AN EVALUATION OF RELATIVITY THEORY

After A Half-Century

by

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"More than half of logic consists in clarity of thought, especially in the matter of fundamental concepts."

I

THE KINDS OF EFFECT OBSERVED

An Outline

We shall first list those groups of physical observations or effects which are usually held to be relevant to relativity theory, and which are included in all complete discussions of it.

A. Effects Involving Distance and Time Interval

Namely, distance and/or time interval distortions referring to an object in motion with respect to an observer using light signals to measure those intervals, the effects becoming more pronounced as the speed of the motion increases.

When measured under these conditions length intervals appear shortened. Time intervals, as given by the clocked sending and receipt of a light signal, are observed to be increased under those conditions.

It was this group of effects that was first noticed by H.A.Lorentz in 1895 as an explanation of the Michelson-Morley experiment. Lorentz expanded his ideas in 1904 with the concept of 'local time' and the celebrated Lorentz transformations, which were incorporated by Einstein in the special relativity theory, put forth in 1905.

These effects, largely because of insufficiently clarified thinking about them, have led to a great deal of unnecessary misinterpretation, largely in the form of generalizations quite unsupported by the phenomena themselves and the physical basis for them.

Discussion of the matter in greater detail will be found in the second chapter.

B. Observed Mass Increase of a Charged Particle With Increase of Particle Velocity

The beginnings of the study of this phenomenon were made by J.J.Thomson in the last century.

The physical basis for this effect is stated at the conclusion of chapter II, and there is nothing more that is required to be said about it in this outline.

C. The Involvement of Energy in Mass and of Mass in Energy, given approximately by $\frac{E}{c^2} = m$, c being the velocity of an electromagnetic wave *in vacuo*.

The above statement is considerably more physically correct than the more usual and more imprecise form, 'the equivalence of energy and mass'.

This mutual involvement and interchangeability of radiant energy and mass gives rise to a group of extremely important physical effects.

As a matter of the history of science, neither those effects nor the above equation connecting them were discovered by A.Einstein. Indeed, as we shall see in chapter III, the equation $E = mc^2$ long ante-dates relativity theory, and, even more important for purposes of logical clarification, does not arise within the theory because of the relativistic assumptions *per se*.

How the equation gets to be in relativit[illegible] theory at all sheds most interesting ana[illegible]tical light upon the foundations of the th[illegible]ry and on the definition of the scope of [illegible]

competency in physical phenomena.

A discussion of the matter is to be found in chapter III.

D. The Effect of the Presence of a Gravitational Field in Lowering the Frequency of an Electromagnetic Wave

In the range of visible light, this would mean a shift toward the red end of the spectrum - the so-called 'Einstein shift'.

As we shall find, this effect is simply one of the consequences of C, as well as is the bend of the path of light in gravitational fields.

E. The Equivalence of Inertial and Gravitational Mass

This view was first urged by E.Mach [*Die Mechanik in ihrer Entwickelung*, ch.II,sec.viii par.1, Prague, 1883] as part of his general theory of phenomenological relativity which later so heavily influenced Einstein.

However, the above heading states and embodies not a 'principle' but simply a physical fact that stems directly from the connection between applied force, of whatever sort, and acceleration. It is this connection, first clearly stated in Newton's *Principia*, which constitutes the actual principle involved. Mach, when indeed he is not in actual error (as in his unsuccessful attempts to extricate his theory of absolute and dogmatic relativity from the grave difficulties and impasse occasioned in it by the fact of the simple existence of the phenomena of rotation) is invariably found to arrive finally at some reformulation of Newton's mechanics and dynamics, often in more ambiguous and hence less usable form. See supra, chapter IV, op. cit.

F. The Constancy of the Velocity of an Electromagnetic Wave Independent of the Speed of the Source, the wave being in an isotropic, homogeneous medium

This statement is often imprecisely given and incorrectly termed a 'principle'; namely, 'the principle of the constancy of the velocity of light'. It is not a principle, but a single illustration of a universal fact of wave motion, viz. a moving source may change the frequency and wavelength of a wave, but the product of the two, the velocity, must remain constant for a given isotropic homogeneous medium. This fact enters in a fundamental way also into the discussion of the effects of Group A. Discussion in chapter I.

G. Electromagnetic Induction

The attempt to reduce this group of physical effects to 'merely a matter of relative motion' is discussed in chapter IV.

This attempt is related to the more general one to substitute hyper-geometric fields for forces. Since it is quite obvious that abstract geometry apart from some physical entity could impose no metric upon the physical world, it comes as no surprise that when such fields are scrutinized they turn out to be force fields, inseparably associated with matter in some form, and thus with the ineradicable singularities that arise in the field equations and that present a fundamental, physical antithesis to any dogmatic form of relativistic speculation.

All these facts are intimately connected with the physical implications of Cerenkov radiation that will be discussed in the Appendix.

II

MEASUREMENT AND MEASURABILITY IN RELATIVITY THEORY

The essentially distinctive part of relativity theory, defining it as such, is concerned with the measurement of length and time intervals by signals of finite speed, the sending and receiving points of such signals being in motion with respect to each other and the measuring observer being at the receiving end. The signals are electromagnetic radiation.

When the exact conditions are thus clearly enunciated and born in mind, a host of troublesome and misleading confusions is straightway obviated - confusions that right up to the time of this writing have persistently dogged expositions, even the most technical and advanced, in relativity theory.

First, there are some basic consequences stemming from two of the above conditions taken together:

Namely,
1. Finite signal speed
2. Motion of the sending and receiving points with respect to each other.

This combination immediately means that length and time intervals, as arrived at by such a system of measurement would naturally be subject to illusory distortions, inherent and inescapable in the defects of such a system of measurement.

The time distortion arises from nothing more esoteric than the elementary relation that distance equals speed times time, or, in this instance

$$s = ct,$$

where c is the velocity of light, assumed constant in special relativity theory (See F, page 4). We may rewrite the equation as

$$s/t = c \text{ (constant)}$$

Obviously, if s, the distance, is measured as shortened, the time interval, t, must be measured as lengthened in order to preserve the constancy of c, - and so it proves, in the Lorentz transformations.

These transformations are simply the mathematical expression for the observed distortions of lengths and times (and hence, of course, of unit lengths and times) occasioned by the above-described optical illusion, inevitably arising from the method of measurement employed. If sound signals had been used instead of light, the distortions of measured lengths and times would have been correspondingly greater on account of the lower velocity of sound. The Lorentz transformations, adopted by Einstein in his special theory of relativity, would exactly apply, requiring only the substitution of the velocity of sound in, say, air for the symbol c. Then the sensory illusion would be easily apparent. For the high speed of light masks it. [See Chapter III also, discussion on page 11f.]

The elevation by a host of writers, learned and otherwise, of the optical sensory illusion here involved into an asserted intrinsic relativity of space and time themselves has been perhaps the most gigantic hoax, probably because largely unconscious, ever to arise in the history of science. It was not perceived for what it was because of men's habitual and preferential regard for the sense of sight, blurring their otherwise normal tendency to check an illusion by the use of another sense. Performing the same mathematics on sound signals under like conditions would instantly have proven the point.

But what actually happened was like a man saying that because a pencil appeared bent and foreshortened if half-immersed in water, that therefore 'space' was curved, twisted and 'relative'. Or that because a watch slowed down in a magnetic field, that therefore 'time was relative'. Indeed, a closely related form of this absurdity was actually put forth in general relativity theory, where part of the relativity of time is derived from the slowing down of electromagnetic vibrations in a gravitational field, the rest of the 'relativity' being due to the optical illusion above discussed. Why not add to this pot pourri the

'magnetic relativity of time' observable in vibrating systems made of iron when they are in the presence of a magnetic field? There no reason not to, for it is on the same footing as the gravitationally induced 'relativity'. In fact, because of its very vaunted accuracy and attention to minutiae, the theory should include it. What difference would it make if a few more heterogeneous effects of quite definite and clear physical explanation were conglomerated together under the catch-all, 'relativity'? [See Chapter IV, section on gravitation.]

Indeed, that very procedure of conglomeration instead of explanation has all-too-often been the modus operandi of the theory. The ideal mathematical instrument for such descriptive conglomeration instead of explanation lay close at hand: the tensor analysis - which by its very nature can lump together all sorts of heterogeneous phenomena, the relations between which are not known, and present them in one format with a disarming degree of formal mathematical elegance. We are not here disputing in the least the place and usefulness of the tensor analysis in deriving and presenting, say, field equations. But it is not either physical unification or explanation to lump electromagnetic and gravitational factors together in one tensor and then put this contrived conglomerate forth as a proof, demonstration, or explanation of unification between the two sorts of phenomena. The tensor analysis, and not always in its best sense, pervades general relativity theory.

The great search in science is toward the greater unity of nature, as reflected in a minimum of assumptions. We would all like to see gravitational and electromagnetic phenomena unifiedly explained, and that in particular is an old quest, arising shortly after Newton. But Einstein's suggestion, first made in 1929, and recently revived, is merely to make a tensor conglomeration, with no genuine explantion or fundamental concept whatsoever. It is perhaps not insignificant that Einstein received the Nobel Prize for his work with the photoelectric effect rather than for his relativity theory, which nevertheless has performed a useful historical function.

