***, the angular velocity, is given by the vector (cross) product:

$$\frac{d\hat{\mathbf{u}}}{dt} = \frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{2.3}$$

The angle ψ is the inclination of the radius vector ***OP*** from ***OX***, ***k*** is a constant unit vector, in the **Z**-direction, perpendicular to the orbital plane (out of the page in Figure 2.1) and $(d\psi/dt)\boldsymbol{k}$ is the angular velocity. The velocity ***v*** is:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{2.4}$$

The velocity in the radial direction is:

$$v_r\hat{\mathbf{u}} = \frac{dr}{dt}\hat{\mathbf{u}} \tag{2.5}$$

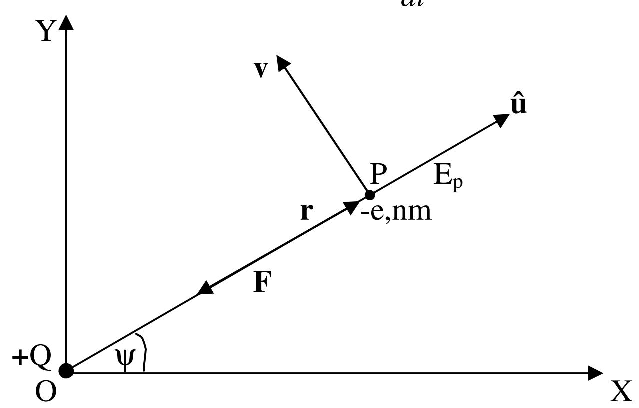

Figure 2.1. A particle of charge $-e$ and mass *nm* at a point *P* revolving anticlockwise, with angular displacement ψ, in an orbit at velocity ***v*** under the attraction of a stationary positive charge *Q* at a centre of attraction *O*.

The acceleration (noting that ***k*** is a constant unit vector) is obtained, a vector in two orthogonal directions, as:

$$\frac{d\mathbf{v}}{dt} = \left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} + \left(2\frac{dr}{dt}\frac{d\psi}{dt} + r\frac{d^2\psi}{dt^2}\right)\mathbf{k}\times\hat{\mathbf{u}} \tag{2.6}$$

The acceleration, in the direction of force of attraction, is $a_r\hat{\boldsymbol{u}}$ in the radial direction only, so that:

$$\frac{d\mathbf{v}}{dt} = \left\{\frac{d^2 r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} = a_r\hat{\mathbf{u}} \tag{2.7}$$

2.2.2 Angular momentum

From equation (2.6), the force perpendicular to the radial direction $\hat{\boldsymbol{u}}$ is zero and, therefore, the acceleration of mass nm is also zero in this direction, so that the equation gives:

$$nm\left(2\frac{dr}{dt}\frac{d\psi}{dt} + r\frac{d^2\psi}{dt^2}\right)\mathbf{k}\times\hat{\mathbf{u}} = 0$$

This equation can be expressed in terms of angular momentum, as:

$$\frac{nm}{r}\frac{d}{dt}\left(r^2\frac{d\psi}{dt}\right)\mathbf{k}\times\hat{\mathbf{u}} = \frac{n}{r}\frac{d}{dt}\mathbf{L}\times\hat{\mathbf{u}} = 0$$

where

$$n\mathbf{L} = nmr^2\frac{d\psi}{dt}\mathbf{k} \tag{2.8}$$

Here, $\boldsymbol{L}$ is the constant angular momentum with respect to the first orbit. Equation (2.7) and equation (2.8) will be used to derive the equation of the orbit of motion of the particle at P in Figure 2.1.

2.2.3 Forces on a revolving charged particle

In Figure 2.1, the positively charged particle at O is considered to be very much more massive such that it could be taken as almost stationary. In this case, we have central motion where a particle at P carrying the electronic charge $-e$ and a multiple nm of the electronic mass m, revolves in an orbit, round a stationary particle of charge $+Q$, as nucleus at O. The particle moves in an electrostatic field E_p under a force of attraction (Coulomb force) and a radiation reaction force.

In elliptic motion, there is a component of velocity in the radial $\hat{\mathbf{u}}$ direction. Equation (1.8) [11] gives the accelerating force $\boldsymbol{F}$, as:

$$\mathbf{F} = \frac{eE_p}{c}(\mathbf{c}-\mathbf{v}) = \frac{eE_p}{c}(\mathbf{c}.\hat{\mathbf{u}} - \mathbf{v}.\hat{\mathbf{u}})\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt}$$

The scalar products with $\hat{\mathbf{u}}$, with reference to Figure 1.1 [11], gives:

$$\mathbf{F} = \frac{eE_p}{c}(-c\cos\alpha + v\cos\theta)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt}$$

As α is a small angle, $c\cos\alpha \approx c$ and with $v\cos\theta = -v_r$, we get:

$$\mathbf{F} = -eE_p\left(1+\frac{v_r}{c}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{2.9}$$

where v_r is the speed in the radial direction, $-eE_p\hat{\boldsymbol{u}}$ is the electrostatic force and $-(eE_pv_r/c)\hat{\boldsymbol{u}}$ is the radiation reaction force. The radiation reaction force, akin to a frictional force or damping force in dynamics, results in energy radiation. Radiation always comes into play when a charged particle is accelerated or decelerated by an electrostatic field.

Substituting for v_r from equation (2.5) into equation (2.9) gives the accelerating force on a charged particle of mass *nm*, as:

$$\mathbf{F} = -eE_p\left(1+\frac{1}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{2.10}$$

Substituting for the acceleration $a_r\hat{\boldsymbol{u}}$ from equations (2.7) into equation (2.10), gives the accelerating force $\boldsymbol{F}$ on a charged particle of mass *nm*, moving in an electrostatic field E_p, as:

$$\mathbf{F} = -eE_p\left(1+\frac{1}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} \tag{2.11}$$

Putting $E_p = Q/4\pi\varepsilon_o r^2$, gives the magnitude of the force as:

$$F = \frac{-eQ}{4\pi\varepsilon_o r^2}\left(1+\frac{1}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \tag{2.12}$$

$$F = \frac{-\chi}{nmr^2}\left(1+\frac{1}{c}\frac{dr}{dt}\right) = \frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2 \tag{2.13}$$

where $\chi = eQ/4\pi\varepsilon_o$ is a constant. Equation (2.13) is the mixed differential equation of motion of the particle revolving in an orbit through angle ψ and with instantaneous radius r at time t. We need to reduce it to an equation of r as a function of one variable, ψ.

2.2.4 Equation of the unipolar orbit of motion

In equation (2.13), taking the angle ψ as the variable, making the substitution $r = 1/u$ to give $dr/du = -1/u^2$ and with $(d\psi/dt) = L/mr^2$ (equation 2.8), we get:

$$\frac{dr}{dt} = \frac{dr}{du}\frac{d\psi}{dt}\frac{du}{d\psi} = \frac{-L}{m}\frac{du}{d\psi} \tag{2.14}$$

$$\frac{d^2r}{dt^2}=\frac{d}{dt}\frac{dr}{dt}=\frac{d\psi}{dt}\frac{d}{d\psi}\left(-\frac{L}{m}\frac{du}{d\psi}\right)=\frac{-L^2u^2}{m^2}\frac{d^2u}{d\psi^2} \tag{2.15}$$

Substituting equations (2.15) and (2.14) into equation (2.13) gives:

$$-\frac{L^2u^2}{m^2}\frac{d^2u}{d\psi^2}-\frac{L^2u^3}{m^2}=-\frac{\chi u^2}{nm}\left(1-\frac{L}{mc}\frac{du}{d\psi}\right)$$

$$\frac{d^2u}{d\psi^2}+\frac{\chi}{ncL}\frac{du}{d\psi}+u=\frac{m\chi}{nL^2} \tag{2.16}$$

This is a 2nd order differential equation with constant coefficients. A solution for the nth orbit is $u = (A/n)exp(x\psi)$, the *transient*, if the *auxiliary equation*, $x^2 + 2qx + 1 = 0$ and $q = \chi/2ncL$. This gives:

$$x=-q\pm\sqrt{q^2-1}=-q\pm j\alpha$$

where α is the 'rotation factor' and $\alpha^2 = 1 - q^2$, is positive. The general solution is:

$$u=\frac{1}{r}=\frac{A}{n}\exp\left(j\alpha-q\right)\psi+\frac{m\chi}{nL^2} \tag{2.17}$$

The particular or appropriate solution of equation (2.16) is:

$$u=\frac{1}{r}=\frac{A}{n}\exp\left(-q\psi\right)\cos\left(\alpha\psi+\beta\right)+\frac{m\chi}{nL^2} \tag{2.18}$$

where the excitement amplitude A/n and phase angle β are obtained from the initial conditions and $m\chi/nL^2$ is the *steady state*.

Equation (2.18) gives the path or the nth unstable orbit of the particle (at P in Fig.2.1) with O as the fixed centre of revolution. For $q > 0 < 1$ and $\alpha < 1$, the orbit is an unclosed (aperiodic) ellipse whose major axis (line joining the points of farthest of the particles) rotates about an axis through the centre, perpendicular to the orbital plane. A particle completes a cycle of $2\pi/\alpha$ radians while the major axis goes through $2\pi/\alpha - 2\pi$ radians.

The exponential decay factor, $exp(-q\psi)$, is due to energy radiation. As a result of radiation of energy, after a great number of revolutions in the angle ψ, $(A/n)\ exp(-q\psi)$, the *transient*, decreases to zero and the radius increases to the *steady state* $nL^2/m\chi$, as long as q is greater than zero. This is the radius of the stable orbit when the radiator settles down from the excited state with the particle revolving in the nth stable orbit, a circle of radius $nL^2/m\chi$. = nr_1. The radius r_1 is with respect to the innermost orbit where $n = 1$.

In the stable orbit, there is only motion in a perfect circle, perpendicular to a radial electric field. No radial motion of the charged particle, no change of potential or kinetic energy and, therefore, no radiation of energy. The author [11] showed that radiation occurs provided there is a component of velocity of a charged particle in the direction of an electric field.

2.3 Bipolar motion under a central force

A bipolar orbit consists of two oppositely charged particles, of equal mass, revolving under mutual attraction, round a common centre of mass. The two particles form the two poles of the orbit.

2.3.1 Description of bipolar orbit

A bipolar orbit consists of two particles of equal mass but oppositely charged, each carrying the electronic charge of magnitude e and a multiple nm of the electronic mass m, under mutual attraction, revolving round their common centre of mass, the common centre of revolution, at a point O as depicted in Figure 2.2. The centripetal electrostatic force of attraction $\boldsymbol{F}$, on a charged particle, is balanced by the centrifugal force due to acceleration.

The two oppositely charged particles at P and S in Figure 2.2, separated by distance $2r$, make up the two poles of the bipolar orbit, each particle being one pole in the orbit. Thus the bipolar orbit (in contrast to the unipolar orbit) has no nucleus but an empty point as the centre of mass, the centre of revolution, located halfway between the two particles.

2.3.2 Velocity and acceleration in a bipolar orbit

In Figure 2.2 the particle (of mass nm and charge $-e$) at point P, of position vector $\boldsymbol{r}$, is moving with velocity $\boldsymbol{v}_p$, at an angle ψ in the electrostatic field $\boldsymbol{E}_p$ of the other particle (of mass nm and charge $+e$) at S. The particle at S of position vector $-\boldsymbol{r}$, is moving with velocity $\boldsymbol{v}_s$ in the electrostatic field $\boldsymbol{E}_s$ of the particle at P. The velocities $\boldsymbol{v}_p$ and $\boldsymbol{v}_s$ are respectively given by the vector equations:

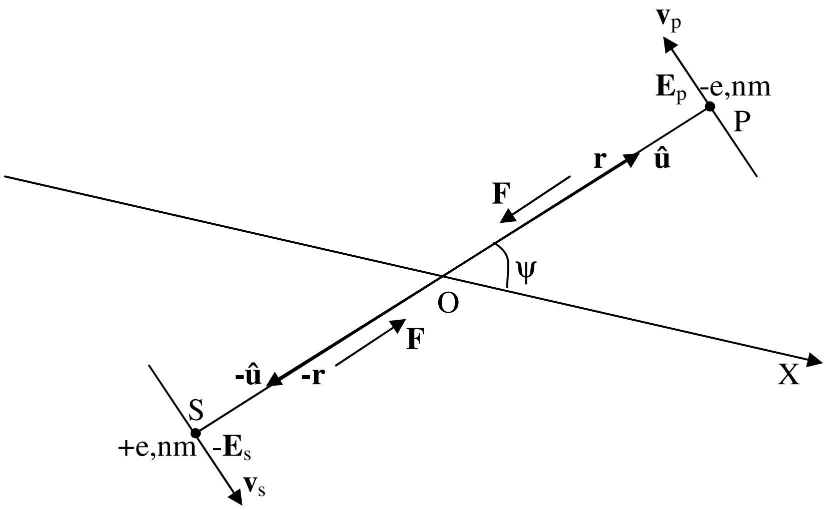

Figure 2.2. Two equal and oppositely charged particles at P and S having electronic charges $-e$ and $+e$ and the same mass nm (n being an integer and m is the electronic mass) revolving anticlockwise in angle ψ, under mutual attraction, in an orbit of radius r, round the centre of revolution at O.

$$\mathbf{v}_p = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{2.19}$$

$$\mathbf{v}_s = -\frac{d\mathbf{r}}{dt} = -\frac{dr}{dt}\hat{\mathbf{u}} + r\frac{d\psi}{dt}\mathbf{k}\times\hat{\mathbf{u}} \tag{2.20}$$

The two particles, of equal mass, move with the same angular velocity, in a plane orbit, but with relative linear velocity in the radial direction. The relative velocity $\boldsymbol{v}_r$ of the moving particle at P with respect to the moving particle at S, is:

$$\mathbf{v}_r = v_r\hat{\mathbf{u}} = \mathbf{v}_p - \mathbf{v}_s = 2\frac{dr}{dt}\hat{\mathbf{u}} \tag{2.21}$$

It is shown in section 2.2.3 that the accelerating force $\boldsymbol{F}$, due to attraction, on a particle of charge $-e$ and mass nm revolving in an ellipse, at time t, with speed v_r in the direction of an electrostatic field $\hat{\boldsymbol{u}}E_p$ of magnitude E_p (Figure 2.1), is given by equation (2.9):

$$\mathbf{F} = -eE_p\left(1+\frac{v_r}{c}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{2.22}$$

Substituting for v_r from equation (2.21) into equation (2.22) gives the accelerating force on a charged particle of mass *nm*, as:

$$\mathbf{F} = -eE_p\left(1+\frac{2}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\frac{d\mathbf{v}}{dt} \tag{2.23}$$

Substituting for the acceleration $a_r\hat{\boldsymbol{u}}$ from equation (2.7) into equation (2.23), gives the accelerating force $\boldsymbol{F}$ on a particle of mass *nm*, as:

$$\mathbf{F} = -eE_p\left(1+\frac{2}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\}\hat{\mathbf{u}} \tag{2.24}$$

Putting $E_p = e/16\pi\varepsilon_o r^2$, gives the magnitude of the force as:

$$F = \frac{-e^2}{16\pi\varepsilon_o r^2}\left(1+\frac{2}{c}\frac{dr}{dt}\right) = nm\left\{\frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2\right\} \tag{2.25}$$

$$F = \frac{-\kappa}{nmr^2}\left(1+\frac{2}{c}\frac{dr}{dt}\right) = \frac{d^2r}{dt^2} - r\left(\frac{d\psi}{dt}\right)^2 \tag{2.26}$$

where $\kappa = e^2/16\pi\varepsilon_o$. Equation (2.26) is the mixed differential equation of revolution of the particle in an orbit through angle ψ.

2.3.3 Equation of bipolar orbit of motion

In equation (2.26), taking the angle ψ as the variable and making the substitution $r = 1/u$ and with $(d\psi/dt) = L/mr^2$ (equation 2.8) we get equations (2.14) and (2.15) and the differential equation:

$$\frac{d^2u}{d\psi^2} + \frac{2\kappa}{ncL}\frac{du}{d\psi} + u = \frac{m\kappa}{nL^2} \tag{2.27}$$

If $u = (A/n)exp(y\psi)$ is a solution for the nth orbit, the *auxiliary equation* $y^2 + 2by + 1 = 0$, with $b = \kappa/ncL$. The general solution is:

$$u = \frac{1}{r} = \frac{A}{n}\exp\left(j\alpha - b\right)\psi + \frac{mK}{nL^2} \tag{2.28}$$

The appropriate solution of equation (2.27) is:

$$u = \frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi + \beta) + \frac{m\kappa}{nL^2} \tag{2.29}$$

where the amplitude of the excitement *A/n* and phase angle β are determined from the initial conditions α is the' rotation factor' and $\alpha^2 = 1 - b^2$. The *steady state* is $m\kappa/nL^2$, obtained after many cycles of revolution.

Equation (2.29) gives the path or the nth unstable orbit of the particles (at *P* or at *S* in Fig.2.2) with *O* as the centre of revolution. For $b > 0 < 1$ and $\alpha < 1$, the orbit is an unclosed (aperiodic) ellipse where a particle completes one cycle of revolution in $2\pi/\alpha$ radians, with radiation and relative motion in the radial direction.

The exponential decay factor, $exp(-b\psi)$, is due to radiation. After a great number of revolutions in the angle ψ, the *transient* $(A/n)\ exp(-b\psi)$, decreases to zero and the radius settles at the *steady state* $nL^2/m\kappa$, as long b is greater than zero. This is the radius of the stable orbit when the radiating particle settles down from the excited state with the two particles revolving in the nth stable orbit, a circle of radius $nL^2/m\kappa = nr_1$, shown as *WCYD* in Figure 2.3.

In the stable bipolar orbit there is only revolution in a perfect circle. There is no motion of a particle along the electric field, only perpendicular to the electric field and, therefore, no radiation.

2.4 Free ellipse and stable orbit of revolution of a radiating charged particle

Equation (2.29), giving the bipolar orbit of a radiating particle in the nth orbit, with the phase angle β being 0, may be written as:

$$\frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi) + \frac{1}{nr_1}$$

where $r_1 = L^2/m\kappa$ is the radius of the first orbit. If the decay factor b is negligible, $\alpha \approx 1$, the equation of the nth orbit becomes:

$$\frac{1}{r} = \frac{A}{n}\cos\psi + \frac{1}{nr_1} = \frac{1}{nr_1}(1 + Ar_1\cos\psi) \tag{2.30}$$

The orbit, shown as *WXYZ* in Figure 2.3, an ellipse of eccentricity $\eta = Ar_1 = A/B$, is the **free ellipse**. This ellipse is a hypothetical orbit that the particle would have taken if there were no radiation, i.e. if $b = 0$.

Revolution of a radiating particle is in an unclosed (aperiodic) ellipse, with a decreasing period (increasing frequency). After a great number of revolutions ($\psi \to \infty$), the nth orbit reduces to a circle, the **stable orbit** of radius nr_1, shown as *CDEF* in Figure 2.3. The frequencies of revolution of a radiating particle, in the unstable orbits, are very nearly equal to that of revolution in the nth stable orbit. So, radiation from a charged particle, in a bipolar orbit or unipolar orbit, is a narrow band of frequencies, very nearly equal to the frequency of revolution in the nth stable orbit. This leads to a spread of frequencies of revolution, as discussed in section 2.6.

2.5 Energy radiated by a revolving charged particle

The accelerating force on a particle of mass *nm* and charge $-e$ revolving at time *t* and at a point distance *r* from the centre of a force of attraction due to an electric field of magnitude E_p of an electric charge, is given by equation (2.23). The radiation force is:

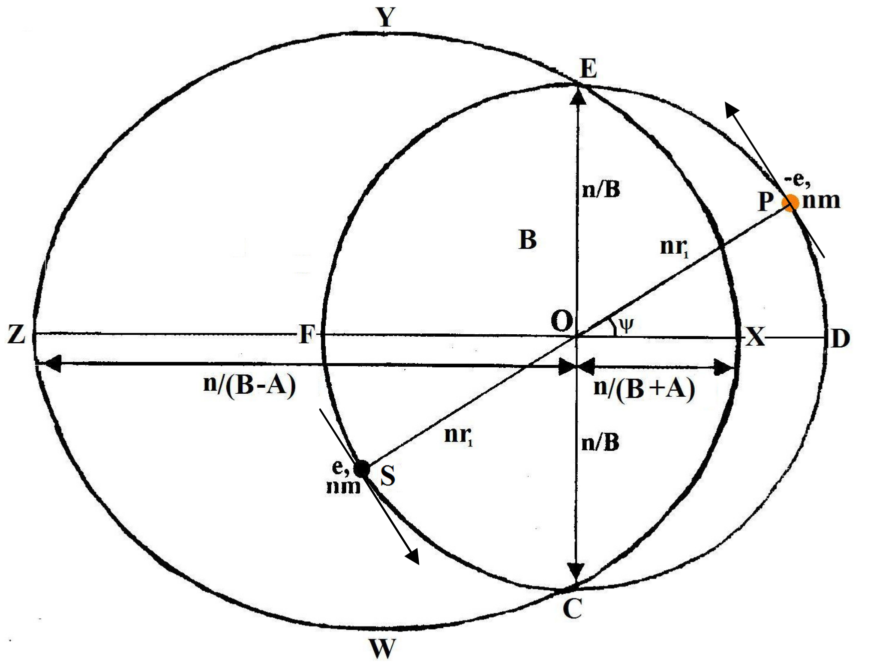

Figure 2.3. **Free ellipse** W*XYZ* of eccentricity *A/B* and **steady orbit** *CDEF* of revolution of a radiating particle, at *P* or S, in the nth circle of radius n/B = nr_1 with centre at *O* as one focus of the free ellipse.

$$-\mathbf{R}_f = eE_p\left(\frac{2}{c}\frac{dr}{dt}\right)\hat{\mathbf{u}} = \frac{2\kappa}{c}\frac{1}{r^2}\left(\frac{dr}{dt}\right)\hat{\mathbf{u}}$$

where $\kappa = e^2/16\pi\varepsilon_o$. The energy radiated is obtained by integrating the radiation force with respect to displacement *(dr)* in one cycle, *s* to *(s+1)*, through $2\pi/\alpha$ radians, of the nth orbit, to give:

$$s_r = \frac{2\kappa}{c}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \frac{1}{r^2}\left(\frac{dr}{dt}\right)(dr)$$

Substituting for *(dr/dt)* from equation (2.14) and with *(dr)* = $-1/u^2(du)$, we obtain:

$$s_r = \frac{2\kappa L}{mc}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \left(\frac{du}{d\psi}\right)^2 (d\psi) \qquad (2.31)$$

Substituting for $(du/d\psi)$ from equation (2.28), gives the integral in complex form. The energy radiated in the (s+1)th cycle of the nth orbit, is given by the [Real Part] of the integral:

$$s_r = \frac{2\kappa L}{mc}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \frac{A^2}{n^2}\exp 2\{(j\alpha-b)\psi\}(j\alpha-b)^2(d\psi) \qquad (2.32)$$

$$s_r = \frac{2\kappa LA^2}{mcn^2}\left[\frac{\exp 2\{(j\alpha-b)\psi\}(j\alpha-b)^2}{2(j\alpha-b)}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

$$s_r = \frac{\kappa LA^2}{mcn^2}\left[\frac{\exp(-2b\psi)\{\cos(2\alpha\psi)+j\sin(2\alpha\psi)\}(j\alpha-b)}{1}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

The [Real Part] is obtained as:

$$s_r = \frac{-\kappa LA^2}{mcn^2}\left[\frac{\exp(-2b\psi)\{b\cos(2\alpha\psi)+\alpha\sin(2\alpha\psi)\}}{1}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

$$s_r = \frac{\kappa LA^2 b}{mcn^2}\exp\left(\frac{-4\pi bs}{\alpha}\right)\left\{1-\exp\left(\frac{-4\pi b}{\alpha}\right)\right\} \tag{2.33}$$

In the final cycle *($s \to \infty$)*, the energy radiated is 0. The total energy radiated, after many cycles, is the sum of geometric series:

$$E_r = \sum_{s=0}^{\infty} s_r = \frac{\kappa LA^2 b}{mcn^2} = \frac{A^2\kappa^2}{mc^2n^3} \tag{2.34}$$

Here A/n is the excitement amplitude in the nth orbit and $b = \kappa/ncL$.

2.6 Period of oscillation of a radiator

Equation (2.29) gives the bipolar orbit of a charged particle of mass nm revolving through angle ψ, in the nth orbit, with constant angular momentum nL and with phase angle $\beta = 0$, as:

$$\frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi) + \frac{m\kappa}{nL^2} \tag{2.35}$$

$$\frac{1}{r} = \frac{m\kappa}{nL^2}\left\{1+\frac{AL^2}{m\kappa}\exp(-b\psi)\cos(\alpha\psi)\right\}$$

$$\frac{1}{r} = \frac{B}{n}\left\{1+\frac{A}{B}\exp(-b\psi)\cos(\alpha\psi)\right\}$$

where $B = m\kappa/L^2$.

$$\frac{1}{r} = \frac{B}{n}\{1+\eta\exp(-b\psi)\cos(\alpha\psi)\} \tag{2.36}$$

This is the equation of an unclosed (aperiodic) ellipse, in the polar coordinates, with eccentricity $\eta = A/B$. Equation (2.36) gives r as:

$$r = \frac{n}{B}\{1+\eta\exp(-b\psi)\cos(\alpha\psi)\}^{-1} \qquad (2.37)$$

The angular momentum *nL* of the particle, of mass *nm*, gives:

$$dt = \frac{mr^2}{L}(d\psi)$$

$$dt = \frac{mn^2}{LB^2}\{1+\eta\exp(-b\psi)\cos(\alpha\psi)\}^{-2}(d\psi) \qquad (2.38)$$

The period of revolution $T_{(s+1)}$ in the (s + 1)th cycle (s = 0, 1, 2, 3...∞) of the nth orbit (n = 1, 2, 3...N_h), is obtained by integrating equation (2.38) for ψ through an angle $2\pi/\alpha$ radians, to obtain:

$$T_{(s+1)} = \frac{mn^2}{LB^2}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}\{1+\eta\exp(-b\psi)\cos(\alpha\psi)\}^{-2}(d\psi) \qquad (2.39)$$

Expanding the integrand, in equation (2.39), into an infinite series, by the binomial theorem, we obtain:

$$\{1+\eta\exp(-b\psi)\cos(\alpha\psi)\}^{-2}$$
$$= \sum_{p=0}^{\infty}(-p)^p(1+p)\eta^p\exp(-pb\psi)\cos^p(\alpha\psi)$$

where *p* is a positive integer, 0 – ∞. Equation (2.39) then becomes:

$$T_{(s+1)} =$$
$$\frac{mn^2}{LB^2}\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}\sum_{p=0}^{\infty}(-p)^p(1+p)\eta^p\exp(-pb\psi)\cos^p(\alpha\psi)(d\psi) \qquad (2.40)$$

$$T_{(s+1)} = \frac{mn^2}{LB^2}\sum_{p=0}^{\infty}Q_p$$

where

$$Q_p = (-1)^p(1+p)\eta^p\int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi\{s+1\}}{\alpha}}\exp(-pb\psi)\cos^p(\alpha\psi)(d\psi) \qquad (2.41)$$

Expressing $cos^p(\alpha\psi)$ as a sum of cosines of multiples of $(\alpha\psi)$, let us take the first five terms of Q_p.

$$Q_0 = \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} (d\psi) = \frac{2\pi}{\alpha} \tag{2.42}$$

$$.Q_1 = -2\eta \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-b\psi)\cos(\alpha\psi)(d\psi)$$

Putting $exp(-b\,\psi)cos(\alpha\psi)$ = [Real Part] of $\{exp(j\alpha - b)\psi\}$, the integral, Q_1, is obtained as:

$$Q_1 = -2\eta\left[\frac{\exp(-b\psi)\{-b\cos\alpha\psi + \alpha\sin(\alpha\psi)\}}{a^2 + b^2}\right]_{\psi=\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}}$$

Noting that $\alpha^2 + b^2 = 1$, we get:

$$Q_1 = -2\eta b\exp\left(\frac{-2\pi bs}{\alpha}\right)\left\{1 - \exp\left(\frac{-2\pi b}{\alpha}\right)\right\} \tag{2.43}$$

$$Q_2 = -3\eta^2 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-2b\psi)\cos^2(\alpha\psi)(d\psi)$$

$$Q_2 = \frac{-3\eta^2}{2} \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-2b\psi)\{1 + \cos(2\alpha\psi)\}(d\psi)$$

$$Q_2 = \frac{3\eta^2}{2}\left(\frac{1}{2b} + \frac{b}{2}\right)\exp\left(\frac{-4\pi bs}{\alpha}\right)\left\{1 - \exp\left(\frac{-4\pi b}{\alpha}\right)\right\} \tag{2.44}$$

$$35\,Q_3 = -4\eta^3 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-3b\psi)\cos^3(\alpha\psi)(d\psi)$$

$$Q_3 = -\eta^3 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-3b\psi)\{3\cos(\alpha\psi)+\cos(3\alpha\psi)\}(d\psi)$$

$$Q_3 = \eta^3\left(\frac{9b}{\alpha^2+9b^2}+\frac{b}{3}\right)\exp\left(\frac{-6\pi bs}{\alpha}\right)\left\{1-\exp\left(\frac{-6\pi b}{\alpha}\right)\right\} \qquad (2.45)$$

$$Q_4 = 5\varepsilon^4 \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-4b\psi)\cos^4(\alpha\psi)(d\psi)$$

Expressing $cos^4(\alpha\psi)$ in terms of $cos(2\alpha\psi)$ and $cos(4\alpha\psi)$, gives:

$$Q_4 = \frac{5\eta^4}{8} \int_{\frac{2\pi s}{\alpha}}^{\frac{2\pi(s+1)}{\alpha}} \exp(-4b\psi)\{3+\cos(4\alpha\psi)+4\cos(2\alpha\psi)\}(d\psi)$$

$$Q_4 =$$

$$\frac{5\eta^4}{8}\left(\frac{3}{4b}+\frac{b}{4}+\frac{4b}{\alpha^2+4b^2}\right)\exp\left(\frac{-8\pi bs}{\alpha}\right)\left\{1-\exp\left(\frac{-8\pi b}{\alpha}\right)\right\} \qquad (2.46)$$

Note that as $s \to \infty$ or $b = 0$, only Q_0 remains, since $Q_1 = Q_2 = Q_3 = \ldots Qp = 0$. Where $b \neq 0$, the period of the (s + 1)th cycle in the nth orbit of revolution, is obtained as the sum:

$$\begin{aligned} T_{(s+1)} &= \frac{mn^2}{LB^2}\sum_{p=0}^{\infty} Q_p \\ &= \frac{mn^2}{LB^2}\left(Q_0 + Q_1 + Q_2 + Q_3 + \ldots + Q_p + \ldots + Q_\infty\right) \end{aligned} \qquad (2.47)$$

Since the eccentricity η is small and b is smaller, neglecting powers of η greater than 2 and powers of b greater than 1, we obtain $Q_1 \approx Q_3 \approx Q_5 \approx ... \approx Q_{2p+1} \approx 0$. An approximate expression for the period of revolution in the first cycle (with $s = 0$), in the nth orbit, is obtained as the sum:

$$T_1 = \frac{mn^2}{LB^2}\sum_{p=0}^{\infty} Q_p \approx Q_o + Q_2 + ... + Q_{2p} + ... + Q_\infty \approx Q_0 + Q_2 \approx \frac{mn^2}{LB^2}\left[\frac{2\pi}{\alpha} + \frac{3\eta^2}{2}\left(\frac{1}{2b} + \frac{b}{2}\right)\left\{1 - \exp\left(-\frac{4\pi b}{\alpha}\right)\right\}\right] \tag{2.48}$$

With $\alpha^2 = 1 - b^2$, we obtain: $\alpha \approx 1$ and:

$$T_1 \approx \frac{mn^2}{LB^2}\left(\frac{2\pi}{\alpha} + 3\eta^2\frac{2\pi}{\alpha}\right) \approx \frac{mn^2}{LB^2}\left(2\pi + \frac{3\eta^2}{2}2\pi\right)$$

$$T_1 \approx \frac{2\pi mn^2}{LB^2}\left(1 + \frac{3\eta^2}{2}\right) \tag{2.49}$$

Equation (2.49) gives the time taken, in the first cycle, for the particle, in the nth orbit, to go through $2\pi/\alpha$ radians, due to the rotation of the major axis that goes through $(2\pi/\alpha - \pi)$ radians. After a great number of cycles $(s \to \infty)$, the period of revolution, through 2π radians, in the steady circle of the nth orbit, is:

$$T_n = \frac{2\pi mn^2}{LB^2} \tag{2.50}$$

T_n is the period of revolution of the particle in the steady orbit, a circle of radius $r_n = n/B = 16\pi n\varepsilon_o L^2/me$ with speed $v_n = BL/nm = e^2/16\pi n\varepsilon_o L$ and frequency $f_n = LB^2/2\pi mn^2 = me^4/2\pi L^3(16\pi n\varepsilon_o)^2$.

The period of the first cycle T_1, (equation (2.49), is the highest while that in the steady orbit T_n (equation 2.50) is the least. The wavelength, λ_1 of radiation, during the 1st cycle of the nth orbit, is:

$$\lambda_1 = cT_1 = \frac{2\pi mn^2 c}{LB^2}\left(1 + \frac{3\eta^2}{2}\right) \tag{2.51}$$

where c is the speed of light in a vacuum. The separation, splitting or increment of the wavelengths, $\Delta\lambda$, is:

$$\Delta\lambda = \frac{3\eta^2 \pi m n^2 c}{LB^2} \tag{2.52}$$

The ratio of the separation of wavelengths and the wavelength, with respect to revolution in the nth stable orbit, the same as the magnitude of separation of frequencies, is:

$$\frac{\Delta\lambda}{\lambda_n} = -\frac{\Delta f}{f_n} = \frac{3\eta^2}{2} \tag{2.53}$$

This ratio is related to the splitting, spread or "fine structure" of a spectral line due to frequency of radiation from the hydrogen atom. The radiation is not of precise frequencies, but has a spread around the frequency of revolution of a particle in the nth stable orbit.

2.7 Conclusion

The orbit of revolution of a neutral body, round a gravitational force of attraction, is a closed ellipse, without radiation of energy. Such a periodic motion may continue *ad infinitum*. On the other hand, a charged particle revolves in an unclosed (aperiodic) elliptic orbit, with emission of radiation at the frequency of revolution, before settling into a stable circular orbit. This is the source of atomic radiation.

The unipolar revolution of an electron round a nucleus, as discussed in section 2.2, leads to the development of unipolar model or nuclear model of the hydrogen atom. Similarly, bipolar motion of two oppositely charged particles round a centre of revolution, as discussed under section 2.3, leads to the development of bipolar model or non-nuclear model of the hydrogen atom. The developments of unipolar and bipolar models of the hydrogen atom are the subjects of chapters 3 and 4 of this book, respectively.

It was shown by the author [11] that the mass of a particle is independent of its speed. Therefore, in the treatment of the motion of electrons round a centre of revolution, in the unipolar or bipolar model of the hydrogen atom, relativistic effects were not taken into consideration.

Neither was the spin of a revolving particle regarded in the emission of radiation from the hydrogen atom.

The paper concludes that the revolution of a charged particle in a circular orbit, round a centre of force of attraction, is inherently stable. Radiation takes place only if the particle is excited by being dislodged from the stable circular orbit. An excited particle revolves in an aperiodic elliptic orbit, emitting energy, as given by equation (2.34), before reverting back to the stable circular orbit.

The narrow spread of frequencies, with respect to revolution in the nth orbit, as given by equation (2.53), may explain the “fine structure” [12] of the spectral lines of radiation from the hydrogen atom, without considering relativistic effects, electron spin or quantum mechanics. This is a new and exciting development.

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3. A NUCLEAR MODEL OF THE HYDROGEN ATOM OUTSIDE QUANTUM MECHANICS

Abstract

A nuclear model of the hydrogen atom is devised consisting of N_h coplanar orbits each with a particle of mass nm and electronic charge $-e$ revolving in the nth orbit round a nucleus of charge $+N_h e$. A particle revolves through angle ψ in an unclosed elliptic orbit, at a distance r from the nucleus, as:

$$\frac{1}{r} = \frac{A}{n}\exp(-q\psi)\cos(\alpha\psi + \beta) + \frac{m\chi}{nL^2}$$

where A and β are determined from the initial conditions, q, α, and χ are constants, nL is a constant angular momentum in the nth orbit and m is the electronic mass. The number n (1, 2, 3...N_h) leads to quantisation of the orbits. The decay factor, $exp(-q\psi)$, is the result of radiation as a particle revolves and settles into the nth stable circle of radius $r_n = nL^2/m\chi$ with speed $v_n = \chi/nL$. It is shown that relative motions between the orbiting particles and the nucleus and between the particles themselves, give rise to radiation of discrete frequencies.

Keywords: Angular momentum, hydrogen atom, nucleus, orbit of revolution, radiation.

3.1 Introduction

Various models of the hydrogen atom have been introduced. The most successful of these models is the Rutherford's nuclear model introduced in 1911 [1].

3.1.1 Rutherford's nuclear model of the hydrogen atom

Lord Earnest Rutherford [1] proposed a nuclear theory of the atom consisting of a heavy positively charged central nucleus around which a cloud of negatively charged electrons revolve in circular orbits. The hydrogen atom is the simplest, consisting of one electron of charge $-e$ and mass m revolving in a circular orbit round a much heavier central nucleus of charge $+e$. This model, conceived on the basis of experimental results, has sufficed since, although with some difficulties regarding its stability and emitted radiation. This paper introduces a new nuclear model of the hydrogen atom for the liquid or solid state. The model is stabilized without recourse to Bohr's quantum mechanics.

According to classical electrodynamics [2] the electron of the Rutherford's model, in being accelerated towards the positively charged nucleus of the atom, by the centripetal force, should:

(i) emit radiation over a continuous range of frequencies with power proportional to the square of its acceleration and

(ii) lose potential energy and gain kinetic energy as it spirals into the nucleus, leading to collapse of the atom.

The second prediction is contradicted by observation as atoms are the most stable objects known in nature. The first effect is contradicted by experiments as a detailed study of the radiation from hydrogen gas, undertaken by J. J. Balmer as early as 1885, as described by Bitter [3], showed that the emitted radiation had discrete frequencies. The spectral lines in the Balmer series of the hydrogen spectrum satisfy the formula (Balmer formula):

$$\nu_{2q} = \frac{1}{\lambda_{2q}} = R\left(\frac{1}{2^2} - \frac{1}{q^2}\right) \tag{3.1}$$

where λ_{2q} is the wavelength, ν_{2q} is the wave number, R is the Rydberg constant and q is an integer greater than 2. The first of the four visible lines $(q = 3)$ is red. The spectral series limit $(q \rightarrow \infty)$, lying in the violet (not visible) region of the spectrum, is $\nu_2 = R/4$.

Niels Bohr [4] brilliantly rescued the atom from radiating and collapsing by invoking the quantum theory and making two postulates that prevented the atom from radiating energy. Otherwise the atom should emit radiation resulting in its collapse. Bohr's postulates are:

(i) In those (quantum) orbits where the angular momentum is $nh/2\pi$, n being an integer and h the Planck constant, the energy of the electron is constant.

(ii) The electron can pass from an orbit of total energy E_q to an inner orbit of total energy E_n in a quantum jump, the difference being released in the form of radiation of frequency f_{nq}, with energy given by $E_q - E_n = hf_{nq}$

The first postulate quantized the angular momentum with respect to (quantum) number n. With these postulates Bohr was able to derive a formula for the wave numbers of the lines of the spectrum of the hydrogen atom in exactly the same mathematical form as obtained by Balmer and generalized by J.R. Rydberg in 1889. Bohr's model gives:

$$\nu_{nq} = \frac{1}{\lambda_{nq}} = \frac{me^4}{8c\varepsilon_o^2 h^3}\left(\frac{1}{n^2} - \frac{1}{q^2}\right) per\ metre \qquad (3.2)$$

$$\frac{1}{\lambda_{nq}} = R\left(\frac{1}{n^2} - \frac{1}{q^2}\right) per\ metre \qquad (3.3)$$

where n and q are integers greater than 0, with q greater than n, c is the speed of light in a vacuum, ε_o is the permittivity of a vacuum and h is the Planck constant. Equation (3.3) is the Balmer-Rydberg formula giving the Rydberg constant R as:

$$R = \frac{me^4}{8c\varepsilon_o^2 h^3}\ per\ metre \qquad (3.4)$$

Substituting the values of the physical quantities in equation (3.4), R is found as 1.097×10^7 *per metre*, in agreement with observation.

In equation (3.3), if $n = 1$ we have the Lyman series, in the far ultra-violet region of the spectrum. R is the spectral limit for the Lyman series. For $n = 2$, we have the Balmer (4 visible lines) series and where $n = 3$ we get the Paschen series in the near infra-red region. In a series the lines crowd together and their intensities should decrease to zero as the series limit $(q \rightarrow \infty)$ is approached. Other series, in the infrared region, are obtained for $n = 4, 5, 6\ldots$.

The fact that a purely chance agreement between the quantities in equation (3.4) is highly improbable, lends plausibility to Bohr's theory of the hydrogen atom. This theory gave great impetus to quantum mechanics and was recognized as a remarkable triumph of the human intellect. However, the quantum jump and the absence of a direct link between the frequency of the emitted radiation and the frequency of revolution of the electron, in its orbit, leaves a question mark on Bohr's quantum theory.

Subsequently, Bohr's quantum theory was modified, notably by Sommerfield [5], for elliptic orbits and more complex atoms. This imposed extra ad-hoc conditions justifiable only by the (not always) correctness of their consequences.

3.1.2 An alternative nuclear model of the hydrogen atom

An alternative nuclear model of the hydrogen atom is introduced in this paper. The new nuclear model is for the solid or liquid state of the hydrogen atom. It is inherently stable outside quantum mechanics.

The new nuclear model consists of N_h coplanar orbits. A particle of charge $-e$ and mass nm revolves in the nth circular orbit with speed v_1/n and angular momentum nL at a distance nr_1 from a nucleus of charge $+N_he$. The radius r_1, speed v_1 and angular momentum L are with respect to the first orbit, the innermost orbit where $n = 1$.

In the stable state, a particle of mass nm revolves in the nth circular orbit. If a particle is disturbed from the circular orbit, it revolves as a radiator emitting a burst of radiation of increasing frequency and decreasing intensity as it spirals out towards the stable circular orbit. The frequencies of emitted radiation are very nearly equal to that of revolution of the particle in the nth stable circular orbit.

3.2 New nuclear model of the hydrogen atom

The new nuclear model of the hydrogen atom consists of a concentric arrangement of N_h coplanar orbits. The orbits are equally spaced, each with a particle of charge $-e$ and mass nm revolving in a circle round a nucleus of charge $+N_he$.

3.2.1 Equation of the orbit of motion

The equation of motion of a particle of mass nm revolving, in the nth orbit, with constant angular momentum nL, at a point distance r from the nucleus, is derived in equation (2.18) [6] as:

$$\frac{1}{r} = \frac{A}{n}\exp(-q\psi)\cos(\alpha\psi + \beta) + \frac{m\chi}{nL^2} \tag{3.5}$$

where the amplitude of the excitement A and phase angle β are determined from the initial conditions and q, α and χ are constants.

The exponential decay factor $(-q\psi)$, in equation (3.5), is as a result of radiation of energy. The negatively charged particle will revolve, round the positively charged nucleus, in an unclosed (aperiodic) elliptic orbit with many cycles of revolutions, radiating energy at the frequency of revolution, before settling down into the stable circle of radius $nL^2/m\chi$, ready to be excited again.

3.2.2 Radiation from the new nuclear model

Figure 3.1 represents the new nuclear model of the hydrogen atom. It consists of a concentric arrangement of N_h coplanar orbits. The orbits are equally spaced, each with a particle of charge $-e$ and mass nm revolving round a nucleus of charge $+N_he$, n being an integer: $1, 2, 3, 4.....N_h$. A circular orbit has radius $r_n = nr_1 = nL^2/m\chi$ in which a particle revolves with speed $v_n = v_1/n = \chi/nL$, where $\chi = N_he^2/4\pi\varepsilon_o$ and $n = 1$ for the innermost (first) orbit.

The frequency of revolution of a particle in the nth stable orbit, a circle of radius r_n, is:

$$f_n = \frac{v_n}{2\pi r_n} = \frac{1}{2\pi}\frac{\chi}{nL}\frac{m\chi}{nL^2} = \frac{m\chi^2}{2\pi n^2 L^3} = \frac{mN_h^2e^4}{2\pi\left(4\pi\varepsilon_o\right)^2 L^3}\frac{1}{n^2}$$

$$f_n = \frac{mN_h^2e^4}{2\pi\left(4\pi\varepsilon_o\right)^2 L^3}\frac{1}{n^2} = \frac{cS}{n^2} \qquad (3.6)$$

$$S = \frac{mN_h^2e^4}{2\pi c\left(4\pi\varepsilon_o\right)^2 L^3} \qquad (3.7)$$

where S is a constant. If an electron is disturbed or dislodged from the stable circular orbit, it revolves in an unclosed elliptic orbit, emitting a burst of radiation in a narrow band of frequencies nearly equal to the frequency f_n of the circular revolution (equation 3.6)

Let us now follow the motion of two particles at positions P and Q with radii nr_1 and qr_1 respectively, revolving in anticlockwise sense round the centre O as in Figure 3.1. The frequencies of revolution at P and Q are given by equation (3.6) for the respective orbital numbers n and q.

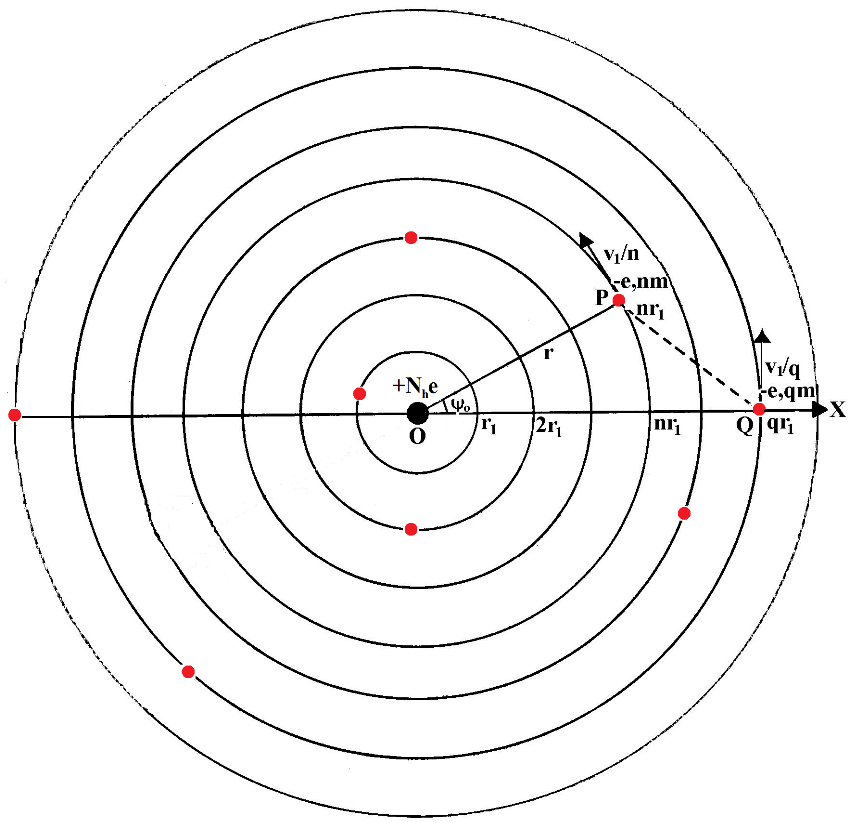

Figure 3.1. New nuclear model of the hydrogen atom, consisting of N_h coplanar orbits each with a negatively charged particle revolving anticlockwise, in angle ψ, under the attraction of a nucleus of charge $+N_he$. A particle in the nth orbit has a multiple of the electronic mass nm and charge $-e$, n being an integer. The particle in the nth orbit revolves in a circle of radius nr_1 with velocity v_1/n and angular momentum $nmv_1r_1 = nL$

In Figure 3.1, let the particles at positions P and Q be as shown at the initial time $t = 0$. The relative positions of the points O, P and Q are as shown, with $\boldsymbol{OP}$ and $\boldsymbol{OQ}$ in an angular displacement ψ_o at the initial stage. In time t the line $\boldsymbol{OP}$ moves to $\boldsymbol{OP}_t$ through an angle ψ_n and line $\boldsymbol{OQ}$ moves to $\boldsymbol{OQ}_t$ through an angle ψ_q. The difference in angular displacement, the instantaneous angle $\boldsymbol{P}_t\boldsymbol{OQ}_t$, is:

$$\psi_t = \psi_o + \psi_n - \psi_q$$

The angular frequency of oscillation of the particles at P and Q, is:

$$\frac{d\psi_t}{dt} = \frac{d\psi_n}{dt} - \frac{d\psi_q}{dt} = \omega_n - \omega_q = 2\pi f_n - 2\pi f_q = 2\pi f_{nq} \qquad (3.8)$$

Combining equation (3.8) above with equation (3.6) where $f_n = cS/n^2$ and $f_q = cS/q^2$, gives:

$$f_{nq} = cS\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \qquad (3.9)$$

The wave number is:

$$\frac{f_{nq}}{c} = \frac{1}{\lambda_{nq}} = S\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \textit{ per metre} \qquad (3.10)$$

Radiation from the new nuclear model of the hydrogen atom is of two sources. The first source is interaction between the electron revolving in the nth unstable orbit, and the nucleus, producing radiation of frequency f_n given by equation (3.6). The second source is interaction between electrons revolving in the nth and qth orbits, producing radiation of frequency f_{nq} given by equation (3.9), with the limit $(q \to \infty)$ given by equation (3.6).

3.2.3 Number of orbits in the new nuclear model

Measurements with a mass spectrometer showed that the hydrogen atom is about 1836 times the mass m of the electron. The total number N_h of orbits, each containing one particle of mass nm in the hydrogen atom, is obtained from the sum of the natural numbers: $n = 1, 2, 3, \ldots N_h$. This sum, which carries half of the mass of the atom, comes to: $N_h(N_h + 1)m/2 = 1836m/2$. Solving the quadratic equation gives $N_h = 42.35$. Obviously, N_h should be an integer, *42* or *43* or some other number. Further investigation and experimental work is required here to ascertain the actual number N_h of orbits.

3.3 Conclusion

The particles, in two orbits, emit radiation in a narrow band of frequencies and wave numbers as given by equations (3.9) and (3.10) respectively. At the same time, there is radiation of frequency given by equation (3.6) as a result of interaction between the orbiting particles and the nucleus. This is what this paper has set out to derive without recourse to quantum mechanics. In the process, the frequencies of emitted radiation are directly related to the frequencies of revolutions of the charged particles; something which quantum mechanics failed to do.

Equation (3.10) is similar to the Balmer-Rydberg formula (equation 3.3), but the Rydberg constant R (equation 3.4) is different from S (equation 3.7). The Balmer-Rydberg formula for the emitted radiation from the hydrogen atom, being the result of experimental observations, must stand for something. It is suggested here that the new nuclear model obtains with the liquid or solid phase of hydrogen.

The new nuclear model of the hydrogen atom is like the solar system with the planets revolving round a much heavier centre of attraction (the Sun). Should the centre fall apart, the particles of the nuclear model (in the solid or liquid state) would disperse to become a non-nuclear model (in the gaseous state). This issue is treated in paper 4 [8].

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4. A NON-NUCLEAR MODEL OF THE HYDROGEN ATOM

Abstract

A non-nuclear model of the atom of hydrogen gas is devised consisting of N_h planar orbits each with two particles of the same mass *nm* and opposite charges, of magnitude equal to the electronic charge *e*, revolving in the nth orbit round a common centre. A particle revolves through angle ψ, in an unclosed elliptic orbit, at a distance *r* from the centre, as:

$$\frac{1}{r} = \frac{A}{n}\exp(-b\psi)\cos(\alpha\psi + \beta) + \frac{m\kappa}{nL^2}$$

where *A* and β are determined from the initial conditions, *b*, α, and κ are constants, *nL* is a constant angular momentum in the nth orbit and *m* is the electronic mass. The number *n* (1, 2, 3...N_h) leads to quantisation of the orbits. The decay factor, $exp(-b\psi)$, is the result of radiation as a particle revolves and settles in the nth stable orbit, a circle of radius $r_n = nL^2/m\kappa$ with speed $v_n = \kappa/nL$. Interactions between revolving particles, in each of the N_h orbits, give rise to radiation of discrete frequencies in accordance with observations.

Keywords: Angular momentum, hydrogen atom, centre of revolution, radiation.

4.1 Introduction

The paper introduces a non-nuclear model of the hydrogen atom for the gas state. This model is different from the Rutherford's nuclear model [1] and also different from the nuclear model described by the author [2]. The non-nuclear model is stabilized and the Balmer-Rydberg formula [3], for discrete frequencies of emitted radiation, is derived, without recourse to Bohr's quantum mechanics [4].

The Balmer-Rydberg formula [3] gives the wave number of emitted radiation from the hydrogen atom as:

$$\frac{1}{\lambda_{nq}} = R\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \textit{ per metre} \tag{4.1}$$

where R is the Rydberg constant and *n* and *q* are integers greater than 0, with *q* greater than *n*. Niels Bohr [4] derived the formula as:

$$\nu_{nq} = \frac{1}{\lambda_{nq}} = \frac{me^4}{8c\varepsilon_o^2 h^3}\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \textit{per metre} \tag{4.2}$$

where *m* is the electronic mass, *e* the magnitude of the electronic charge, *c* is the speed of light in a vacuum, ε_o is the permittivity of a vacuum and *h* the Planck constant. The Rydberg constant *R*, is:

$$R = \frac{me^4}{8c\varepsilon_o^2 h^3} \textit{ per metre} \tag{4.3}$$

4.1.1 Alternative model of the hydrogen atom

An alternative model of the hydrogen atom is introduced in this paper. The non-nuclear model, called *bipolar model*, is for the gas state of the hydrogen atom. The bipolar model consists of a number N_h of circular orbits in one plane. The nth stable circular orbit consists of a particle of charge $-e$ and mass nm at a distance $2nr_1$ from another particle of the same mass nm but charge $+e$ revolving with speed v_1/n and constant angular momentum nL, under mutual attraction. Here, n is an integer: $1, 2, 3 \ldots N_h$. The radius r_1, speed v_1 and angular momentum L are for the inner orbit, the first orbit with n = 1. The pair of oppositely charged particles makes up the two poles of a bipolar orbit.

In the stable state, a particle of mass *nm*, in the bipolar or non-nuclear model, revolves in the nth circular orbit. If a particle is disturbed from the nth stable circular orbit, it revolves as a radiator emitting a burst of radiation of increasing frequency and decreasing intensity as it spirals out towards the stable circular orbit. The frequencies of emitted radiation are very nearly equal to that of revolution in the nth circular orbit.

In the bipolar model, an excited hydrogen atom will consist of a number of radiators with the charged particles oscillating in (unclosed) coplanar elliptic orbits. In the following section, it is shown that interaction between a particle of the bipolar orbit in the nth orbit and another in the qth orbit results in emission of radiation of discrete frequencies in accordance with the Balmer-Rydberg formula for the spectrum of the hydrogen atom in the gas state.

The derivation of Balmer-Rydberg formula, without recourse to Bohr's quantum theory, is the most remarkable result of this paper. It avoids the ad-hoc restrictions and removes the quantum jump of an electron as a necessary condition for emission of radiation from the hydrogen atom. It also relates the frequencies of radiation to the frequencies of circular revolutions of the electrons, something which the quantum theory failed to do.

4.2 Non-nuclear model of the hydrogen atom

The non-nuclear model of the hydrogen atom consists of a concentric arrangement of N_h coplanar orbits. The orbits are equally spaced, each with two particles of charges $-e$ and $+e$ and same mass nm revolving in the nth orbit round a centre of revolution, the centre of mass of the two particles.

4.2.1 Equation of the orbit of motion

The equation of the orbit of motion of a particle of mass nm revolving, in the nth orbit, at a point distance r from the centre, is derived (equation 2.29) in paper 2 [5], as:

$$\frac{1}{r}=\frac{A}{n}\exp(-b\psi)\cos(\alpha\psi+\beta)+\frac{m\kappa}{nL^2} \tag{4.4}$$

where the excitement amplitude A and phase angle β are determined from the initial conditions, b, α are constants, $\kappa = e^2/16\pi\varepsilon_o$ and nL is a constant angular momentum in the nth orbit..