Let there be no doubt about what we are saying. We are making this treatise as short as possible in order to keep it as clear as possible, in contrast to the multitude of published wordiness on this subject, attesting only to the confusion surrounding it. For example, even Eddington and Milne, though they disposed of it mathematically, did not realize the notorious 'twin paradox' of relativity to be the trivial thing that it is, arising only from the simple fact that the so-called 'paradoxical' time 'lengthening' was an optical illusion due to reading a clock dial by means of a light signal of necessarily finite speed under conditions of observational motion when such finite speed needs must introduce an erroneous experimental distortion of measurement. Using a sound signal under like conditions would introduce a much greater 'time retardation' because of the much lower velocity of sound as compared with light. If Eddington and Milne had realized the trivially simple basis of the 'paradox' they would have pointed it out instead of merely showing that when the traveling twins are again together, the time discrepancy disappears (as of course it must, for then there is no chance for signal error) in a manner consistent with the mathematics of the Lorentz transformations.

Illustrating the point further, it is not generally realized that even the ordinary Doppler effect will introduce a 'time retardation' when signals subject to Doppler's effect are used to read clock dials on two mutually receding systems of observation. Similarly there would be an illusion of a 'speeding up of time' if the two systems were approaching each other. What is more important, since light signals are subject to Doppler's effect, such time-effects must be algebraically added to these stemming from the Lorentz optical illusion, since the causes of the two illusions are independent, the Lorentz effects arising solely from the fact of the finite speed of light, in connection with motion during observation. There is absolutely no question of an intrinsic distortion of 'space' or 'time' being involved. Long before relativity theory, the well-known Doppler effect was giving us distorted 'local time' if any had been superficial enough to accept an optical illusion for intrinsic

physical reality, much as an unsophisticated savage would view any experimental or sensory illusion. In a reference discovered after this study was undertaken, we are happy to note that as far back as December, 1938, J.R. Colthurst remarked in 'The Mathematical Gazette' (p.460): "The so-called Lorentz contraction..., the Lorentz transformations, and the relativistic and classical formulae for the Doppler Shift effect, are all obtainable from a consideration of two photographs, and an abstract discussion on units. This interesting result is...ultimately dependent on the simple fact that it takes light a finite time to travel a finite distance." This reference will prove to the reader, as it did to us, an excellent confirmation of the results already established.

If we allowed every sensory illusion to elicit the irresponsible statement that thus the non-objectivity and relativity of 'space' and/or 'time' were 'proved', there would be confusion worse confounded, and not science. A by no means negligible tendency toward such confusion has been evidenced by the hitherto uncritical acceptance of an erroneous interpretation of the distortion effects called to attention by relativity theory. Worse still, as a factor working for such confusion, are the semantic misrepresentations, arising from misinterpretation, to be found in the theory itself as it has been put forth. No less than Einstein has said ['The Meaning of Relativity', 1946, p.36]: "This follows from the second of equations (29) and shows that the clock goes slower than if it were at rest relatively to K'." The correct statement, which is a very different matter, should of course have been that the clock appears to go more slowly when observed from the moving system K' by means of light signals, than when so observed when K' is not moving in respect of the clock. This illustration is typical of the sort of misleading statements (whether unintentional or not is beside the point) that have been put forth, so much so as to have become characteristic.

An even more striking and scientifically inexcusable instance exists of the naive acceptance by Einstein of the Lorentz effects as physical realities in themselves rather than the simply optical effects of the measuring situation that they are. He says in effect ['The Meaning of Relativity',1953,p.59-60] that an observer at the center of a rotating wheel would, by light signals measure the ratio of the circumference to the diameter of the wheel as $> \pi$ since the Lorentz effect appears only along, and not perpendicular to the direction of motion. Einstein characteristically does not refer to the optical cause of the measurement distortion, and says merely equivocally that: [ib.,p.60] "With respect to K(the observer at the center) the rods upon the periphery experience the Lorentz contraction." (italics ours)

Actually they do nothing of the kind, and the contraction appears only in the observer's measurements and no where else,- certainly not in the rods. But, more surprisingly, because so unwarrantedly, Einstein goes on to conclude that [ib.,p.60]: "It therefore follows that the laws of configuration of rigid bodies with respect to K'(the uniformly rotating circumferential system) do not agree with the laws of configuration of rigid bodies that are in accordance with Euclidean geometry."

Unfortunately, neither Euclidean geometry nor the 'laws of the configuration of rigid bodies' are involved, and nothing even remotely like the above-quoted conclusion follows from such an experiment. The 'laws of the configuration of rigid bodies', or anything even half so pompous and dogmatic, are not in the least involved here, but only an optical measurement-effect on the same physical level of significance as the Doppler effect. Since the above argument is the basis of Einstein's validation of 'general relativity' it is already patent that the general and special relativity theories, as they have been put forth, have been egregiously misinterpreted and mis-presented, while the facts on which they are primarily based are those of actually the most elementary kind in the theory of physical observation.

The entire matter is strongly reminiscent of 'the new clothes' of the emperor in Hans Andersen's revealing little story. The emperor in this instance has for too long been the science of physics. The new clothes are relativity theory, as put forth and interpreted. It is high time that someone call attention to the fact that the emperor actually [illegible]d nothing on. Yet in the story nearly all so feared being called 'unintelligent' for not seeing the 'fine texture' of the non-existent cloth, that no one spoke out except a street-boy, who named the subterfuge for what it was. If any might wish to call us 'a mere street-boy in the field' we gladly accept the appelation.

It will be useless for the school of relativists to say after this publication that they were aware of these points, for their mountain of published papers and books on the subject prove they were not. Indeed a concerted albeit unwitting tendency has existed to obscure, ignore and pass over the very basic and vital points that would once for all have made the subject clear, and so increased both its usefulness and understanding in science. We frankly admit that in the last two paragraphs there has been a bit of eminently justified lack of patience with the notational and verbal obscurantism that has in such quantities for almost five decades been stifling correct interpretation in this field, and hindering the progress of fundamental concepts, with respect to which physics is at such an impasse today, reduced on so many fronts to a practically empirical, trial-and-error affair.

Dr. P.W. Bridgman ['The Logic of Modern Physics, p.171] may well have had "reason for wondering whether Einstein's special theory may not be regarded merely as a very convenient way of tying together a large group of important physical phenomena, but not as being by any means a full or complete statement of natural relations." This is profoundly true.

The distortions of length and time according to special relativity theory (which is to say, according to Lorentz' transformations) are quite usefully correct if regarded and interpreted as what they are; viz. corrections to be introduced into measurement systems involving motion between observed and observer,- to offset the effect upon measurement of employing signals of finite speed. But when such distortions are taken literally instead of simply as heuristic aids in our utilization of the necessary optical illusions involved in our systems of measurement,- then we are indulging in fantasy, with necessarily twisted notions.

What makes the matter more difficult to disentangle is that the so-called 'relativistic' mass-increase of a charged particle with velocity, and the equally misnamed 'relativistic' magnetic effect of moving charges, are phenomena of an entirely different order and different physical origin and explanation than the Lorentz effects. They arise in relativity theory only because the signal chosen - light - is an electromagnetic wave, and for reasons unconnected with relativistic assumptions. Indeed, it is only the Lorentz, Doppler, and similarly occasioned effects which can rightly be called 'relativistic', and they all have in common that they are observational illusions of measurement, stemming from the signal situation under conditions of motion between the observed and observer. The aberration of light also belongs in this class of effects.

But the mass-increase of a charged particle of high velocity, the magnetic field surrounding a moving charge, and the bending of the path of light in a gravitational field, are phenomena of a very different sort, and they will be dealt with in the following two chapters.

It remains only to say that we can indeed get numerically correct results from completely misinformed interpretations, up to the point where that dream-world is crashed by veridical phenomena and valid conclusions which it had not thought to take into account. If we regarded a pencil as bending whenever we partially immersed it in water, and then unbending as it was removed,- certainly our results would fit observation as long as we confined our inquiry to the use of light signals. For the illusion was caused by the natural limitations of the technique of inquiry. But let us use the sense of touch while the pencil is semi-immersed, and at once our illusion is dispersed and its partial validity, based on the nature of light, is revealed. The light bent, not the pencil. To equate a physical entity with merely a contemporary means of its measurability is to confuse and obstruct inquiry in the name of nothing more sacred than a demonstrated dearth of experimental ingenuity. To eulogize such dearth into a principle is disas[illegible]

III

THE DERIVATION OF $E = MC^2$ AND THE POYNTING AND POINCARE VECTORS

Prior to proceeding to the main point of this discussion it is necessary to state an important shortcoming of relativity theory not mentioned thus far, one that affects it even within the bonafide sphere of its competency, i.e. in the matter of accounting and correcting for optical illusions of the measuring situation. This shortcoming is that the c of the Lorentz transformations is taken to mean in relativity theory the velocity of light in vacuo, whereas c actually must be taken to mean the velocity of light in the medium in which the observation takes place. If the observation cuts across a boundary between two media, two Lorentz transformations are required: the first using the speed of light from the observed point to the medium boundary; and the second using the speed of light in the second medium over the distance from the boundary to the observer, refraction also being taken into account in the ordinary fashion.