The exponential decay factor $(-b\psi)$, in equation (4.4), is as a result of radiation of energy. The two charged particles will revolve, round their centre of mass, in an unclosed (aperiodic) elliptic orbit with many cycles of revolutions, radiating energy, before settling down into the nth stable orbit, a circle of radius $nL^2/m\kappa$.

4.2.2 Radiation from the non-nuclear model

The non-nuclear or bipolar model of the hydrogen atom consists of a concentric arrangement of N_h coplanar orbits. Each orbit has two particles revolving under mutual attraction, round a common centre of revolution. A particle revolves in a stable circular orbit of radius $r_n = nr_1 = nL^2/m\kappa$ with velocity $v_n = v_1/n = \kappa/nL$, where $\kappa = e^2/16\pi\varepsilon_o$ and $n = 1$ for the innermost orbit. Such a configuration of N_h orbits is shown in Figure 4.1. The non-nuclear model of the hydrogen atom, in contrast to the nuclear model, has no particle as the nucleus, but an empty common centre of mass as the centre of revolution for all the particles in the N_h orbits.

The frequency of revolution of a particle moving with constant speed v_n in the nth stable orbit, a circle of radius r_n, is:

$$f_n = \frac{v_n}{2\pi r_n} = \frac{me^4}{16\pi n\varepsilon_o L}\frac{1}{2\pi\left(16\pi n\varepsilon_o L^2\right)} = \frac{me^4}{2\pi\left(16\pi\varepsilon_o\right)^2 L^3}\frac{1}{n^2} \qquad (4.5)$$

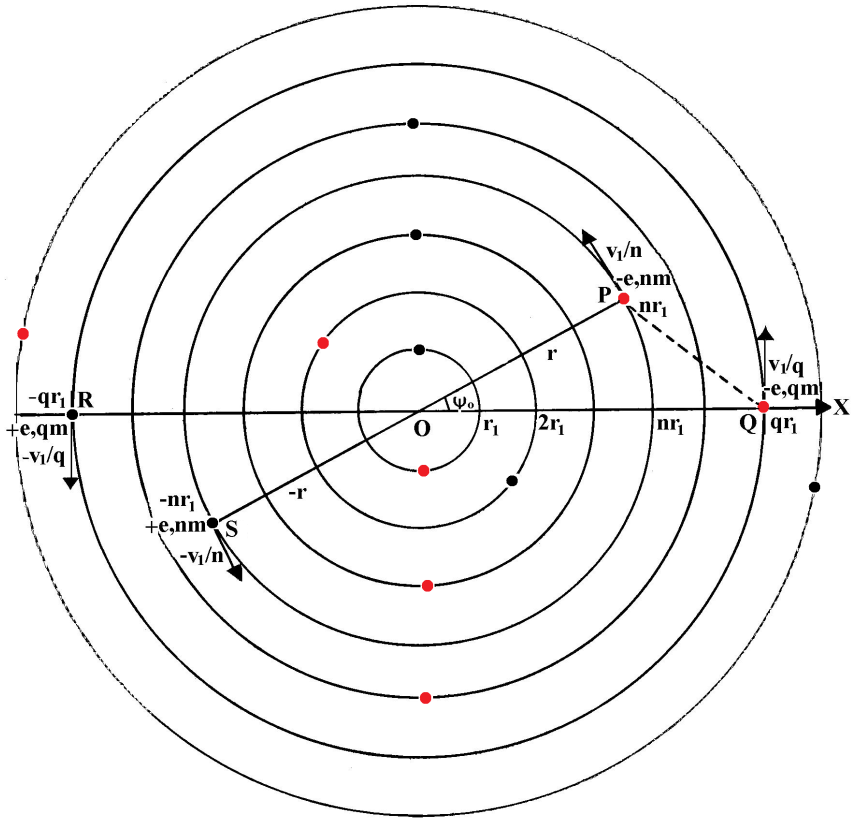

Figure 4.1. Non-nuclear model of the hydrogen atom, consisting of a number N_h of coplanar orbits each with two equal but oppositely charged particles revolving, anti clock-wise, in angle ψ, under mutual attraction. Each of the two particles in the nth orbit has mass *nm*, one carries charge $-e$ and the other $+e$, n being an integer from 1 - N_h, m is the electronic mass and $-e$ the electronic charge. The nth pair of particles revolves in a circular orbit of radius nr_1 with velocity v_1/n and constant angular momentum $nmv_1r_1 = nL$

Putting $L = h/4\pi$ gives equation (4.5) as:

$$f_n = \frac{me^4}{2\pi\left(16\pi\varepsilon_o\right)^2 L^3}\frac{1}{n^2} = \frac{me^4}{8\varepsilon_o{}^2 h^3}\frac{1}{n^2} = \frac{cR}{n^2} \tag{4.6}$$

where the Rydberg constant $R = me^4/8c\varepsilon_o{}^2h^3$, has the value *(1.097 $\times 10^7$ per metre)* as obtained by Bohr (equation 3.2) [4] and confirmed by observation. The Planck constant h appears here in a manner reminiscent of Bohr's first postulate which makes $nL = nh/2\pi$ as the angular momentum in the nth orbit, in contrast to the angular momentum, $nL = nh/4\pi$, as advanced here in order to arrive at equation (4.6).

If a particle, in the nth orbit, is dislodged from the stable circular orbit, it revolves as a radiator in an unclosed (aperiodic) elliptic orbit. It emits a burst of radiation of frequencies very nearly equal to that of revolution given by equation (4.6), before reverting back into the nth stable circular orbit.

4.2.3 The Balmer-Rydberg formula

We shall now follow the motion of particles in two bipolar orbits with the particles at positions P and Q of radii nr_l and qr_l respectively, revolving in anticlockwise sense round the centre O as in Figure.4.1. The frequencies of revolution at P and Q are given by equation (4.5) for the respective orbital numbers n and q.

In Figure 4.1, let the particles at P and Q both have negative charges at the initial stage. The relative positions of the points S, R, O, P and Q are as shown, with ***OP*** and ***OQ*** at an angular displacement ψ_o at the initial stage, time $t = 0$. In time t let the line ***OP*** move to $\boldsymbol{OP_t}$ through an angle ψ_n, and let the line ***OQ*** move to $\boldsymbol{OQ_t}$ through an angle ψ_q. The difference in angular displacement, the instantaneous angle $\boldsymbol{P_tOQ_t}$, is:

$$\psi_t = \psi_o + \psi_n - \psi_q$$

The angular frequency of oscillation of the particles at P and Q, is:

$$\frac{d\psi_t}{dt} = \frac{d\psi_n}{dt} - \frac{d\psi_q}{dt} = \omega_n - \omega_q = 2\pi f_n - 2\pi f_q = 2\pi f_{nq} \tag{4.7}$$

Combining equation (4.7) above with equation (4.6) where $f_n = cR/n^2$ and $f_q = cR/q^2$, gives:

$$f_{nq} = cR\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \tag{4.8}$$

The four particles in two bipolar radiators, of the hydrogen atom, behave like oscillating pairs, emitting radiation in a narrow band of frequencies, with wave numbers ν_{nq} as:

$$\nu_{nq} = \frac{f_{nq}}{c} = R\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \tag{4.9}$$

The atomic particles revolve in their respective orbits as radiators. Interactions between the $2N_h$ particles, in all the N_h number of orbits, result in the emission of radiation of discrete frequencies and wave numbers given by equations (4.8) and (4.9) respectively. Also, the atom can absorb radiation of the same frequencies as it emits. The arrangement of the particles, their revolutions and interactions between them, giving rise to emission or absorption of radiation, determines the physical and chemical properties and thermal condition of the atom,

Equation (4.9), identical to the Balmer-Rydberg formula (equation 4.1), is the result of interactions between excited charged particles as they revolve in their different unstable orbits. This is what this paper has set out to derive without recourse to quantum mechanics. In the process, the frequencies of emitted radiation are directly related to the frequencies of revolutions of the charged particles; something which quantum mechanics failed to do.

4.2.4 Number of orbits in the non-nuclear model

The hydrogen atom is found to be about 1836 times the mass m of the electron. The total number N_h of orbits, each containing two particles either of mass nm, is obtained from the sum of the natural numbers: $n = 1, 2, 3, \ldots N_h$. Twice this sum, which carries the mass of the atom, gives $N_h(N_h + 1)m = 1836m$. This gives $N_h = 42.35$. N_h should be an integer.

4.3 Bipolar model versus nuclear model

The bipolar model of the hydrogen atom has no nucleus but an empty centre round which particles revolve in N_h coplanar orbits. Two positive and negative particles ($+e$ and $-e$), each bearing a multiple nm of the electronic mass m, revolve in the nth orbit, n being an integer: $1, 2, 3 \ldots N_h$.

The total mass of particles in the N_h orbits, with two particles in each orbit, is $N_h(N_h + 1)m$. The positive and negative electrical charges, in an atom, cancel out exactly.

The new nuclear model has a nucleus of charge $+N_he$ and N_h coplanar orbits in each of which particles of charge $-e$ and mass nm revolve. The total charge of the negative particles is $-N_he$, equal and opposite of the charge on the nucleus. The total mass of the N_h revolving particles is $\frac{1}{2}N_h(N_h + 1)m$, same as the mass of the nucleus. Thus the bipolar model and unipolar or nuclear model of the hydrogen atom have the same number of orbits N_h and the same mass, equal to $N_h(N_h + 1)m$. The question now is: "What is the significance of these two different models of the hydrogen atom?"

It is suggested here that the non-nuclear (bipolar) model is what obtains with the gas phase of hydrogen while the new nuclear model exists with respect to the liquid and solid phases, depending on the ambient temperature. Let us now determine the relationship between the constant S as given by equation (3.6) [2] for the nuclear model and the Rydberg constant R, obtained from equation (4.6), for the non-nuclear model. The expression obtained, equal to the ratio of frequency g_n in the nuclear model and the frequency f_n in the non-nuclear nuclear model, is:

$$\frac{S}{R} = \frac{g_n}{f_n} = 16N_h^2 \qquad (4.10)$$

The ratio s_n of radius of revolution in the nth orbit of the nuclear model and the radius r_n of the non-nuclear model, is obtained as:

$$\frac{s_n}{r_n} = \frac{1}{4N_h} \qquad (4.11)$$

The ratio u_n of speed of revolution in the nth orbit of the nuclear model and the speed v_n in the non-nuclear obtained, is obtained as:

$$\frac{u_n}{v_n} = 4N_h \qquad (4.12)$$

It is assumed that the constant angular momentum nL, for the nth stable orbit, is the same for the nuclear and non-nuclear models of the atom.

4.4 Conclusion

Expressions for the Balmer-Rydberg formula and the Rydberg constant (equations 4.1 and 4.2), for the non-nuclear model or bipolar model of the hydrogen atom, are derived without recourse to Bohr's second postulate but with a modification of the first postulate. The modification is to the effect that *the magnitude of the angular momentum of a particle of mass nm, revolving in the nth circular orbit, is equal to* $nL = nh/4\pi$, where $h = 6.626 \times 10^{-34}$ J-sec. Quantisation of angular momentum (nL) and radius of revolution $(nL^2/m\kappa)$ and inverse quantisation of velocity (κ/nL) appear naturally as a consequence of discrete masses (nm), being multiples of the electronic mass m, with n as the orbital number or "quantum number", an integer greater than 0.

The angular momentum of a particle in the first orbit of the bipolar model, is $L = h/4\pi$. This is a fundamental quantity of value equal to $L = 5.273 \times 10^{-35}$ *J-sec*. It defines the Planck constant h in terms of angular momentum rather than "unit of action". Even though the Planck constant is featuring prominently, Bohr's quantum mechanics is not necessary in describing the discrete frequencies of radiation from the hydrogen atom.

The radius of the first orbit $(n = 1)$ of the non-nuclear model of the hydrogen atom is obtained as $r_1 = \varepsilon_o h^2/\pi me^2 = 5.292 \times 10^{-7}$ *m*. This is the same as the **first Bohr radius** [4] obtained, through quantum mechanics, for the nuclear model of the hydrogen atom. The speed of revolution in the first orbit of the non-nuclear model is obtained as $v_1 = e^2/4\varepsilon_o h = 1.094 \times 10^6$ *m/s*. This is different from the speed of revolution in the **first Bohr orbit** of the nuclear model, which is $u_1 = N_h e^2/\varepsilon_o h$ *m/s*. A knowledge of the charge $+N_h e$ in the nucleus, as may be obtained from experiment, is required in order to determine the radius $s_1 = \varepsilon_o h^2/4N_h \pi me^2$ and speed u_1 of revolution (equations 4.11 and 4.12) in the new nuclear model.

The Rutherford-Bohr nuclear model of the hydrogen atom does not distinguish between the models in the gaseous state and in the liquid or solid state. In paper 3 [2], the new nuclear model is for the liquid or sold state and this paper gives the non-nuclear model for the gaseous state. The Balmer-Rydberg formula, which explains the frequencies of radiation in the hydrogen atom spectrum [3], should be for the gaseous state.

This paper gives two sources of radiation from the hydrogen atom. The first is from interaction between the two radiators in the nth orbit. A particle revolves in an unclosed (aperiodic) elliptic orbit emitting radiation of increasing frequencies and decreasing amplitude before settling in the nth stable circular orbit. Thus radiation from a revolving charged particle is a narrow band of frequencies very nearly equal to the frequency of revolution f_n given by equation (4.6). The second source of radiation is from interaction between particles revolving in the nth and qth orbits, resulting in radiation of frequency f_{nq} given by equation (4.8).

The series limit of the frequencies given by equation (4.8), with $q \to \infty$, is the same as the frequency given by equation (4.6). This may explain why the intensity of the series limit is not zero. For the Balmer series [5, 6, 7] the frequency limit, $f_2 = cR/4 = 8.227 \times 10^5\ GHz$, in the violet region (not visible), is present and measurable.

The nth orbit of the hydrogen atom, as well as the atom itself, could be considered as an "intelligent" configuration. Each orbit and the atom as a whole, if disturbed, "remembers" its previous condition or situation and returns to it, with an exhibition of energy. So, hydrogen atoms could combine and produce a manifestation of rudimentary consciousness.

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5. ON THE SPEED OF LIGHT IN A MOVING MEDIUM

Abstract

The speed w of transmitted light, at normal incidence, in a medium of refractive index μ_2 moving with speed v in another medium of refractive index μ_1, is derived as:

$$w = \frac{c}{\mu_2} + v\left(1 - \frac{\mu_1}{\mu_2}\right)$$

where c is the speed of light in a vacuum. The equation for w is applied to give a non-relativistic explanation of the result of Fizeau's experiment without recourse to special relativity.

Keywords: Fizeau's experiment, light, medium, speed.

5.1 Introduction

Euclid (330 – 280 B.C.), who discovered the law of formation of images by mirrors, probably knew the law of reflection of light [1]. Snell discovered the law of refraction of light in about 1620 [1]. When a ray of light, propagated in a *medium 1*, reaches the boundary with another *medium 2*, reflection and refraction occur. Fig. 5.1 depicts a ray ***SP***, emitted with velocity ***s*** from a stationary source *S*, incident at a point *P* on the boundary of two media.

A reflected ray ***PR*** is one propagated with velocity ***u*** from the boundary in the same medium as the incident ray ***SP***. A refracted ray ***PT*** is one transmitted and propagated with velocity ***w*** in the second medium. The angle of incidence is ι, where $(\pi - \iota)$ is the angle between the directions of propagation of the incident ray ***SP*** and the normal ***OPN*** to the boundary at the point of incidence *P*. The angle of reflection ρ is the angle between the directions of propagation of the reflected ray ***PR*** and the normal. The angle of refraction is τ, where $(\pi - \tau)$ is the angle between the directions of propagation of the refracted ray ***PT*** and the normal at *P*.

5.1.1 Laws of reflection of light

Yavorsky and Detlaf [2] recapitulated the laws of reflection and refraction of light, for a stationary medium. The laws of reflection, with reference to Figure 5.1, are:

(i) The incident ray ***SP***, the reflected ray ***PR*** and the normal ***OPN***, at the point of incidence *P*, are coplanar.

(ii) The angle of reflection ρ is equal to the angle of incidence ι.

5.1.2 Laws of refraction of light

The laws of refraction of light, with $v = 0$ in Figure 5.1, are:

(i) The incident ray ***SP***, the refracted ray ***PT*** and the normal ***OPN***, at the point of incidence *P*, are coplanar.

(ii) The ratio of the sine of angle of refraction to the sine of angle of incidence, for light of a given wavelength, is equal to the ratio of the speeds of light *w* and *s* in the media.

The second law of refraction is called Snell's law, which can be deduced from Fermat's principle [3].

The relative indices of refraction μ_1 and μ_2 of *media 1* and *2* respectively, are defined, for a stationary medium, as the ratio:

$$\frac{w}{s} = \frac{\sin\tau}{\sin\iota} = \frac{\mu_1}{\mu_2} \tag{5.1}$$

In a vacuum, $\mu_1 = 1$ and $\mu_1 s = \mu_2 w = c$, the speed of light. If medium 1 is a vacuum, $\mu_2 = \mu$ is the absolute index of refraction.

Where $\mu_1 > \mu_2$ (Figure 5.1), that is transmission from a denser medium to a less dense medium, such as glass to air or vacuum, the angle of refraction τ is greater than the angle of incidence ι,. Total internal reflection occurs if $\tau > \pi/2$ radians. The value υ of ι at which $\tau = \pi/2$ radians is called the *critical angle*, such that $\sin\upsilon = \mu_2/\mu_1 < 1$.

5.1.3 Dispersion of light

The index of refraction depends on the wavelength of light radiation under consideration. The shortened form of Cauchy's empirical formula [4] gives an approximate relationship between the index of refraction μ of a transparent medium and the wavelength λ of light, as:

$$\mu = a + \frac{b}{\lambda^2} \tag{5.2}$$

a and *b* being constants for the medium. The decrease of μ with increase of λ causes the dispersion of white light into the colours of the rainbow.

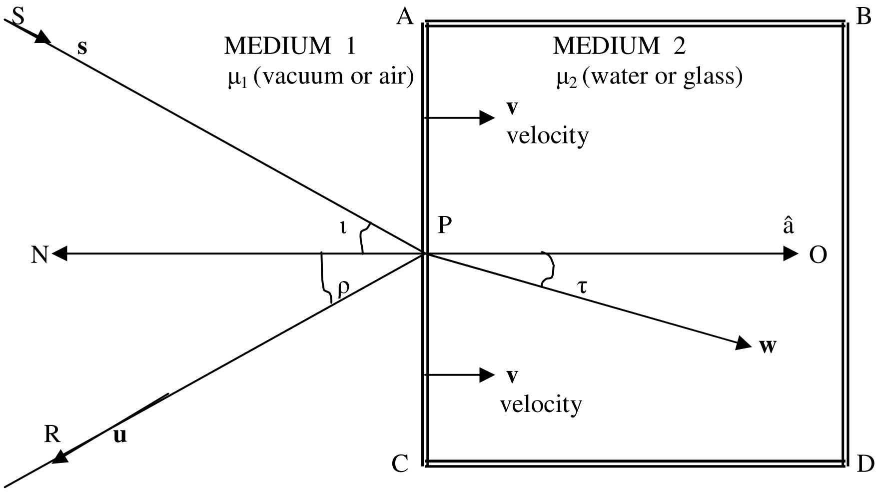

Figure 5.1 Reflection and refraction of a ray of light emitted from a source *S* with velocity ***s*** in medium *1*, incident at a point *P* on the surface of medium *2* moving with velocity ***v***. The ray ***PR*** is reflected with velocity ***u*** and the refracted ray ***PT*** transmitted in medium 2 with velocity ***w***. The velocity vector ***v*** is in the opposite direction of the normal ***OPN*** at *P*.

5.2. Speed of light reflected from a moving medium

Figure 5.1, drawn in accordance with the first law of reflection and refraction, shows a monochromatic ray of light in *medium 1* propagated with velocity ***s*** from a stationary source *S*. The ray is incident at a point *P* on the plane surface of denser *medium 2 (ABCD)*, such as glass or water, moving with velocity ***v*** (relative to a stationary observer) in a direction opposite to the normal ***OPN*** at *P*. The reflected ray is propagated with velocity ***u*** in the same *medium 1* as the incident ray. The angle of incidence is ι and the angle of reflection is ρ. The *medium 2* may be considered as stationary by giving each of the rays a velocity equal to $-\boldsymbol{v}$. The ratio of the magnitudes of the relative velocities of the reflected ray and the incident ray is put as:

$$\frac{|\mathbf{u}-\mathbf{v}|}{|\mathbf{s}-\mathbf{v}|}=\frac{\mu_1}{\mu_1}=1 \qquad (5.2)$$

With reference to Figure 5.1, the ratio is obtained as:

$$\frac{\sqrt{u^2+v^2+2uv\cos\rho}}{\sqrt{s^2+v^2-2sv\cos\iota}}=\frac{u^2+v^2+2uv\cos\rho}{s^2+v^2-2sv\cos\iota}=1 \tag{5.3}$$

If $v = 0$, obviously the incident and reflected rays, being in the same medium, will have the same speed $u = s$.

At normal incidence and the *medium 2* moving with speed v, $\iota = \rho = 0$, we obtain the equation:

$$\frac{u+v}{s-v}=1$$

$$u = s-2v \tag{5.4}$$

Where the first medium is a vacuum, s is the speed of light c. Equation (5.4) is exactly as obtained if the ray of light is considered as a "stream of particles" propagated with speed c. The "particles" or photons would impinge normally on the moving medium at the point of incidence P, on perfectly elastic collisions, and recoil with speed u given by:

$$u = c-2v \tag{5.5}$$

This is the idea behind "ballistic propagation of light", in accordance with Newton's law of restitution [5].

5.3 Speed of light transmitted in a moving medium

A ray of light emitted with velocity $\boldsymbol{s}$ is incident on the surface of a transparent medium moving with velocity $\boldsymbol{v}$ along the normal as in Figure 5.1. The ray is transmitted through the medium with velocity $\boldsymbol{w}$ at the angle of refraction $(\pi - \tau)$. The refractive indices are defined in terms of the ratio of magnitudes of relative velocities of the refracted ray and the incident ray. The ratio of magnitudes of the relative velocity $(\boldsymbol{w} - \boldsymbol{v})$ and relative velocity $(\boldsymbol{s} - \boldsymbol{v})$ is obtained, thus:

$$\frac{|\mathbf{w}-\mathbf{v}|}{|\mathbf{s}-\mathbf{v}|}=\frac{\mu_1}{\mu_2}$$

With reference to Fig. 5.1, the ratio is obtained as:

$$\frac{\sqrt{w^2 + v^2 - 2wv\cos\tau}}{\sqrt{s^2 + v^2 - 2sv\cos\iota}} = \frac{\mu_1}{\mu_2}$$

At normal incidence, $\iota = \tau = 0$ and we obtain:

$$\frac{w - v}{s - v} = \frac{\mu_1}{\mu_2}$$

$$w\mu_2 - v\mu_2 = s\mu_1 - v\mu_1$$

$$w = \frac{s\mu_1}{\mu_2} + v\left(1 - \frac{\mu_1}{\mu_2}\right) \tag{5.8}$$

$$w = \frac{c}{\mu_2} + v\left(1 - \frac{\mu_1}{\mu_2}\right) \tag{5.9}$$

where $s\mu_1 = c$ is the speed of light in a vacuum. For a medium moving in a vacuum where the refractive index $\mu_1 = 1$, we obtain the speed w of transmission of light in the medium, as:

$$w = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu}\right) \tag{5.10}$$

Equation (5.10) is a significant result of this paper. It is used to give a non-relativistic explanation of the result of Fizeau's experiment, which measured the speed of light in moving water, without recourse to the theory of special relativity.

5.4. Fizeau's experiment

A schematic diagram of the apparatus of Fizeau's experiment [6] is shown in Figure 5.2. Carried out in the 1850s, it was one of the most remarkable experiments in physics. In this experiment, light from a source was sent in two opposite directions through transmission and reflection by four half-silvered mirrors M_1 – M_4. One beam travelled downstream (from M_1) through moving water and the other one travelled upstream (from M_4) in the same water. By an ingenious arrangement of the mirrors the two beams were made to recombine and be observed in an interferometer. An interference pattern, as observed in an interferometer, resulted from the difference in the time taken for the two beams to travel the same path, partly in moving water.

Various explanations have been given for the result of Fizeau's experiment. One accepted explanation was that the velocity of light in the moving medium (water) was increased or decreased in accordance with the *relativistic rule for addition of velocities* based on constancy of the speed of light relative to a moving observer or a moving object. A new explanation is proposed in this paper, on the basis of equation (5.10), outside Einstein's theory of special relativity [7, 8].

5.5 Non-relativistic explanation of the result of Fizeau's experiment

According to the Galilean-Newtonian relativity of classical mechanics, the speed of light in a medium moving in a vacuum with velocity $\boldsymbol{v}$, in the opposite direction of the normal, is given in terms of the refractive index, by equation (5.10).

Equation (5.10) gives the speed w of transmission of light (downstream). If the medium is water of index of refraction μ moving with velocity v in a vacuum, the time taken for the beam going downstream to cover the distance $2L$ (where the magnitude v is very small compared with the speed of light c and, therefore, v^2/c^2 can be neglected), is obtained as the equation:

$$t_1 = \frac{2L}{\dfrac{c}{\mu} + v\left(1 - \dfrac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \tag{5.11}$$

For the beam going upstream, with velocity $-v$ (with respect to a stationary observer), the longer transit time is:

$$t_2 = \frac{2L}{\dfrac{c}{\mu} - v\left(1 - \dfrac{1}{\mu}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} \tag{5.12}$$

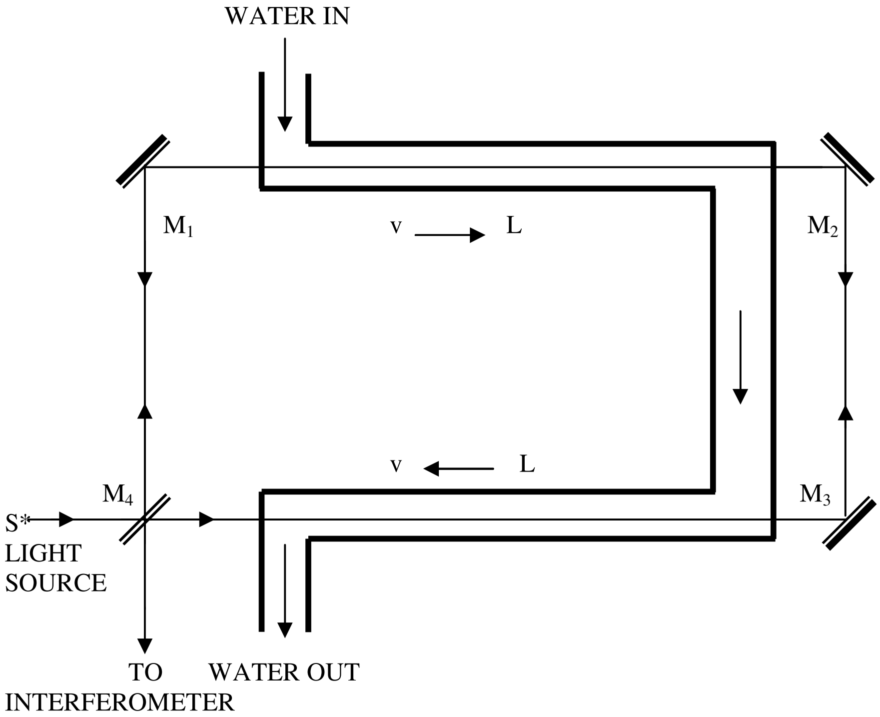

Figure 5.2. Schematic diagram of the apparatus of Fizeau's experiment

The time difference *(Δt) = (t₂ – t₁)* between the two beams, traversing the same path of length *2L* (downstream or upstream) in moving water, is:

$$t_2 - t_1 = \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\} - \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu}\right)\right\}$$

$$\Delta t = \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu}\right) \tag{5.13}$$

The fringe shift δ_x, for light of wavelength λ, is obtained as:

$$\delta_x = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu}\right) \tag{5.14}$$

In the experiments performed by Fizeau [6], $L = 3\ m$, $v = 7\ m/sec.$, $\lambda = 6 \times 10^{-7}\ m$ (yellow light), $c = 3 \times 10^{8}\ m/sec$ and $\mu = 4/3$ (for water). The fringe shift δ_x is obtained as ≈ 0.2, which was easily observable and measurable in the interferometer.

Michelson and Morley in 1886 and later P. Zeeman and associates in 1915 repeated Fizeau's experiment with greater precision in which the interferometer could measure a fringe shift as low as *0.01*. The influence of the motion of the medium on the propagation of light has thus been verified. As to which is the correct explanation, the theory of special relativity, according to Einstein or equation (5.14), in accordance with Galilean relativity of classical mechanics, remains to be seen.

5.6. Relativistic explanation of the result of Fizeau's experiment

According to Galileo's *velocity addition rule*, if you move with velocity ***u*** relative to a medium moving with velocity ***v*** relative to an observer, (passenger jogging with velocity **u** in a ship moving with velocity **v**) your velocity, relative to the observer, is vector sum ***s***, given by:

$$\mathbf{s} = \mathbf{u} + \mathbf{v} \tag{5.15}$$

Here the velocities ***u*** and ***v*** are vector quantities that can be of any magnitude and in any direction. If the velocities are in the same direction, the speed relative to the observer is the magnitude $s = u + v$. This is the Galilean principle of relativity, which is in agreement with observation and natural sense.

According to Einstein's *velocity addition rule,* if you move with velocity ***u*** relative to a medium moving with velocity ***v*** relative to an observer (as a jogger running with velocity **u** in a ship moving with velocity **v**), magnitude of your velocity, relative to the observer, is:

$$s = \frac{u + v}{1 + \frac{uv}{c^2}} \tag{5.16}$$

How the speed of light c in a vacuum comes into (5.16), is perplexing. In vector form, equation (5.16) may be expressed as:

$$\mathbf{s} = \frac{\mathbf{u} + \mathbf{v}}{1 + \frac{\mathbf{u}.\mathbf{v}}{c^2}} \tag{5.17}$$

where ***u.v*** is a scalar product.

In equation (5.17), ***u*** and ***v*** must be collinear to give equation (5.16). In reality, the jogger should be able to run with velocity ***u*** in any direction relative to ***v***. If $u = c$ (speed of light) or $u = v = c$, the speed s remains as c. Equation (5.16), more than anything else, had lent support to the principle of constancy of the speed of light, in accordance with the theory of special relativity. For speeds much less than c, or if c is infinitely large, Einstein's relativistic formula (equation 5.16) reduces to the classical formula (equation 5.15).

According to the theory of special relativity, the velocity of light (jogger), relative to the surface of a medium (ship) moving in a vacuum with velocity ***v***, remains as a constant ***c***. The velocity of light (jogger) within and with respect to the medium (ship) of refractive index μ is $\boldsymbol{c/\mu}$. Einstein's *velocity addition rule*, with $u = c/\mu$, gives the magnitude of velocity ***w*** of light in the moving medium (with respect to a stationary observer) as:

$$w = \frac{\dfrac{c}{\mu} + v}{1 + \dfrac{cv}{\mu c^2}} = \frac{\dfrac{c}{\mu} + v}{1 + \dfrac{v}{\mu c}} \approx \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right) \tag{5.18}$$

The relativistic equation (5.18), compared with (5.10), is used to obtain the transit time difference between the two beams in Fizeau's experiment, and thereby explain the result from the relativistic point of view.

The transit time of the beam going downstream, with speed v very small compared with c, is:

$$t_1 = \frac{2L}{\dfrac{c}{\mu} + v\left(1 - \dfrac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 - \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_1 \approx \frac{2L\mu}{c}\left(1 + \frac{v}{\mu c} - \frac{\mu v}{c}\right)$$

The transit time of the beam going upstream (with speed $-v$) is:

$$t_2 = \frac{2L}{\frac{c}{\mu} - v\left(1 - \frac{1}{\mu^2}\right)} \approx \frac{2\mu L}{c}\left\{1 + \frac{\mu v}{c}\left(1 - \frac{1}{\mu^2}\right)\right\}$$

$$t_2 \approx \frac{2L\mu}{c}\left(1 - \frac{v}{\mu c} + \frac{\mu v}{c}\right)$$

The time difference between the beam going downstream and the other going upstream is obtained as:

$$\Delta t = t_2 - t_1 \approx \frac{4Lv\mu^2}{c^2}\left(1 - \frac{1}{\mu^2}\right) \tag{5.19}$$

The fringe shift δ_y, for light of wavelength λ, is obtained as:

$$\delta_y = \frac{c\Delta t}{\lambda} = \frac{4Lv\mu^2}{\lambda c}\left(1 - \frac{1}{\mu^2}\right) \tag{5.20}$$

This δ_y is larger than the fringe shift δ_x as given by equation (5.14).

5.7 Conclusion

A ray of light has the effect of a "stream of particles" transmitted at the speed of light c in the direction of propagation. The "stream of particles", which impinge on the surface of a medium exert pressure upon reflection from the surface. Reflection of light can be treated as the recoil of "moving particles", under perfectly elastic conditions, obeying Newton's law of restitution [5].

The treatment of reflection and refraction of light in this paper, clearly demonstrate the relativity of the velocity of light with respect to a moving medium, in accordance with Galileo's relativity of classical mechanics. The result of Fizeau's experiment [6] is not a direct consequence of the relativistic *velocity addition rule* but due to the effect of motion of the transmission medium on the speed of light.

In equations (5.6) and (5.7) it is shown that the law of reflection, that is angle of incidence being equal to the angle of reflection, applies irrespective of the speed of the reflecting medium moving along the normal direction. Equation (5.7) is a simple and new expression subject to experimental verification.

In equations (5.10) and (5.18) the speed v, of the moving medium, can take any value between 0 and $\pm c$. For $v = 0$, both equations give the speed of light in the medium $w = c/\mu$, as expected. For $\mu = 1$ or $v = c$, both equations give $w = c$, also as expected. For $v = -c$, equation (5.10) gives $w = c(2/\mu - 1)$, which is reasonable, but the relativistic equation (5.18) gives $w = c(1/\mu^2 + 1/\mu - 1)$, which may be negative as $1 < \mu < 2$.

Equation (5.20) is another good example of Beckmann's *correspondence theory* [9], where the desired result is correctly obtained Mathematically, but based on the wrong underlying principles. Fizeau's experiment might as well have verified the fringe shift in equation (5.14), rather than the relativistic equation (5.20), for the transmission of light in a moving medium. Curt Renshaw [10] analysed the results of several experiments conducted to measure the effect of the speed of a medium on the speed of transmission of light in the medium and he concluded that the results could be explained without invoking special relativity.

5.8 References

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6. ON THE ENERGY AND MASS OF ELECTRIC CHARGES IN A BODY

Abstract

It is shown that the mass M of a distribution of N positive and negative electric charges $(\pm Q_i)$, is given by the sum:

$$M = \mu_o \varepsilon_o \sum_{i=1}^{N} Q_i U_i = 2\mu_o \varepsilon_o W = \frac{2W}{c^2}$$

where μ_o is the permeability, ε_o the permittivity and c the speed of light in a vacuum, U_i is the electrostatic potential at the position of Q_i the ith charge and W the electrostatic energy. The total energy E of mass M moving at speed v, is:

$$E = W + \frac{1}{2}Mv^2 = \frac{1}{2}M\left(c^2 + v^2\right)$$

This is in contrast to the relativity theory which makes $E = Mc^2$. The derivation of mass in terms of electric charges constituting a body is related to an explanation of the origin of inertia.

Keywords: Electric charge, energy, mass, relativity.

6.1 Introduction

Einstein's [1, 2] most famous formula of special relativity, the mass-energy equivalence law, gives the total energy content E of a particle of mass m and rest mass m_o moving with speed v, as:

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{6.1}$$

In this formula, the mass m of the particle and total energy content E becoming infinitely large at the speed of light c is a difficulty. In spite of this problem, equation (6.1), is a most celebrated formula.

Equation (6.1) is used by the proponents of special relativity, to explain why an electron, the lightest particle known in nature, cannot be accelerated beyond the speed of light c. In fact, electrons are easily accelerated and have been accelerated to the speed of light, through a potential energy of 15 MeV or higher, as demonstrated by Bertozzi using a linear accelerator [3]. Cyclic electron accelerators (betatrons and electron synchrotrons) of over 100 BeV [4] have been built and operated, with electron speed equal to that of light c for all practical purposes. Such "massive" electrons would have increased the weight of the accelerator and, on impact, should have crushed through the target to cause a catastrophe. It is most likely that something else is responsible for restraining the electron, accelerated by an electrostatic field, from going beyond the speed of light c.

The author [5] showed that the mass m of a moving electron remains constant at the rest mass m_o and that it is the accelerating force exerted by an electrostatic field, on a moving charged particle, which decreases with speed, becoming zero at the speed of light c. In this respect we have the ultimate speed without infinite mass.

The author [5] also showed that, for an electron of mass m and charge $-e$, revolving with constant speed v in a circle of radius r, under a central electrostatic field of magnitude E, the quantity "m" in equation (6.1) is the ratio $(eE/v^2)r$ of the magnitude of the force (eE) on a stationary electron, to the centripetal acceleration (v^2/r). The centripetal acceleration reduces to zero at the speed of light c. The radius r and the quantity $(eE/v^2)r$ may then become infinite, as for motion in a circle of infinite radius (rectilinear motion in a straight line), without any problem of infinitely large masses at the speed of light.

The purpose of this paper is to derive expressions for the electrostatic energy and physical mass of a distribution of equal number of positive and negative electric charges constituting a neutral body of mass M. The total energy of the body of mass M moving with speed v, relative to an observer, is then deduced as the electrostatic energy of the mass plus its kinetic energy. The total energy content is then compared with Einstein's [1, 2] mass-energy equivalence law of the theory of special relativity.

6.2 Energy content of an electric charge distribution

If an isolated electric charge Q assumed any shape or configuration, it is most likely to be a spherical shell of radius a, with all the charges on the surface at the same potential, as in Figure 6.1(a). Such a figure has "force of explosion" as well as self energy or intrinsic energy due to the charge being situated in its own potential. The intrinsic energy of the charge Q in Figure 6.1(a) is the work w done by an external force in assembling the

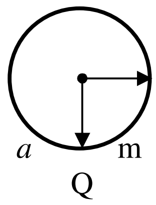

Figure 6.1(a). Uniformly charged spherical shell of fixed radius a, total charge Q and mass m

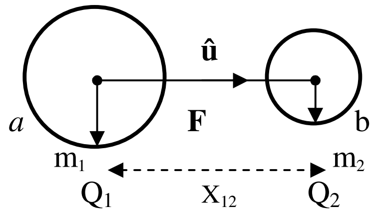

Figure 6.1(b). Two charges Q_1 and Q_2 distance X_{12} apart with force of repulsion $\boldsymbol{F}$ in the $\hat{\boldsymbol{u}}$ direction

magnitude of the charge from *0* to Q as a spherical shell of fixed radius a. The intrinsic energy is always positive.

For points outside a uniformly charged spherical surface, it is as if the whole charge is concentrated at the centre. The potential inside and on the spherical surface of radius a carrying a charge q, is $q/4\pi\varepsilon_o a$ and the work done in increasing the potential from *0* to U and building the charge from *0* to Q at a fixed radius a, by equal infinitesimal amounts (dq), is the intrinsic energy w, given by:

$$w = \int_0^Q \frac{q}{4\pi\varepsilon_o a}(dq) = \frac{Q^2}{8\pi\varepsilon_o a} = \frac{QU}{2} \qquad (6.2)$$

Equation (6.2) shows that the intrinsic energy w is *½Q times the electrostatic potential U in which the charge Q is located*. The energy content is always positive and proportional to the square of the (positive or negative) charge.

Figure 6.1(b) shows two positive electric charges Q_1 and Q_2 of fixed internal radii a_1 and a_2 respectively, separated by a distance X_{12} in a body. The electrostatic energy w_2 of the charges is the sum of the intrinsic energy of each charge being in its own potential and the extrinsic energy due to one charge being in the electrostatic potential of the other, which is expressed in the equation:

$$w_2 = \frac{Q_1^2}{8\pi\varepsilon_o a_1} + \frac{Q_2^2}{8\pi\varepsilon_o a_2} + \frac{Q_1 Q_2}{4\pi\varepsilon_o X_{12}}$$

$$w_2 = \frac{Q_1}{2}\left(\frac{Q_1}{4\pi\varepsilon_o a_1} + \frac{Q_2}{4\pi\varepsilon_0 X_{12}}\right) + \frac{Q_2}{2}\left(\frac{Q_2}{4\pi\varepsilon_o a_2} + \frac{Q_1}{4\pi\varepsilon_o X_{12}}\right) \qquad (6.3)$$

For 3 charges, Q_1, Q_2 and Q_3, of respective radii a_1, a_2, and a_3, separated by distances X_{12}, X_{13}, and X_{23} in a body, the total of the intrinsic and extrinsic energies is:

$$w_3 = \frac{Q_1^2}{8\pi\varepsilon_o a_1} + \frac{Q_2^2}{8\pi\varepsilon_o a_2} + \frac{Q_3}{8\pi\varepsilon_o a_3} + \frac{Q_1 Q_2}{4\pi\varepsilon_o X_{12}} + \frac{Q_1 Q_3}{4\pi\varepsilon_o X_{13}} + \frac{Q_2 Q_3}{4\pi\varepsilon_o X_{23}}$$

$$w_3 = \frac{Q_1}{2}\left(\frac{Q_1}{4\pi\varepsilon_o a_1} + \frac{Q_2}{4\pi\varepsilon_0 X_{12}} + \frac{Q_3}{4\pi\varepsilon_o X_{13}}\right) +$$
$$\frac{Q_2}{2}\left(\frac{Q_2}{4\pi\varepsilon_o a_2} + \frac{Q_1}{4\pi\varepsilon_o X_{12}} + \frac{Q_3}{4\pi\varepsilon_o X_{23}}\right) +$$
$$\frac{Q_3}{2}\left(\frac{Q_3}{4\pi\varepsilon_o a_3} + \frac{Q_1}{4\pi\varepsilon_o X_{13}} + \frac{Q_2}{4\pi\varepsilon_o X_{23}}\right)$$

The extrinsic energies are positive and negative and may cancel out, thus:

$$w_3 = \frac{1}{2}\left\{Q_1\left(U_1 + \Lambda_1\right) + Q_2\left(U_2 + \Lambda_2\right) + Q_3\left(U_3 + \Lambda_3\right)\right\}$$

$$w_3 = \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right)$$

For a number N of charges in a body, the total electrostatic energy is the sum W given by:

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i\left(U_i + \Lambda_i\right) = \frac{1}{2}\sum_{i=1}^{N} Q_i V_i \tag{6.4}$$

where U_i is the intrinsic potential due the ith charge at the point of location of the ith charge, Λ_i is the total extrinsic potential due to all the other charges (excluding the ith charge) at the position of the ith charge and V_i is the electrostatic potential due to all the charges (including the ith charge) at the point of location of the ith charge. It should be noted that the product $(Q_i V_i)$ at any point, outside a charge $(Q_i = 0)$, is zero.

In a neutral body, containing an equal number of positive and negative electric charges, the products, $Q_i U_i$, are all positive and add up to constitute the energy of the mass. The extrinsic potentials Λ_i are positive or negative and the sum of the products $Q_i \Lambda_i$ may be zero, so that equation (6.4) becomes:

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i \left(U_i + \Lambda_i\right) = \frac{1}{2}\sum_{i=1}^{N} Q_i U_i \tag{6.5}$$

Equations (6.4) and (6.5) will be used to derive an expression for the energy content of a body of mass *M*.

6.3 Mass of an isolated electric charge and charge distribution

An isolated positive electric charge of magnitude *Q* moving in a straight line with velocity ***v*** and acceleration *(d**v**/dt)* at time *t*, is associated with a circular magnetic field of intensity ***H*** and an electrodynamic field of intensity $\boldsymbol{E}_a$, as shown in Figure 6.2.

The magnetic flux intensity, $\boldsymbol{B} = \mu_o \boldsymbol{H}$, due to an electric charge of magnitude *Q*, with its electrostatic field of intensity ***E***, (Figure 6.2) moving at velocity ***v*** in a vacuum, is given as a vector (cross) product, by Biot and Savart law of electromagnetism [6], as a vector equation, thus:

$$\mathbf{B} = \mu_o \varepsilon_o \mathbf{v} \times \mathbf{E} = -\mu_o \varepsilon_o \mathbf{v} \times \nabla \phi \tag{6.6}$$

where μ_o is the permeability and ε_o the permittivity of free space or vacuum, ϕ (a scalar) is the instantaneous electric potential at any point due to the charge and $\boldsymbol{E} = -\nabla\phi$, is the electrostatic field intensity, as given by Coulomb's law of electrostatics:

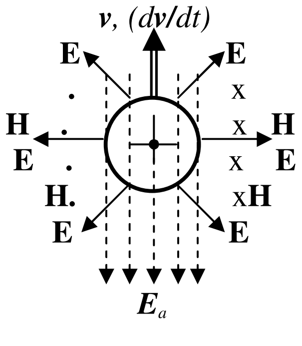

Figure 6.2. An isolated electric charge *Q* and its electrostatic field ***E*** moving in a straight line with velocity ***v*** and acceleration *(d**v**/dt)* at time *t*, generating magnetic field of intensity ***H*** and electrodynamic field of intensity $\boldsymbol{E}_a$.

In Figure 6.2, the magnetic field ***H*** is out of the page on the left and into the page on the right. The electrodynamic field $\boldsymbol{E}_a$ points downwards, opposite to the direction of acceleration. A charge in acceleration is always associated with an electrodynamic field, which opposes the acceleration.

Vector transformation of equation (6.6) gives:

$$\mathbf{B} = -\mu_o \varepsilon_o \mathbf{v} \times \nabla \phi = \mu_o \varepsilon_o \nabla \times (\phi \mathbf{v})$$

Here, the symbol ∇ denotes the "gradient" of a scalar quantity. The notation $\nabla\times$ depicts the "curl" of a vector quantity. The curl of velocity $\boldsymbol{v}$, $(\nabla\times \boldsymbol{v}) = 0$. Faraday's law of electromagnetic induction [7] gives the vector equation:

$$\nabla \times \mathbf{E}_a = -\frac{d\mathbf{B}}{dt} = -\mu_o \varepsilon_o \nabla \times \left(\phi \frac{d\mathbf{v}}{dt} \right)$$

$$\mathbf{E}_a = -\mu_o \varepsilon_o \phi \frac{d\mathbf{v}}{dt} \tag{6.7}$$

The idea put forward here is that the electrodynamic field $\boldsymbol{E}_a$ (equation 6.7) acts internally on the self-same charge Q to produce the reactive force, inertial force or reverse effective force, equal and opposite to the accelerating force. For an isolated uniform spherical shell of electric charge Q and mass m, equation (6.7) and Newton's Second Law of Motion give the reverse effective force as:

$$\mathbf{E}_a Q = -\mu_o \varepsilon_o Q U \frac{d\mathbf{v}}{dt} = -m \frac{d\mathbf{v}}{dt}$$

$$m = \mu_o \varepsilon_o Q U \tag{6.8}$$

where U is the electrostatic (intrinsic) potential due the charge Q at the point of location of the same charge. The derivation of mass m in equation (6) gives an explanation of inertia as experienced by a body under acceleration or deceleration.

For a uniform spherical shell of charge Q, radius a and mass m, equations (6.2) for intrinsic energy and equation (6.8), give:

$$w = \frac{Q^2}{8\pi\varepsilon_o a} = \frac{QU}{2} = \frac{m}{2\mu_o \varepsilon_o} = \frac{1}{2} mc^2 \tag{6.9}$$

$$m = \frac{\mu_o Q^2}{4\pi a} \tag{6.10}$$

where μ_o is the permeability, ε_o the permittivity and $c = (\mu_o\varepsilon_o)^{-1/2}$ is the speed of light in a vacuum, as discovered in 1873 by Maxwell [8]. Equations (6.8) and (6.10), which give the mass of an isolated electric charge Q, should be noted as it has implications in the gravitational force of attraction between bodies or masses composed of equal number of positive and negative.

For a rigid neutral body containing $N/2$ positive electric charges and N/2 negative charges moving with the same acceleration, $(d\mathbf{v}/dt)$, the total electrodynamic field generated at an external point, comes to zero (0). This is obvious as the constituent electric charges generate equal and opposite fields.

For a distribution of N positive and negative electric charges, constituting a body, equation (4.8) gives the mass M of the body as:

$$M = \mu_o\varepsilon_o \sum_{i=1}^{N} Q_i \left(U_i + \Lambda_i\right) = \mu_o\varepsilon_o \sum_{i=1}^{N} Q_i U_i \tag{6.11}$$

where U_i is the intrinsic potential due the ith charge at the point of location of Q_i the ith charge and Λ_i is the extrinsic potential due to all the other charges (excluding the ith charge) at the position of the ith charge. The products Q_iU_i are all positive but the products Λ_iP_i are positive or negative and their sum may be zero, to give equation (6.11).

For a body made up of N electric charges, equations (6.4) for W and equation (6.11) for the mass M, give:

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i U_i = \frac{M}{2\mu_o\varepsilon_o} = \frac{1}{2}Mc^2 \tag{6.12}$$

According to equation (6.12), the work done W in creating a distribution of charges constituting a body of mass M, is $W = ½Mc^2$, equal to the electrostatic energy of the mass. Where mass is independent of speed of a body, the kinetic energy of the body of mass M, moving with speed v, in accordance with classical (Newtonian) mechanics, is $K = ½Mv^2$. The total energy content E, of the body, is:

$$E = W + K = \frac{M}{2}\left(c^2 + v^2\right) \tag{6.13}$$

Equation (6.13) is in contrast to Einstein's mass-energy equivalence law of special relativity [1, 2] as expressed in equation (6.1). Equation (6.13) is what this paper set out to derive with mass of a moving particle remaining constant.

6.4 Conclusion

The derivation of equation (6.8) gives an explanation of the origin of inertia as electrical and internal to an accelerated body. This is a new discovery in physics, particularly electrodynamics.

Equation (6.10), expressing the mass of a charged particle, in the form of a spherical shell of radius a, is instructive. If the electric charge remains constant with speed of the particle, it is reasonable to conclude that the mass should similarly remain constant, contrary to special relativity as expressed in equation (6.1).

Equations (6.1) and (6.13), for a stationary particle, differ by a factor of *2*. However, each equation gives a body of rest mass M_o as the source of a tremendous amount of energy locked up in the particles. If equation (6.13) is correct, it would have a tremendous impact in redirecting the course of modern physics.

In equation (6.1) the kinetic energy is contained in the increase of mass of the particle, which becomes infinitely large at the speed of light c. In equation (6.13), mass M remains constant at the rest mass M_o and the kinetic energy reaches a maximum value equal to $½M_oc^2$ at the speed of light c. Thus, bodies may be accelerated to the speed of light, without mass becoming infinitely large.

The mass-energy equivalence formula, as given by equations (6.9) and (6.13), is more realistic than equation (6.1) for a particle, such as an electron, that can easily be accelerated to the speed of light c. According to equation (6.13), the maximum energy content of an electron of mass m, moving with the speed of light c, is $E_m = mc^2$. Such an electron is easily brought to rest, losing kinetic energy equal to $½mc^2$. The electron can impinge on a target, impart energy and may recoil without causing any damage. If the mass were infinite, its impact would be destructive.

We conclude that mass (equation 6.10) is not a fundamental quantity. The four fundamental quantities are better put as *Length (L)*, *Time (T)*, *Electric Charge (Flux) (Q)* and *Electric Potential (V)*. In this system, *(Metre–Second–Coulomb–Volt)* system, the dimension of *Area (A)* is $[L^2]$, *Volume* ($\forall$) is $[L^3]$, *mass (M)* is $[L^{-2}T^2QV]$, *Permittivity (ε)* is $[L^{-1}QV^{-1}]$, *Permeability (μ)* is $[L^{-1}T^2Q^{-1}V]$, M*agnetic Field (H)* is obtained as $[L^{-1}T^{-1}Q]$, *Magnetic Flux* (Ψ) is $[TV]$, *Resistance (R)* is $[TQ^{-1}V]$, *Resistivity* (ρ) is $[LTQ^{-1}V]$, *Conductivity* (σ) is obtained as $[L^{-1}T^{-1}QV^{-1}]$, *Capacitance (C)* is $[QV^{-1}]$, *Inductance (L)* is $[T^{2}Q^{-1}V]$, *Force (F) is* $[L^{-1}QV]$, *Energy (E)* is $[QV]$, *Power (P)* is *Momentum (p)* is $[L^{-1}TQV]$ and *Angular Momentum (L)* is $[TQV]$, *Gravitational Constant (G)* is $[L^{5}T^{-4}Q^{-1}V^{-1}]$ and *Gravitational Field* (Γ) is $[LT^{-2}]$. There is no fractional exponent of a fundamental quantity in the dimension of any derived quantity.

6.5 References

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7. A UNIFICATION OF ELECTROSTATIC AND GRAVITATIONAL FORCES

Abstract

The gravitational force of attraction $\boldsymbol{F}_G$ between two bodies, distance Z apart, one of mass M_1 containing N_1 positive and negative electric charges $(\pm Q_i)$ and the other of mass M_2 containing N_2 electric charges $(\pm K_i)$, is derived as:

$$\mathbf{F}_G = -\frac{G}{Z^2} M_1 M_2 \hat{\mathbf{u}} = -\frac{G}{Z^2}\left(\mu_o \varepsilon_o \sum_{i=1}^{N_1} Q_i V_i\right)\left(\mu_o \varepsilon_o \sum_{i=1}^{N_2} K_i P_i\right)\hat{\mathbf{u}}$$

where G is the gravitational constant, μ_o the permeability and ε_o the permittivity of a vacuum, V_i and P_i are the total potentials at the respective locations of the charges Q_i and K_i and $\hat{\boldsymbol{u}}$ is a unit vector. This explains Newton's universal law of gravitation, as an electrical phenomenon, outside Einstein's theory of general relativity.

Keywords: Electric charge, force, gravity, mass, relativity.

7.1 Introduction

The force of attraction between two bodies is given by the universal law of gravitation discovered by Sir Isaac Newton around 1687 [1]. The force of attraction or repulsion between two stationary electric charges is expressed by the law of electrostatics enunciated by Charles Coulomb in 1784. It is natural that physicists would think that there should be a connection between gravitational and electrostatic forces. So, for several decades now, physicists have been searching for a theory that would combine gravitational forces between masses and electrostatic forces between charges. In 1915, Einstein [2] came out with the general relativity theory. This, it was believed, explained gravitation in the context of curvature or warping of four-dimensional space-time.

The idea of four-dimensional space-time continuum, where the three spatial coordinates and the time dimension pinpoint an event in space, was first suggested by the German physicist Herman Minkowski, in 1908.

Minkowski's idea was subsequently adopted by Albert Einstein in formulating the theory of general relativity.

According to the general relativity theory, gravity is not a force like other forces, but the result of curvature or warping of four-dimensional space-time due to the presence of matter; not some effect emanating from matter itself. Bodies, like the earth, follow the path of least resistance, a straight line in curved space-time, which, in reality, is an elliptic orbit in three-dimensional space. In other words, curved space pushes (accelerates) objects to follow a particular path in the universe. Where the space-time curvature is infinite, there is what is called "singularity", resulting in a "black hole" - matter so dense and massive that not even light can escape from its vicinity, boundary or event horizon. The idea of warping of space-time, referred to as "ripples of space" (due to time effect), in the presence of matter, giving rise to gravitation, was a revolutionary, brilliant and most appealing theory.

The general relativity theory or Einstein's theory of gravitation, describes the large-scale structure of the universe and the force of gravity or acceleration between bodies, like the sun, planets and moons. Quantum mechanics based on the works of Planck, Bohr, De Broglie and others [3, 4, 5], on the other hand, treats extremely small-scale phenomena down to the atomic particles. Unfortunately, the quantum and relativity theories, the dominant theories of modern physics, being incompatible with one another at high speeds, cannot both be correct, but do coexist peacefully. This inconsistency has informed the search for a unified theory to incorporate general relativity and quantum mechanics – a quantum theory of gravity or quantum field theory. Einstein spent many years searching for a link between general relativity and quantum theory, up to the end of his life, without success. Many physicists are now engaged in the search for a unified field theory or Grand Unified Theory (GUT).

Just as Einstein introduced *time* making a fourth (non spatial) dimension in general relativity, to explain gravitation, in 1919 the Polish mathematician, Kaluza, proposed that a fourth spatial dimension was needed to incorporate electrical forces. This proposal was subsequently refined in 1926 by Swedish physicist, Klein. Kaluza and Klein suggested that electromagnetism was due to "ripples" in four spatial dimensions and one time dimension. The idea of space having more than three dimensions (the usual three extended dimensions and several "curled up" dimensions too small to be observed) was a brilliant and very attractive theory. This theory has stretched physicists' dimensions of imagination too far, but so far, it has not been demonstrated by any experiment.