Failing thus properly to define c is a serious shortcoming in relativity theory, particularly in view of its insistence on the refinements in accuracy of its deductions as regards length and time intervals.

Indeed, the fact that c must be taken as light's velocity in whatever medium the measurements are made overthrows at once the over-bloated and quite incorrect phrase "the principle of the constancy of the velocity of light." There was neither constancy nor a principle.

If the entire system of observations between two mutually moving systems were carried on underwater, then in all the Lorentz equations c is not equal to $3 \cdot 10^{10}$ cm./sec., but we must use $c = 2.25 \cdot 10^{10}$ cm/s. and likewise for any other experimental medium.

The only real meaning of some 'principle' of the constancy of the velocity of light is one that, oddly enough, went quite unperceived and unstated by Einstein. This meaning consists in the physical fact that the velocity of a wave in a medium is independent of either the motion of the source or the observer, but depends only upon the density and elasticity of the medium. It may be here parenthetically remarked that the fact that light behaved in this manner should immediately have suggested the proven presence of some medium, from our experience with all other wave motion.

But the thoughts of Einstein and the relativistic school were in a very different direction, oriented toward the pursuit of measurement illusions as realities, and thus naturally failed to see the significance of a rather simple physical fact literally before their very eyes. Heaping Ossia upon Pelion they had included a result of that unrecognized fact as an arbitrary and unexplained assumption: "the principle of the constancy of the velocity of light", which, as above shown, is incorrect in the terms under which it is conceived, used, and stated in relativity theory.

One of the quickest and clearest ways to demonstrate the serious errors of interpretation and the deep-seated artificiality and *ad hoc* structure of relativity theory as it has been presented, is to set up an ideal experiment, a device which the relativistic school is so fond of propounding. All we must do to accomplish the demonstration is to consider the observational situation with light as the signal in a medium such as water in which the origin of one coordinate system, say a particle energized by a Mev photon, is moving with a velocity greater than that of light in the medium. Thus let us say such a particle is traveling (as has been experimentally proven possible) at a speed of $2.75 \cdot 10^{10}$ cm./sec., whereas the speed of light in water is only $2.25 \cdot 10^{10}$ cm./sec.

Bearing in mind the results of page 13, it is seen that under those circumstances the entire Lorentz transformations of relativity theory break down and are revealed for the artificiality of measurement they are - useful when rightly interpretd within the scope of their competency, useless beyond that point. Such particles can and do exist, and their length do

exist, and their lengths do not become zero. For further interesting and most relevant consequences of this situation see the Appendix on Cerenkov radiation.

But there is an effect of the light signal's being an electromagnetic wave that did creep into relativity theory through the back door (that is, not through the relativistic assumptions. Entirely unsuspected as to its origin, it became misrepresnted as 'the relativistic mass-increase' of a charged particle with velocity. It is not generally realized that the relativistic expressions for energy and for the increase of mass with kinetic energy, are fundamentally derived solely from the consideration that the energy be of electromagnetic origin, and the mass that of a charged particle [v. pp. 42-6, 'The Meaning of Relativity', A. Einstein, cit. sup.]. On the basis of such a derivation (which was easily managed because the signal used in relativity theory is physically speaking an electromagnetic wave, possessing electromagnetic momentum and mass), in which was masked the entire independence of the result from relativistic assumption, the electromagnetic mass-energy interrelation given approximately by the now well-known equation $E = mc^2$. This equation is part and parcel of Maxwell's electromagnetic theory.

It arises most simply from the direct equivalence of Poynting's and Poincaré's (sometimes called Abraham's) vectors. The first (formulated in 1884) refers to the rate of flow of electromagnetic energy, and the second (first formulated in 1904) represents the density of the electromagnetic momentum associated with that energy in an electromagnetic phenomenon. The relation of equivalence between the two vectors on the basis of Maxwell's theory states that Poynting's vector is equal to c^2 times Poincaré's vector, where c is the velocity of light *in vacuo*. If the first vector be $\underline{P}$ and the second, $\underline{p}$, we have hence

$$\underline{P} = \underline{p}\, c^2$$

which immediately, upon considerations of energy density, yields the relation $E = m c^2$ between electromagnetic energy and mass.

It is extremely important to note, in view of the obvious confusion surrounding the point, that this equation warrants no general equivalence of mass and energy. On the contrary, under the conditions of its derivation its says quite specifically that mass can convert directly only into electromagnetic energy - which then in turn may be converted to other types of energy (e.g. heat) by the conditions affecting the primary release. Thus, if a nucleus fissions there is suddenly a huge amount of electromagnetic field energy released, some of which is observed as gamma rays, and some as the kinetic energy of the emitted particles. This energy is indicated, on the basis of the Poynting-Poincaré relationship, as the difference between the mass of the fissioning nucleus and the sum of the masses of the fission-product nuclei and other emitted particles.

What is even more interesting is that the classical density of the luminiferous ether, $1.63889 \cdot 10^{10}$ gm/cm^3 can divide the elastic energy density of the ether, $1.47280 \cdot 10^{31}$ $ergs/cm^3$, to give the elastic etheric energy per gram. This quotient is easily computed to be 8.98657 ergs/gm, which is none other than c^2, the electromagnetic mass-energy equivalent. Thus, the inherent energy in a piece of matter is shown primarily to be the elastic energy of the ether which can be released by the disintegration of that matter. The deduction follows from Maxwell.

Dirac from other considerations was apparently being led to think along the same lines when he said in 1951 that the structure of a 'vacuum' is more complicated than what we have been led to believe, and that an ether is necessary for electromagnetic wave propagation. Relativity theory had all along been denying an ether on the poor grounds of equating a contemporary stage of limitation on measurability with non-existence. As stated before, the very independence of the velocity of light, of the speed of either the observer or the source rendered it more than very probable that light, whatever else it might be, was wave motion in a medium. That medium entered the [illegible]ture again - as electromagnetic waves - in the derivation of the mass-energy relation. Finally, the

elastic energy of that medium appeared as the basis of the energy of that relation. It is important again to stress that in the $E = mc^2$ relation it is electromagnetic energy that is basically and specifically involved. (All along had been known the more general mass-energy relation, indicated by the presence of the mass term in the expression for kinetic energy, $mv^2/2$.) The basis of the electromagnetic mass referred to by the Poynting-Poincaré relation is now seen to be the inertial mass of the medium, manifesting with wave motion. In Chapter IV we shall confront the difficulties facing relativity theory in its illogical attempt to deny the existence of a medium for electromagnetic waves, while talking at the same time of 'field' and of the 'curvature of space' in which, absurdly enough, it was nothingness that was supposed to be curved and have structure.

As regards $E = mc^2$, it is a historical fact that its pseudo-appearance in relativity theory (pseudo because not arising out of the relativistic assumptions at all), diverted from Maxwellian electromagnetic theory where it belonged, - has held back a great deal of progress in physics, and specifically constituted a block to the understanding of the exact nature of electromagnetic mass and of the fundamental details of the primary energy release in atomic fission. Without such obstacles, we are now in a position to understand why measurements should show the increase of electromagnetic mass with velocity.

We know from countless experiments that a mass of neutral atoms cannot disturb the ether. It is only an excess of charge, i.e. charged particles or particles of charge, that can and does disturb the ether without exception, in the form of electromagnetic wave effects. Since magnetic fields are ultimately the result of the motion of charged particles, the foregoing statement includes the electromagnetic effects (i.e. ether disturbances) caused by magnetic fields.

The critical nature of the velocity of light <u>in vacuo</u> (i.e. in ether) with respect to the apparent increase of the mass of a charged particle with increasing velocity, is no different from the critical nature of the wave-speed in any medium with respect to the velocity of a body traveling through that medium and to the attendant dynamical phenomena upon increase of that velocity. Every jet pilot is well acquainted with the 'wall of air' that builds up before him as he approaches the speed of sound, the wave velocity for the medium here involved - air. If a ball in air approached the speed of sound, it would be the inertial mass of the air piling up in front of the ball on its path that would make it seem to the casual observer that the ball had itself increased in mass, since it would now require more than the previous force to drive the ball on at its previous speed. There is nothing sacred about the speed of light any more than there is about the sonic barrier. Projectiles in air have passed far beyond the sonic barrier and we have sent particles through water and other media at velocities greater than that of light waves in these media.

In fact, there is a direct and, in the light of the foregoing discussion, an immediately understandable relation between the Cerenkov and the Mach angle. The reader is referred to the Appendix for the appropriate discussion.

Stemming partly from the inspiration of the remarkable pioneering work of J.J. Thomson in the 1880's the discussion of the mass-increase of an electron with kinetic energy occurred in the first expressions of Lorentz' electron theory , as early as 1904 and before. Indeed, the increase of e/m with v/c was first formulated by Kaufmann and Abraham in 1901-04. As Lorentz so honestly admitted ['The Theory of Electrons 2nd ed.,1915,p.198], his transformations, unbeknownst to him, were anticipated in substance by the physicist Voigt in 1887. The contribution of Lorentz' derivation of his transformations was that he did so not by means of any arbitrary postulates, but discovered them by dint of excellent analysis based on the actual data concerning electrons, that were known at the time. H.A.Lorentz is a far greater figure than even the degree of contemporary praise shows. Impeccably

honest in a simple manner at times reminiscent of Heaviside, he never indulges in mere notational semantics, but gives one solid deductions from fundamentals all the way through. His remarkable theory of electrons contained all of special relativity, including the mass-energy relations, and [p.50,op.cit.] unmistakably anticipated the Heisenberg principle.

III

GRAVITATION AND ELECTROMAGNETISM IN RELATIVITY THEORY

We have just seen how the very important Maxwellian concepts (made explicit notably by Abraham and Poincaré) of electromagnetic momentum and electromagnetic mass lead to the relation $E = mc^2$, which is inherent in electromagnetic theory and which enters into the relativistic equations by an indirect route, quite independent of relativistic assumptions, through the use of an electromagnetic signal - light. Close examination of Einstein's derivations shows how he unwittingly introduces the electromagnetic momentum of the signal into the formulae. After that, of course $E = mc^2$ followed, but not as a result of relativity, as it was presented.

Closely related to this topic is the increase of electromagnetic mass (e.g. the mass of a charged particle) at high velocity, known since the brilliant researches of Sir J.J. Thomson in the 1880's. We have suggested in the foregoing chapter(the reader is also referred to the Appendix, Cerenkov radiation)that the increasingly high resistance of a medium to projectiles traveling with velocities approaching the wave speed for that medium. By direct implication,the first derivative of the medium's resistance as a function of the projectile's velocity must have a maximum $< \infty$; that is, $dR/dv = 0$ must produce a value of $v = k$, where $k \neq \infty$. Hence, the graphed function must be some sort of modified sigmoid curve. From empirical data obtained by Bairstow, Fowler, and Hartree in 1920 and by Cranz in 1925 (v. Bibliography), as well as from von Kármán's and Moore's mathematical analysis of medium-resistance to the motion of projectiles we obtain a striking confirmation that the curve (resistance as function of velocity) is of a sigmoid type of shape, showing the required critical value of velocity, after which resistance decreases. This critical velocity is of course greater than the wave velocity for the medium. And the ratio of the two might well apply to other media than air by reason of the similarity of the dynamic structure of the experimental situation. If c be taken as the wave-speed for the medium, we obtain for the critical ratio in air, $v/c = 1.25$. We note also that the expression $(1-b-\frac{v^2}{c^2})$ where b is a constant, enters as a critical discriminant in von Kármán and Moore's theory of medium-resitance. (1932)

One of the effects of such resistance is, of course an apparent increase of the mass of the projectile, since the inertia of the medium, as well as its resistance to shearing stresses are involved. Even with ordinary artillery possessing considerable muzzle velocity, the apparent mass of a projectile can increase to over 20 times its 'rest mass' because of the resistance of the medium, in this case, air. A much greater factor applies to the ratio of apparent to rest mass' in the case of an airplane propeller in motion. What complicates the matter somewhat is that such increases in effective mass on account of real physical causes are combined with measurement-distortion effects of the Lorentz type if distance-time ratios are used to measure mass by way of acceleration.

We can easily prove this from the fundamental Newtonian equation $m = f/a = ft^2/s$, where s is distance and t, time. Since in optical illusions of the Lorentz type in the measuring situation we have a distortion of decreased length measurements and increased

measurements of time intervals due to finite signal speed as before shown, it is evident that any mass measured in such a manner would show a purely fictitious increase in addition to the actual effective increase due to medium-resistance. It is entirely a fortuitous occurrence that a function of the square of the ratio of the projectile speed to the wave velocity of the medium should be included in the mathematical expressions for both these types of effect. The reason for its double presence is that in both effects, for very different reasons, the relation of the speed through the medium to the wave speed in the medium(the signal speed in the Lorentz effect) is important. The two effects - one fictitious and the other actual - are entirely distinct, though they have been extremely and almost inextricably confused in existing relativistic interpretation, - the first not being realized for what it was, and the second not realized at all.

The preceding discussion of certain significant considerations regarding electromagnetic mass forms the necessary prelude to a discussion of the place of gravitation in relativity theory. For general relativity states that measured time intervals (i.e. by light signals) are subject to an increase which involves not only the usual Lorentz effect, but also the Newtonian gravitational potential m/r. This is no more than saying that the speed of light becomes less in a gravitational field, thus producing a red shift (the so-called Einstein shift effect). But any wave motion in any material medium would become decelerated in an intense gravitational field on solely Newtonian principles. As we have seen, electromagnetic mass possesses inertial properties, no different from any other mass, and there is no reason to exclude it from gravitational action. Thus, the introduction of electromagnetic mass into the relativistic equations led directly to the result that light would slow down in a gravitational field, and this quite independent of any of the relativistic assumptions. Likewise for the bending of the path of light in a gravitational field, though here we have a refractive effect also, due to a variation in medium-density, because of the gravitational field.

The introduction of the electromagnetic mass, as the essential element in all these effects, is hidden and masked to a great extent by Einstein's Hamiltonian approach and tensor notation, which, as said before, can be used to conceal as well as to reveal. As used in relativity theory, these methods have all too often resulted only in a fictitious unification of phenomena(with no added fundamental physical concept or insight) and/or in an ambiguity less useful than the original unconglomerated 'classical' result. This is particularly true in the attempt to combine in a purely formal manner the results of electromagnetic and gravitational theory, without any new fundamental physical insight that would actually unify them. That insight, it is now apparent, of course lay in the direction of a medium for electromagnetic and gravitational waves (a change in gravitation is communicated with the speed of light) - a direction and insight entirely lacking in relativity theory; which lack has occasioned much of the theory's ad hoc and procrustean nature, and the self-reductio ad absurdum of the necessity of using the meaningless phrases referring to a curvature or field-structure of dogmatically asserted 'empty space'. [The 'curvature' of Minkowskian space-time (v. Appendix, Ch.II) is a horse of another color. This ambiguous phrase can mean any accelerated motion at all, or any motion along a curved path in three dimensions.] Finally, the Hamiltonian treatment of gravitation, via Euler's theorem of 1736, that when no forces act upon a particle its motion is a geodesic, - such treatment by no means eliminates forces as physical realities, as so often and erroneously stated. First, the geodesic is traveled at constant velocity, the initiation of which required forces of acceleration. Second, the geodesics of relativity are the ambiguous geodesics of the fictitious Minkowskian continuum, with the coordinate ict used to produce a mere formal resemblance to another Cartesian coordinate. Under these ad hoc conditions a 'geodesic' simply means the old and well-known Hamiltonian principle of least action, perfectly true and not relativistic in the least in its assumptions. The sphere of relativity per se is rapidly shrinking to measurement-distortions only.

A word now as to the relativistic 'principle of equivalence. In " The Meaning of Relativity," p.56, Einstein writes: " The ratio of the masses of two bodies is defined in mechanics in two ways which differ from each other fundamentally; in the first place, as the reciprocal ratio of the accelerations which the same motional force imparts to them(inert mass), and in the second place, as the ratio of the forces which act on them in the same gravitational field(gravitational mass)."

Both the idea and the manner of expression are derived from Mach's Mechanik(cited in section E, Ch.I), but the thought is very confused. First of all, the two ways do not "differ from each other fundamentally." If Einstein had written that the first way of defining mass experimentally was through the reciprocal acceleration imparted to a body by forces of impact, and that the second way was through the reciprocal acceleration imparted by forces of gravitation, there would have been no problem (again, a purely semantic one), for it would vanish in the face of the simple fact that the same acceleration, no matter how produced, is the same physical phenomenon. The two definitions do not differ at all, much less "fundamentally." Since they do not, it is obvious that experiments using gravitation to produce the acceleration of a body must yield the same measurement for mass as experiments using impact, magnetic force, electrostatic force, or any other kind of force, to produce acceleration in the body. If the last two types of forces were used, naturally corrections because of Lenz' law might have to be applied, depending on the experimental conditions, but the principle of measuring mass by means of a produced acceleration(however produced) remains exactly the same. Mass = Force/acceleration is the only principle needed to demonstrate the necessary equivalence of "gravitational" and "inertial" mass. Any experiments "confirming" the equivalence of the two were misconceived and completely uncalled for, as there was nothing to confirm but what the ordinary Newtonian mechanics had already confirmed long since.

However, that equivalence does not mean (as it has been misused to mean) that every accelerated body may hence be considered to be in a gravitational field, for a gravitational field must bend a ray of light and do many other things which by no means can occur if the cause of the acceleration was simple impact. The equivalence of two effects by no means asserts the equivalence of their causes, for the same effect may be produced in several ways. The very principle of relativity itself is deficient logically for neglecting this sound experimental principle over and over again(see also Appendix to Ch.II). In the case of the 'principle of equivalence', first a false dichotomy was created where none existed(semantic fallacy) and then a correct principle of equivalence of effect was misapplied to mean equivalence of cause(logical fallacy).