In the early 1980s the search for a unified field theory assumed extra dimensions with the introduction of the superstring theories as described by Hawking [6], Greene [7] and Barbour [8]. According to the union of five types of superstring theories, called the M-theory (*'M'* for *mysterious*), the fundamental constituents of the physical world are not point particles but infinitesimal one-dimensional open strings (lines) or closed strings (loops). All the forces and energies in the universe arise from the strings' vibrations at different frequencies in ten spatial dimensions and one time dimension; an elementary particle, like the electron, being a string vibrating at a resonant frequency. The advocates of the M-theory take it as the Grand Unified Theory (GUT) of everything.

Some other physicists have been engaged in the pursuit for a unified field theory but not along the lines of reconciling general relativity and quantum mechanics. It may as well be that either the general relativity theory or quantum mechanics is incorrect or that both theories are wrong. It is not so much like looking for the proverbial "needle in the haystack" but searching for something that was not a "needle" or never in "the haystack". What is needed is to go back to the first principles and conduct the search on the basis of the well-known Newton's universal law of gravitation and Coulomb's law of electrostatics.

7.2 Newton's universal law of gravitation

The law of force of attraction between masses in space was discovered by the great English physicist and mathematician, Sir Isaac Newton around 1687 [1]. Newton's law of gravitation states:

> *"Every object in the universe attracts every other object with a force that is proportional to the product of their masses and inversely proportional to the square of the separation between the two objects".*

Mathematically, the gravitational force of attraction $\boldsymbol{F}_G$ between two objects of masses M_1 and M_2 in space separated by a distance Z, between their centres of mass, is expressed as:

$$\mathbf{F}_G = -G\frac{M_1 M_2}{Z^2}\hat{\mathbf{u}} \tag{7.1}$$

where $\hat{\boldsymbol{u}}$ is a unit vector pointing in the direction of the force of repulsion, opposite to the force $\boldsymbol{F}_G$ and G is the gravitational constant. For now, we don't know why $\boldsymbol{F}_G$ is always attractive.

7.3 Coulomb's law of electrostatics

The American politician, scientist and inventor, Benjamin Franklin (1706 – 1790) had established that there were two kinds of electricity, positive charges and negative charges. He demonstrated that electric charges always existed in equal and opposite amounts. Franklin's discovery was of great significance in physics. Thus, for neutral bodies the forces of repulsion and attraction, due to the equal number of positive and negative charges, cancel out exactly. There is no electrostatic force of repulsion or attraction between two neutral bodies, but the gravitational force of attraction remains and persists.

Forces between stationary electric charges are expressed in the law enunciated by the French physicist, Charles Coulomb, in 1785. Coulomb's law of electrostatics, a significant law in physics, states:

> *"The force of repulsion or attraction between two electric charges is proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them".*

An addition to Coulomb's law is to the effect that

> *"Like charges repel and unlike charges attract"*,

This is in contrast to gravitation where the forces are always attractive.

The electrostatic force $\boldsymbol{F}_{E,}$ of repulsion or attraction, between two stationary electric charges of magnitudes Q and K distance Z apart, in space, is expressed as a vector:

$$\mathbf{F}_E = \pm \frac{QK}{4\pi\varepsilon_o Z^2}\hat{\mathbf{u}} \tag{7.2}$$

where the force between two electric charges is positive (repulsive) for like charges and negative (attractive) for unlike charges, ε_o is the permittivity of a vacuum and the unit vector $\hat{\boldsymbol{u}}$ points in the direction of the force of repulsion.

7.4 Electrostatic and gravitational forces between two isolated charges

Figure 7.1 shows two isolated electric charges of magnitudes Q and K respectively in the form of spherical shells of radii a and b. The masses m_1 and m_2 are separated by a distance Z in space. Z is very much larger than the radius a or b. The force $\boldsymbol{F}$ between the stationary charges, which is a combination of electrostatic force of repulsion or attraction and gravitational force of attraction, is expressed as:

$$\mathbf{F} = \pm \frac{QK}{4\pi\varepsilon_o Z^2}\hat{\mathbf{u}} - G\frac{m_1 m_2}{Z^2}\hat{\mathbf{u}} \tag{7.3}$$

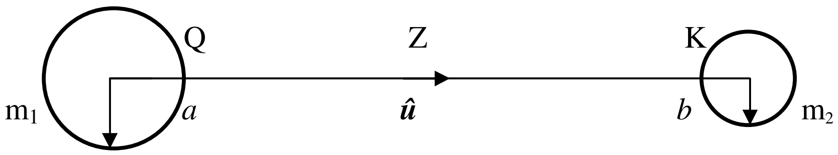

Figure 7.1. Electrostatic and gravitational forces between two charges Q and K.

where $\hat{\boldsymbol{u}}$ is a unit vector in the direction of force of repulsion and G is the gravitational constant. The first term on the right-hand side of equation (7.3) is the electrostatic force of repulsion or attraction between isolated electric charges of magnitudes Q and K and the second term is the gravitational force of attraction $\boldsymbol{F}_G$, between masses m_1 and m_2, given by Newton's universal law of gravitation. Equation (7.3) gives the gravitational force, equation (7.1), as:

$$\mathbf{F}_G = -\frac{G}{Z^2} m_1 m_2 \hat{\mathbf{u}} \tag{7.4}$$

The author [9] showed that the mass m of an isolated electric charge (positive or negative) of magnitude Q, in the form of a spherical shell of radius a in free space of permeability μ_o, is given by the equation:

$$m = \frac{\mu_o Q^2}{4\pi a} \tag{7.5}$$

Substituting for the mass m_1 (of electric charge Q) and mass m_2 (of another electric charge K) from equation (7.5) into (7.4), gives the gravitational force of attraction. The gravitational force of attraction between two isolated electric charges, where the separation Z is much larger than the radius a or b, is given by the vector equation:

$$\mathbf{F}_G = -\frac{G}{Z^2} m_1 m_2 \hat{\mathbf{u}} = -\frac{G}{Z^2}\left(\frac{\mu_o Q^2}{4\pi a}\right)\left(\frac{\mu_o K^2}{4\pi b}\right)\hat{\mathbf{u}} \tag{7.6}$$

Thus $\boldsymbol{F}_G$, the force of attraction due to gravity, is proportional to the product of square of the charges or proportional to the square of product of the charges. This force is always positive because the square of any quantity (positive or negative) is always positive.

7.5 Electrostatic force of repulsion or attraction between two neutral bodies

Figure 7.2 shows a neutral body of mass M_1 containing $N_1/2$ positive electric charges and $N_1/2$ negative electric charges, N_1 being an even number. Each charge is of quantity $\pm Q_i$ = $(-1)^i Q_i$, in the form of a spherical shell of radius a_i (i = 1, 2, 3, 4....N). The potential p_i at a point R, due to a charge Q_i distance Z_i from R, is:

$$p_i = \frac{(-1)^i Q_i}{4\pi\varepsilon_o Z_i}$$

The electrostatic potential P, at a point R, due to all the $N_1/2$ positive electric charges and $N_1/2$ negative electric charges in a neutral body of mass M_1, is the sum:

$$P = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{N_1}(-1)^i \frac{Q_i}{Z_i} = \frac{1}{4\pi\varepsilon_o Z}\sum_{i=1}^{N_1}(-1)^i Q_i \tag{7.7}$$

where Z is the distance of the centre of charge O from the point R in space. Since a neutral body contains an equal number of positive and negative electric charges, the potential at R, is:

$$P = \frac{1}{4\pi\varepsilon_o Z}\sum_{i=1}^{N_1}(-1)^i Q_i = \frac{1}{4\pi\varepsilon_o Z}\left(\sum_{i=1}^{\frac{N_1}{2}} Q_{2i} - \sum_{i=1}^{\frac{N_1}{2}} Q_{(2i-1)}\right) = 0 \tag{5.8}$$

where $\sum_{i=1}^{N_1/2} Q_{2i}$ is the total amount of positive charge equal to $\sum_{i=1}^{N_1/2} Q_{(2i-1)}$

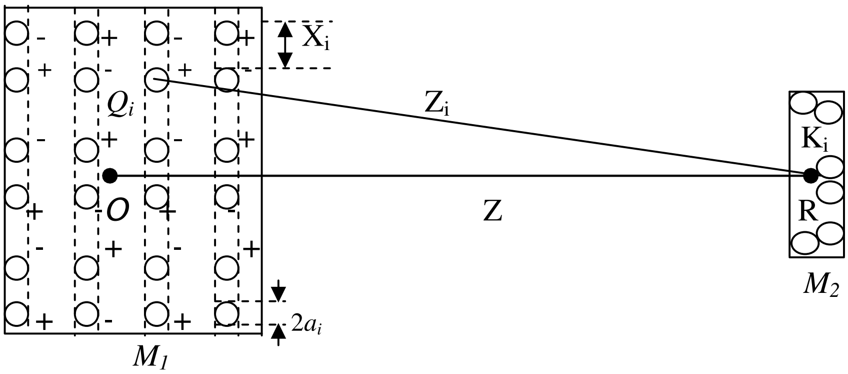

Figure 7.2. A body of mass M_1 containing N_1 positive and negative charges each of magnitude Q_i at a distance Z_i from a point R with mass M_2 in space. Two constituent charges, each of radius a_i, making a pair of particles, are separated by a distance X_i, much lager than a_i. The distance Z_i is much larger than X_i.

the total negative charge in a neutral body. Equation (5.8), making the potential at any point, due to a neutral body, zero, is stating the obvious. This is obvious for electric charges equal in magnitude. A second neutral body of mass M_2 placed at R (Fig. 7.2) experiences no potential and no force, as the forces of repulsion and attraction cancel out exactly, leaving the gravitational forces.

7.6 Gravitational force of between two bodies

The author [9] showed that the mass M of a distribution of N positive and negative electric charges $(\pm Q_i)$ constituting a neutral body, as in Figure 7.2, is given by the sum of products, thus:

$$M = \mu_o \varepsilon_o \sum_{i=1}^{N} Q_i V_i \qquad (7.9)$$

where V_i is the total potential at the point of location of charge Q_i.

The gravitational force of attraction $\boldsymbol{F}_G$ between the body of mass M_1 and another body at R (Figure 7.2), of mass M_2 containing N_2 charges $(\pm K_i)$, is obtained as the product of two sums, thus:

$$\mathbf{F}_G = -\frac{G}{Z^2} M_1 M_2 \hat{\mathbf{u}} = -\frac{G}{Z^2}\left(\mu_o \varepsilon_o \sum_{i=1}^{N_1} Q_i V_i\right)\left(\mu_o \varepsilon_o \sum_{j=1}^{N_2} K_i P_i\right)\hat{\mathbf{u}} \qquad (7.10)$$

where Z is the separation between the bodies, G is the gravitational constant and V_i and P_i are the total potentials at the respective locations of the charges Q_i and K_i.

In Figure 7.2, pairs of opposite charges Q_i, of radius a_i, separated by distance X_i, the total potential V_i at the location of charge Q_i, is:

$$V_i = \frac{Q_i}{4\pi\varepsilon_o}\left(\frac{1}{a_i} - \frac{1}{X_i}\right)$$

Other potentials at the position of Q_i cancel out. Equation (9) gives

$$M = \frac{\mu_o}{4\pi}\sum_{i=1}^{N} Q_i^2\left(\frac{1}{a_i} - \frac{1}{X_i}\right) \qquad (7.11)$$

Where the constituent electric charges are of the same magnitude Q, same radius a and same separation X, equation (7.11) becomes:

$$M = \frac{\mu_o}{4\pi}\sum_{i=1}^{N} Q_i^2\left(\frac{1}{a_i} - \frac{1}{X_i}\right) = \frac{\mu_o N Q^2}{4\pi}\left(\frac{1}{a} - \frac{1}{X}\right) \qquad (7.12)$$

Let us find the force of attraction $\boldsymbol{F}_G$ between the body of mass M_1 and another of mass M_2 at R (Figure 7.2). The mass M_2 contains $N_2/2$ pairs of opposite charges, each of magnitude K in the form of a spherical shell of radius b, distance Y apart. Force $\boldsymbol{F}_G$ is the product:

$$\mathbf{F}_G = -\frac{GM_1M_2}{Z^2}\hat{\mathbf{u}} =$$

$$-\frac{G}{Z^2}\left(\frac{\mu_o N_1 Q^2}{4\pi}\right)\left(\frac{1}{a} - \frac{1}{X}\right)\left(\frac{\mu_o N_2 K^2}{4\pi}\right)\left(\frac{1}{b} - \frac{1}{Y}\right)\hat{\mathbf{u}} \qquad (7.13)$$

In equation (7.13), if the charges are in two bodies where *1/X* and *1/Y* can be neglected compared with *1/a* or *1/a*, equation (7.13) gives $\boldsymbol{F}_G$, as:

$$\mathbf{F}_G = -G\frac{M_1M_2}{Z^2}\hat{\mathbf{u}} = -G\left(\frac{\mu_o}{4\pi}\right)^2 \frac{N_1N_2Q^2K^2}{abZ^2}\hat{\mathbf{u}} \qquad (7.14)$$

Equation (7.13) and (7.14), giving $\boldsymbol{F}_G$ as proportional to the product of squares of the charges, are what this paper set out to derive.

7.7 Conclusion

The import of equation (7.14) is that gravitational force of attraction is electrical in nature. This result explains the force of gravity without recourse to four-dimensional space-time continuum of Einstein's theory of general relativity. It should put to rest the present quest, by physicists [10], for the Grand Unified Theory (GUT) of everything.

The space, of constant potential, inside a spherical electric charge, is different from the space outside where the potential decreases with distance from the charge. The spherical surface may be regarded as a boundary tantamount to "warpjng" of space, giving rise to gravitation in a manner envisaged by Einstein, but without the "time" dimension.

Space, outside matter, is not empty but crisscrossed by electric fields emanating from bodies. The electric fields are ever present, but cancel out at a point in space, to leave the gravitational fields. A gravitational field is present everywhere in space as continuous pressure towards a body, producing an effect decreasing with distance from the body and indepengent of relative velocity between two bodies. Change in position or magnitude of a body should be transmitted at the speed of light.

We, following the greats, Galileo Galelei and Isaac Newton, contend that *space* and *time* are absolute quantities, independent of the position or motion of the observer and that *space* is of one dimension, ***Length***, a vector quantity with three orthogonal components.

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8. EXPLANATIONS OF THE RESULTS OF ROGER'S AND BERTOZZI'S EXPERIMENTS WITHOUT RECOURSE TO SPECIAL RELATIVITY

Abstract

The results of Roger's experiment with electrons revolving at defined speeds in circular orbits and Bertozzi's experiment with high-speed electrons moving in a linear accelerator, can be explained as due to accelerating force on an electron decreasing with speed, reducing to zeo at the speed of light. This is in contrast to special relativity where the results are ascribed to the mass of a moving electron increasing with speed, becoming infinitely large at the speed of light. The result of Roger's experiment is also shown to be in agreement with *radiational electrodynamics* for circular revolution of electrons round a centre of force. The explanations lead to the speed light as the ultimate speed without infinite mass.

Keywords: Rectilinear and circular motion, special relativity, speed.

8.1 Introduction

In classical electrodynamics [1, 2], the mass of a particle is independent of its speed and a charged particle, such as an electron, can be accelerated by an electrostatic force, beyond the speed of light. But observations on accelerated electrons, the lightest particles known in nature, showed that their speeds could not exceed that of light. Relativistic electrodynamics [3, 4] and *radiational electrodynamics* [5] deal with the issues that restrain accelerated particles from going beyond the speed of light.

Relativistic electrodynamics explaines the speed of light being a limit by positing that the mass m of a moving particle increases with its speed v, becoming infinitely large at the speed of light c. The mass-velocity formula, of special relativity, is:

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_o \tag{8.1}$$

where m_o is the rest mass. This position is apparently plausible as an infinite mass cannot be pushed any faster by any finite force.

In equation (8.1), the difficulty with infinite masses, at the speed of light $(v = c)$, is avoided by insisting that the speed v may be as near as possible, but it never really becomes equal to c. Photons, as "particles" supposed to move at the speed of light, are given zero rest mass. Also, the speed of light c in a vacuum is made an absolute constant, independent of the speed of the source of light or the speed of the observer.

Radiational electrodynamics proposes that the speed of light c is an ultimate limit because the accelerating force exerted by an eletrostatic field, on a moving charged particle, decreases with the speed of the particle. The accelerating force reduces to zero at the speed of light and the particle continues to move at that speed, with the rest mass m_o, in accordance with Newton's second law of motion.

In *radiational electrodynamics*, decrease in accelerating force with speed in the revolution of an electron round a centre of force, gives the same effect as apparent increase in mass with speed in accordance with equation (8.1). Therefore, Roger's experiment (1939) [6], supposed to have proved increase of mass with speed, might as well have confirmed decrease of accelerating force with speed, as far as circular motion round a central force is concerned.

In rectilinear motion, it was found that electrons cannot be accelerated beyond the speed of light, no matter the magnitude of the accelerating potential in a linear accelerator. The existence of a limiting speed, eqaul to the speed of light, was clearly demonstrated in Bertozzi's experiment (1964) [7]. Here, again, the limiting speed was attributed to mass increasing with speed, becoming infinitely large at the speed of light, as per the relativistic equation (8.1). An accelerating force decreasing with speed, becoming zero at the speed of light, in accordance with *radiational electrodynamics*, should also lead to that speed being an ultimate limit, in accordance with Newton's laws of motion.

In this paper, the motion of electrons, in an electrostatic field, is treated under classical, relativistic and *radiational electrodynamics*. It is found that the results of Roger's and Bertozzi's experiments are in agreement with predictions of *radiational electrodynamics*, but not on the basis of mass increasing to become infinitely large at the speed of light. Under *radiational electrodynamics*, it is shown that the speed of light is a limit not because the mass becomes infinitely large at that speed but as a result of accelerating force exerted by an electrostatatic field, on a moving electron, decreasing with speed, reducing to zero at the speed of light.

8.2 Classical electrodynamics

8.2.1 Potential energy lost by an accelerated electron

In classical electrodynamics, the accelerating force $\boldsymbol{F}$ on an electron of charge $-e$ and constant mass m, moving with velocity $\boldsymbol{v}$ and acceleration $d\boldsymbol{v}/dt$ at time t, in an electrostatic field of intensity $\boldsymbol{E}$, is given, in accordance with Newton's second law of motion, by the vector equation:

$$\mathbf{F} = -e\mathbf{E} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{8.2}$$

For rectilinear motion, in the direction of a displacement $\boldsymbol{x}$, equation (8.2), with E as the magnitude of $\boldsymbol{E}$, becomes:

$$-eE\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} = -mv\frac{dv}{dx}\hat{\mathbf{u}} \tag{8.3}$$

where $\hat{\boldsymbol{u}}$ is a unit vector in the direction of the electrostatic field $\boldsymbol{E}$ and the displacement $\boldsymbol{x}$. The scalar equation is:

$$eE = mv\frac{dv}{dx} \tag{8.4}$$

The potential energy P lost by the moving electron or work done on the electron, in being accelerated with constant mass m, through a distance x, to a speed v from rest, is gven by the definite integral:

$$P = \int_0^x eE(dx) = m\int_0^v v(dv) \tag{8.5}$$

Integrating, equation (8.5) becomes:

$$P = \frac{1}{2}mv^2 = \frac{1}{2}m_o v^2$$

$$\frac{P}{m_o c^2} = \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{8.6}$$

Here, the potential energy P lost is equal to the kinetic energy gained, as there is no consideration of energy radiation.

8.2.2 Potential energy gained by a deccelerated electron

In classical electrodynamics, an electron, moving at the speed of light *c*, can be decelerated to a stop and may be accelerated in the opposite direction to reach a speed greater than **–***c*. The potential energy *P* gained in decelerating an electron from the speed of light *c* to a speed *v*, is:

$$P = \frac{1}{2}m(c^2 - v^2) \tag{8.7}$$

8.2.3 Circular revolution of an electron

Figure 8.1 shows an electron of mass *m* and charge –*e* revolving in a circle of radius *r* in an electrostatic field of intensity ***E*** due to a point charge *Q* at the centre *O*. The accelerating force ***F***, in accordance with Newton' second law of motion, is given by the vector equation:

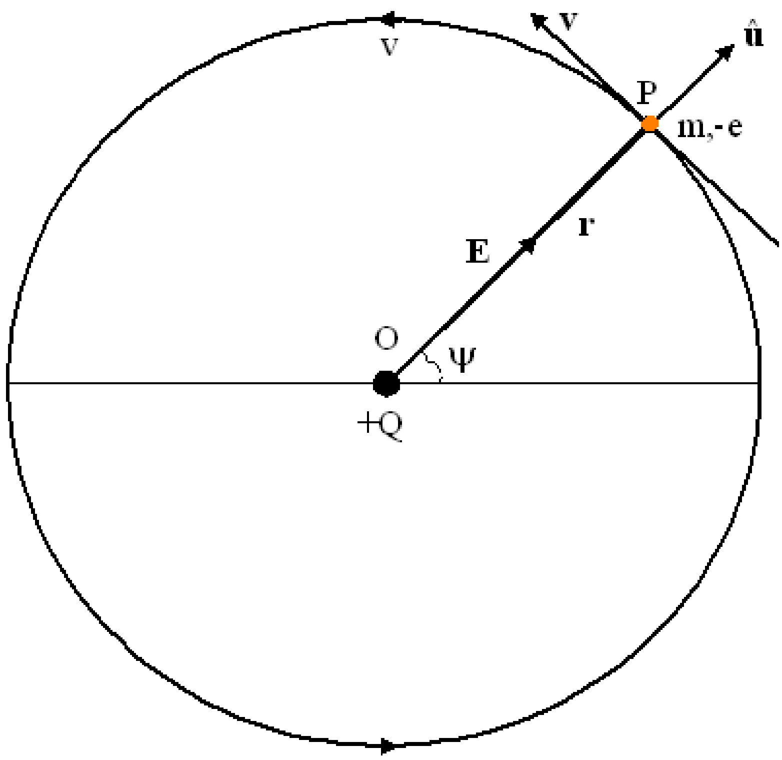

Figure 8.1. An electron of mass *m* and charge –*e* revolving with speed *v* in a circle of radius *r*, in a radial field of intensity ***E*** due to a positive charge *Q* at the centre *O*.

$$\mathbf{F} = -e\mathbf{E} = m\frac{d\mathbf{v}}{dt} = -m\frac{v^2}{r}\hat{\mathbf{u}} \tag{8.8}$$

where $(-v^2/r)\hat{\mathbf{u}}$ is the centripetal acceleration due to the accelerating force. The scalar equation is:

$$eE = m\frac{v^2}{r} \tag{8.9}$$

$$\frac{eEr}{m_o v^2} = \frac{m}{m_o} \tag{8.10}$$

where m_o is the rest mass. In classical electrodynamics, $m = m_o$ is a constant and equation (8.10) should give a unity, for all values of E, r and v. This is not what was observed in laboratory experiments.

8.3 Relativistic electrodynamics

8.3.1 Potential energy lost by an accelerated electron

In relativistic electrodynamics, the kinetic energy K gained by an electron or the work done, in being accelerated to a speed v from rest, is the potential energy P lost. The kinetic energy K of a particle of mass m and rest mass m_o moving with speed v, is given by the relativistic equation:

$$K = P = mc^2 - m_o c^2$$

$$P = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_o c^2$$

$$\frac{P}{m_o c^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \tag{8.11}$$

where m_o is the rest mass (at $v = 0$) and c the speed of light in a vacuum. Bertozzi's experiment was conducted to verify equation (8.11) and it appeared to have done so in a remarkable way.

8.3.2 Potential energy gained by a deccelerated electron

In relativistic electrodynamics, an electron moving at the speed of light c (with infinite mass), cannot be stopped by any decelerating force. The electron continues to move at the same speed of light c, gaining potential energy without losing kineticl energy, contrary to the principle of conservation of energy.

8.3.3 Circular revolution of an electron

In relativistic electrodynamics, the accelerating force $\boldsymbol{F}$ on an electron is independent of its velocity $\boldsymbol{v}$ at time t in an electrostatic field of intensity $\boldsymbol{E}$, but the mass m increases with speed v in accordance with the mass-velocity formula, equation (8.1). In Figure 8.1, constant centripetal acceleration $(-v^2/r)\hat{\boldsymbol{u}}$, gives the force $\boldsymbol{F}$, in accordance with Newton's second law of motion, as the vector:

$$\mathbf{F} = -e\mathbf{E} = -m\frac{v^2}{r}\hat{\mathbf{u}} \tag{8.12}$$

Combining equation (8.12) with equation (8.1), gives:

$$eE = m\frac{v^2}{r} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} \tag{8.13}$$

$$\frac{eEr}{m_o v^2} = \frac{m}{m_o} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8.14}$$

Roger's experiment set out to verify equation (8.14) and it did so convincingly. It provided an evidence of apparent increase in mass with speed in accordance with special relativity.

8.4 *Radiational electrodynamics*

8.4.1 Motion of an electron in an electrostatic field

Figure 8.2 depicts an electron of charge $-e$ and constant mass $m = m_o$, moving at a point P with velocity $\boldsymbol{v}$ at time t, in an electrostatic field of intensity $\boldsymbol{E}$ due to a stationary source charge $+Q$ at the origin O. The velocity $\boldsymbol{v}$ is at an angle θ to the accelerating force $\boldsymbol{F}$, which is a force of attraction in the **PO** direction. The relative velocity between the accelerating force (propagated with velocity of light $\boldsymbol{c}$) and the electron moving with velocity **v**, is the vector ($\boldsymbol{c} - \boldsymbol{v}$). The velocity of light $\boldsymbol{c}$, is inclined at the aberration angle α to the accelerating force $\boldsymbol{F}$, such that:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{8.15}$$

where v and c are the magnitudes of $\boldsymbol{v}$ and $\boldsymbol{c}$ respectively.

In *radiational electrodynamics*, the accelerating force $\boldsymbol{F}$, with reference to Figure 8.2, is given by the vector equation:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c} - \mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{8.16}$$

where E is the magnitude of the electrostatic field of intensity $\boldsymbol{E}$.

Expanding equation (8.16) by taking the *modulus* of ($\boldsymbol{c} - \boldsymbol{v}$), with respect to the angles θ and α in Figure 8.2, gives the equation:

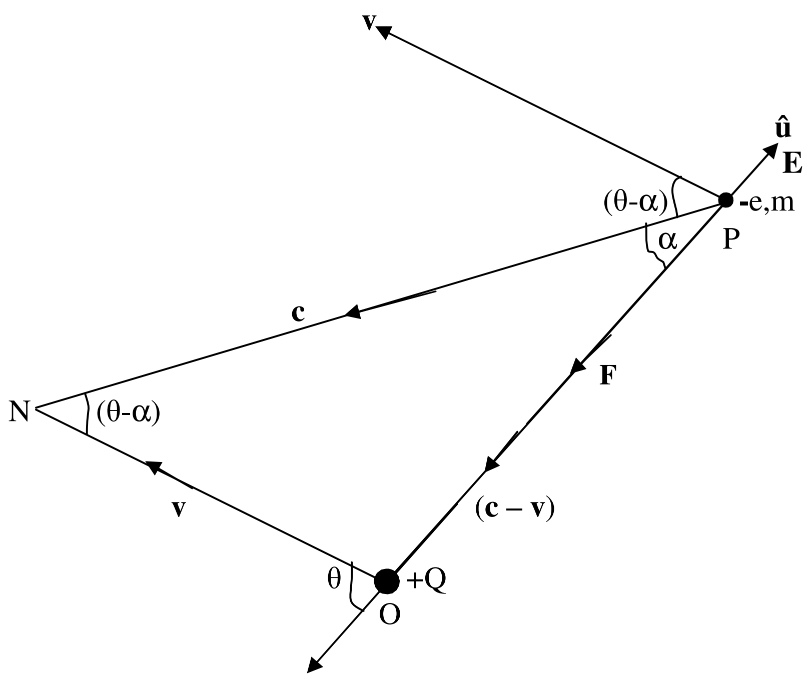

Figure 8.2 An electron of charge $-e$ and mass m moving, at a point P, with velocity $\boldsymbol{v}$, at an angle θ to the accelerating force $\boldsymbol{F}$. The unit vector $\hat{\boldsymbol{u}}$ is in the direction of the electrostatic field **E** due to a positive charge Q at O.

$$\mathbf{F} = \frac{-eE}{c}\sqrt{c^2 + v^2 - 2cv\{\cos(\theta - \alpha)\}}\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt} \qquad (8.17)$$

where $(\theta - \alpha)$ is the angle between $\boldsymbol{c}$ and $\boldsymbol{v}$ and $\hat{\mathbf{u}}$ is a unit vector in the direction of the field **E,** opposite to the direction of $(\boldsymbol{c} - \boldsymbol{v})$. The electron can move in a straight line, in the direction of the force, with acceleration where $\theta = 0$ or against the force with deceleration where $\theta = \pi$ radians or it can revolve in a circle, with constant speed v, if θ is equal to $\pi/2$ radians. Motion in a circle is at right angle to a radial electric field without change in potential or kinetic energy,

8.4.2 Potential energy lost by an accelerated electron

For an accelerated electron, equations (8.15) and (8.17), with $\theta = 0$, give the vector equation:

$$\mathbf{F} = -eE\left(1 - \frac{v}{c}\right)\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} \qquad (8.18)$$

The scalar equation is:

$$eE\left(1 - \frac{v}{c}\right) = m\frac{dv}{dt} = mv\frac{dv}{dx} \qquad (8.19)$$

The potential energy P lost in accelerating the electron, through a distance x, to a speed v from rest, is given by the integral:

$$P = \int_0^x eE(dx) = \int_0^v mv\frac{dv}{1 - \frac{v}{c}} \qquad (8.20)$$

Resolving the right-hand integral into partial fractions, we obtain:

$$P = mc\int_0^v \left(\frac{1}{1 - \frac{v}{c}} - 1\right)dv \qquad (8.21)$$

$$P = -mc^2 \ln\left(1 - \frac{v}{c}\right) - mcv \qquad (8.22)$$

$$\frac{p}{mc^2} = -\ln\left(1 - \frac{v}{c}\right) - \frac{v}{c} \qquad (8.23)$$

Equation (8.23) for *radiational electrodynamics*, should be compared with equation (8.11) for relativistic electrodynamics and equation (8.6) for classical electrodynamics.