Another case of sophistical semantics is the relativistic statement that electromagnetic effects are due 'merely' to 'relative motion', which is sometimes even more confusedly stated as 'the relativity of motion'. We shall dispose of this matter briefly, for it is trivial. But just because they are trivial it is important that sophistries be not allowed to be considered as otherwise in scientific inquiry. If any consider that it might be unfair to conclude that sophistry was at work in these instances, then let him further consider that the only alternative to sophistry in these instances would have had to be extreme naiveté. The choice is herewith left quite open.

Ever since Arago's discovery in 1824 of the damping effect of a copper plate on the vibrations of a compass needle, it was steadily realized that far more than the mere relative motion of the two bodies was at work to produce such effects. If a charged pith ball and a compass needle are placed both in the same cardboard box the needle shows no deflection. But if the box is moved (either uniformly or not) the needle will be defelected according to the magnetic field generated around the moving charge. In both instances the compass needle and the pith ball were at rest relative to each other, but obviously not at rest to the medium which transmits electromagnetic waves. The wave medium, consistently overlooked by relativity is again the crux of the matter. Another consequence of this point, one devolving upon elastic hysteresis will be mentioned at the close of the next chapter.

V

ASSEMBLED CONCLUSIONS

A simple though basic critical-methodological principle emerging out of the foregoing analyses of relativity theory can, perhaps be most appropriately stated first: Namely, that since the concepts and assumptions of a theory underlie and determine its mathematical expression, it is upon a scrutiny of the nature and the degree of clarity of these foundational concepts that any discussion or true evaluation of a theory must be based in the last analysis. It is the rationale rather than the costume of a theory which must be carefully examined, and which will finally be used in the irresistable judgment of history, quite irrespective of the merely notational aspects of the matter, whether these be simple or complex.

Specifically, the following conclusions have been reached:

1. Measured length and time intervals, using signals of finite speed with which to measure, would necessarily and quite clearly have to vary with the mutual velocity of observed and observer. It is the finite speed of the signal used for measuring that is the ultimate cause of such distortion effects, as is easily seen and as may be easily demonstrated.

Using a light signal, relativity theory is quite incorrect to interpret such effects as anything else but an optical illusion for which it can provide precise corrections. The sphere of competency of bona-fide relativity theory is limited to such corrections.

Such effects are due to errors induced by a mode of measurement and have nothing whatsoever to do with the nature of space and time as such, as has been so regrettably and widely mis-stated, even in original sources. If sound be substituted for light signals this basic point about the Lorentz effects is easily apprehended. A slower signal magnifies Lorentz effects, a faster signal reduces their magnitude.

2. Moreover, it may without ado be shown[v. Appendix to Ch.II] that there never was any inherent obstacle to the knowledge of the individual vector-contributions of two or more bodies in uniform motion with respect to each other, but only difficulties arising from procedural and equipment deficiencies, or lack of experimental ingenuity. Relativity theory has been notoriously and pessimistically ready to eulogize such purely anthropomorphic failings into alleged 'universal facts' of nature; and to confuse experimental appearance with physical reality, much as a 'green' desert scout would accept a mirage at face value.

3. Even when restricted within its own sphere of competency, that of measurement illusions, relativity theory as hitherto presented is incorrect on its own terms, for its serious neglect to specify that the signal speed used in the Lorentz equations is not invariant, but must be taken as the speed of the signal in whatever medium (or mediums) the measuring is done, as in vacuo, air, water, etc. This neglect is all the more serious because of relativity's insistence upon its precision. Experiments carried on underwater would be 25% in error if the ordinary Lorentz transformations presented by Einstein were used. This aspect of the matter again points up the artificial and illusory nature of the Lorentz effects, so misinterpreted by relativity theory.

4. Einstein's rather unrestrained dictum that the covariance of general relativity theory "takes away from space and time the last remnant of objectivity"[Annalen der Physik, vol.49] is seen not only to be unfounded with reference to its own premises in fact, but to rest on a basic miscomprehension of the meaning of the Lorentz transformations in the experimental situation, to put it very mildly.

5. The so-called 'principle of equivalence' of inertial and gravitational mass (one might as well add cohesionally, adhesionally, electrostatically, or magnetically, etc., accelerated mass too) is wholly inherent in Newton, and requires no assumptions other than his definition of mass via produced acceleration, viz. $m = F/a$, where F may be any type of accelerating force, and a is the acceleration produced, whether F was gravitational force, impact, or any other kind of force acting to accelerate the mass m.

6. $E = mc^2$ arises directly out of Maxwell's electromagnetic theory from considerations of electromagnetic mass and momentum (as also does the radiation pressure predicted by Maxwell), and specifically from the relation between Poynting's and Poincaré's vectors. The expression enters into the relativity equations simply because the signal there used, i.e. light, is an electromagnetic wave.

It turns out that the energy inherent in mass is the elastic energy of the electromagnetic wave-medium the activation of which a mass may effect in various ways. This elastic energy per gram (i.e. E/m) is found to be numerically equal to c^2, thus again resulting in $E = mc^2$ by another derivation, both derivations quite independent of the relativistic assumptions. Relativity theory obtains the result by surreptitiously introducing electromagnetic mass into its derivations. This was comparatively easy to do since it had chosen to begin with an electromagnetic signal - light.

7. Similarly, it is electromagnetic mass and momentum, and not relativity theory, that explain why the path of light should bend or its speed decrease in a gravitational field. Again the electromagnetic mass is introduced in a masked manner into the relativistic derivations, the results obtained stemming directly out of that circumstance alone, and not from any relativistic assumptions as claimed.

8. The measured mass-increase of a charged particle at high velocities is (aside from the contribution of

Lorentz effects) is the familiar great apparent increase in mass of a body moving through a medium at speeds approaching the wave-speed for that medium, such effects being a combination of head pressure of the medium, piling up in front, and the viscosity of the medium. Hence, if charged particles traveled at a speed greater than that of light there should be shock wave effects, and something comparable to Mach angle. An analysis of Cerenkov radiation actually demonstrates this to be the case (see Appendix).

9. The reality of electromagnetic mass and momentum, the independence of the velocity of light of the velocity of either source or observer, the apparent mass increases of charged particles approaching the wave velocity for the medium, the Cerenkov or shock-wave effects at and beyond the barrier, and last but by no means least, the demonstrated generation, reception and existence of electromagnetic and gravitational wave-disturbances,— all directly and unequivocally show the existence of the wave-medium.

Aside from certain essentially trivial semantic sophistries, the nature of which is purely verbal, all the difficulties into which relativity theory falls are due to its complete overlooking of, and indeed arbitrary denial of, (while it made use of electromagnetic mass to its own advantage) the substantial medium for electromagnetic wave-propagation. We add here parenthetically that even within its own proper derivation, from Maxwellian electromagnetic theory, the relation $E= mc^2$ says that mass is energy no more nor less than one could say that a piece of coal is the energy of the heat and light produced by its burning. Even the pre-electromagnetic equation for energy, $E = mv^2/2$, had mass as one component. Energy and mass are never dissociated effects.

10. Matter in any but its statistically abnormal state of electrical neutrality (and even then if its atoms are magnetically aligned) is constantly setting up electromagnetic waves as it moves through the medium for their transmission. In any event it is setting up gravitational waves in so moving.

11. In any manifold with physical characteristics, to speak of structure without substance is an absurdity: i.e. a contradiction in terms.

Also, in any such manifold, curvature is neither producible, maintainable, nor variable without forces (whether acting singularly, or quasi-continuously by being distributed through a medium); and this - as one proof among several - by reason of the connection between force (via acceleration) with the 2nd derivative of a function, and of curvature with the same derivative. Thus even uniform motion is physically impossible without originally initiating forces.

12. The unity of gravitation and electromagnetism is not demonstrated by notational treatment of the gravitational field so as to present it as a formal mathematical analogy to Maxwell's electromagnetic field. First of all, such presentation has nothing to do with relativity per se, and besides the analogy was long since (in 1893) exploited for all it was worth by Oliver Heaviside ['Electromagnetic Theory" Vol. I, Ch. IV]. Indeed, Heaviside's exploitation of the analogy possesses more value for physical science than Einstein's, in that Heaviside was admirably aware of the formalistic blind alleys which later were to entrap relativity theory. For he warned [Preface, Vol. I, op. cit.] against too easily being satisfied with an indiscriminate use of field or "potential functions, which are such powerful aids to obscuring and complicating the subject, and hiding from view useful and sometimes important relations." And the same may be applied likewise to invariant theory in physics, when it attempts to set itself up as genuine explanation.

Heaviside was on the right track. For the unity of electromagnetism and gravitation is a physical one, and consists in the fact that the same medium transmits the disturbances responsible for both kinds of effects. The true answer is elegantly simple.

13. The various cosmological speculations which have been so misleadingly included as relativity theory have nothing to do with relativistic assumptions as such. Thus, to take one fundamental example, the Riemannian or positive curvature of 'space' is simply a direct consequence of the shape of the lines of force in the Newtonian gravitational field, with respect to the center of mass of the universe. The physics of the medium is again the real question.