8.4.3 Potential energy gained by a decelerated electron

For a decelerated electron, equations (8.15) and (8.17), with $\theta = \pi$ radians, give the vector equation:

$$\mathbf{F} = -eE\left(1+\frac{v}{c}\right)\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \tag{8.24}$$

$$eE\left(1+\frac{v}{c}\right) = -m\frac{dv}{dt} = -mv\frac{dv}{dx} \tag{8.25}$$

Potential energy P gained in deceleration from the speed of light c to v, is:

$$P = \int_0^x eE(dx) = \int_c^v -mv\frac{dv}{1+\frac{v}{c}} \tag{8.26}$$

$$P = -mc\int_c^v \left(1-\frac{1}{1+\frac{v}{c}}\right)dv \tag{8.27}$$

$$P = mc^2 \ln\frac{1}{2}\left(1+\frac{v}{c}\right) + mc^2\left(1-\frac{v}{c}\right) \tag{8.28}$$

8.4.4 Accelerating force in circular revolution

With the angle $\theta = \pi/2$ radians, the electron revolves in a circle of radius r with constant speed v and centripetal acceleration $(-v^2/r)\hat{\mathbf{u}}$. Noting that $\cos(\pi/2 - \alpha) = \sin\alpha$, equations (8.15) and (8.17) give:

$$\mathbf{F} = -eE\sqrt{1-\frac{v^2}{c^2}}\hat{\mathbf{u}} = -m\frac{v^2}{r}\hat{\mathbf{u}} \tag{8.29}$$

$$eE = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} \qquad (8.30)$$

Equation (8.30), with $m = m_o$, is identical to (8.13), as confirmed by Roger's experiment performed in 1939.

8.5 Roger's experiment

An experiment by M. Roger [6], seem to support the relativistic mass-velocity formula. In this experiment, an electron of mass m, moving at a well defined speed v, was made to enter a radial electrostatic field. The electron was deflected to move in a circle of radius r under a centripetal accelerating force of magnitude F, to give equation (8.14) for m/m_o. The results of Roger's experiment are shown in Table 8.1.

TABLE 8.1. RESULTS OF ROGER'S EXPERIMENT FOR THREE DIFFERENT SPEEDS
($c = 2.998 \times 10^8$ m/sec, $e/mo = 1.759 \times 10^{11}$ C/kg)

SPEED v m/sec	v/c	Er $\times 10^5$ V	OBSERVED m/mo	CALCULATED $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
1.900×10^8	0.634	2.671	1.302	1.293
2.087×10^8	0.696	3.487	1.408	1.393
2.247×10^8	0.750	4.341	1.512	1.511
2.998×10^8	1.000	?	?	∞

The observed values of the ratio m/m_o, obtained for three different speeds, by measuring Er, and the calculated values from equation (8.14), were in close agreement, as shown in Table 8.1. Roger's experiment has seemingly verified the theory of special relativity, which predicted that the mass m of an electron increases with its speed v, becoming infinitely large at the speed of light c. However, the speed, in equation (8.1) is never allowed to reach that of light. But Bertozzi's experiment showed that electrons are easily accelerated to the speed of light through a potential energy of 15 MeV or over.

8.6 Bertozzi's experiment

A remarkable demonstration of the speed of light being a universal limiting speed, was in an experiment conducted by William Bertozzi, at the Massachusetts Institute of Technology in 1964 [7]. In this experiment, the speed *v* of high-energy electrons was determined by measuring the time *T* required for them to traverse a distance of 8.4 metres after having been accelerated through a potential energy *P* inside a linear accelerator. Bertozzi's experimental data is reproduced in Table 8.2. It was clearly demonstrated, in this experiment, that electrons accelerated through energies of 15 MeV or more, attain, for all practical purposes, the speed of light *c* as a limit.

TABLE 8.2. RESULTS OF BERTOZZI'S EXPERIMENTS WITH ELECTRONS ACCELERATED THROUGH ENERGY P
(m_oc^2 = 0.5 MeV, v = 8.4/T m/sec)

P MeV	P/m_oc^2	T x 10^{-8} sec.	v x 10^{8} m/sec	v/c	$(v/c)^2$
0.5	1	3.23	2.60	0.87	0.76
1.0	2	308	2.73	0.91	0.88
1.5	3	2.92	2.88	0.96	092
4.5	9	2.84	2.96	0.99	0.97
15.0	30	2.80	3.00	1.00	1.00

A graph of P/m_oc^2 (potential energy in units of m_oc^2) against $(v/c)^2$ (speed squared in units c^2), is shown in Figure 8.3; the solid line (A) in accordance with classical electrodynamics (equation 8.6), the dashed curve (B) according to relativistic electrodynamics (equation 8.11) and the dashed curve (C) according to radiational electrodynamics (equation 8.23). The solid squares are the results of Bertozzi's experiment (Table 8.2). Bertozzi's experiment appears to be in agreement with relativistic and radiational electrodynamics but away from classical electrodynamics.

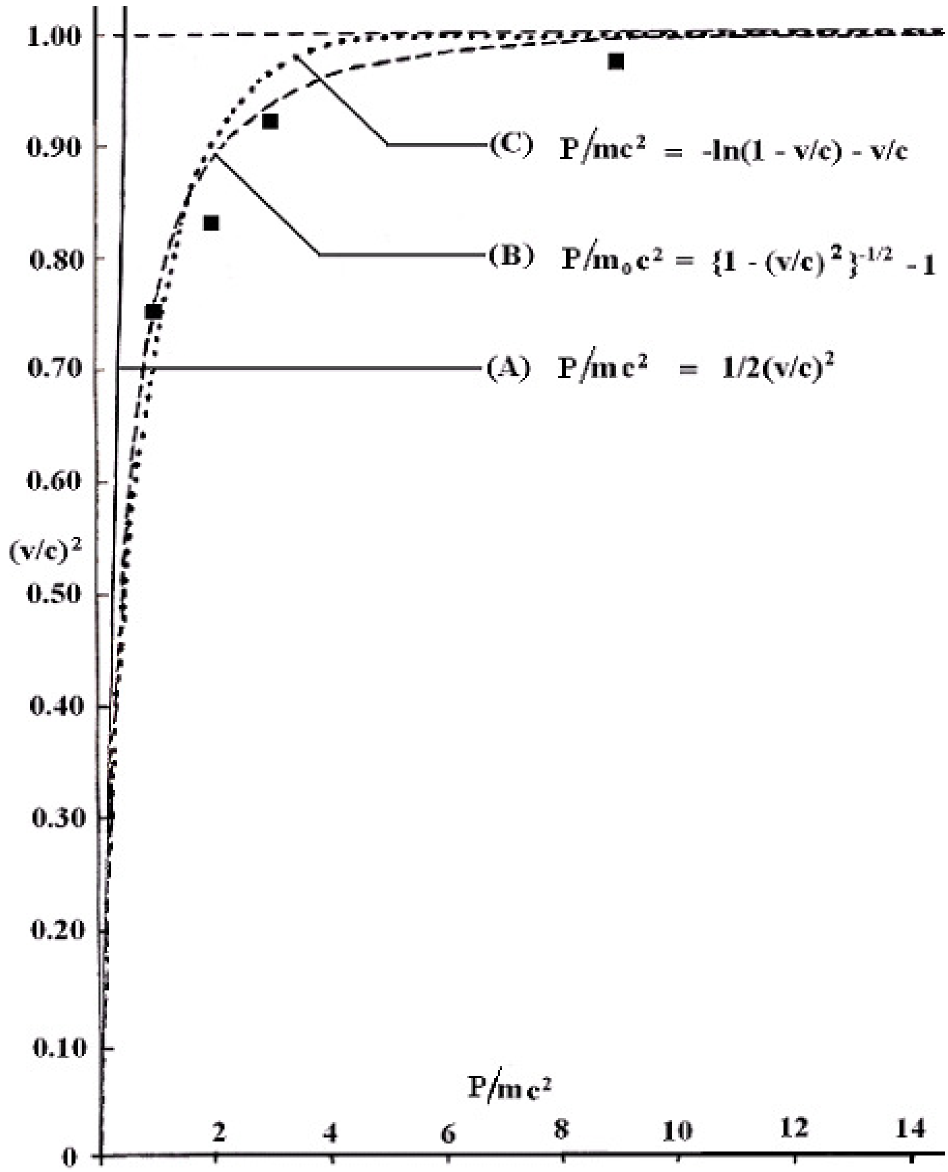

Figure 8.3. v^2/c^2 (speed squared in units of c^2) against P/mc^2 (potential energy in units of mc^2) for an electron of mass m accelerated from zero initial speed (c is speed of light); the solid line (A) according to classical electrodynamics equation (8.6), the dashed curve according to relativistic electrodynamics (equation 8.11) and the dotted curve (C) according to *radiational electrodynamics* (equation 8.23). The solid squares are the result of Bertozzi's experiment (Table 8.2).

8.7 Radius of revolution in a circle

In classical electrodynamics, the radius of circular revolution of an electron of charge $-e$ and mass $m = m_o$ in an electrostatic field of magnitude E due to a central source charge, as given by equation (8.9), is:

$$r_o = \frac{m_o v^2}{eE} \tag{8.31}$$

where r_o is the classical radius. In relativistic electrodynamics, the radius of circular revolution, as given by equation (8.13), is:

$$r = \frac{mv^2}{eE} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{8.32}$$

Radiational electrodynamics (equation 8.30) gives the radius of circular revolution as:

$$r = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{8.33}$$

8.8 Conclusion

Roger's experimental results (Table 8.1) are in agreement with relativistic electrodynamics (equation 8.11) and *radiational electrodynamics* (equation 8.23) for an electron of charge $-e$ and mass m revolving with speed v in a circle of radius r, under the influence of a radial electrostatic field of magnitude E. It could, therefore, be concluded that the predicted increase of mass of a particle with its speed, might have been confirmed.

The relativistic mass *"m"* in equation 1, is not a physical mass, but the ratio of the force *(eE)* on a stationary charged particle to the centripetal acceleration (v^2/r) in circular motion. This ratio may be infinite, for motion in a circle of infinite radius (a straight line). Thus, we have the speed of light as the limit, without infinite mass.

Relativistic electrodynamics and *radiational electrodynamics* merge to classical at very low speeds. Relativistic electrodynamics and *radiational electrodynamics* give zero acceleration at the speed of light. In relativistic electrodynamics, an electron cannot attain the speed of light

c, no matter the magnitude of accelerating potential. In *radiational electrodynamics* an electron is easily accelerated to the speed of light by a potential energy of 15 Mev or over.

Relativistic electrodynamics (equation 8.32) gives the same expression, for radius of circular revolution of an electron, as *radiational electrodynamics* (equation 8.33). While the radius can increase and become infinite for motion in a straight line, the mass (equation 8.1), a physical quantity, cannot expand and become infinitely large while its dimension reduces to zero.

Decrease of accelerating force with speed, as predicted by *radiational electrodynamics*, gives the same effect as apparent increase of mass with speed, as far as circular revolution of an electron is concerned. What actually increases with speed is the radius of revolution in circular motion as expressed in equation 8.33. This radius can become infinite with speed, for rectilinear motion. So, if *radiational electrodynamics* is valid, the relativistic mass-velocity formula is applicable only to circular revolution of an electron round a centre of force of attraction. Applying this formula, to rectilinear motion of electrons in a linear accelerator, is questionable.

Bertozzi's experimental results (Table 8.2) are in agreement with relativistic electrodynamics (equation 8.11) and *radiational electrodynamics* (equation 8.23) for an electron in rectilinear motion, as depicted in Fig. 8.3. The two systems of electrodynamics demonstrate the speed of light c as a limit; relativistic electrodynamics on the basis of mass, of a moving particle, becoming infinitely large at the speed of light and *radiational electrodynamics* on the basis of accelerating force, on a charged particle, reducing to zero at that speed.

The question now is: "Which one of the electrodynamics is correct?" The answer may be found in the motion of electrons decelerated from the speed of light c. According to relativistic electrodynamics, an electron, moving at the speed of light (with infinite mass and infinite kinetic energy), cannot be stopped by any finite force. In *radiational electrodynamics*, an electron moving at the speed of light, is easily brought to rest (equation 8.28), on entering a decelerating field, after gaining potential energy equal to $0.307mc^2$ and radiating energy equal to $0.193mc^2$. The electron may then be accelerated to reach a speed equal to $-c$. An electron moving at the speed of light, being stopped at all, contradicts the theory of special relativity.

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9. LONGITUDINAL AND TRANSVERSE ELECTRIC WAVE PROPAGATION IN A MEDIUM

Abstract

An electric field of flux density D may be considered as an "elastic medium" having "density ρ" = $\mu_o D^2$ and exerting "pressure P" = D^2/ε_o, where μ_o is the permeability and ε_o the permittivity of vacuum. Longitudinal waves, excited by oscillations of charged particles in a medium, are propagated at the speed of light $c = \sqrt{\dfrac{P}{\rho}} = \sqrt{\dfrac{1}{\mu_o \varepsilon_o}}$. The longitudinal waves, between the charged particles, are absorbed by the oscillating particles. Transverse waves, set up by the oscillating particles, are emitted as electromagnetic radiation, propagated at the speed of light c.

Keywords: density, light, polarization, pressure, velocity, waves

9.1 Introduction

A wave is a process by which an influence reaches out from one body to another without transfer of matter, mass or electric charge. This aspect of wave motion was elucidated by C.G. Darwin (*New Conceptions of Matter*, 1931, p. 29) [1], Professor of Physics at the University of Edinburgh, who wrote:

"The most elementary way in which I can attract anyone's attention is to throw a stone at him. Another is to poke him with a stick, without transfer of matter from me to him - a small motion that I produce in the stick at my end, turns into a small motion at his end".

A small motion at one end of a medium producing a small motion at the other end, without transfer of matter, takes time to travel in or along the medium. The small motion is transmitted as a succession of pulses or a wave of displacements or compressions and rarefactions along the length of the medium which may be solid, liquid, gas or an electric field. The speed of transmission depends on some physical properties, such as elasticity and density of the medium.

The small displacements, oscillations or vibrations in a wave motion may be in the direction of propagation, in which case we have a longitudinal wave, like vibrations in a rigid rod or like sound waves in an air column. Where the displacements are perpendicular to the direction of propagation, we get a transverse wave, like waves in a stretched string, waves on the surface of water or electromagnetic waves.

James Clerk Maxwell (1865) [2], in his epoch-making treatise, showed that waves from oscillating electric currents consisted of transverse vibrations of electric and magnetic fields propagated at the speed of light c, hence the term electromagnetic waves. He concluded that electromagnetic radiation is a vast spectrum from radio frequencies at the lower end, through microwave, infrared light, visible light and ultraviolet light to x-rays and gamma rays at the higher end, all propagated at speed $c = \sqrt{\frac{1}{\mu_o \varepsilon_o}} = 2.998 \times 10^8\ m/s$) in free space or vacuum, where μ_o is the permeability and ε_o the permittivity of a vacuum. .

The spectrum of visible (white) light range from wavelength (red) $4 \times 10^{-7}\ m$ to (violet) $7 \times 10^{-7}\ m$. Found below the visible spectrum are the infrared rays ($10^{-6}\ m$ to $10^{-3}\ m$). Beyond the visible region are the ultraviolet radiation ($1.5 \times 10^{-8}\ m$ to $10^{-7}\ m$), the x-rays and the gamma rays. The light radiations are produced from the motions of charged particles in the electric fields [3, 4] of the atoms of a material.

This paper considers electric field as an "elastic medium" having "density ρ" and exerting "pressure P". As such an electric field supports the propagation of a longitudinal wave with the speed of light $c = \sqrt{\frac{1}{\mu_o \varepsilon_o}} = \sqrt{\frac{P}{\rho}}$. Such a wave may be excited by the motion of electric charges in an electric field. A longitudinal wave, which exists between the electric charges in a body, is not, in any way, polarized.

A transverse wave, on the other hand, is always polarised. Transverse electromagnetic waves are quickly attenuated in a conducting medium. The attenuation is due to a phenomenon called Skin Effect [5, 6, 7].

In a transverse wave the oscillations of the fields are perpendicular to the direction of propagation. The electric vector gives the polarization. Where the transverse oscillations of the electric fields are not random but more in one plane than the other, the wave is said to be polarized.

9.2 "Pressure" in an electric field

Figure 9.1 shows an electric field of intensity $\boldsymbol{E}$ cutting normally through an element of (shaded) surface area $(\delta \boldsymbol{A})$ at a point X distance $(x + l)$ from an origin. The quantity of electric charge (δq) encompassed by the (shaded) area $(\delta \boldsymbol{A})$ is given by Gauss's law as the scalar product:

$$(\delta q) = \varepsilon_o \mathbf{E}.(\delta \mathbf{A}) = \varepsilon_o E(\delta A) \tag{9.1}$$

where ε_o is the permittivity of free space. The force $(\delta \boldsymbol{F})$ exerted over the surface area $(\delta \boldsymbol{A})$ is:

$$(\delta \mathbf{F}) = \varepsilon_o E(\delta A)\mathbf{E} = \varepsilon_o E^2 (\delta A)\hat{\mathbf{u}} \tag{9.2}$$

where $\hat{\boldsymbol{u}}$ is a unit vector in the direction of the electric field as well as the area. Equation (9.2) gives the force per unit area, equal to the pressure P or stress at R, as:

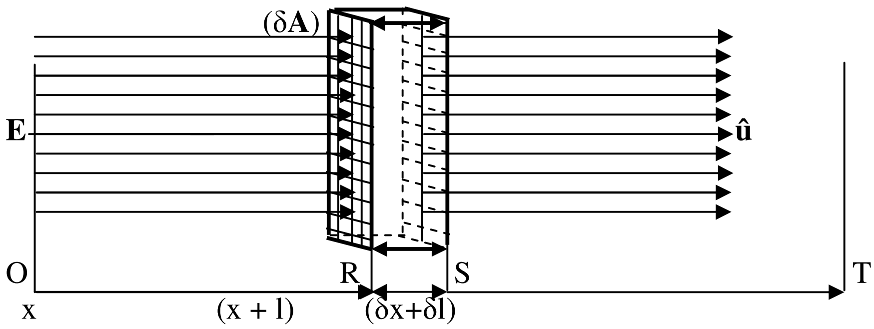

Figure 9.1. Electric field $\boldsymbol{E}$ across an element of surface area $(\delta \boldsymbol{A})$ at R

$$P = \varepsilon_o E^2 = \frac{D^2}{\varepsilon_o} \tag{9.3}$$

where $D = \varepsilon_o E_o$ is the electric flux density.

9.3 "Elasticity" of an electrostatic field

An electric field may be stressed and strained, like a column of air or an elastic rod. Thus an electric field is "elastic". So, a stream of compressions and rarefactions may propagate, at the speed of light c, along an electrostatic field, to constitute a longitudinal electric wave.

In Figure 9.1, if a segment of an electric field, of intensity $\boldsymbol{E}$ and length (δx) across surface area $(\delta \boldsymbol{A})$ at R, suffers an elongation (rarefaction) or shortening (compression) of length (δl), the strain is $(\delta l)/(\delta x)$. The total force $\boldsymbol{f}$ across the (shaded) surface area $(\delta \boldsymbol{A})$ at R, as a result of the electrostatic force (equation 9.2) and the strain in addition, is proposed as:

$$\mathbf{f} = \varepsilon_o E^2 (\delta \mathbf{A})\left(1 + \frac{\delta l}{\delta x}\right) = P(\delta \mathbf{A})\left(1 + \frac{\delta l}{\delta x}\right) \tag{9.4}$$

The validity of equation (9.4) lies on its leading to the correct result in order to arrive at the wave equation and derive an expression for the speed of a longitudinal wave in an electrostatic field.

9.4 "Density" of an electric field

In Figure 9.1, the potential V at X is given by the integral:

$$V = \int \mathbf{E}.(d\mathbf{x}) = \int E(dx)$$

If the electric field at R is increased by a small amount $(\delta \boldsymbol{E})$. The corresponding increase in the electric charge is $\boldsymbol{\varepsilon_o(\delta E).(\delta A)}$. Work done in increasing the charge, by infinitesimal amounts, is:

$$w = \iiint \varepsilon_o (d\mathbf{E}).(d\mathbf{A}) E(dx) = \iint \varepsilon_o E(dE)(d\tau) \tag{9.5}$$

where $(\delta\tau)$ is an element of volume. Integrating equation (9.5) gives:

$$w = \iint \varepsilon_o E(dE)(d\tau) = \frac{1}{2}\int \varepsilon_o E^2 (d\tau) \tag{9.6}$$

The energy per unit volume υ, is:

$$\upsilon = \frac{1}{2}\varepsilon_o E^2 = \frac{D^2}{2\varepsilon_o} \tag{9.7}$$

It should be noted that equation (9.7) for energy per unit volume, is not the same as equation (9.3) for pressure P.

The author [8] showed that the energy content of a mass m at rest, is

$$w = \frac{m}{2\mu_o \varepsilon_o} \tag{9.8}$$

where μ_o is the permeability of free space. If "ρ" is the "density" of the electric field, the energy per unit volume, is:

$$\upsilon = \frac{D^2}{2\varepsilon_o} = \frac{\rho}{2\mu_o \varepsilon_o}$$

$$\rho = \mu_o D^2 \tag{9.9}$$

Speed of wave propagation along the electric field is obtained as:

$$c = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{D^2}{\mu_o \varepsilon_o D^2}} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} \tag{9.10}$$

9.5 Longitudinal wave equation

Figure 9.1 shows an element of (shaded) surface area *(δ**A**)* at a point *R* distance *(x +* 1*)* from an origin. A segment of the electric field with surface area *(δ**A**)* and initial width *(δ**x**)* oscillates between two points *O* and *T*, through a displacement *l*, with speed *(dl/dt)* at time *t*.

At the point *O*, the segment has original volume *(δA)(δx)*. On moving from *O* through a distance *l*, with speed *(dl/dt)*, let the segment increase in width from the initial *(δx)* to *(δx + δl)* so that the volume increases by *(δA)(δl)*. The segment, of mass *ρ(δA)(δx)*, oscillates with speed *(dl/dt)* and acceleration (d^2l/dt^2) such that the accelerating force on the mass is equal to the impressed force on the segment *XR* of original volume *(δA)(δx)*.

The segment *RS*, in getting its width increased from *(δx)* to *(δx+δl)*, suffers a strain equal to *(δl/δx)*. The force ***f*** on (shaded) surface area *(δ**A**)*, with stress P, at *R*, is given by equation (9.4) as:

$$\mathbf{f} = P(\delta\mathbf{A})\left(1 + \frac{\delta l}{\delta x}\right) \tag{9.11}$$

The difference in force between the (shaded) surfaces at *R* and *S*, within a distance *(δx)*, is:

$$\frac{d\mathbf{f}}{dx}(\delta x) = P(\delta\mathbf{A})\left(\frac{d^2 l}{dx^2}\right)(\delta x) \tag{9.12}$$

Equation (9.12) gives the impressed force on the segment *RS*, which gives acceleration (d^2l/dt^2) on mass *ρ(δA)(δx)*. Newton's second law of motion gives the equation:

$$P(\delta\mathbf{A})\left(\frac{d^2 l}{dx^2}\right)(\delta x) = \rho(\delta\mathbf{A})(\delta x)\left(\frac{d^2 l}{dt^2}\right)$$

$$\left(\frac{d^2 l}{dx^2}\right) = \frac{\rho}{P}\left(\frac{d^2 l}{dt^2}\right) \tag{9.13}$$

$$\left(\frac{d^2l}{dx^2}\right)=\frac{\rho}{P}\left(\frac{d^2l}{dt^2}\right)=\mu_o\varepsilon_o\left(\frac{d^2l}{dt^2}\right)=\frac{1}{c^2}\left(\frac{d^2l}{dt^2}\right) \tag{9.14}$$

This is the wave equation, for a longitudinal wave with displacement l at time t, propagated with speed c in the **x**-direction.

9.6 Transverse wave equation

Consider Figure 9.2 showing an electric charge of magnitude Q oscillating in the vertical direction due to an externally applied force:

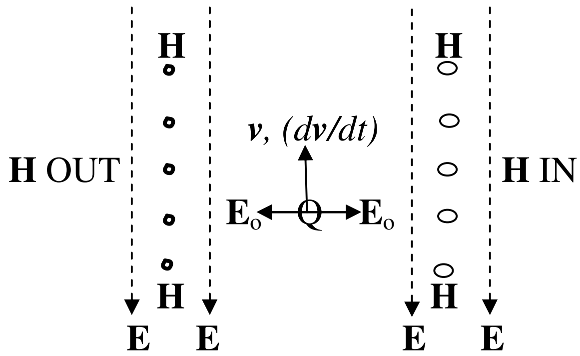

Figure 9.2 An electric charge Q moving with velocity $\boldsymbol{v}$ and acceleration *(dv/dt)* at time t producing a magnetic field of intensity $\boldsymbol{H}$ (out of the page on the left and into the page on the right) and electrodynamic field of intensity $\boldsymbol{E}$ downwards.

The charge Q moving with velocity $\boldsymbol{v}$ (in the **z**-direction) vertically and acceleration *(dv/dt)* at time t, creates a magnetic field $\boldsymbol{H}$ given by:

$$\mathbf{H}=\varepsilon_o\mathbf{v}\times\mathbf{E}_o \tag{9.15}$$

where $\boldsymbol{E}_o$ is the radial electrostatic field of the charge and ε_o the permittivity of vacuum. The electrodynamic field of intensity $\boldsymbol{E}$ in the downward direction (the –**z** direction) is as shown in Figure 9.2. The dynamic configuration gives rise to an electromagnetic wave propagated horizontally, away from the charge.

A changing magnetic field $\boldsymbol{H}$ induces a voltage by creating electric field $\boldsymbol{E}$, according to Faraday's law [9] as given by Maxwell's equation:

$$\nabla \times \mathbf{E} = -\mu_o \frac{\partial \mathbf{H}}{\partial t} \tag{9.16}$$

where $\nabla\times$ denotes the curl of a vector and μ_o is permeability of vacuum.

In free space (vacuum) and in the absence of any conduction current, Ampere's law gives the Maxwell equation [10]:

$$\nabla \times \mathbf{H} = \varepsilon_o \frac{\partial \mathbf{E}}{\partial t} \tag{9.17}$$

whee ε_o is the permittivity of vacuum. Equations (9.15) and (9.17) give:

$$\mathbf{E} = -\mu_o \varepsilon_o \varphi \frac{d\mathbf{v}}{dt} \tag{9.18}$$

where φ is the electrostatic potential at a point due to the charge and $\boldsymbol{E_o} = -\nabla\varphi$ is the electrostatic field.