14. There is a well-known non-Doppler red-shift observed in the light from very distant nebulae, and proportional to the distance. Because of the inherent weaknesses and inapplicability of its assumptions when extended beyond their very limited sphere of competency (that of measurement illusions of the sort illustrated in the Lorentz effects), relativity theory was forced to a well-nigh ridiculous extreme of imposition on logic and credulity by the simple fact of the nebular red-shift. It was forced to speak of an expanding universe, at the outposts of which the nebulae, for no good or apparent dynamic reason were rushing apart from each other at speeds approaching that of light.

Again, the overlooking of the physical medium for wave-transmission is at the root of the trouble. For the nebular red-shift, when the medium of the light waves is taken into account, becomes simply the straightforward effect of elastic hysteresis.

If electromagnetic waves are based upon some kind of elasticity (as all waves must be) then there is elastic hysteresis, whereby if the energy of displacement is E_d and the energy of recovery is E_r, then $E_d > E_r$ by an amount depending upon the elastic constants involved. It is worth noting that these constants must involve the viscosity of the medium, a property also of critical importance in shock-wave phenomena. It is also worth noting that elastic hysteresis of a medium, other things being equal, serves to measure the viscosity of the medium. This relationship may be expressed by a partial differential equation. In elastic hysteresis we have a damped oscillation of also constantly decreasing frequency. It is known that 1.75×10^9 light-years is the critical distance for light. Since the energy of the average light wave = $6h \times 10^{-15}$ ergs, where h is Planck's constant, we find that the hysteresis loss, on the average, for visible light is only 2.41×10^{-36} ergs per cm. of wave travel. This extremely low figure agrees very well with the immense elastic energy of the wave-medium. It also agrees with the approximate law which we may write as $H_1M_2 = H_2M_1$, where H and M are the hysteresis and the elastic modulus, the subscripts de-

noting two different substances.

There is now an even greater significance in the critical value 1.75×10^9 light-years, for thermodynamically it relates to a state of 100% entropy with respect to a disturbance of the electromagnetic wave-medium. Since the same medium is involved, as before observed, this result applies also to gravitational disturbances of the medium, as well as to the possible (and with high speed particle-particle collisions, very probable) longitudinal disturbances therein.

Since neither electromagnetic nor gravitational disturbances could go beyond that distance (1.75×10^9 light-years), then neither could the changes which must be sustained by such waves, and hence all communication of physical effects from one part of the universe to another must cease after that distance, including gravitational effects directed from the center of mass of the physical universe. Thus the upper limit of the diameter of the material universe is placed at 3.5×10^9 light-years.

16. The misconceptions regarding relativity theory, evident in even technical treatises of the last few years, have not been lessened or mitigated by the often over-dressed and overwrought semantics, both verbal and mathematical, in which the theory has all-too-frequently been presented from its very inception. That presentation actually worked to obscure what was useful in the theory and to furnish a misleading impression of its nature and scope, which in actuality is seen to be quite circumscribed when a correct analysis of the physical meaning of the Lorentz effects is carried out.

In addition, there was found to exist a deep-seated lack of uniformity of basis in the theory, a great many of its claimed results having nothing to do with the relativistic assumptions, but arising from the actual introduction of electromagnetic mass and momentum in the derivations, the claimed results thus springing directly from Maxwell's electromagnetic theory based on stresses and strains in a wave-medium. Indeed the proven experimental existence of the profoundly important concept of electromagnetic mass and momentum directly involves the inertia of the very wave-medium relativity theory attempted to deny, and had to deny in order to retain any consistency with its own assumptions. Consequently, those assumptions fail with the failure of that denial. What further confused the surreptitious introduction of electromagnetic momentum and mass into the derivations of relativity theory was the fact that the measuring signal it employed was a form of electromagnetic energy.

It is not difficult to see that such a situation as described in these sixteen conclusions demands a searching and fundamental clarification, leading to the establishment of clearly conceived and unambiguously stated interpretations and fundamental principles. The preceding pages provide an introduction to that task. An entirely new path, as the foregoing chapters have suggested in detail, will have to be taken.

Relativity theory as presented to date has been misleading to the extent that it has been seriously confused.

APPENDIX

On Čerenkov Radiation

In 1934 P.A.Cerenkov, bombarding non-fluorescent, non-radioactive liquids and solids with beams of Compton-effect and beta-ray electrons, reported the production of a visible radiation when the electrons traveled through the various media at velocities greater than that of light in those media.

He proved that the radiation was not fluorescence, that it was not emitted spherically, but conically, and that its intensity depended only upon the index of refraction of the medium. Three years later I.Frank and I.Tamm were able to show that such radiation was a consequence of Maxwell's electromagnetic equations and that the semi-vertex angle of the Cerenkov-cone could be calculated from the velocity of the electrons and the refractive index of the medium. A new physical process for producing radiation had been discovered, and a new physical effect as well.

If we call the semi-vertex angle of the cone (the axis of which is the electron's path, the base being directed in the same way as the motion of the electron) $\underline{C}$, for Cerenkov angle, then Frank and Tamm showed that

$$\cos C = c/vn$$

where $\underline{n}$ is the refractive index involved, $\underline{c}$ is the velocity of light *in vacuo*, and $\underline{v}$ ($>c'$) is the velocity of the electron.

However, this important formula is not as revealing as it can be made by a simple transformation, as follows. Letting $\underline{c'}$ be the speed of light in a medium of refractive index $\underline{n}$, we have $n = c/c'$, where $\underline{c}$ is as before the speed of light *in vacuo*. Substituting for $\underline{n}$ in Frank and Tamm's formula, we obtain the more analytically useful equation

$$\cos C = c'/v$$

It is now immediately apparent that the cosine of the Cerenkov angle is simply the ratio between the speed of light in the medium and the greater speed of the electron in the medium. But it is this identical ratio of physical quantities that can furnish us the generalized Mach angle (here called M) in the case of sound. We have

$$\sin M = s'/v$$

where v is the supersonic speed of a body through a medium, and s' is the speed of sound in that medium. If s' = s, the speed of sound in air, we obtain the ordinary Mach angle as the arc-sine s/v. Hence, we see that the cosine of the Cerenkov angle is obtained from the same sort of physical relationships used to determine the sine of the Mach angle. The complementary nature of the two angles arises from the fact that longitudinal waves (e.g. sound) are propagated along the direction of excitation, while transverse waves (e.g. light) are propagated at 90° to the direction of excitation. The Cerenkov angle is thus seen to be the physical equivalent of the Mach angle when superluminal instead of supersonic speeds are attained. The importance of this finding is that it brings to bear a particularly tangible experimental prove of the physical medium propagating electromagnetic waves. Cerenkov radiation had to be included in the mathematical consequences of Maxwell's theory, as Maxwell's basic assumptions were stresses and strains in the medium. Cerenkov cones are shock-wave cones. Moreover, there is now seen to be every physical reason to believe that particles can travel at speeds >c, the apparent mass decreasing after v/c reached about 1.25, as previously calculated. (p.20, supra)

In the light of the present discussion and those in Chapters II and IV, one is in a position to perceive the obviousness of the extreme superficiality of Einstein's statement ["The Meaning of Relativity," 1953, p.38]: "That material velocities exceeding that of light are not possible, follows from the appearance of the radical $\sqrt{1-v^2/c^2}$ in the special Lorentz transformation." Einstein's naive acceptance of the Lorentz optical illusion arising simply from the measuring situation employed, is only too apparent.

In addition, as we showed on page 13, the "c" in the Lorentz transformations must be taken as the speed of light in the medium in which the measuring is done, and cannot at all be restricted to the speed of light *in vacuo*. Thus, on this second count, Einstein's statement is completely disqualified by the experimental proof of particle speeds greater than the speed of light in the same medium. His failure to grasp the physical significance of the Lorentz transformations is thus confirmed by the fact that such particle speeds were known at least since the early 1930's, while his statement, above quoted, was made in 1953. We have no interests other than service to historical and scientific truth, and the rectification of gross misconceptions, in thus setting down these impartial facts, painful though we realize they may be in some quarters. The lack of progress engendered by misconceptions is much more painful, especially when such misconceptions are voiced as dogmas, and set up to be what they are not.

Appendix to Chapter II

We shall first examine the fallacy of equating difficult measurability (or non-measurability under certain imposed conditions) with non-existence. This is very relevant as it is the underlying fallacy of the relativistic assumptions in general.

As we have stated it, the reason for the fallacy is apparent. We have now to cite examples. We shall see that very real and effective physical differences are under the realtivistic assumptions cited as either immaterial or non-existent, both conclusions being equally false.

Let us first consider the case of two bodies in uniform motion with respect to each other, say at 900 mph. Furthermore, let these two bodies be rocket ships at great distances from any gravitational masses whatsoever, and let one rocket be traveling at 900 mph while the motors of the second ship are entirely shut off. An observer in the instrument room of the second ship, observing the first ship through a window, could obviously tell from a glance at his own instrument panel and at the light from the first rocket's exhaust, that it was the first ship that was moving and contributing the total vector to the mutual velocity.

If now he started the motors of his own ship(the *second*) attaining a steady speed of 500 mph, he would now know that he was contributing that much, and the mutual velocity of the two bodies would now be 400 mph. Now let the *first* ship, previously traveling at 900 mph, attain another steady speed at 1400 mph. The mutual velocity will again be 900 mph, as in the beginning. The relativist would say that since anything other than 'relative motion' is not demonstrable or measurable, that the first and last condition of the two ships are exactly the same physical phenomenon - obviously untrue and absurd in the light of this example.

Since one exception serves to overthrow a universal proposition, the contention that the individual vector contribution in the mutual uniform motion of two or more bodies cannot be ascertained and does not exist for physics, is hereby not only disproved, but shown to rest on nothing more substantial than a lack of experimental resourcefulness, and a eulogy of ambiguity for reasons nothing better than its own sake.

Another simple disproof of the relativistic assertion will be given. Let a wide rubber band be stretched with, say, a finger, while held at the other end by a hook. Newton's third law still applies, and the forces exerted by the finger and the hook are equal; but they are not at all distributed equally throughout the rubber band, particularly if the rubber is losing its elasticity and has correspondingly higher than normal viscosity. It required time for the initial pull of the finger to be transmitted to the end held by the hook. As a consequence, there is greater stretch (as shown by passing light through the band) toward the finger-end of the rubber band than toward the hook-end. As the finger and hook move away from each other at uniform velocity, the region of greatest stretch indicates that the finger is contributing more to the mutual velocity of recession than the hook. Again, it is only a matter of finesse in experimental arrangemements and conditions that is necessary in order to analyze relative motion into its component parts. There is no inherent obstacle in nature at all.

It is worthy of note that all one is obliged to do is prove existence in these matters, and not, in addition, susceptibility to measurement, which we are also and gratuitously doing in the interests of completeness. For the existence of a state of affairs and its susceptibility to measurement can be two very different things under different imposed or chosen conditions. It is notable in mathematics as well as in physics, that one may have established existence theorems long before more detailed theorems pertaining to the analytical nature of the existent fact have been proved. There is every reason in logic for mere existence theorems first to be established even before measurability can be analytically expressed. To say that the first cannot be established before the second can be expressed is to commit a gross fallacy,- one of the main forms of false reductivism.

This fallacy is at the basis of relativistic thinking, which actually encourages and delights in a lack of unambiguous measurability, and which thus operates actively to halt progress toward sounder and more resourceful and revealing experimental techniques. In this sense relativistic thinking as expressed in relativity theory belongs rather with the arts of the ancient and modern Sophists than in the laboratory or the chair of science.

We shall now furnish proofs of the existence of unambiguous direction and unambiguous simultaneity. Consider two circularly revolving bodies with a common center of revolution, and with uniform angular velocities of 120° and 72°/sec. respectively. As may be easily calculated, there will be only two conjunction points for the two bodies, with respect to their common center of revolution, and these two places are diametrically opposed, thus establishing an unambiguous direction in space, namely along the line between the two conjunction points in the common orbital plane. This direction arises entirely out of the system itself and requires no external system of coordinates.

Next consider three revolving bodies, A, B and C, about a common center, each possessing a different uniform angular velocity. At any given moment A will be a certain angular distance from B, say d_{AB}; and B will be a certain distance from C, d_{BC}. The attaining of d_{BC} and d_{AB} by the pairs of bodies BC and AB are absolutely unambiguous simultaneous events.

The possibility of optical illusions of the Lorentz and/or Doppler type under certain measuring conditions imposed or selected, is entirely irrelevant to the fact of the existence of unambiguous direction and unambiguous simultaneity. Even within their own sphere of relevance and competency, such effects simply denote deficiencies and difficulties within the method of measurement, and permit no conclusions outside the province of measurement-distortions produced under various experimental conditions. Even without the simple existence proofs above given, it should have been unequivocally clear that the Lorentz effects had nothing whatsoever to do with space or time <u>per se</u>, any more than any other sensory illusion.

It should be finally noted that the existence of an electromagnetic wave-medium renders the measurability aspect of unambiguous direction and assignment of motion that much more experimentally approachable. (If the reader has not yet seen Chapters III and IV he is referred to them herewith <u>in re</u> this matter.)

One last observation remains to complete this part of the discussion; namely the non-relativistic and unambiguous character of rotational motion well-nigh irregardless of particular experimental conditions. This fact has driven the more dogmatic relativists since Ernst Mach to the contention, in itself a <u>reductio ad absurdum</u>, that the cause of a fast rotating fly-wheel's bursting apart lies in the gravitational effect of the surrounding fixed stars in space. Not only (and this should not have been necessary to state) is such a contention unsupported by all we know of gravitation; but, as any engineer knows, it is the angular velocity and type of material and construction of the fly-wheel that alone determine the point at which it will burst. Centrifugal forces arise -and it is they that cause the bursting of the wheel - from the very forces that initiated and maintained the rotation. We do not have to go even outside the wheel, much less traverse all the light years to the fixed stars, to know and prove

why it bursts apart when rotated too rapidly. The fact that Mach <u>et al</u>. were forced to appeal to the fixed stars in itself bears direct evidence of the poverty of relativistic theory when incorrectly and unwarrantedly extended beyond its rightful domain of allowing for the errors of measurement occasioned through the use of signals of finite speed.

We now arrive at another aspect of measurement in relativity theory. We already have had occasion(p.22) to refer to the heuristic Minkowskian 4-space adopted by Einstein, wherein the coordinate <u>ict</u> is arbitrarily chosen in order to obtain a formal resemblance to a Cartesian distance-element, and still embody the physical fact(though now in considerably masked form) that the measurements were performed by the use of light signals.

It should be emphasized - as it has not been sufficiently, if at all in relativistic treatises - that this space is simply a mathematical working convenience for the purposes of relativistic discussion, and that its use as a tool in such discussions does not at all endow it with any intrinsic physical reality apart from its employment as a formal convenience. Indeed, when we examine the Minkowskian element

$$ds_4^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

where ds_4 is the 4-space element and where x is x_1; y, x_2; z, x_3; and ict, x_4(the <u>i</u> being necessary so that squaring produces the negative sign),- we find that it amounts to saying nothing more than

$$ds_3^2 = c^2dt^2 = dx^2 + dy^2 + dz^2.$$

In other words, $cdt = ds_3$, where ds_3 is the ordinary distance element in three spatial dimensions. Integrating this last equation, we have

$$ct = s,$$

which represents nothing more abstruse than the familiar " rate times time equals distance" of secondary school mathematics, put into terms of the speed of a light signal traversing the distance <u>s</u>. In this connection another fertile source of misleading notions and another demonstration of internal confusion in the theory is Einstein's very objectionable use of the phrase 'light-time' to refer to the quantity ct. This misleading phrase confuses on two counts:

1. The phrase 'light-time' has long been and is properly used in astronomy to denote the time interval between the sending and receiving of a light signal from a heavenly body. Astronomy has for long corrected for measurement errors induced by the optical illusion caused by light-time in its true sense.

2. ct is not a time at all, but a distance — the distance light travels in time t at speed c.

Resuming discussion of Minkowskian 4-space, we note that despite its other formal conveniences, its interval is very unusefully ambiguous. Thus, a curved Minkowskian 4-space interval may refer to either uniform linear velocity along a curved path in three spatial dimensions, or just as well to an accelerated motion along a straight path in three spatial dimensions. A 'curvature of space-time,' in the over-colored terminology of relativity theory, thus boils down to nothing more than some kind of accelerated motion of the garden variety. 'Space-time' is motion.

Before leaving the subject of Minkowskian arbitrary 4-space, it is necessary to say a few words on geodesics, or paths of minimum distance on a given surface. [Note that if we are not restricted to remain on the given surface or, in general, in a certain type of region, then irrespective of the number of dimensions, a straight line is still the minimal distance between two points in any space.]

The geodesics of relativity theory are nothing but the geometric form which the Hamiltonian principle of least action assumes when the Minkowskian coordinates are employed. This fact constitutes the real source of the convenience of the heuristic Minkowskian 4-space, quite aside from relativistic assumptions.

Finally, the Minkowskian 4-space, as aforesaid, presents no actual physical unity of space and time. That fact is additionally evident in its non-homogeneity. The non-homogeneity of the Minkowskian space shows not only in the special factor ic pr- fixed to the time coordinate, but also is shown

the fact that in the Lorentz transformations, to which the Minkowskian type of arbitrary 'space-time' addresses itself, time distortions behave quite differently from space distortions, since their combination in the velocity of the light-signal makes them inversely related to each other, as explained on pp. 5 and 6, supra. Bridgman [" The Logic of Modern Physics," p.74] most accurately sensed the difficulties arising from the artificiality of the Minkowskian 4-space, for he wrote: " Although mathematically the numbers measuring space and time enter into the relativity formulas symmetrically, nevertheless the physical operations by which these numbers are obtained are entirely distinct and never fuse, and I believe it can lead only to confusion to see in the possibility of a four dimensional treatment anything other than a purely formal matter." Again (see also p.11, supra) Bridgman's sound physical intuition was right, as regards the Minkowskian space of relativity theory.

As the concluding section of this Appendix there will be considered another aspect of the 'curvature of space' spoken of in relativity theory. As pointed out in the 11th conclusion of Chapter V, the phrase amounts to a contradiction in terms as far as physics is concerned, structure without substance being impossible. Similarly, to talk of curved geodesics in space is to make the unanalyzed assumption of a field of force. And such a configuration of forces can be exerted only by a substance, a medium. In no other way could a field in physics guide, or impose a metric upon, the path of a body or material particle.