Taking the curl of equation (9.16) gives:

$$\nabla \times \nabla \times \mathbf{E} = -\mu_o \frac{\partial}{\partial t} \nabla \times \mathbf{H}$$

$$\nabla \nabla . \mathbf{E} - \nabla^2 \mathbf{E} = -\mu_o \varepsilon_o \frac{\partial}{\partial t} \frac{\partial \mathbf{E}}{\partial t}$$

where ∇**.** denotes the divergence of a vector. Since there is no distribution of charges, ∇**.E** is zero and we obtain:

$$\nabla^2 \mathbf{E} = \mu_o \varepsilon_o \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{9.19}$$

$$\frac{\partial^2 E_z}{\partial x^2} = \mu_o \varepsilon_o \frac{\partial^2 E_z}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} \tag{9.20}$$

This is the one-dimensional transverse wave equation with electric field E_z at time t, propagated with speed c in the **x**-direction.

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9.7 Skin Effect

A transverse electromagnetic wave is polarized with the electric and magnetic field vectors orthogonal to one another and perpendicular to the direction of propagation. An electromagnetic wave, normally incident on the surface of a medium, has the electric and magnetic fields lying on the surface. The wave may be reflected, absorbed or transmitted depending on the nature of the medium. For a conducting medium the wave is attenuated and absorbed within a short distance of penetration due to a phenomenon called Skin Effect. The attenuation is a result of induced current flow and dissipation of energy in the medium. The Skin Thickness or Skin Depth is the distance of penetration within which the current density is reduced to *1/e* (about *1/2.783*) of its magnitude at the surface.

Salty sea water, a poor conductor of electricity, has conductivity of about $5\ (ohm\text{-}m)^{-1}$. The Skin Thickness for an electromagnetic wave of radio frequency $3 \times 10^7\ Hz$ (wavelength *10 m*), falling on sea water, is $4.1 \times 10^{-2}\ m$. Thus radio communication should not be possible between two aerials, some metres apart, submerged in the sea. For yellow light of wavelength $6 \times 10^{-7}\ m$ (frequency $5 \times 10^{14}\ Hz$), in sea water, the Skin Thickness might be $10^{-5}\ m$, a very thin thickness indeed. So, sea water should have been opaque to light radiation, but it is not. Is there any difference between light and radio waves as electromagnetic radiations? This issue needs to be resolved.

9.8 Polarization of light

Figure 9.3 depicts a ray of polarized light *PO* from a source at a point *P*, incident on a plane surface *AB* of a medium. *ON* is the normal at the point of incidence *O* and ι is the angle of incidence. *OQ* is the reflected ray at angle ρ to the normal. *OR* is the refracted ray and τ is the angle of refraction. Transverse electric field oscillations, in the plane of incidence (surface of the paper) are indicated as double arrows and oscillations perpendicular to the plane of incidence are shown as small circles.

The law of reflection makes $\rho = \iota$ and Snell's law gives:

$$\frac{\sin \iota}{\sin \tau} = \mu \tag{9.21}$$

At an angle of incidence, other than 0 or π/2 radians, less of vibrations in the plane of incidence are reflected, while components perpendicular to the plane are not affected. So, reflected light is polarized perpendicular to the plane of incidence. At a particular angle of incidence β, called *rewster angle* or *Polarization angle*, the reflection of components in the plane of

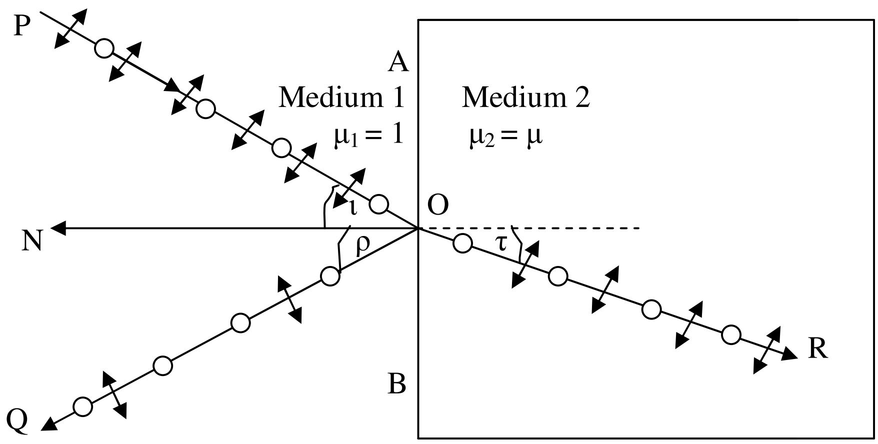

Figure 9.3 A ray of light *PO* reflected from a plane surface *AB*, with *OQ* as the reflected ray and *OR* the refracted ray. The transverse oscillations in the plane of incidence (surface of the paper) are shown as double arrows and oscillations perpendicular to the plane of incidence as small circles along the ray.

incidence is zero. At this angle ($\iota = \beta = \rho$), transverse vibrations (in the plane of incidence), in the refracted ray, should be in the direction of the reflected ray; the reflected ray being perpendicular to the refracted ray, making $(\tau + \beta) = \pi/2$ radians, to give the equation:

$$\frac{\sin\beta}{\sin\tau} = \frac{\sin\beta}{\sin\left(\dfrac{\pi}{2} - \beta\right)} = \frac{\sin\beta}{\cos\beta} = \tan\beta = \mu \qquad (9.20)$$

Fresnel's equations [11] give the ratio of intensities of each of the two polarization components of light which is reflected or refracted between two media with different indices of refraction, in terms of the angle of incidence and angle of refraction..

9.9 Conclusion

An electric field may be considered as an elastic medium which can be stressed and strained in relation to moving charged particles. As such, there could be propagation of longitudinal waves, at the speed of light, between the charged particles. The longitudinal oscillations are absorbed by the atomic particles, with a manifestation of emission of radiation as heat or light,

Motions of electric charges also give rise to transverse waves with oscillations perpendicular to the direction of propagation. The transverse are emitted and propagated in space as electromagnetic radiation.

The fact that fish can be seen swimming in clear sea water and divers use search lights to find their way in the depth of the sea indicates that light waves are transmitted in sea water, whereas radio waves are readily attenuated even in a poor conductor like salty sea water. Perhaps, the difference in polarization may explain why radio waves are quickly attenuated in sea water while light waves may be transmitted. It may as well be that light radiation is more of a longitudinal wave than a transverse one.

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APPENDIX 1

"The experts in a particular field can become so indoctrinated and so committed to the current paradigm that their critical and imaginative powers are inhibited and they cannot see 'see beyond their own noses'. In these circumstances, scientific progress may come to a halt – knowledge may even regress – until intellectual intruders come through the interdisciplinary frontiers and look at the field without preconceptions".

Joseph Ziman, *Reliable Knowledge: An Explanation of the Grounds for Belief in Science*, Cambridge University Press (1978), p. 134.

A COMPARISON OF SOME EQUATIONS IN CLASSICAL, RELATIVISTIC AND RADIATIONAL ELECTRODYNAMICS

There are now three systems of electrodynamics that happen to be applicable under different situations. Classical electrodynamics is applicable to electrically charged particles moving at very low speeds compared to the speed of light, relativistic electrodynamics is for charged particles moving at speeds comparable to that of light and quantum electrodynamics is for atomic particles moving at high speeds. Radiational electrodynamics is a consistent system of electrodynamics applicable to all charged particles moving at speeds up to that of light, with emission of radiation and the mass of a moving particle remaining constant. A comparison of some important equations in A classical, B relativistic and C radiational electrodynamics is given in the Table below.

(***Vectors*** are indicated in **boldface** type and *scalars* in ordinary type)

A. CLASSICAL ELECTRODYNAMICS	**B. RELATIVISTIC ELECTRODYNAMICS**	**C. RADIATIONAL ELECTRODYNAMICS**
1. Mass m of a particle is independent of its speed v at time t: $m = m_o$ where m_o is the rest mass (at speed $v = 0$, with respect to an observer).	**1.** Mass m of a particle increases with its speed v at time t: $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_o$ m becomes ∞ at $v = c$	**1.** Mass m of a particle is independent of its speed v at time t: $m = m_o$ where m_o is the rest mass (at peed $v = 0$, with respect to an observer)

2. Magnitude of electric charge q is independent of its speed v at time t: $q = q_o$ (rest charge)	**2.** Magnitude of electric charge q independent of its speed v at time t: $q = q_o$ (rest charge)	**2.** Magnitude of electric charge q independent of its speed v at time t: $q = q_o$ (rest charge)
3. NEWTON'S 2nd LAW OF MOTION: **Force** $\mathbf{F} = m\frac{d\mathbf{v}}{dt}$, ***v*** is the velocity and $\frac{d\mathbf{v}}{dt}$ the acceleration of constant mass *m*, at time *t*.	**3.** NEWTON'S 2nd LAW OF MOTION: **Force** $\mathbf{F} = \frac{d}{dt}(m\mathbf{v})$, ***v*** is the velocity and $(m\mathbf{v})$ the momentum of mass *m*, which depends on velocity at time *t*	**3.** NEWTON'S 2nd LAW OF MOTION: **Force** $\mathbf{F} = m\frac{d\mathbf{v}}{dt}$, ***v*** is the velocity and $\frac{d\mathbf{v}}{dt}$ the acceleration of constant mass *m*, at time *t*.
4. COULOMB'S LAW ON A MOVING CHARGED PARTICLE **4.1** A particle of charge *q* and mass *m*, moving with velocity ***v***, at time t, under the acceleration *(d**v**/dt)* of electrostatic field of intensity ***E***...... For a stationary source charge Q, accelerating force ***F***, is $\mathbf{F} = \frac{Qq}{kZ^2}\hat{\mathbf{u}} = \mathbf{E}q$ $\mathbf{E} = \frac{Q}{kZ^2}\hat{\mathbf{u}}$ is the field. where *k* is a constant, *Z* is the distance between the charges (Q and q) and **û** a unit vector in the direction of force of repulsion and $\mathbf{F} = \mathbf{E}q = m\frac{d\mathbf{v}}{dt}$	**4.** COULOMB'S LAW ON A MOVING CHARGED PARTICLE **4.1** A particle of charge *q* and mass *m*, moving with velocity ***v***, at time t, under the acceleration *(d**v**/dt)* of an electrostatic field of intensity ***E***........ For a stationary source charge Q, accelerating force ***F***, is: $\mathbf{F} = \frac{Qq}{kZ^2}\hat{\mathbf{u}} = \mathbf{E}q$ $\mathbf{E} = \frac{Q}{kZ^2}\hat{\mathbf{u}}$ is the field**.** where *k* is a constant, *Z* is the distance between the charges (Q and q) and **û** a unit vector in the direction of force of repulsion and $\mathbf{F} = \mathbf{E}q = \frac{d}{dt}(m\mathbf{v})$	**4.** COULOMB'S LAW ON A MOVING CHARGED PARTICLE **4.1** A particle of charge *q* and mass *m*, moving with velocity ***v***, at time t, under the acceleration *(d**v**/dt)* of an electrostatic field of intensity ***E***..................... For a stationary source charge Q, accelerating force ***F***, is: $\mathbf{F} = \frac{Qq}{ckZ^2}(\mathbf{c} - \mathbf{v}) =$ $E = \frac{Q}{kZ^2}$ the magnitude of the electrostatic field. where *k* is a constant, *Z* is the distance between the charges (Q and q), **c** is the velocity of light at the aberration angle and $\mathbf{F} = \frac{Eq}{c}(\mathbf{c} - \mathbf{v}) = m\frac{d\mathbf{v}}{dt}$

In rectilinear motion ***E*** and ***v*** are collinear and $Eq = m\frac{dv}{dt}$. For motion in a uniform electrostatic field of intensity *E* with $m = m_o$ as a constant, **Speed**: $v = at$ where $a = \frac{Eq}{m}$ is a constant and speed *v* = *0* at time *t* = *0*. Maximum attainable speed, as $t \to \infty$, is infinitely large, contrary to observation on accelerated particles.	In rectilinear motion, ***E*** and ***v*** are collinear and $Eq = \frac{d}{dt}(mv)$ For motion in a uniform field of intensity *E* with *m* dependent on speed *v*, **Speed**: $v = \frac{at}{\sqrt{1+\left(\frac{at}{c}\right)^2}}$, where $a = \frac{Eq}{m_o}$ is a constant and *v* = *0* at *t* = *0*. Speed of light *c* is the maximum attainable, as time $t \to \infty$	In rectilinear motion, ***c*** and ***v*** are collinear so that $\frac{Eq}{c}(c - v)\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}}$ $Eq(1 - \frac{v}{c}) = m\frac{dv}{dt}$, For motion in a uniform field of intensity *E* with $m = m_o$ as a constant. **Speed**: $v = c\left(1 - e^{\frac{-at}{c}}\right)$ where $a = \frac{Eq}{m}$ is a constant and *v* = *0* at *t* = *0*. Speed of light *c* is the maximum attainable, as time $t \to \infty$
4.2 For a particle of charge *q* and mass *m* decelerated from the speed of light *c* by a uniform electrostatic field ***E***, the decelerating force ***F***, with velocity ***v*** at time *t*, is: $\mathbf{F} = \mathbf{E}q = -m\frac{d\mathbf{v}}{dt}$ For rectilinear motion: $Eq = -m\frac{dv}{dt}$ **Speed:** $v = c - at$ with $a = \frac{Eq}{m}$ constant. Maximum speed as time $t \to \infty$, is minus infinity ($-\infty$), contrary to observations	**4.2** For a particle of charge *q* and mass *m* decelerated from the speed of light *c* by a uniform electrostatic field ***E***, the decelerating force ***F***, with velocity ***v*** at time t, is: $\mathbf{F} = \mathbf{E}q = -\frac{d}{dt}(m\mathbf{v})$ For rectilinear motion, $Eq = -\frac{d}{dt}(mv)$ *m* dependent on speed. **Speed:** $v = c$ A particle moving at the speed of light *c*, cannot be stopped by any finite force. It continues to move with the speed c.	**4.2** For a particle of charge *q* and mass *m* decelerated from the speed of light *c* by a uniform electrostatic field of intensity ***E***, the decelerating force ***F*** is: $\mathbf{F} = \frac{Eq}{c}(\mathbf{c} + \mathbf{v}) = m\frac{d\mathbf{v}}{dt}$ where ***v*** is velocity at time t and ***c*** is velocity of light at the aberration angle. For rectilinear motion: $Eq(1 + \frac{v}{c}) = -m\frac{dv}{dt}$ **Speed**: $v = c\left(2e^{\frac{-at}{c}} - 1\right)$ Speed as $t \to \infty$, is $-c$

5. NEWTON'S UNIVERSAL LAW OF GRAVITATION Force of attraction **F** between two bodies is proportional to the product of their masses M_1 and M_2, inversely proportional to the square of their separation Z in space and independent of their relative velocity. $\mathbf{F} = -G\frac{M_1 M_2}{Z^2}\hat{\mathbf{u}}$ where G is the gravitational constant and $\hat{\mathbf{u}}$ a unit vector in the direction of force of repulsion.	**5.** NEWTON'S UNIVERSAL LAW OF GRAVITATION $\mathbf{F} = -G\frac{M_1 M_2}{Z^2}\hat{\mathbf{u}}$ **5.1** In the Theory of General Relativity (Einstein's theory of gravitation), gravitation is the result of curvature, warping or distortion of (four-dimensional) *space-time continuum* due to the presence of matter. Bodies move along a path of least resistance, a straight line in (four-dimensional) *space-time*, but, in reality, a curved trajectory in (three-dimensional) *space*.	**5.** NEWTON'S UNIVERSAL LAW OF GRAVITATION $\mathbf{F} = -G\frac{M_1 M_2}{Z^2}\hat{\mathbf{u}}$ **5.1** The electrostatic forces of repulsion and attraction between the electric charges in two (neutral) masses M_1 and M_2, cancel out exactly. **5.2** The mass M_1 or M_2 of a body is proportional to the <u>sum of squares of the constituent charges</u> **5.3** Gravitational force between two masses M_1 and M_2, being proportional to <u>product of the sum of squares of the constituent electric charges</u>, remain positive and attractive.
6. Relative velocity ***w*** between two bodies moving with velocities ***u*** and ***v*** (relative to a frame of reference), is: $\mathbf{w} = \mathbf{u} - \mathbf{v}$ where ***u*** and ***v*** are vectors in any direction.	**6.** Relative velocity ***w*** between two bodies moving with velocities ***u*** and ***v*** (relative to a frame of reference), is: $\mathbf{w} = (\mathbf{u} - \mathbf{v})\left(1 - \frac{uv}{c^2}\right)^{-1}$ ***u*** and ***v*** are collinear.	**6.** Relative velocity ***w*** between two bodies moving with velocities ***u*** and ***v*** (relative to a frame a reference), is: $\mathbf{w} = \mathbf{u} - \mathbf{v}$ ***u*** and ***v*** are vectors in any direction.
7. Velocity of light ***z*** from a source moving with velocity ***u***, as seen by an observer moving with velocity ***v*** (relative to a frame of reference), is: $\mathbf{z} = \mathbf{c} + (\mathbf{u} - \mathbf{v})$ where ***c*** is the velocity of	**7.** Velocity of light ***z*** from a source moving with velocity ***u***, as seen by an observer moving with velocity ***v*** (relative to a frame of reference), is: $\mathbf{z} = \mathbf{c}$ where ***c*** is the velocity of	**7.** Velocity of light ***z*** from a source moving with velocity ***u***, relative to an observer moving with velocity ***v***, is: $\mathbf{z} = \mathbf{c} + (\mathbf{u} - \mathbf{v})$ where ***c*** is the velocity of light relative to the source.

light relative to the source. **7.1** Velocity of propagation of an electrostatic force is ∞ (infinite) **7.2** Velocity of transmission of electrostatic force, relative to a charged particle moving with velocity **v**, is: ∞ of infinite magnitude	light in a vacuum, an absolute constant. **7.1** Velocity of propagation of an electrostatic force is ∞ (infinite) **7.2** Velocity of transmission of electrostatic force, relative to a charged particle moving with velocity **v**, is: ∞ of infinite magnitude	**7.1** Velocity of propagation of an electrostatic force is **c** **7.2** Velocity of transmission of electrostatic force, relative to a charged particle moving with velocity **v**, is: $\mathbf{s} = \mathbf{c} - \mathbf{v}$ **7.3** Velocity of light ***c*** is a vector quantity of constant magnitude $c = 2.998 \times 10^8$ *m/s*, relative to the source.
8. Kinetic energy K of a body of mass $m = m_o$ moving with speed v, is: $K = \frac{1}{2}mv^2$ (Classical mechanics does not reckon with the internal energy content of a body of mass m)	**8.** Total energy E_m of a body of rest mass m_o moving at speed v, is: $E_m = mc^2 = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ where *c* is the speed of light in a vacuum.	**8.** Total energy E_m of a body of mass $m = m_o$ moving at speed v, is: $E_m = \frac{1}{2}m(c^2 + v^2)$ where *c* is the speed of light, which is a constant relative to the source only. Energy content $E = 1/2\, mc^2$
9. Radiation reaction force $\boldsymbol{R_f}$ due to a particle of charge q moving with velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$, at time t. $\mathbf{R}_f = \frac{q^2}{6\pi\varepsilon_o c^3}\frac{d\mathbf{a}}{dt}$ *(Abraham-Lorentz formula)* *where c is* the speed of light in a vacuum. {There should be no radiation force in circular motion with constant speed as (d**a**/dt) = 0	**9.** THE FORMULA FOR RADIATION REACTION FORCE, IN RELATIVISTIC ELECTRODYNAMICS, HAS SOME DIFFICULTIES. See David Griffith, *Introduction to Electrodynamics*, Prentice-Hall Inc., New Jersey, 1981, p. 382 Without radiation force, there should be no radiation power	**9.** Radiation reaction force $\boldsymbol{R_f}$ due to a particle of charge q moving with speed v in the direction of an electrostatic field of intensity $\boldsymbol{E}$: $\mathbf{R}_f = -\frac{qv}{c}\mathbf{E} = -Eq\frac{\mathbf{v}}{c}$ *where c* is the speed of light, which is a constant relative to the source only. (At the speed of light, a moving charged particle takes the form of light radiation)

10. RADIATION POWER **10.1** Radiation power R_p of a particle of charge *q* moving with speed *v* and acceleration a, is: $R_p = \frac{q^2 a^2}{6\pi\varepsilon_o c^3}$ *(Larmor formula)* where *c* is the speed of light (*v* « *c*) in a vacuum. **10.2** Radiation power R_p of a particle of charge *q* moving with speed *v* *and* centripetal acceleration $a = v^2/r$, in a circle of radius *r*, is $R_p = \frac{q^2 v^4}{6\pi\varepsilon_o r^2 c^3}$ *(Larmor formula)* (This formula influenced physics, early in the 20th century, leading to Bohr's quantum theory of the hydrogen atom).	**10.** RADIATION POWER **10.1** Radiation power R_p of a particle of charge *q* moving with speed *v* and acceleration *a* in the same direction, is: $R_p = \frac{q^2 a^2 \gamma^4}{6\pi\varepsilon_o c^3}$ *(Lienard formula)* where *c* is speed of light and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ **10.2** Radiation power R_p of a particle of charge *q* moving with speed *v and* acceleration $a = v^2/r$, in a circle of radius *r*, is $R_p = \frac{q^2 v^4 \gamma^4}{6\pi\varepsilon_o r^2 c^3}$ *(Lienard formula)* (Radiation power R_p explodes at speed *v* = *c*).	**10.** RADIATION POWER **10.1** Radiation power of particle of charge *q* moving with velocity ***v***, is the scalar product: $R_p = -\mathbf{v}.\mathbf{R}_f$ For collinear motion ***v, E*** and $\boldsymbol{R_f}$ are in the same direction and radiation power is: $R_p = Eq\frac{v^2}{c}$ Radiation power R_p is 0 if ***v*** is orthogonal to ***E*** and $\boldsymbol{R_f}$ as in circular motion of an electron round a nucleus. **10.2** Radiation power of a particle of charge q revolving with speed *v* and acceleration $a = v^2/r$, in a circle of radius *r*, is zero (Rutherford's nuclear model of the hydrogen atom is, therefore, stable without recourse to Bohr's quantum theory).
11. Potential energy *P* lost by an electron, of mass $m = m_o$, accelerated from rest to speed *v* by an electrostatic field: $P = \frac{1}{2}mv^2$	**11.** Potential energy *P* lost by an electron, of rest mass m_o, accelerated from rest to speed *v* by a field. $P = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_o c^2$	**11.** Potential energy *P* lost by an electron of mass $m = m_o$, accelerated from rest to speed *v* by an electrostatic field: $P =$ $-mc^2 \ln\left(1-\frac{v}{c}\right) - mcv$

11.1 Kinetic energy K gained $K = \frac{1}{2}mv^2 = P$ **11.2** Energy radiated: *R = P – K = 0* *(There must always be energy radiation)*	**11.1** Kinetic energy gained is *K = P* **11.2** Energy radiated: *R = P – K = 0* *(There must always be energy radiation)*	**11.1** Kinetic energy *K* gained is: $K = \frac{1}{2}mv^2$ **11.2** Energy radiated: *R = P – K > 0* *(There must always be energy radiation)*
12. Potential energy *P* gained by an electron, of *mass m* = m_o, decelerated, by an electrostatic field, from the speed of light *c* to speed *v is*: $P = \frac{1}{2}m(c^2 - v^2)$ **12.1** Kinetic energy K lost is: $K = \frac{1}{2}m(c^2 - v^2)$ **12.2** Energy radiated: *R = K – P = 0* *(There must always be energy radiation)*	**12.** An electron moving at the speed of light *c* (with infinite kinetic energy), cannot be stopped by any decelerating field; it continues to move at the same speed of light, gaining potential energy *P* without losing kinetic energy. **12.1** Kinetic energy lost is *K = 0*: **12.2** Energy radiated: *R = 0 – P = –P?* *(The energy radiated, must always be positive)*	**12.** Potential energy *P* gained by an electron, of *mass m* = m_o, decelerated, by an electrostatic field, from the speed of light *c* to speed *v is*: $P = mc^2 \ln\frac{1}{2}\left(1+\frac{v}{c}\right) + mc^2\left(1-\frac{v}{c}\right)$ **12.1** Kinetic energy K lost, is: $K = \frac{1}{2}m(c^2 - v^2)$ **12.2** Energy radiated: *R = K – P > 0* *(There is energy radiation)*
13. Potential energy *P* gained by an electron, of mass *m* = m_o, decelerated by an electrostatic field, from the speed of light *c* to rest is: $P = \frac{1}{2}mc^2$ **13.1** Kinetic energy *K* lost is: $K = \frac{1}{2}mc^2$	**13.** An electron moving at the speed of light *c* (with infinite kinetic energy), cannot be stopped by any decelerating field; it continues to move at the same speed of light, gaining potential energy *P* without losing kinetic energy. **13.1** Kinetic energy *K* lost is *K = 0*	**13.** Potential energy *P* gained by an electron, of mass *m* = m_o, decelerated by an electrostatic field, from the speed of light *c* to rest *(v = 0)*, is: $P = mc^2 \ln\frac{1}{2} + mc^2$ $P = mc^2(1 – 0.693)$ $P = 0.307mc^2$ **13.1** Kinetic energy *K* lost is: $K = \frac{1}{2}mc^2$

13.2 Energy radiated: *R = K – P = 0* *(There must always be energy radiation)*	**13.2** Energy radiated: *R = 0 – P = –P?* *(R is always positive)*	**13.2** Energy radiated: $R = K - P = 0.193mc^2$ *(There must always be energy radiation)*
14. Classical **r**adius *r* of revolution of an electron of charge *–e* and mass $m = m_o$ moving with speed *v* in a radial electrostatic field of magnitude *E* due to a positively charged nucleus: $r = \frac{mv^2}{eE} = r_o$ Mass $m = m_o$ **Mass *m* is independent *of* speed *v of a charged particle such as an electron.***	**14.** Relativistic **r**adius *r* of revolution of an electron of charge *–e* and mass *m* moving with speed *v* in a radial electrostatic field of magnitude *E* due to a positively charged nucleus: $r = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o$ $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_o$ $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ **Mass *m* of a charged particle such as an electron, depends on γ, mass becoming infinitely large at $\gamma = \infty$ or *v* = *c*.**	**14.** Radiational **r**adius *r* of revolution of an electron of charge *–e* and mass $m = m_o$ moving with speed *v* in a radial electrostatic field of magnitude *E* due to a positively charged nucleus: $r = \frac{mv^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o$ Mass $m = m_o$ Radius *r* can become infinitely large as is the case in rectilinear motion. **For a charged particle such as an electron, the variation of radius of circular revolution *r*, with γ, was mistaken, in Special Relativity, as being the result of mass *m* depending on γ,** **For circular revolution of a charged particle as an electron, Relativistic and Radiational Electrodynamics are in agreement but for different reasons.**

APPENDIX 2

"All good things that exist are the fruits of originality".
John Stewart Mill (1806 – 1873)

BORH'S DERIVATION OF BALMER-RYDBERG FORMULA THROUGH QUANTUM MECHANICS

Niels Borh (1885 – 1962), in a superbly original thought, derived the Balmer-Rydberg formula, for the spectral lines of radiation from the hydrogen atom, by invoking the quantum theory and making two postulates. The first postulate is that the electron, in the Rutherford nuclear model of the hydrogen atom, can revolve, without radiation, round the nucleus in allowed, quantum or stable orbits for which the angular momentum L_n is quantized, so that:

$$L_n = \frac{nh}{2\pi} \tag{A2.1}$$

where n, the quantum number, is an integer greater than zero, and h is the Plank constant.

The second of Bohr's postulate is that an excited electron translates from a stable orbit of radius r_q, corresponding to quantum number q and total energy (kinetic and potential) E_q, to an inner orbit of radius r_n, corresponding to quantum number n and total energy E_n. The electron loses potential energy and gains kinetic energy and, in the process, it emits radiation of frequency f_{nq}, in accordance with de Brogilie's hypothesis, such that:

$$E_q - E_n = hf_{ng} \tag{A2.2}$$

where n is a number greater than zero but less than q.

Let us apply the two postulates of Bohr to the hydrogen atom whose electron of mass m and charge $-e$ at a point P revolves with velocity v_n about a stationary nucleus of mass M and charge $+e$ in a circular orbit of radius r_n, as shown in Figure A2.1 below. The angular momentum of the electron, in the nth orbit, is

$$L_n = \frac{nh}{2\pi} = mv_n r_n \tag{A2.3}$$

Equating the centripetal forces on the electron, we get:

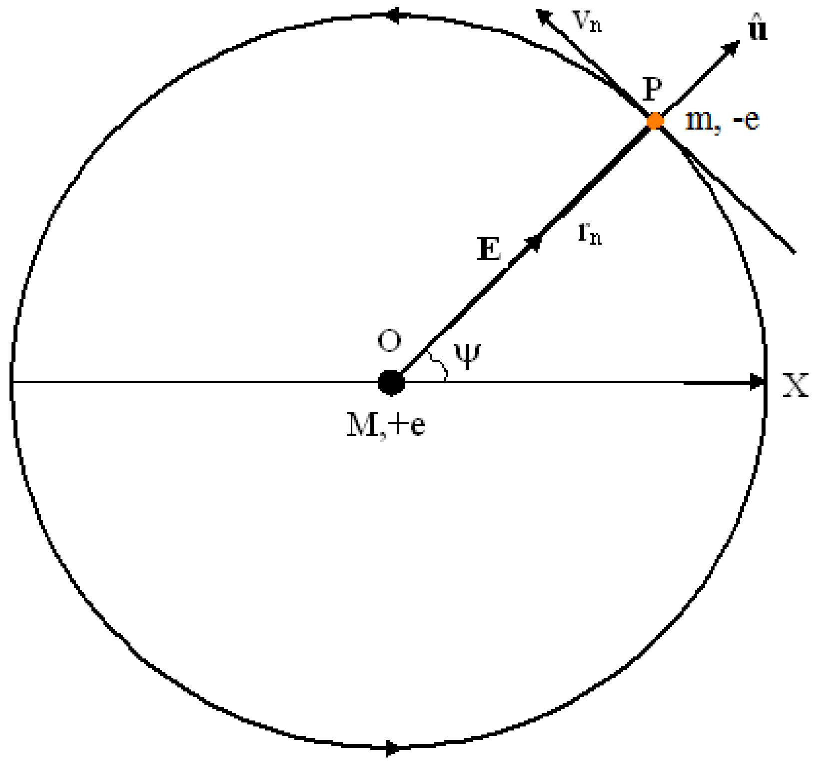

Figure A2.1. An electron of charge $-e$ and mass m at a point P revolving with speed v_n, through an angle ψ, in a circle of radius r_n under the attraction of a heavy nucleus of mass M and charge $+e$ at the central point O.

$$\frac{mv_n^{\ 2}}{r_n} = \frac{e^2}{4\pi\varepsilon_o r^2} \tag{A2. 4}$$

Equations (A2.3) and (A2.4) give the speed and radius of revolution as:

$$v_n = \frac{e^2}{2\varepsilon_o nh} \tag{A2.5}$$

$$r_n = \frac{\varepsilon_o n^2 h^2}{\pi m e^2} \tag{A2.6}$$

With n = 1, we get the Bohr radius $r_1 = 5.292 \times 10^{-7}$ m.

The total energy in the nth quantum state is obtained as:

$$E_n = \frac{1}{2} m v_n^2 - \frac{e^2}{4\pi\varepsilon_o r_n} \tag{A2.7}$$

Substitute for v_n and r_n from equations (A2.5) and (A2.6) gives:

$$E_n = -\frac{me^4}{8\varepsilon_o^2 n^2 h^2} \tag{A2.8}$$

The total energy in the qth quantum state is:

$$E_q = -\frac{me^4}{8\varepsilon_o^2 q^2 h^2} \tag{A2.9}$$

Equation (A2.2) then becomes:

$$E_q - E_n = \frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n^2} - \frac{1}{q^2}\right) = hf_{nq} \tag{A2.10}$$

$$\frac{f_{nq}}{c} = \frac{1}{\lambda_{nq}} = \frac{me^4}{8c\varepsilon_o^2 h^3}\left(\frac{1}{n^2} - \frac{1}{q^2}\right) \tag{A2.11}$$

Equation (A2.11) is the Balmer-Rydberg formula (with n>0<q) for the spectral lines of radiation from the hydrogen atom.

The stabilization of Rutherford's nuclear model of the hydrogen atom, by Bohr, was recognized as a brilliant achievement of the human mind. It gave an additional impetus to the development of quantum mechanics. However, the transition from one orbit to another, in zero time, as a necessary condition for radiation of energy, is a drawback on Bohr's quantum theory. So also is the failure to relate the frequency of emitted radiation to the frequency of revolution of the electron, round the positively charged nucleus.

Equation (A2.11) is an example of Bechmann's *Correspondence Theory*, whereby the expected, desired or correct result is obtained mathematically, but is based on wrong underlying principles. Wrong theories in learning and knowledge are like weeds in a farm. Unless good, fruit-bearing fruits are cultivated, weeds will take over. Once weeds have gained ground, it is very difficult to uproot them. But uprooted they must be if knowledge is to advance.

APPENDIX 3

"Everything that happens, happens as it should, and if you observe carefully, you will find this to be so".

Marcus Aurelius Antoninus (180 – 121 B.C.)

Revolution of neutral body in a closed ellipse

The force of attraction ***F*** between two bodies of masses *M* and *m*, distance *r* apart, is given by Newton's universal law of gravitation:

$$\mathbf{F} = \frac{-GMm}{r^2}\hat{\mathbf{u}} \tag{A3.1}$$

where *G* is the gravitational constant and $\hat{\mathbf{u}}$ is a unit vector in the radial direction. This force is extremely feeble that it is only noticeable in respect of huge masses like the planets and the Sun.

Two bodies under mutual attraction will revolve round their centre of mass with the larger mass *M* revolving in an inner orbit and the lighter mass *m* revolving in an outer orbit. The two bodies will revolve with the same angular velocity under equal and opposite forces of attraction. If *M* is very much larger than *m* as in the case of the Sun $(M = 2 \times 10^{30}\ kg)$ and the Earth $(6 \times 10^{24}\ kg)$, the Sun may be considered as almost stationary at a point while the Earth revolves at a distance *r* from the point (the centre of the Sun).

The gravitational force of attraction, on a moving body, is independent of its speed in a gravitational field. Newton's second law of motion, on a body of mass *m* moving at time *t* with speed *v* and acceleration *dv/dt* in the radial direction, gives:

$$\mathbf{F} = \frac{-GMm}{r^2}\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \tag{A3.2}$$

The centripetal acceleration on a body revolving through angle φ, under a central force, gives the accelerating force as:

$$\mathbf{F} = \frac{-GMm}{r^2}\hat{\mathbf{u}} = m\left\{\frac{d^2 r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2\right\}\hat{\mathbf{u}}$$

$$\frac{-GM}{r^2} = \frac{d^2 r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2 \tag{A3.3}$$

This is a differential equation of motion in r and φ as functions of time t. We can reduce it to an equation of r as a function of φ.

The angular momentum K of m, being a constant (Kepler's second law), gives:

$$K = mr^2 \frac{d\varphi}{dt} \tag{A3.4}$$

Making the substitution $r = 1/u$ and with equation (A3.4), gives:

$$\frac{dr}{dt} = \frac{dr}{du}\frac{d\varphi}{dt}\frac{du}{d\varphi} = \frac{-K}{m}\frac{du}{d\varphi} \tag{A3.5}$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\frac{dr}{dt} = \frac{d\varphi}{dt}\frac{d}{d\varphi}\left(\frac{-K}{m}\frac{du}{d\varphi}\right) = \frac{-K^2u^2}{m^2}\frac{d^2u}{d\varphi^2} \tag{A3.6}$$

Substituting equations (A3.6) and (A3.5) in equation (A3.3) gives:

$$-\frac{K^2u^2}{m^2}\frac{d^2u}{d\varphi^2} - \frac{K^2u^3}{m^2} = -GMu^2$$

$$\frac{d^2u}{d\varphi^2} + u = \frac{GMm^2}{K^2} \tag{A3.7}$$

It should be noted that in equation (A3.7), the radiation component containing $du/d\varphi$, in the case of a charged particle revolving under a central force, is missing.

Equation (A3.7) is a second order differential equation with constant coefficients. Trying $u = Aexp(z\varphi)$ as a solution, we obtain:

$$z^2 + 1 = 0$$

.

$$z^2 = \sqrt{-1} = \pm j$$

The general solution of equation (A3.7) is:

$$u = \frac{1}{r} = A\exp\left(j\varphi\right) + \frac{GMm^2}{K^2} \tag{A3.8}$$

An appropriate solution is:

$$\frac{1}{r} = A\cos\left(\varphi + \beta\right) + \frac{GMm^2}{K^2} \tag{A3.9}$$

where the amplitude A and phase angle β are determined from the initial conditions. If $\beta = 0$, equation (A3.9) may be written as:

$$\frac{1}{r} = \frac{GMm^2}{K^2}\left(1 + \frac{AK^2}{GMm^2}\cos\varphi\right) \tag{A3.10}$$

$$\frac{1}{r} = B\left(1 + \frac{A}{B}\cos\varphi\right) = B\left(1 + \varepsilon\cos\varphi\right) \tag{A3.11}$$

where $B = GMm^2/K^2$. Equation (A3.11) gives an ellipse, in the polar coordinates, with eccentricity $\eta = A/B$. The ellipse is shown as $XYZW$ in Figure A3.1.

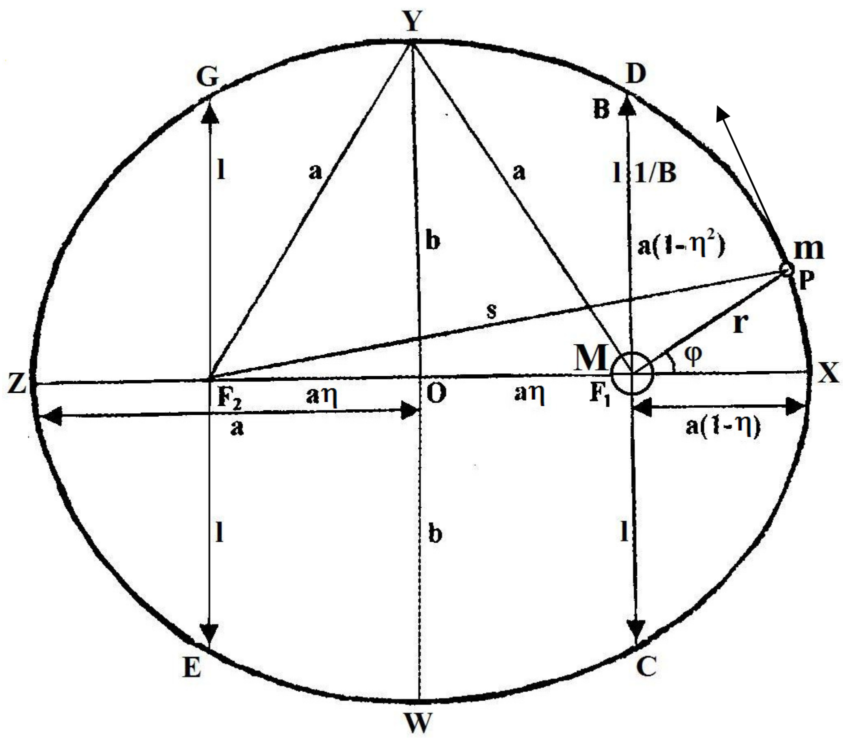

Figure A3.1 A body of lighter mass m revolving in an elliptic orbit, of eccentricity η, round a much heavier body of mass M at the centre of mass, the focus F_1, In Figure A3.1, the lighter mass m revolves, in angle φ, at a point P, in a closed ellipse, at a distance r from a much heavier mass M at one focus F_1. The other focus F_2 is at a distance s from P. A property of an ellipse is that the distances $s + r = 2a$, the length of the major axis ZX. Other properties are obtained as the angle φ takes values 0, $\pi/2$ and π radians. The line ZX is the major axis, WY is the minor axis and the chords CD and EG are the latus rectums, with half length $l = a(1 - \eta^2) = 1/B$. The eccentricity of the ellipse is ratio of the distance between the foci (F_1F_2) to the length of the major axis (ZX). For a circle, the two foci coincide and the eccentricity η is equal to zero.

In planetary motion, X, the point of closest approach to the Sun, is called the perihelion and Z, the point of farthest separation, is the aphelion. A planet moves faster as it approaches the Sun; it is fastest at the perihelion and slowest at the aphelion. At any time, a planet moves in such a way that the difference in kinetic energy between two points is equal to the difference in potential energy between the same points.

The reader should show that change in kinetic energy from the perihelion to the aphelion is the same as change in potential energy between the same points.

Period of revolution in closed ellipse

The period of revolution T_e is obtained from equation (A3.4) as the definite integral:

$$T_e = \int_0^{2\pi} \frac{mr^2}{K}(d\varphi)$$

Substituting for r from equation (A3.11), we obtain:

$$T_e = \frac{m}{KB^2}\int_0^{2\pi} (1+\eta\cos\varphi)^{-2}(d\varphi)$$

Expanding the integral by the binomial theorem, gives:

$$T_e = \frac{m}{KB^2}\sum_{p=0}^{\infty}(-1)^p(1+p)\eta^p\int_0^{2\pi}\cos^p\varphi(d\varphi) = \frac{m}{KB^2}\sum_{p=0}^{\infty}Q_p$$

where p is an integer taking the values 0, 1, 2, 3…∞. As the terms for all odd values of p are zero, we shall take only the even p's to give:

$$Q_{2p} = (1+2p)\eta^{2p}\int_0^{2\pi}\cos^{2p}\varphi(d\varphi)$$

$$Q_0 = \int_0^{2\pi}(d\varphi) = 2\pi \qquad \text{(A3.12)}$$

$$Q_2 = 3\eta^2\int_0^{2\pi}\cos^2\varphi(d\varphi)$$

$$Q_2 = \frac{3\eta^2}{2}\int_0^{2\pi}(1+\cos 2\varphi)(d\varphi) = \frac{3\eta^2}{2}(2\pi) \qquad \text{(A3.13)}$$

$$Q_4 = 5\eta^4\int_0^{2\pi}\cos^4\varphi(d\varphi)$$

$$Q_4 = \frac{5\eta^4}{8}\int_0^{2\pi}\{3+\cos(4\varphi)+4\cos(2\varphi)\}(d\varphi)$$

Q_4 is obtained as:

$$Q_4 = \frac{15\eta^4}{8}(2\pi) \qquad \text{(A3.14)}$$

$$T_e = \frac{m}{KB^2}\sum_{p=0}^{\infty}Q_{2p} = \frac{m}{KB^2}(Q_0+Q_2+Q_4+Q_6+\ldots+Q_\infty)$$

$$T_e = \frac{2\pi m}{KB^2}\left\{1+\frac{3\eta^2}{2}+\frac{15\eta^4}{8}+\frac{35\eta^6}{16}+\ldots+(-1)^p\frac{\left(\frac{-3}{2}!\right)}{\left(\frac{-3}{2}-p\right)!\,p!}\eta^{2p}\right\} \text{(A3.15)}$$

Here, the definition of factorial (!) of a number is extended to include negative fractions. An inspection of equation (A3.15) reveals that it is identical to:

$$T_e = \frac{2\pi m}{KB^2}\left(1-\eta^2\right)^{-\frac{3}{2}} \tag{A3.16}$$

The period of revolution can also be obtained from equation (A3.4) and the area S of the ellipse. This is Kepler's second law:

$$\frac{dS}{dt} = \frac{S}{T_c} = \frac{r^2(d\varphi)}{2(dt)} = \frac{K}{2m}$$

$$T_e = \frac{2mS}{K} \tag{A3.17}$$

Comparing equations (A3.17) and (A3.16) the area S is obtained as:

$$S = \frac{\pi}{B^2}\left(1-\eta^2\right)^{-\frac{3}{2}} = \pi ab \tag{A3.18}$$

It is left as an exercise to the reader to show, from the geometry of an ellipse, Figure A3.1, that the product $ab = a^2(1 - \eta^2)^{1/2}$ and πab is the area of an ellipse of eccentricity η, semi major axis a and semi minor axis b.

APPENDIX 4

"Nothing is too good to be true if it be consistent with the laws of nature, and in such things as these, experiment is the best test of such consistency".

Michael Faraday (1791 - 1867)

A PROPOSED EXPERIMENT TO TEST *RADIATIONAL ELECTRODYNAMICS*

A schematic diagram of an experiment, proposed to test *radiational electrodynamics*, is shown below (Figure A4.1). It consists of a long evacuated glass tube with a cathode at each end and an anode at the centre. The hot cathode, on the left, is kept at a high negative voltage $-V_{HC}$, the anode at zero *(0)* volt and the cold cathode at *0* or $-V_{HC}$ volts. The hot cathode, heated by a heater *H*, emits electrons which pass through a grid to move toward the anode. An accelerating electrostatic field is maintained, within a separation *L*, between the hot cathode and the anode. A decelerating field is provided, also within a separation *L*, between the anode and the cold cathode.

The grid is maintained at a positive or negative voltage relative to the hot cathode. If the grid is negative, electrons are repelled by it and no charges reach the anode. If the grid is positive, the electrons are accelerated through it to the anode. Some of the accelerated electrons pass through a small hole in the anode and may proceed to reach the cold cathode.

In the first place, the cold cathode is maintained at the same zero (0) volt as the anode. The accelerated electrons pass through the anode and may drift on, with constant speed, to hit the cold cathode. Electrons striking the cold cathode have a heating effect on it.

A narrow pulse of electrons, from the hot cathode, produces a signal as it passes through the anode and another signal as it strikes the cold cathode. The speed of the electrons is determined from the length L and the time interval between the two signals. This time interval and speed, measured with a fast oscilloscope, may be varied.

The next step is to maintain the cold cathode at $-V_{HC}$, the same high voltage as the hot cathode, so that a decelerating field exists between the anode at 0 Volt and the cold cathode. Thus, relative to the anode, the high

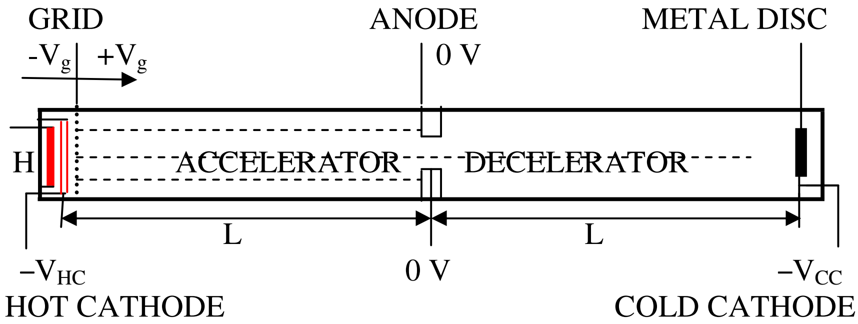

Figure A4.1 Apparatus for testing *radiational electrodynamics*

accelerating potential is equal to the decelerating potential. As a result of energy radiation, electrons passing the anode will be stopped and turned back, before reaching the cold cathode.

In classical, relativistic and *radiational electrodynamics*, moving electrons decelerated by an electric field, radiate energy at the expense of their kinetic energy. Therefore, the potential energy gained is less than the kinetic energy lost, the difference being the energy radiated. Narrow positive and negative-going voltages, on the grid, should give a train of pulses of electrons, from the hot cathode to the anode, with a repeat rate of *T seconds*. The electrons would oscillate, with decreasing amplitude about the anode. They should emit light/heat and electromagnetic radiations, dissipating energy at the anode, before another pulse arrives within *T seconds*.

In classical and relativistic electrodynamics, heat is dissipated on the cold cathode when moving electrons lose kinetic energy by striking it. Where electrons do not reach the cold cathode, there should be no heating as might be indicated by a rise in temperature.

In *radiational electrodynamics*, there is light/heat radiation in the direction of motion of the accelerated or decelerated electrons. Radiation is transmitted, at the speed of light, ahead of the moving electrons, to impinge on the cold cathode. As such, there should be heat dissipation even where the electrons do not reach and hit the cold cathode. The incidence of radiation and rise in temperature of the cold cathode, verifies *radiational electrodynamics*.

APPENDIX 5

"He who adds not to his learning diminishes it".
Jewish Talmud (3rd Century A.D.)

The physical constants constitute a body knowledge garnered over the years for the purpose of describing the physical world. These constants are being constantly corrected or added to through experiments and observations. Some physical constants are given in the Table below:

SOME PHYSICAL CONSTANTS

QUANTITY	SYMBOL	DIMENSION*	VALUE#
Charge of the electron	$-e$	Q	-1.602×10^{-19} C
Mass of the electron	m.	$L^{-2}T^{2}QV$	9.110×10^{-31} kg
Charge of the proton	$+e$	Q	1.602×10^{-19} C
Mass of the proton	m_p	$L^{-2}T^{2}QV$	1.673×10^{-27} kg
Mass of the neutron	m_n	$L^{-2}T^{2}QV$	1.750×10^{-27} kg
Mass of the hydrogen atom	m_h	$L^{-2}T^{2}QV$	1.673×10^{-27} kg
Radius of the electron	r_e	L	2.817×10^{-15} m
Permittivity of vacuum	ε_o	$L^{-1}QV^{-1}$	8.854×10^{-12} F/m
Permeability of vacuum	μ_o	$L^{-1}T^{2}Q^{-1}V$	$4\pi \times 10^{-7}$ H/m
Speed of light in space	c	LT^{-1}	2.998×10^{8} m/s
Refractive index of water	μ.	1	1.333 (at 300 K)
Planck constant	h.	TQV	6.626×10^{-34} J-s
Rydberg constant (Limit of Lyman series)	R $\nu_1 = 1/\lambda_1$	L^{-1}	1.097×10^{7} /m (3.291×10^{15} Hz)
First bipolar radius First Bohr radius	r_1	L	5.292×10^{-7} m
Speed of revolution in the first bipolar orbit	v_1	LT^{-1}	1.094×10^{6} m/s
Angular momentum in the first bipolar orbit	$L = \frac{h}{4\pi}$	TQV	5.272×10^{-34} J-s
Frequency of revolution in the first bipolar orbit (Limit of Lyman series, cR)	f_1	T^{-1}	3.291×10^{15} Hz (Ultra-violet)
Wavelength of limit of Lyman series, 1/R	λ_1	L	9.116×10^{-8} m (Ultra-violet)

Wavelength of limit of Balmer series, 4/R	H_∞	L	3.646×10^{-7} m (Near ultra-violet)
Wavelength of fourth line of Balmer series, 9/2R	H_δ	L	4.102×10^{-7} m (Near ultra-violet)
Wavelength of third line of Balmer series, 100/21R	H_γ	L	4.341×10^{-7} m (Violet, visible)
Wavelength of second line of Balmer series, 16/3R	H_β	L	4.862×10^{-7} m (Blue-green, visible)
Wavelength of fist line of Balmer series, 36/5R	H_α	L	6.563×10^{-7} m (Red, visible)
Surface electrostatic field of the electron	σ_e	$L^{-1}V$	1.817×10^{20} V/m
Electrostatic potential of the electron	V_e	V	5.118×10^{5} V
Intrinsic energy of the electron	e_i $= 0.5mc^2$	QV	4.100×10^{-14} J ≈ 0.25 MeV
Electron kinetic energy at the speed of light c.	e_k $= 0.5mc^2$	QV	4.100×10^{-14} J ≈ 0.25 MeV
Gravitational constant	G	$L^5T^{-4}Q^{-1}V^{-1}$	6.673×10^{-11} $Nm^2/(kg)^2$
Gravitational field	Γ	LT^{-2}	Nm/(kg)
Acceleration due to gravity	g	LT^{-2}	9.807 m/s^2
Faraday constant	F	$L^2T^{-2}V^{-1}$	9.652×10^{4} C 6.025×10^{23} C/mole
Avogadro's number	N_o	1	6.025×10^{23} /mol
Volume of 1 mol at STP	V_o	L^3	22.421×10^{-3} m^3
Average radius of the Earth's orbit. Astronomical Unit (AU)	O_e	L	1.496×10^{11} m = 1 AU 8.311 light minutes
Average radius of the Moon's orbit	O_m	L	3.84×10^{8} m 1.28 light seconds
Approx. distance between Sun and nearest Star (Proxima Centauri)	S_s	L	4.2 light years (LY) = 2.71×10^{4} AU. I LY $\approx 9.46 \times 10^{15}$ m
Closest distance to the nearest planet (Venus)	V_e	L	4.1×10^{10} m 0.274 AU
Radius of the Sun	R_s	L	6.96×10^{8} m

Average radius of the Earth	R_e	L	6..378 × 10^6 m
Average radius of the Moon	R_m	L	1.737 x 10^6 m
Mass of the Sun	M_s	$L^{-2}T^2QV$	1.99 x 10^{30} kg
Mass of the Earth	M_e	$L^{-2}T^2QV$	5.97× 10^{24} kg
Mass of the Moon	M_m	$L^{-2}T^2QV$	7.35 x 10^{22} kg
Speed of rotation of the Earth at the equator	E_r	LT^{-1}	1.670 x 10^6 m/h 4.639 x 10^2 m/s
Speed of rotation of the Moon at the equator	M_r	LT^{-1}	3.680 x 10^6 m/h 4.592 m/s
Period of rotation of the Earth	D_e	T	24 hr = 9.64 x 10^4 s (Mean solar day)
Period of rotation of Moon	D_m	T	27.32 days
Average speed of orbital revolution of the Earth	S_e	LT^{-1}	1.075 x 10^8 m/h 2.986 x 10^4 m/s
Average speed of orbital revolution of the Moon	S_m	LT^{-1}	3.70 x 10^6 m/h 1.03 x 10^3 m/s
Period of revolution (orbital period) of the Earth	Y_e	T	1 solar year ≈ 365.26 days
Period of revolution (orbital period\) of the Moon	Y_m	T	27.32 days
Lunar month (Period between same phases)	L_m	T	29.53 days
Radiation (heat/light) power generated by the Sun	W_s	$T^{-1}QV$	3.38 x 10^{26} W (At about 5500^o C)
Solar constant (Intensity at outer edge of atmosphere)	C_s	L^{-2} $T^{-1}QV$	1.4 kW/m^2
Constant of aberration	α.	1	20.47 arc sec.
Solar parallax	π_s	1	8.8 arc sec.
Sound in air speed at STP	v_s	LT^{-1}	3.34 x 10^2 m/s
Standard pressure	P_s	$L^{-3}QV$	≈ 10^5 newton/m^2
Standard temperature	T_s	-	0^o C (273.16 K)
Escape velocity (neglecting air friction)	v_e	LT^{-1}	1.12 x 10^4 m/s

*The Fundamental Quantities are Length *[L]*, Time *[T]*, Electric Charge *[Q]* and Voltage *[V]*.

#Source: Encarta Encyclopaedia

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.....ABOUT THE BOOK

The book introduces *radiational electrodynamics* in 9 Papers, by an extension of Coulomb's law, to a moving charged particle. It is shown that an electrical force is propagated at the speed of light and the accelerating force on a charged particle depends on its speed in an electrostatic field. A particle can be accelerated to the speed of light, as a limit, with its mass remaining constant. This is contrary to special relativity where mass increases with speed becoming infinitely large at the speed of light. Radiation force is found as the difference between the force on a stationary charged particle and the accelerating force on a moving particle and radiation power is shown to be the scalar product of radiation force and velocity The source of inertia of a body, composed of positive and negative electric charges, is identified as a consequence of the electrodynamic field generated by an accelerated charged particle acting on the same charges. The book gives a simple explanation of the source of radiation from accelerated charged particles, in contrast to classical and relativistic electrodynamics. Rutherford's nuclear model of the hydrogen atom is shown to be stable without recourse to quantum mechanics. The book will appeal to mathematicians, physicists and engineers who are not comfortable with the relativity theory and quantum mechanics.

.....ABOUT THE AUTHOR

Musa Daji Abdullahi was born in Katsina, Nigeria, in 1941. He obtained Bachelor of Science Degree in Physics, in 1965, from the University of Manchester, England and Masters Degree in Electronics and Telecoms, in 1968, from A.B.U., Zaria. He is a Fellow of the Institution of Engineering and Technology, Fellow of the Nigerian Society of Engineers and Fellow of the Academy of Engineering. Currently he is Senior Lecturer at Federal University of Technology, Minna, Nigeria.

Engr. M.D. Abdullahi was conferred with the National Honour of Member of Federal Republic (MFR) in 2005, in recognition of his public service from 1965 - 2000. In addition to teaching and research in renewable energy, the author has devoted many years in developing *radiational electrodynamics* where an electrical force is propagated at the speed of light and a charged particle can be accelerated up to the speed of light with constant mass and with emission of radiation. He is also engaged in the development of a new writing *(Tafi)* for Hausa and other Nigerian languages. He is happily married to Hajia Rabi Remawa.