Interestingly enough, the actual relativity involved, and one entirely overlooked by relativity theory, is what might be called the relativity of vacua. On one level of observation, say to the unaided eye, a room full of air appears empty. It is empty "relative" to the eye, though even here such an artificial way of speaking helps not a whit, nor explains anything. The eye perceives what it is equipped to see: anything more than this in the way of verbalization is word-mongering. We must find better and added means of observation, that is all.

Now some comparatively simple tests, one so simple as merely flailing the arms about, proves that the room is not empty to the sense of touch. Moistening the flailing arms confirms this conclusion by added effects. A fan confirms it, and by means of the medium, air, in the room, the fan can even send bits of paper scurrying over a table. What appears to be a magnetic " wind" when we swing a magnet about, can also move iron filings.

Rising to the next level of vacuum, we now deaerate the room or enclosure. All the phenomena dependent on the previously visually non-existent, although quite real and now vanished substance, air, disappear. However, gravitational and electromagnetic effects of apparent action-at-a-distance still take place. Obviously, our vacuum is not the absolute one we had thought it to be. From our first lesson with the air, we are ready to admit that a still more impalpable substance observable only from its effects (now consisting solely of gravitation and electromagnetism) is still in the room.

All apparent action-at-a-distance implies the existence and activity of some physical medium.

The reasoning has been made purposely as elementary as possible to show how simply the relativistic errors could have been avoided.

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viscosity, a definite type of three-dimensional eddy can form in the medium, which can be mathematically described as

$$Re^{\Theta i}, \quad \Theta = f(t)$$

(see "The Imaginary as a Dimensionality Operator," an unpub - lished lecture by C.A.Muses copyright 1952).

Such conical-spherical eddies thus constitute charged particles, and "charge" becomes simply three-dimensional swirl in the medium. The sense of this 3-space rotation(4-space if its time coordinate be treated in Minkowskian fashion), which is invariant for coordinate transformations, determines the sign of the charge. Further, there is a definite relation between turbulent flow in the wave-medium and the formation and maintenance of such three-dimensional eddies, or particles of matter.

It is now clear why fundamental particles are found in quantum mechanics to have no definite boundary, as well as why a particle has a wave nature. Also, the fundamental physical importance of spinors becomes obvious, as well as do the links between field and particle theory. It can, however, be shown by actual calculation that the Dirac spin is comparatively quite slow, like a precessional spin, while the swirl-speed that is the basis of charge is of very much greater angular velocity.

It is now also evident why electric and magnetic components are perpendicular, for the Coriolis and centrifugal forces of the swirl are "electric," while the much lesser vortical forces created by pressure of the surrounding medium, and which are perpendicular to the first set of forces, are "magnetic" as we observe them in their effects.

The general relation of turbulent flow to the wave-medium include also gravitational effects around swirls and swirl-clusters(ponderable matter) by Bernoulli's principle, appearing thus in a new form as the basic reason for gravitation.

But there is no space or reason here to present any further developments. It is sufficient that we have established the significance of the spinor solutions and their relation to the preceding text. It remains only to be said, in connection with the technical difficulties encountered in attempts at a formally unified field theory - quite aside from the inevitable confusion stemming from the lack of physical insight of a merely formal approach, - that the reason for the imperviousness in this respect of the electromagnetic field to non-spinor treatment lay in the curl vectors of the Maxwellian electromagnetic field equations; while the gravitational field vectors, on the other hand, contain no curl operator for the unperturbed field.

SUPPLEMENT ON SPINORS

Dr. Vaclav Hlavaty's public release, on July 29, 1958, of his spinor solutions to the Einsteinian unified field equations, appeared after this book had gone to press, but we managed to have this supplement inserted, to include the latest possible relevant data. Since Hlavaty's method utilized spinors very essentially, we shall first briefly summarize the history of spinors.

The theory began in physical conception, if not in mathematical realization, with P. Dirac's first paper on electron spin in 1927. J. von Neumann in 1928 pointed out a hyper-geometrical interpretation of Dirac's matrices. Independently of each other, in 1929, H. Weyl and V. Fock mathematically related Dirac's work to generalized field theory, E. Schroedinger and others clarifying this relation further.

In the meantime the eminent mathematical physicist Ehrenfest had first specifically called for a "spinor" on physical grounds in order to have the spinning electron mathematically described. In the very eventful year 1929 in this respect, van der Waerden put forth the first mathematical theory of a two-component spinor. Van der Waerden's representations obtained further refinements from J. A. Schouten soon after.

In 1933 O. Veblen related spinors to the mathematical space used by relativity theory. He did this in terms of projective geometry of a type initially suggested, on wholly mathematically theoretical grounds by C. Segre and C. Juel at the close of the last century. We may remark in passing that Veblen's geometric emphasis was a very sound one for physical theory. On the same principle, without Schroedinger's wave mechanics, quantum theory would have floundered in an endlessly awkward sea of clumsy matrices. It was also Schroedinger's fruitful concept that sharply highlighted in quantum theory the importance of complex conjugates, which is one of the routes whereby spinors enter the quantum picture.

Finally, in 1958, V. Hlavaty (of the Graduate Institute For Applied Mathematics, Indiana University) solved the Einsteinian tensor equations combining electromagnetic and gravitational potential fields, by means of a complex generalization of a tensor, i.e. by spinor theory.

Spinor theory might be cursorily defined as the utilization of complex variable theory in generalized tensor analysis. By adding the complex domain spinors generalize tensors more than even the very general gauge-invariant tensors required to represent electromagnetic potential fields. Reduced to simplest terms, anything which spins or rotates as it proceeds on a given path of motion may have a spinor to describe it. As is generally the case in invariance-theory developments, spinor theory may assume matrix form. Thus Dirac's electron spin theory was originally conceived in matrix form, though it most conveniently lends itself to spinor notation.

Aside from their fundamental relation to the probability phase-spaces of quantum mechanics (where conjugate complex components play an essential role), spinors possess a formal mathematical ambivalence relating them also to generalized Maxwellian field theory, wherein electromagnetic and gravitational potentials are tensorially combined. For partial differential equations in generalized Minkowskian space, where accelerated motion allowed, may too be accurately and compactly rendered in spinor notation.

The physical reason for the validity of this last application is that accelerations appear as curvatures in a Minkowskian, pseudo-Euclidean space, and all curved paths of motion can ultimately logically be related to a general spin or rotation concept in such a space.

Hlavaty's spinor solutions of the generalized field equations of general relativity only make now utterly explicit the consequences of the implicit inclusion of the fact of electromagnetic mass of its signal in special relativity theory (see Ch. III supra).

Naturally, the final explicit solutions must demonstrate electromagnetic mass in the fundamental though unacknowledged rôle it has played in relativity theory from the start, without the slightest obligation to relativistic assumptions as such.

Hlavaty finds that gravitation may exist "without matter." By this is meant By this is meant that gravitational effects can exist without those collections and combinations of electrical particles we call ponderable matter. But, as the foregoing has demonstrated (Ch. V supra), matter also embraces, and extremely importantly, the fundamental substantial wave-medium in which both electromagnetic and gravitational disturbances are propagated, the wave-medium that relativity theory had omitted and even denied ostensibly although it had lain there all along in the unwitting inclusion in relativity theory of the electromagnetic mass of its signal.

Now the attraction of electromagnetic mass by a gravitational field (e.g. the bending of the path of light by a massive body) also demonstrates the converse by Newton's law of the mutual gravitational influence of two masses as Gmm'/d^2. That is, the electromagnetic mass has also very slightly deflected the body whose gravitational field it entered. *Whatever is susceptible to a gravitational field must possess a gravitational field of its own.* Thus it is possible for two moving electromagnetic masses to exercise gravitational effects (slight though they may be) on each other, thus yielding gravitational effects "without matter." This conclusion is simply one of the necessary consequences of the physical fact of electromagnetic mass. It emerged explicitly in the Hlavaty spinor solutions of general relativity only because the theory had from the very beginning carried electromagnetic mass as an unrealized stowaway.

Hlavaty's solutions only make it that much clearer that actually concrete triumphs (such as $E = mc^2$) were in fact due to the mathematical consequences of the Maxwell-Poynting-Heaviside-Thomson-Abraham-Poincaré concept of electromagnetic mass, based in turn on the substantial nature of the wave-medium for gravitational and electromagnetic disturbances.

For similar reasons as discussed we may have in a given physical context neither gravitational disturbances in the macroscopic sense nor particled matter, and yet still have electromagnetic disturbances in the medium, for it possesses a very low viscosity and high elasticity.

As long as we have come this far, we may as well complete the journey. The last major unknown factor lying before us is obviously the nature of what we call "charge." One will not solve this problem by removing it – as even the keen G. D. Birkhoff did – one step further from solution by postulating an electric fluid that can charge matter.

Again the final physical solution must be sought in terms of the wave-medium. Subjected to certain internal rotations in three dimensions which can easily be maintained because of the low